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Theoretical Bounds for the Influence of Tissue-Level Ductility on the Apparent-Level Strength of Human Trabecular Bone

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Abstract

The role of tissue-level post-yield behavior on the apparent-level strength of trabecular bone is a potentially important aspect of bone quality. To gain insight into this issue, we compared the apparent-level strength of trabecular bone for the hypothetical cases of fully brittle versus fully ductile failure behavior of the trabecular tissue. Twenty human cadaver trabecular bone specimens (5 mm cube; BV/TV = 6–36%) were scanned with micro-CT to create 3D finite element models (22-micron element size). For each model, apparent-level strength was computed assuming either fully brittle (fracture with no tissue ductility) or fully ductile (yield with no tissue fracture) tissue-level behaviors. We found that the apparent-level ultimate strength for the brittle behavior was only about half the value of the apparent-level 0.2%-offset yield strength for the ductile behavior, and the ratio of these brittle to ductile strengths was almost constant (mean $\pm$ SD = 0.56 ± 0.02 ; $n=20$; $R^2 = 0.99$ between the two measures). As a result of this small variation, although the ratio of brittle to ductile strengths was positively correlated with the bone volume fraction ($R^2=0.44$, $p=0.01$) and structure model index (SMI, $R^2=0.58$, $p<0.01$), these effects were small. Mechanistically, the fully ductile behavior resulted in a much higher apparent-level strength because in this case about 16-fold more tissue was required to fail than for the fully brittle behavior; also, there was more tensile- than compressive-mode of failure at the tissue level for the fully brittle behavior. We conclude that, in theory, the apparent-level strength behavior of human trabecular bone can vary appreciably depending on whether the tissue fails in a fully ductile versus fully brittle manner, and this effect is largely constant despite appreciable variations in bone volume fraction and microarchitecture.

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Conflict of interest statement

Dr. Keaveny has a financial interest in O.N. Diagnostics and both he and the company may benefit from the results of this research. All other authors have no conflict of interest.

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Keywords

finite element analysis; bone quality; brittle; ductile; microarchitecture

1. Introduction

One fundamental issue in bone biomechanics is the influence of tissue-level ductility on the apparent-level strength behavior of trabecular bone. Tissue-level ductility is most easily characterized by the magnitude of the post-yield strain to fracture. While this post-yield strain decreases with aging for cortical tissue (Burstein et al. 1976; Courtney et al. 1996; McCalden et al. 1993; Reilly and Burstein 1975) and is thought to influence whole-bone fragility (Seeman and Delmas 2006), many details for trabecular bone remain unclear. In the only study of its type we are aware of, Hernandez *et al.* (Hernandez et al. 2005) tested 231 individual trabeculae from 32 cadavers to failure in tension, and found relatively large values of ultimate strain (mean $\pm$ SD = 8.8% $\pm$ 3.7%). Compared to average ultimate strains for human cortical tissue on the order of about 2% (McCalden et al. 1993), this limited literature suggests that trabecular tissue may be much more ductile than cortical tissue. However, it is difficult to interpret the relevance of such findings without understanding the influence of tissue-level ductility on the apparent-level strength of trabecular bone. This multi-scale issue, which relates ductility at one scale to strength at a higher scale, has been well investigated for composites (Llorca and Gonzalez 1998; Mortensen and Llorca 2010) and cellular solid materials (Gibson and Ashby 1997; Maiti et al. 1984), but less so for trabecular bone. Verhulp (Verhulp et al. 2008b) used micro-CT-based finite element analysis to study the micro-mechanics of bovine trabecular bone and found no appreciable effect of variations in the assumed shape of the post-yield behavior of the tissue on the apparent-level strength. However, they did not address the degree of ductility, imposing no limit on the magnitude of the tissue-level strains. Thus, we investigated how the apparent-level trabecular bone strength and the underlying failure mechanisms are altered when the tissue-level failure behavior is changed from fully ductile to fully brittle — the two extremes of possible tissue-level ductility; we also assessed how these effects depend on typical population variations in bone volume fraction and microarchitecture. In this way, we provided theoretical bounds for the influence of tissue-level ductility on the apparent-level strength of human trabecular bone.

