Delay-dependent stability of neutral system with mixed time-varying delays and nonlinear perturbations using delay-dividing approach

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Abstract This paper studies delay-dependent robust stability problem for neutral system with mixed time-varying delays and nonlinear perturbations. Based on the delay-dividing approach, a novel Lyapunov functional is constructed, and a novel delay-dependent stability criterion is derived to guarantee the robust stability of the neutral system. Expressed in terms of linear matrix inequalities, the stability condition can be checked using the numerically efficient Matlab LMI Control Toolbox. Two numerical examples are provided to demonstrate the effectiveness and the reduced conservatism of the analysis result.

Keywords Neutral system · Robust stability · Time-varying delay · Nonlinear perturbation

Introduction

A neutral system that involves time delay in both state and state derivative simultaneously is encountered in various areas,

including population ecology [1], distributed networks containing lossless transmission lines [2], heat exchangers, robots in contact with rigid environments [3], etc. Due to its wider application, stability analysis of neutral systems with time-delay is thus the focus of theoretical and practical importance in the last two decades, with many scholars recently paying attention to delay-dependent stability criteria [4–23], generally less conservative than delay-independent ones.

It is well known nonlinearities, as time delays, may cause instability and poor performance of practical systems, which have driven many researchers to investigate the problem of nonlinear perturbed systems with mixed delays [7, 9–18]. In [12–14], based on a descriptor model transformation combined with an integral inequality approach, the robust stability of neutral systems with nonlinear parameter perturbations is studied. However, these model transformations often introduce additional dynamics which leads to relatively conservative results. Moreover, in [13], the derivative of the discrete delay is <1 which limits its bigger application. The free-weighting matrix method was proposed in He et al. [4–6] to study the delay-dependent stability, in which the bounding techniques on some cross product terms are not involved. By using the free-weighting matrix approach, the authors in [15] have studied the robust stability of time-delay neutral systems with nonlinear parameter perturbations. Although this method produces better results in comparison to some existing ones, it is often associated with an increment in variables. In addition, the utilization of augmented-type Lyapunov–Krasovskii functionals presented in [19, 20] have provided significant improvements on the stability results for neutral systems. Recently, a piecewise delay method introduced in [18] and [22] have made their stability criteria less conservative. While the proposed method in the above literatures needs to decide more possible

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variables and LMIs which may increase the complexity of the computation. In [16–21], by dividing the delay interval into multiple segments, a new Lyapunov–Krasovskii functional is constructed with different weighting matrices corresponding to different segments in the Lyapunov–Krasovskii functional. The delay-dividing method is useful for reducing conservatism of the analysis result.

In this paper, the problem of robustly delay-dependent stability for neutral systems with mixed delays and nonlinear perturbations is studied. Motivated by [21], a novel Lyapunov functional is introduced with the time-varying delay divided into different segments, and a new delay-dependent stability criterion is obtained without employing free-weighting matrices. The proposed criterion is both delay-dependent and rate-delay dependent, and the method in this paper have turned out to be less conservative than some existing methods. Finally, a numerical example is given to demonstrate the effectiveness of the proposed method.

Notation: Throughout this paper, T stands for matrix transposition. $\mathbf{R}^n$ is the n -dimensional Euclidean space. $\mathbf{R}^{n \times m}$ is the set of all $n \times m$ real matrices. $\mathbf{C}([-d, 0], \mathbf{R}^n)$ stands for the Banach space of continuous functions mapping the interval $[-d, 0]$ into $\mathbf{R}^n$, with the topology of uniform convergence. I denotes the identity matrix of appropriate dimensions. $P > 0$ means that P is positive definite matrix. $*$ represents the elements below the main diagonal of a symmetric matrix.

Problem statement

Consider the following neutral system with mixed time-varying delays and nonlinear perturbations

$$\begin{cases} \dot{x}(t) = Ax(t) + Bx(t-h(t)) + C\dot{x}(t-d(t)) + f_1(x(t), t) + f_2(x(t-h(t)), t) + f_3(\dot{x}(t-d(t)), t), \\ x(\theta) = \phi(\theta), \quad \dot{x}(\theta) = \varphi(\theta), \quad \forall \theta \in [-\max(h, d), 0], \end{cases} \quad (1)$$

where $x(t) \in \mathbf{R}^n$ is the state vector, $A, B, C \in \mathbf{R}^{n \times n}$ are constant matrices with appropriate dimensions. $h(t)$ and $d(t)$ are time-varying discrete delay and neutral delay respectively, which satisfy

$$0 \leq h(t) \leq h < \infty, \quad |\dot{h}(t)| \leq \mu, \quad 0 \leq d(t) \leq d < \infty, \quad \dot{d}(t) \leq v < 1, \quad (2)$$

where h, μ, d and v are constants. $\phi(\theta), \varphi(\theta)$ are the initial condition functions that are continuously differentiable on $[-\max(h, d), 0]$. $f_1(x(t), t), f_2(x(t-h(t)), t), f_3(\dot{x}(t-d(t)), t)$

are unknown nonlinear perturbations. They satisfy that $f_1(0, t) = 0, f_2(0, t) = 0, f_3(0, t) = 0$, and

$$\begin{aligned} f_1^T(x(t), t)f_1(x(t), t) &\leq \alpha^2 x^T(t)x(t), \\ f_2^T(x(t-h(t)), t)f_2(x(t-h(t)), t) &\leq \beta^2 x^T(t-h(t))x(t-h(t)), \\ f_3^T(\dot{x}(t-d(t)), t)f_3(\dot{x}(t-d(t)), t) &\leq \gamma^2 \dot{x}^T(t-d(t))\dot{x}(t-d(t)), \end{aligned} \quad (3)$$

where $\alpha \geq 0, \beta \geq 0$ and $\gamma \geq 0$ are given constants, for simplicity, $f_1 := f_1(x(t), t), f_2 := f_2(x(t-h(t)), t), f_3 := f_3(\dot{x}(t-d(t)), t)$. In order to derive the maximum allowable delay for stability, we divide $h(t)$ into several components by an integer N with the length of each component denoted by $\tau(t)$, i.e.,

$$\tau(t) = \frac{h(t)}{N}.$$

Therefore, $\tau(t)$ represents a partition of the time-varying delay $h(t)$, which satisfies

$$0 \leq \tau(t) \leq \tau < \infty, \quad |\dot{\tau}(t)| \leq h_d, \quad (4)$$

$$\text{where } \tau = \frac{h}{N}, \quad h_d = \frac{\mu}{N}.$$

The following lemmas and definition are introduced, which will be used in the proof of the main results.

Lemma 1 [23] For any constant matrix $0 < R = R^T \in \mathbf{R}^{n \times n}$, scalar $\gamma > 0$, vector function $\omega : [0, \gamma] \rightarrow \mathbf{R}^n$ such that the integrations concerned are well defined, then

$$\left(\int_0^\gamma \omega(s) ds \right)^T R \left(\int_0^\gamma \omega(s) ds \right) \leq \gamma \int_0^\gamma \omega^T(s) R \omega(s) ds.$$

Lemma 2 (Schur Complement) Given one positive definite matrix $S_2 > 0$ and constant matrices S_1, S_3 , where $S_1 = S_1^T$, then $S_1 + S_3^T S_2^{-1} S_3 < 0$ if and only if

$$\begin{pmatrix} S_1 & S_3^T \\ S_3 & -S_2 \end{pmatrix} < 0 \quad \text{or} \quad \begin{pmatrix} -S_2 & S_3 \\ S_3^T & S_1 \end{pmatrix} < 0.$$

The operator $\mathcal{D} : \mathbf{C}([-d, 0], \mathbf{R}^n) \rightarrow \mathbf{R}^n$ is defined to be $\mathcal{D}(t) = x(t) - Cx(t-d)$. Its stability is defined as follows

Definition 1 [24] The operator $\mathcal{D}$ is said to be stable if the zero solution of the homogeneous difference equation $\mathcal{D}(t) = 0, t \geq 0, x_0 = \psi \in \{\phi \in \mathbf{C}([-d, 0]) : \mathcal{D}\phi = 0\}$ is uniformly asymptotically stable.

Stability analysis

In this section, we present a delay-dependent robustly stable criterion for the system (1) with uncertainty (3) by using a delay-dividing approach.

Theorem 1 For given scalars α, β and γ , system (1) with uncertainty (3) and mixed time-varying delays satisfying (2) is robustly stable if $\mathcal{D}$ is stable and there exist positive definite matrices $P > 0, R_j > 0, W_j > 0, Q_i > 0, Z_i > 0 (i = 1, 2, \dots, N; j = 1, 2)$, and scalars $\varepsilon_j \geq 0 (j = 1, 2, 3)$, such that the following symmetric linear matrix inequality holds:

$$\Theta = \begin{pmatrix} \Theta_1 & \Theta_2 & \Gamma^T \Lambda \\ \Theta_2^T & \Theta_3 & 0 \\ \Lambda^T \Gamma & 0 & -\Lambda \end{pmatrix} < 0, \quad (5)$$

where

$$\Gamma = (A, B, 0, C, 0, I, I, I), \quad \Lambda = W_1 + d^2 W_2 + \tau^2 \sum_{i=1}^N Z_i,$$