2. Materials and Methods

2.1 Specimen Preparation and Imaging

We analyzed 20 specimens of human trabecular bone, taken from four anatomic sites, 16 cadavers, both sexes, and spanning a wide range of age and bone volume fraction (Table 1). The specimens were originally machined as 8 mm-diameter cylindrical cores along their principal trabecular orientation, and were scanned at a spatial resolution of 10–22 microns using micro-CT (n=15; Scanco Medical AG, Switzerland) or serial milling (n=5) (Beck et al. 1997). For this analysis, we virtually extracted out a 5 mm cube from the central portion of each image and resampled all images to a 22-micron voxel size. Measures of bone volume fraction (BV/TV) and standard microarchitecture parameters (Hildebrand et al.

1999; Laib et al. 2001) were obtained from these specimens, the microarchitecture data available only for the 15 specimens scanned with micro-CT. These 15 specimens displayed substantial heterogeneity in the bone volume fraction and microarchitecture (Mean $\pm$ SD: BV/TV=0.16 $\pm$ 0.06, SMI=1.00 $\pm$ 0.73, Conn.Den. = 5.78 $\pm$ 2.86, Tb.N = 1.39 $\pm$ 0.29, Tb.Sp = 0.70 $\pm$ 0.14, Tb.Th = 0.17 $\pm$ 0.03, Tb.(1/N).SD = 0.26 $\pm$ 0.05, Tb.Sp.SD = 0.24 $\pm$ 0.06, Tb.Th.SD = 0.06 $\pm$ 0.01, and DA = 1.88 $\pm$ 0.35).

2.2 Finite Element Modeling

Voxel-based finite element models were generated for each specimen by converting each image voxel in the 5 mm cube to an 8-noded brick element (22 micron element size). This element size fulfilled the convergence requirements (Guldberg et al. 1998; Niebur et al. 1999). In each model, all finite elements were assigned the same hard tissue material properties, having an isotropic elastic modulus of 18.0 GPa (Bayraktar et al. 2004), a Poisson's ratio of 0.3, and failure properties as described below. Displacement roller-type boundary conditions were applied to impose unconstrained uniaxial compression loading.

For each model, two separate non-linear finite element analyses were performed to simulate fully ductile and fully brittle tissue-level behaviors (Figure 1). For both types of analyses, kinematic large-deformation geometric non-linearity was included in the constitutive model (Bevill et al. 2006; Stolken and Kinney 2003). All analyses were performed using a highly scalable, implicit parallel finite element framework, Olympus (Adams et al. 2004) on a Sun Constellation Linux Cluster (Ranger; TACC, TX, USA). On average, each fully ductile analysis required about 100 CPU hours and each fully brittle analysis required about 450 CPU hours.

For the fully ductile case, as described in detail elsewhere (Bevill et al. 2006), we assumed tissue-level failure by yielding, using a rate-independent elasto-plasticity type model (Papadopoulos and Lu 1998). Such yielding comprised a modified von-Mises criterion with tension-compression strength asymmetry (Bayraktar et al. 2004; Niebur et al. 2000), which was achieved by introducing pseudo-kinematic hardening to shift the yield envelope. The tissue-level yield strains of 0.33% in tension and -0.81% in compression were chosen (Bevill et al. 2006). The apparent-level yield strength was obtained from the computed apparent-level stress-strain curves using a 0.2%-offset criterion (Figure 2). Such measures of apparent-level yield strength have been well validated previously in experiments (Bevill et al. 2006; Sanyal et al. 2012) and are about 83% of, and highly correlated ($R^2=0.96$) with, ultimate strength (Crawford et al. 2003).

For the fully brittle case, element yielding was not allowed and instead we used a quasi-nonlinear approach to simulate brittle fracture of any elements that exceeded the assumed tissue-level yield strengths in either tension or compression. In particular, an elastic analysis was performed to a specified apparent-level strain; stresses were computed at each element centroid, as was the overall apparent-level stress for the whole specimen. Once either the maximum or minimum principal stress at any element centroid exceeded the assumed respective tissue-level tensile or compressive yield strength, that element was assumed to fail in a brittle manner and we reduced its elastic modulus 100-fold for subsequent analyses. Using these reduced properties in all such failed elements, a new elastic analysis was then

performed for the whole specimen to an incrementally higher apparent-level strain. This whole process was repeated until we generated an overall apparent-level stress-strain curve that displayed an ultimate point. Typically for each of these fully brittle analyses, 20 increments of apparent-level strain were applied, and thus 20 geometrically non-linear but otherwise elastic analyses were performed. The apparent-level ultimate strength was defined as the maximum stress on the computed apparent-level stress-strain curve. An apparent-level 0.2%-offset yield strength was not used for these brittle analyses since the corresponding yield strain was greater than the ultimate strain (Figure 2).

To aid in interpretation of the results, we also performed an ancillary parametric sub-study on three trabecular bone specimens wherein the tissue-level compressive strength was altered while keeping the tissue-level tensile strength constant.

2.3 Cellular Solid Analysis

To provide additional insight, we also used cellular solid theory (Gibson and Ashby 1997) to derive an analytical expression for the ratio of brittle to ductile strengths. As described elsewhere (Cowin 2001; Gibson and Ashby 1997; Keaveny 2001), we assumed a 2D honeycomb-like structure with a hexagonal unit cell having side length L and thickness T , and accounted for tissue-level tension-compression strength asymmetry. As in the computer models, the tissue-level fracture strength was assumed to equal the tissue-level yield strength. Any kinematic large-deformation effects were ignored for analytical simplicity, and the dominant failure mode was assumed to be bending-type failure of the oblique struts. This analysis revealed the following equation for the ratio R of brittle, σ_{brit} , to ductile, σ_{duc} , apparent-level strengths:

$$\sigma_{\text{duc}} = \left\{ \frac{T}{L} \right\}^2 \left(\frac{1}{2\cos^2\theta} \right) \left(\frac{2C}{1+C} \right) \sigma_{y-t}^T ; \quad \sigma_{\text{brit}} = \left\{ \frac{T}{L} \right\}^2 \left(\frac{1}{3\cos^2\theta} \right) \sigma_{y-t}^T ; \quad R = \frac{\sigma_{\text{brit}}}{\sigma_{\text{duc}}} = \frac{1+C}{3C} \quad \text{Equation 1}$$

in which θ is the angle of the oblique struts, σ_{y-t}^T is the assumed tissue-level yield strength in tension, and C is the compressive-to-tensile ratio of the tissue-level yield strength.

2.4 Outcomes and Statistical Analyses

The main quantitative outcomes were the apparent-level strengths and corresponding strains, and bone volume fraction and the microarchitectural properties. We also quantified the proportion of failed tissue (at apparent-level strength), defined as the number of Gauss points exceeding the assumed tissue-level yield point divided by the total number of Gauss points in the model. In the ductile analysis, the simulated tissue failure occurred via *yielding* while in the brittle analysis, tissue failure occurred via *fracture*. However, for simplicity, we henceforth adopt the terminology '*failed tissue*' to denote failed tissue regardless of whether it failed by yielding or fracture. We also distinguished tissue failure by its mode, *i.e.* tensile vs. compressive. Nonlinear and general linear regression models (JMP 9.0; SAS Institute, Cary, NC USA) were used to determine the effect of the ductile versus brittle behaviors on the strength outcomes, accounting for variations in bone volume fraction and microarchitecture.

To further aid in interpretation of our results, model predictions of apparent-level yield (and ultimate) strain were compared against the corresponding experimental data for the same 20 specimens used in the simulations. These experimental data were obtained previously (Kopperdahl and Keaveny 1998; Morgan and Keaveny 2001) from mechanical testing of the full cylindrical specimens from which the 5 mm-cube specimens were virtually extracted.

3. Results

Changing the tissue-level failure behavior from fully ductile to fully brittle substantially reduced the apparent-level strength of the trabecular bone, and did so to a similar degree across all specimens regardless of bone volume fraction and microarchitecture. There was a strong non-linear correlation between the apparent strength and the bone volume fraction for both ductile ($Y=0.064X^{1.73}$, $R^2=0.98$, $p<0.0001$) and brittle ($Y=0.033X^{1.73}$, $R^2=0.96$, $p<0.0001$) behaviors. As a result of the similarity of exponents and the high R^2 values, the ratio of brittle to ductile strength (mean $\pm$ SD = 0.56 ± 0.02) was almost constant across cohort ($R^2=0.99$ between the two strength measures). The slight inter-specimen variation in this ratio was moderately explained by SMI ($R^2=0.58$, $p<0.01$) and bone volume fraction ($R^2=0.44$, $p=0.01$). Since the brittle and ductile strengths were so highly correlated with each other, the correlation coefficients for the variation of brittle strength with microarchitecture were identical to those for ductile strength.