$$\Theta_1 = \begin{pmatrix} \Theta_{11} & PB & 0 & PC & W_2 & P & P & P \\ * & \Theta_{22} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -(1-v)R_2 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -(1-v)W_1 + \varepsilon_3 \gamma^2 I & 0 & 0 & 0 & 0 \\ * & * & * & * & -R_1 - W_2 & 0 & 0 & 0 \\ * & * & * & * & * & -\varepsilon_1 I & 0 & 0 \\ * & * & * & * & * & * & -\varepsilon_2 I & 0 \\ * & * & * & * & * & * & * & -\varepsilon_3 I \end{pmatrix}$$

with

$$\Theta_{11} = PA + A^T P + R_1 + R_2 - W_2 + Q_1 - Z_1 + \varepsilon_1 \alpha^2 I,$$

$$\Theta_{22} = -(1 - N\mu)Q_N - Z_N + \varepsilon_2 \beta^2 I,$$

$$\Theta_2 = \begin{pmatrix} Z_1 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & Z_N \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 \end{pmatrix},$$

$$\Theta_3 = \begin{pmatrix} M_1 & Z_2 & & & \\ Z_2 & M_2 & \ddots & & \\ & \ddots & \ddots & \ddots & \\ & & \ddots & M_{N-2} & Z_{N-1} \\ & & & Z_{N-1} & M_{N-1} \end{pmatrix},$$

where the unwritten are zero blocks and

$$M_i = -(1 - i\mu)Q_i + (1 + i\mu)Q_{i+1} - Z_i - Z_{i+1}.$$

Proof Choose a Lyapunov functional candidate for the system (1) to be

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t), \quad (6)$$

where

$$V_1(t) = x^T(t)Px(t), \quad (7)$$

$$V_2(t) = \int_{t-d}^t x^T(s)R_1x(s)ds + \int_{t-d(t)}^t x^T(s)R_2x(s)ds, \quad (8)$$

$$V_3(t) = \int_{t-d(t)}^t \dot{x}^T(s)W_1\dot{x}(s)ds + \sum_{i=1}^N \int_{t-i\tau(t)}^{t-(i-1)\tau(t)} x^T(s)Q_i x(s)ds, \quad (9)$$

$$V_4(t) = \int_{-d}^0 \int_{t+\theta}^t d\dot{x}^T(s)W_2\dot{x}(s)dsd\theta + \sum_{i=1}^N \int_{-i\tau}^{-(i-1)\tau} \int_{t+\theta}^t \tau \dot{x}^T(s)Z_i\dot{x}(s)dsd\theta, \quad (10)$$

where $P = P^T > 0, R_j = R_j^T > 0, W_j = W_j^T > 0, Q_i = Q_i^T > 0, Z_i = Z_i^T > 0 (i = 1, 2, \dots, N; j = 1, 2)$ are to be determined.

From (3), we can obtain for any scalars $\varepsilon_1 > 0, \varepsilon_2 > 0$ and $\varepsilon_3 > 0$,

$$\begin{aligned} \varepsilon_1 \left(\alpha^2 x^T(t)x(t) - f_1^T(x(t), t)f_1(x(t), t) \right) &\geq 0, \\ \varepsilon_2 \left(\beta^2 x^T(t-h(t))x(t-h(t)) - f_2^T(x(t-h(t)), t)f_2(x(t-h(t)), t) \right) &\geq 0, \\ \varepsilon_3 \left(\gamma^2 \dot{x}^T(t-d(t))\dot{x}(t-d(t)) - f_3^T(\dot{x}(t-d(t)), t)f_3(\dot{x}(t-d(t)), t) \right) &\geq 0. \end{aligned} \quad (11)$$

Next, taking the derivative of $V(t)$ with respect to t along the solution of system (1) yields

$$\begin{aligned} \dot{V}_1(t) &= 2x^T(t)P\dot{x}(t) = 2x^T(t)P(Ax(t) + Bx(t-h(t)) \\ &\quad + C\dot{x}(t-d(t)) + f_1 + f_2 + f_3) \\ &= x^T(t)(PA + A^TP)x(t) + 2x^T(t)PBx(t-h(t)) \\ &\quad + 2x^T(t)PC\dot{x}(t-d(t)) \\ &\quad + 2x^T(t)Pf_1 + 2x^T(t)Pf_2 + 2x^T(t)Pf_3, \end{aligned} \quad (12)$$

$$\begin{aligned} \dot{V}_2(t) &\leq x^T(t)(R_1 + R_2)x(t) - x^T(t-d)R_1x(t-d) \\ &\quad - (1-v)x^T(t-d(t))R_2x(t-d(t)), \end{aligned} \quad (13)$$