Changing the tissue-level failure behavior from fully ductile to fully brittle also substantially altered the failure mechanisms. At the point of apparent-level failure, on average, only a tiny proportion of the bone tissue had failed ($1.1\% \pm 0.4\%$) for the fully brittle bone, which was almost 16-fold less than for the fully ductile bone ($16.5\% \pm 6.5\%$, Figure 3). In general, the tissue that failed (*i.e.* yielded) in the fully ductile analyses subsumed the tissue that failed (*i.e.* fractured) in the fully brittle analyses (Figure 4). Further, for the fully ductile behavior, the tissue failed primarily in tensile mode at low bone volume fraction ($< 10\%$) and primarily in compressive mode at high bone volume fraction, whereas for the fully brittle behavior, the tissue failed primarily in tensile mode at all bone volume fractions (Figure 5).

Both our parametric finite element study and our cellular solid analysis indicated that increasing the tissue-level compressive strength compared to the tensile strength served to decrease the ratio of apparent-level brittle to ductile strength (Table 2). Further, the uniformity of the brittle-to-ductile strength ratio, as predicted by the finite element analyses for all $n=20$ specimens, was consistent with the cellular solid theory, which prescribed that this ratio depends only on the tissue-level tensile-compressive strength asymmetry ratio C and not at all on bone volume fraction or microarchitecture (see R in Equation 1).

Comparison of the apparent-level failure strains predicted by the model vs. measured by experiment confirmed that the finite element results bounded the real apparent-level trabecular bone behavior (Figure 6). There was good agreement between the experimental measurements of the apparent-level 0.2%-offset yield strain and the fully ductile predictions for those specimens having a bone volume fraction above about 0.20; at the lowest values of bone volume fraction (~ 0.10), the experimental measurements of the apparent-level 0.2%-

offset yield strain fell to about half-way between the predicted yield strain for the fully ductile cases and the ultimate strain for the fully brittle cases.

4. Discussion

These computational results provide theoretical bounds on the effects of tissue-level ductility on apparent-level strength for human trabecular bone. One unexpected observation was the large magnitude of the effect — a two-fold change in the apparent-level strength — and another was its high degree of uniformity across the wide variety of specimens analyzed. The good agreement of the finite element results with those from our cellular solid analysis helps explain both of these unexpected results: if failure of individual trabeculae is dominated by large-deformation-related bending, then the ratio of the fully brittle to fully ductile apparent-level strength is determined by the extension of plastic-type yielding across the cross-section of the individual trabeculae, an effect which is dominated by the ratio of compressive to tensile strength of the trabecular tissue and which is almost insensitive to bone volume fraction and microarchitecture. Taken together, these findings provide new insight into the theoretical effects of tissue-level ductility on the apparent-level trabecular bone strength.

The high degree of uniformity of the observed effect is noteworthy. While the main effect was largely constant across all specimens, the slight variation in the ratio of brittle to ductile apparent-level strength was partially explained by SMI ($r = -0.76$; $p < 0.01$) and bone volume fraction ($r = 0.66$; $p = 0.01$). Since SMI and bone volume fraction are significantly correlated to each other ($R^2 = 0.74$; $p < 0.001$), we speculate that due to its higher correlation, SMI might be the more mechanistic parameter. However, any effect of microarchitecture was small, as demonstrated by the small variation in the ratio of ductile to brittle strength observed across all 20 specimens despite a wide range of bone volume fraction and microarchitecture. Consistent with the finite element results, the analytical cellular solid theory prescribes that the ratio R of brittle to ductile apparent-level strength depends only on the tissue-level strength asymmetry ratio C and not at all on bone volume fraction or microarchitecture. The finite element results show that, for real trabecular microstructures, there is an influence of bone volume fraction and plate- versus rod-like microstructure on the mechanism by which post-yield behavior can influence apparent-level strength — but this is a small effect.