$$\begin{aligned} \dot{V}_3(t) &= \dot{x}^T(t)W_1\dot{x}(t) - (1-\dot{d}(t))\dot{x}^T(t-d(t))W_1\dot{x}(t-d(t)) \\ &\quad + \sum_{i=1}^N \left[(1-(i-1)\dot{\tau}(t))x^T(t-(i-1)\tau(t)) \right. \\ &\quad \left. Q_ix(t-(i-1)\tau(t)) - (1-i\dot{\tau}(t))x^T(t-i\tau(t)) \right. \\ &\quad \left. Q_ix(t-i\tau(t)) \right] \dot{x}^T(t)W_1\dot{x}(t) - (1-v)\dot{x}^T(t-d(t)) \\ &\quad W_1\dot{x}(t-d(t)) + \sum_{i=1}^N \left[(1+(i-1)h_d)x^T(t-(i-1) \right. \\ &\quad \left. \tau(t))Q_ix(t-(i-1)\tau(t)) - (1-ih_d)x^T(t-i\tau(t)) \right. \\ &\quad \left. Q_ix(t-i\tau(t)) \right], \leq \dot{x}^T(t)W_1\dot{x}(t) - (1-v)\dot{x}^T(t-d(t)) \\ &\quad W_1\dot{x}(t-d(t)) + \sum_{i=1}^N \left[(1+(i-1)\mu)x^T(t-(i-1) \right. \\ &\quad \left. \tau(t))Q_ix(t-(i-1)\tau(t)) - (1-i\mu)x^T(t-i\tau(t)) \right. \\ &\quad \left. Q_ix(t-i\tau(t)) \right], \end{aligned} \quad (14)$$

$$\begin{aligned} \dot{V}_4(t) &\leq \dot{x}^T(t) \left(d^2W_2 + \tau^2 \sum_{i=1}^N Z_i \right) \dot{x}(t) - \int_{t-d}^t \dot{x}^T(s)W_2\dot{x}(s)ds \\ &\quad - \sum_{i=1}^N \int_{t-i\tau}^{t-(i-1)\tau} \tau \dot{x}^T(\theta)Z_i\dot{x}(\theta)d\theta \leq \dot{x}^T(t) \\ &\quad \left(d^2W_2 + \tau^2 \sum_{i=1}^N Z_i \right) \dot{x}(t) - \int_{t-d}^t \dot{x}^T(s)W_2\dot{x}(s)ds \\ &\quad - \sum_{i=1}^N \int_{t-i\tau(t)}^{t-(i-1)\tau(t)} \tau \dot{x}^T(\theta)Z_i\dot{x}(\theta)d\theta \end{aligned}$$

$$\begin{aligned} &\leq \dot{x}^T(t) \left(d^2W_2 + \tau^2 \sum_{i=1}^N Z_i \right) \dot{x}(t) - \left(\int_{t-d}^t \dot{x}(s)ds \right)^T \\ &\quad W \left(\int_{t-d}^t \dot{x}(s)ds \right) - \sum_{i=1}^N \left(x^T(t-(i-1)\tau(t)) - x^T(t-i\tau(t)) \right) \\ &\quad Z_i \left(x(t-(i-1)\tau(t)) - x(t-i\tau(t)) \right) \\ &= \dot{x}^T(t) \left(d^2W_2 + \tau^2 \sum_{i=1}^N Z_i \right) \dot{x}(t) - x^T(t)W_2x(t) - x^T(t-d) \\ &\quad W_2x(t-d) + 2x^T(t)W_2x(t-d) - \sum_{i=1}^N \left(x^T(t-(i-1)\tau(t)) \right. \\ &\quad \left. Z_ix^T(t-(i-1)\tau(t)) - 2x^T(t-(i-1)\tau(t))Z_ix(t-i\tau(t)) \right. \\ &\quad \left. + x^T(t-i\tau(t))Z_ix(t-i\tau(t)) \right) \end{aligned} \quad (15)$$

Then combining Eqs. (11–15), we derive

$$\begin{aligned} \dot{V}(t) &= \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) + \dot{V}_4(t) + \varepsilon_1 \left(\alpha^2 x^T(t)x(t) \right. \\ &\quad \left. - f_1^T(x(t), t)f_1(x(t), t) \right) + \varepsilon_2 \left(\beta^2 x^T(t-h(t))x(t-h(t)) \right. \\ &\quad \left. - f_2^T(x(t-h(t)), t)f_2(x(t-h(t)), t) \right) \\ &\quad + \varepsilon_3 \left(\gamma^2 \dot{x}^T(t-d(t))\dot{x}(t-d(t)) - f_3^T(\dot{x}(t-d(t)), t) \right. \\ &\quad \left. f_3(\dot{x}(t-d(t)), t) \right) = x^T(t)(PA + A^TP + R_1 + R_2 \\ &\quad - W_2 + Q_1 - Z_1 + \varepsilon_1\alpha^2I)x(t) + 2x^T(t)PBx(t-h(t)) \\ &\quad + 2x^T(t)PC\dot{x}(t-d(t)) + 2x^T(t)W_2x(t-d) \\ &\quad + 2x^T(t)Pf_1 + 2x^T(t)Pf_2 + 2x^T(t)Pf_3 \\ &\quad + x^T(t-h(t)) \left(-(1-N\mu)Q_N - Z_N + \varepsilon_2\beta^2I \right) x(t-h(t)) \\ &\quad + x^T(t-d(t)) \left(-(1-v)R_2 \right) x(t-d(t)) \\ &\quad + \dot{x}^T(t-d(t)) \left(-(1-v)W_1 + \varepsilon_3\gamma^2I \right) \dot{x}(t-d(t)) \\ &\quad + x^T(t-d) \left(-R_1 - W_2 \right) x(t-d) + f_1^T(-\varepsilon_1I)f_1 \\ &\quad + f_2^T(-\varepsilon_2I)f_2 + f_3^T(-\varepsilon_3I)f_3 + \dot{x}^T(t) \\ &\quad \left(W_1 + d^2W_2 + \tau^2 \sum_{i=1}^N Z_i \right) \dot{x}(t) \\ &\quad + 2x^T(t)Z_1x(t-\tau(t)) + x^T(t-\tau(t)) \\ &\quad \left(-(1-\mu)Q_1 + (1+\mu)Q_2 - Z_1 - Z_2 \right) x(t-\tau(t)) \\ &\quad + 2x^T(t-\tau(t))Z_2x(t-2\tau(t)) + x^T(t-2\tau(t)) \\ &\quad \left(-(1-2\mu)Q_2 + (1+2\mu)Q_3 - Z_2 - Z_3 \right) x(t-2\tau(t)) \\ &\quad + \dots + 2x^T(t-(N-2)\tau(t))Z_{N-1}x(t-(N-1)\tau(t)) \\ &\quad + x^T(t-(N-1)\tau(t)) \left(-(1-(N-1)\mu)Q_{N-1} \right. \\ &\quad \left. + (1+(N-1)\mu)Q_N - Z_{N-1} - Z_N \right) \cdot \\ &\quad x(t-(N-1)\tau(t)) + 2x^T(t-(N-1)\tau(t)) \\ &\quad Z_Nx(t-h(t)) = \xi^T(t) \begin{bmatrix} \Theta_1 + \Gamma^T\Lambda\Gamma & \Theta_2 \\ \Theta_2^T & \Theta_3 \end{bmatrix} \xi(t), \end{aligned} \quad (16)$$

where

$$\xi(t) = \begin{bmatrix} x^T(t), x^T(t-h(t)), x^T(t-d(t)), \dot{x}^T(t-d(t)), \\ x^T(t-d), f_1, f_2, f_3, x^T(t-\tau(t))x^T(t-2\tau(t)), \\ \dots, x^T(t-(N-1)\tau(t)) \end{bmatrix}^T. \quad (17)$$

Using the Schur complement Lemma,

$$\begin{bmatrix} \Theta_1 + \Gamma^T \Lambda \Gamma & \Theta_2 \\ \Theta_2^T & \Theta_3 \end{bmatrix} < 0 \quad (18)$$

is equivalent to (5). It is easy to see that results in $\dot{V}(t) < 0$ from (18). Therefore, according to Hale [24], if the operator $\mathcal{D}$ is stable and there exist symmetric positive definite matrices $P > 0, R_j > 0, W_j > 0, Q_i > 0, Z_i > 0 (i = 1, 2, \dots, N; j = 1, 2)$, and scalars $\varepsilon_j \geq 0 (j = 1, 2, 3)$ such that the LMI (5) is satisfied, then system (1) with uncertainty (3) is robustly stable. This completes the proof.

Remark 1 Theorem 1 gives a delay-dependent and rate-dependent stability criterion for system (1) by employing a delay-dividing approach as in [21]. However, in [21], the author obtained the stability results by dividing the constant time delay into several components. In our paper, we study the robust stability problem for time-varying delay systems. By dividing the time-varying delay into multiple segments, a new Lyapunov–Krasovskii functional is constructed with different weighting matrices corresponding to different segments in the Lyapunov–Krasovskii functional. Although a piecewise delay method introduced in [18] and [22] have made their stability criteria less conservative, more possible variables and LMIs must be decided which make the computation quite complex. In Theorem 1, a delay-dividing approach is proposed to estimate the upper bound of the derivative of Lyapunov functional without using as more as possible decision variables. Compared with those in [13–18], the method in this paper is more useful.