Analysis of the proportion and distribution of failed tissue helps explain the magnitude of the observed two-fold strengthening effect. Our results showed that the fully ductile behavior led to a 16-fold greater proportion of failed tissue compared to the fully brittle behavior, which translated to an almost two-fold higher apparent-level strength. The failure distribution patterns (Figure 5) further indicate that there is more tissue-level failure mode by tension if the bone is fully brittle compared to fully ductile. Since bone tissue is weaker in tension than compression, brittle trabecular bone is therefore weaker than ductile bone because less tissue per unit mass of bone is required to fail in brittle bone, and because most of the failure that does occur in brittle bone is by the weaker tensile failure mode. Previous work has shown that the failure mechanisms for fully ductile tissue-level behavior for low bone volume fraction are strongly influenced by kinematic non-linear “large-deformation”

effects (Bevill et al. 2006; Sanyal et al. 2012; Stolken and Kinney 2003), which render individual trabeculae more susceptible to tensile failure due to excessive bending. In contrast, for high bone volume fraction, the thick, plate-like trabeculae have a tendency to fail primarily in compression. This explains why, going from high to low bone volume fraction, we observed for the ductile bone a shift from compressive to tensile tissue-level failure modes.

Despite this new insight, we acknowledge that our analysis only provides theoretical bounds between two assumed extremes of tissue-level post-yield behavior and does not address the important question of how apparent-level strength could be affected by typical age-, population-, disease-, or treatment-related variations in real post-yield behavior. This remains a topic for future research. Despite our finding of the high degree of uniformity of the ratio of brittle-to-ductile strength, the comparison between our theoretical bounds and the real apparent-level yield strains (Figure 6) suggests a possible interaction between bone volume fraction and ductility. In particular, at high bone volume fraction, the real tissue-level ductility may be sufficiently high as to effectively be fully ductile. This would be consistent with the results from mechanical testing of individual trabeculae in tension which showed an average ultimate strain of about 8% (Hernandez et al. 2005), and also with results from time-elapsd micro-CT imaging studies of whale vertebral trabecular bone under compression testing (Nazarian and Müller 2004), which showed individual trabeculae undergoing large deformations without actually fracturing. However, at low bone volume fraction, our results (Figure 6) also suggest that low levels in the real tissue-level ductility in some individuals may reduce apparent-level strength. Consistent with this, there is much variation in reported measures of the ultimate strain of individual trabeculae, reaching values as low as 1.8% (Hernandez et al. 2005). However, since only one study has been reported for tissue-level ductility for trabecular bone (Hernandez et al. 2005), more studies are needed in this area. In general, the effects of aging on the apparent-level strength of trabecular bone appear to be dominated by reductions in bone volume fraction (and associated changes in microarchitecture) (Keaveny et al. 2001). One could speculate that if the ductility of trabecular tissue were to decrease with age, disease, or treatment in a subset of people — perhaps in those with reduced bone turnover (diminished remodeling rates) and low bone volume fraction who have accumulated mineralization, microdamage or crosslinks — then the data in Figure 6 suggest there might be a potentially important role of such diminished tissue-level ductility in age-related reduction of bone strength, but this effect may only be important at low bone volume fraction.

The study has some limitations. First, the study was purely computational, and thus the model predictions await direct experimental validation. Second, although our models had sufficiently high spatial resolution for numerical convergence (Guldborg et al. 1998; Niebur et al. 1999), they did not include such micro-scale features as intra-specimen variation in mineral density and tissue material properties, or fine details of resorption spaces or microcracks. Although inclusion of such details would likely influence absolute values of any strength estimates (Bourne and van der Meulen 2004; Easley et al. 2010; Jaasma et al. 2001; McNamara et al. 2006; Vaughan et al. 2012; Verhulst et al. 2008a), it is not clear that their exclusion would appreciably influence the direct comparison between the two behaviors. On the one hand, it may be that such micro-scale effects might similarly

influence the brittle and ductile failure mechanisms if failure of individual trabeculae is dominated more by overall load sharing across the trabecular network than by local material behavior; on the other hand, it is possible that fine geometric details of resorption spaces or microcracks might have a more deleterious “stress riser” effect in more brittle bone tissue. Clearly, additional research is required to resolve these issues. Third, to simulate the fully brittle case, we used an iterative quasi-nonlinear approach in which the elastic modulus of failed elements was reduced 100-fold after each step of the analysis. A more detailed way to simulate tissue-level brittle failure might include a 3D constitutive damage model within the framework of continuum mechanics (Garcia et al. 2009; Kosmopoulos and Keller 2008; Zysset and Curnier 1996). Fourth, at the apparent level, we compared a 0.2%-offset yield strength (for ductile behavior) against an ultimate strength (for brittle behavior). Since the 0.2%-offset yield strength for human trabecular bone is highly correlated with the ultimate strength (Crawford et al. 2003), the general trends here should remain valid, and the true ratio of fully brittle to fully ductile strength is likely lower than reported here. It is also possible that a reduction in tissue-level ductility might affect the ratio of apparent-level yield to ultimate strength. Finally, having established bounds of the effects of fully brittle vs. fully ductile tissue-level behavior on the apparent-level strength, it would now be interesting to explore intermediate cases of tissue-level ductility.