Remark 2 In [25], discretized Lyapunov functional method is mainly used to discuss the stability of a class of constant time delay systems. The advantage of this method is that the estimation of the upper bound of delays ensuring system stability can be very close to its real value. However, the algorithm of using this method is complicated. Moreover, it has two limitations: firstly, this method can only be applicable to systems with constant time delays, while it fails for systems with time-varying delays; secondly, it is difficult to be extended to the synthesis of a control system. While in this paper, a delay-dividing approach can be applied to systems with time-varying delays, and it is very important role in reducing conservativeness of our results.

Remark 3 If $d(t) \equiv d$, or $d(t) \equiv d$ and $h(t) \equiv h$, then system (1) reduces to the following system, respectively:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bx(t-h(t)) + C\dot{x}(t-d) + f_1(x(t), t) \\ \quad + f_2(x(t-h(t)), t) + f_3(\dot{x}(t-d), t), \\ x(\theta) = \phi(\theta), \quad \dot{x}(\theta) = \varphi(\theta), \forall \theta \in [-\max(d, h), 0], \end{cases} \quad (19)$$

or

$$\begin{cases} \dot{x}(t) = Ax(t) + Bx(t-h) + C\dot{x}(t-d) + f_1(x(t), t) \\ \quad + f_2(x(t-h), t) + f_3(\dot{x}(t-d), t), \\ x(\theta) = \phi(\theta), \quad \dot{x}(\theta) = \varphi(\theta), \forall \theta \in [-\max(d, h), 0]. \end{cases} \quad (20)$$

Then, according to Theorem 1, we have the following corollaries for the delay-dependent stability of system (19) and (20), respectively.

Corollary 1 For given scalars α, β and γ , system (19) with uncertainty and mixed time delays is robustly stable if $\mathcal{D}$ is stable and there exist positive definite matrices $P > 0, R_1 > 0, W_j > 0, Q_i > 0, Z_i > 0 (i = 1, 2, \dots, N; j = 1, 2)$, and scalars $\varepsilon_j \geq 0 (j = 1, 2, 3)$, such that the following symmetric linear matrix inequality holds:

$$\hat{\Theta} = \begin{pmatrix} \hat{\Theta}_1 & \hat{\Theta}_2 & \hat{\Gamma}^T \Lambda \\ \hat{\Theta}_2^T & \Theta_3 & 0 \\ \Lambda^T \hat{\Gamma} & 0 & -\Lambda \end{pmatrix} < 0, \quad (21)$$

where

$$\hat{\Gamma} = (A, B, C, 0, I, I, I), \quad \Lambda = W_1 + d^2 W_2 + \tau^2 \sum_{i=1}^N Z_i,$$

$$\hat{\Theta}_1 = \begin{pmatrix} \hat{\Theta}_{11} & PB & PC & W_2 & P & P & P \\ * & \Theta_{22} & 0 & 0 & 0 & 0 & 0 \\ * & * & -W_1 + \varepsilon_3 \gamma^2 I & 0 & 0 & 0 & 0 \\ * & * & * & -R_1 - W_2 & 0 & 0 & 0 \\ * & * & * & * & -\varepsilon_1 I & 0 & 0 \\ * & * & * & * & * & -\varepsilon_2 I & 0 \\ * & * & * & * & * & * & -\varepsilon_3 I \end{pmatrix}$$

with

$$\hat{\Theta}_{11} = PA + A^T P + R_1 - W_2 + Q_1 - Z_1 + \varepsilon_1 \alpha^2 I$$

$$\Theta_{22} = -(1 - N\mu)Q_N - Z_N + \varepsilon_2 \beta^2 I$$

$$\hat{\Theta}_2 = \begin{pmatrix} Z_1 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & Z_N \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 \end{pmatrix},$$

$$\Theta_3 = \begin{pmatrix} M_1 & Z_2 & & & & \\ Z_2 & M_2 & \ddots & & & \\ & \ddots & \ddots & \ddots & & \\ & & \ddots & \ddots & M_{N-2} & Z_{N-1} \\ & & & \ddots & Z_{N-1} & M_{N-1} \end{pmatrix},$$

where the unwritten are zero blocks and

$$M_i = -(1 - i\mu)Q_i + (1 + i\mu)Q_{i+1} - Z_i - Z_{i+1}.$$

Proof Choose a Lyapunov functional candidate for the system (19) to be

$$\begin{aligned} V(t) = & x^T(t)Px(t) + \int_{t-d}^t x^T(s)R_1x(s)ds \\ & + \int_{t-d}^t \dot{x}^T(s)W_1\dot{x}(s)ds + \int_{-d}^0 \int_{t+\theta}^t d\dot{x}^T(s)W_2\dot{x}(s)dsd\theta \\ & + \sum_{i=1}^N \int_{t-i\tau(t)}^{t-(i-1)\tau(t)} x^T(s)Q_i x(s)ds \sum_{i=1}^N \int_{-i\tau}^{-(i-1)\tau} \\ & \int_{t+\theta}^t \tau \dot{x}^T(s)Z_i \dot{x}(s)dsd\theta. \end{aligned} \quad (22)$$

Next, similar to Theorem 1, we can get the result of Corollary 1. This completes the proof.