In summary, our computational study demonstrated that human trabecular bone is substantially weaker if the bone tissue is fully brittle compared to if it is fully ductile, reflecting an appreciable difference in the proportion and mode of tissue-level failure but an effect that was otherwise constant across a wide range of bone volume fraction and microarchitecture.

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Figure 1.
 Material models depicting the stress-strain response at the tissue-level for: **A)** fully brittle, and **B)** fully ductile bone tissue.

Figure 2.

An example of a typical apparent-level stress-strain curve for trabecular bone corresponding to the fully ductile and fully brittle tissue-level behaviors, computed from the finite element analyses. For the ductile analysis, a 0.2%-offset criterion was used to define the apparent strength. For the brittle analysis, a 0.2%-offset criterion was not used because it exceeded the ultimate strain and thus an ultimate stress was used. The general shape of these curves did not depend on bone volume fraction.

Figure 3.

The variation in the proportion of failed tissue with bone volume fraction for brittle and ductile tissue-level behaviors.

Figure 4.

The spatial distribution of the failed tissue at the apparent-level strength for fully brittle and fully ductile behaviors for three different specimens: **A)** low bone volume fraction (6.2%); **B)** medium bone volume fraction (17.3%); and **C)** high bone volume fraction (28.2%). **Blue:** tissue failure in compressive mode; **Red:** tissue failure in tensile mode.

Figure 5.

The ratio of the volume of failed tissue in tension to the volume of failed tissue in compression (at the apparent-level failure point) plotted versus the bone volume fraction, for the brittle and ductile behaviors. For brittle behavior, the ratio was greater than one for almost all specimens, reflecting a dominance of tensile failure mode at the tissue level. For ductile behavior, the ratio was less than one for most specimens, reflecting a dominance of compressive failure mode at the tissue level, particularly at bone volume fraction ($>20\%$).

Figure 6.

The variation in the apparent-level failure strain vs. bone volume fraction for simulated fully ductile ($R^2=0.54$) and fully brittle ($R^2=0.68$) behaviors from finite element analysis, and for real specimens from mechanical testing ($R^2=0.77$).

Table 1

Specimen details for the final cohort.

Anatomic Site	No. of Specimens ⁺ (male/female)	Age (years)	Bone Volume Fraction (Mean $\pm$ SD)
<i>Femoral Neck</i>	7 (4/3)	69 $\pm$ 9	0.25 $\pm$ 0.05
<i>Greater Trochanter</i>	3(3/0)	66 $\pm$ 10	0.12 $\pm$ 0.03
<i>Proximal Tibia</i>	4(4/0)	65 $\pm$ 9	0.14 $\pm$ 0.04
<i>Vertebral Body</i>	6(4/2)	66 $\pm$ 8	0.09 $\pm$ 0.03
<i>Pooled</i>	20(15/5)	67 $\pm$ 9	0.16 $\pm$ 0.06

⁺ Number of trabecular bone cube specimens; altogether these specimens were taken from a total of 16 different cadavers

Table 2

The influence of variation in tissue-level strength asymmetry (C) on the ratio of brittle to ductile strength as predicted by both cellular solid theory and finite element analysis ($n=3$ specimens; $BV/TV = 8.8\%$, 17.2% , 26.8%).

Tissue-level			Apparent-level	
Compression (σ -comp)	Tension (σ -tens)	σ -comp/ σ -tens	Brittle-to-Ductile strength	
(MPa)	(MPa)	(C)	FE-prediction	Cellular solid theory
145.8	59.4	2.45	0.53 ± 0.03	0.47
102.6	59.4	1.73	0.57 ± 0.02	0.53
59.4	59.4	1.00	0.63 ± 0.02	0.67