Corollary 2 For given scalars α, β and γ , system (20) with uncertainty and mixed constant delays is robustly stable if the $\mathcal{D}$ is stable and there exist positive definite matrices $P > 0, R_1 > 0, W_j > 0, Q_i > 0, Z_i > 0 (i = 1, 2, \dots, N; j = 1, 2)$, and scalars $\varepsilon_j \geq 0 (j = 1, 2, 3)$, such that the following symmetric linear matrix inequality holds:

$$\tilde{\Theta} = \begin{pmatrix} \hat{\Theta}_1 & \hat{\Theta}_2 & \hat{\Gamma}^T \Lambda \\ \hat{\Theta}_2^T & \hat{\Theta}_3 & 0 \\ \Lambda^T \hat{\Gamma} & 0 & -\Lambda \end{pmatrix} < 0, \quad (23)$$

here

$$\begin{aligned} \hat{\Gamma} = & (A, B, C, 0, I, I, I), \quad \Lambda = W_1 + d^2 W_2 + \tau^2 \sum_{i=1}^N Z_i, \\ \tilde{\Theta}_1 = & \begin{pmatrix} \hat{\Theta}_{11} & PB & PC & W_2 & P & P & P \\ * & \hat{\Theta}_{22} & 0 & 0 & 0 & 0 & 0 \\ * & * & -W_1 + \varepsilon_3 \gamma^2 I & 0 & 0 & 0 & 0 \\ * & * & * & -R_1 - W_2 & 0 & 0 & 0 \\ * & * & * & * & -\varepsilon_1 I & 0 & 0 \\ * & * & * & * & * & -\varepsilon_2 I & 0 \\ * & * & * & * & * & * & -\varepsilon_3 I \end{pmatrix}, \end{aligned}$$

with

$$\hat{\Theta}_{11} = PA + A^T P + R_1 - W_2 + Q_1 - Z_1 + \varepsilon_1 \alpha^2 I$$

$$\tilde{\Theta}_{22} = -Q_N - Z_N + \varepsilon_2 \beta^2 I$$

$$\hat{\Theta}_2 = \begin{pmatrix} Z_1 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & Z_N \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 \end{pmatrix},$$

$$\tilde{\Theta}_3 = \begin{pmatrix} M_1 & Z_2 & & & & \\ Z_2 & M_2 & \ddots & & & \\ & \ddots & \ddots & \ddots & & \\ & & \ddots & \ddots & M_{N-2} & Z_{N-1} \\ & & & \ddots & Z_{N-1} & M_{N-1} \end{pmatrix},$$

where the unwritten are zero blocks and

$$M_i = -Q_i + Q_{i+1} - Z_i - Z_{i+1}.$$

Illustrative examples

In this section, we use two examples and compare our results with the previous ones to show the effectiveness of ours.

Example 1 Consider the following system as in [18] with:

$$A = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 0.4 \\ 0.4 & 0 \end{pmatrix}, \quad C = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.1 \end{pmatrix},$$

where $\alpha = 0.1, \beta = \gamma = 0.05$. Let $v = 0.6, \mu$ is unknown. We apply Theorem 1 to calculate the maximal upper bound of $d = h = 4.9423$ which guarantees the robust stability of the system for $N = 3$. However, applying criteria in [16] and [18], the maximum value of $d = h$ for the above system is 3.9563 and 4.6235, respectively. It can be seen that our proposed stability criterion is less conservative than those in [16–18].

Example 2 Consider the following system as in Han [13] and Zhang [14] with:

$$\begin{aligned} P &= \begin{pmatrix} 11.3444 & 0.2924 \\ 0.2924 & 7.0873 \end{pmatrix}, R_1 = \begin{pmatrix} 0.0100 & 0.0003 \\ 0.0003 & 0.0070 \end{pmatrix}, R_2 = \begin{pmatrix} 0.0100 & 0.0003 \\ 0.0003 & 0.0070 \end{pmatrix}, \\ W_1 &= \begin{pmatrix} 2.4745 & 0.0814 \\ 0.0814 & 2.1716 \end{pmatrix}, W_2 = \begin{pmatrix} 0.5305 \times 10^{-4} & 0.0124 \times 10^{-4} \\ 0.0124 \times 10^{-4} & 0.3830 \times 10^{-4} \end{pmatrix}, \\ Q_1 &= \begin{pmatrix} 0.0138 & 0.0004 \\ 0.0004 & 0.0097 \end{pmatrix}, Q_2 = \begin{pmatrix} 0.0026 & 0.0001 \\ 0.0001 & 0.0018 \end{pmatrix}, \\ Q_3 &= \begin{pmatrix} 0.8741 \times 10^{-3} & 0.0219 \times 10^{-3} \\ 0.0219 \times 10^{-3} & 0.6124 \times 10^{-3} \end{pmatrix}, Z_1 = \begin{pmatrix} 0.5297 \times 10^{-3} & 0.0121 \times 10^{-3} \\ 0.0121 \times 10^{-3} & 0.3707 \times 10^{-3} \end{pmatrix}, \\ Z_2 &= \begin{pmatrix} 0.4931 \times 10^{-3} & 0.0123 \times 10^{-3} \\ 0.0123 \times 10^{-3} & 0.3462 \times 10^{-3} \end{pmatrix}, Z_3 = \begin{pmatrix} 19.7201 & 1.8822 \\ 1.8822 & 14.1308 \end{pmatrix}, \\ \varepsilon_1 &= 468.9253, \quad \varepsilon_2 = 139.0536, \quad \varepsilon_3 = 57.6563. \end{aligned}$$

$$A = \begin{pmatrix} -1.2 & 0.1 \\ -0.1 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} -0.6 & 0.7 \\ -1 & -0.8 \end{pmatrix}, \quad C = \begin{pmatrix} c & 0 \\ 0 & c \end{pmatrix},$$

where $0 \leq |c| < 1, \alpha \geq 0, \beta \geq 0, \gamma \geq 0$.

Case I Let $c = 0.1, \beta = 0.1, d = 1, v = 0, \mu = 0.5$. For $N = 3$ and different values of γ , we apply Corollary 1 to calculate the maximal upper bound of h which guarantees the robust stability of the system. For different γ , Table 1

illustrates the numerical results $\alpha = 0$ and $\alpha = 0.1$, respectively. It is easy to see from Table 1 that the maximum allowable delay h decreases as γ increases. In addition, it can be seen that our proposed stability criterion gives a much less conservative result than those in [13–18].

Case II Next, we will consider this case that the discrete delay and the neutral delay are all time-varying. For $c = 0.1, \beta = 0.1, \mu = v = 0.5$, and different values of γ , the maximal upper bound of $d = h$ obtained from Theorem 1 are listed in Table 2. when $N = 3, \alpha = 0.1$ and $\gamma = 0.3$, by solving LMI (5) of Theorem 1, we obtain the maximum bound $d = h = 1.0422$ with the following solutions

For other cases, we can also obtain the maximum bound $h = d$ with the solutions which we omit here. In addition, if $\mu \geq 1$, for $\alpha = 0.1, \beta = 0.2, \gamma = 0.1, v = 0.5$, and different values of c , the maximum upper bounds on the allowable delay of $d = h$ obtained from Theorem 1 are listed in Table 3. Obviously, the stability criterion in [13] cannot be applied to this case in that the derivative of the discrete delay is < 1 . From Table 3, as c increases, h decreases. When $0 \leq c \leq 0.42$, it can be seen from Table 3 that the

Table 1 Maximum upper bound of h with $\mu = 0.5$ and different values of γ

γ	$\alpha = 0$				$\alpha = 0.1$			
	0	0.1	0.2	0.3	0	0.1	0.2	0.3
Han [13]	0.9328	0.7402	0.5637	0.4042	0.8148	0.6439	0.4864	0.3433
Zhang [14]	0.9488	0.7695	0.6087	0.4667	0.8408	0.6841	0.5420	0.4144
Qiu [15]	0.9839	0.8024	0.6392	0.4941	0.8752	0.7166	0.5727	0.4438
Rakkiyappan [16]	1.4886	1.2437	0.9921	0.7367	1.3244	1.0901	0.8475	0.6300
Lakshmanan [17]	1.6325	1.3386	1.0816	0.8563	1.4440	1.1950	0.9734	0.7760
Cheng [18]	1.6865	1.3721	1.0923	0.8613	1.4721	1.2466	0.9996	0.7804
Corollary 1 (N=3)	2.2587	2.0022	1.7469	1.4925	2.2227	1.9686	1.7136	1.4598

Table 2 Maximum upper bound of $h = d$ with $\mu = \nu = 0.5$ and different values of γ

γ	$\alpha = 0$				$\alpha = 0.1$			
	0	0.1	0.2	0.3	0	0.1	0.2	0.3
Han [13]	0.8524	0.5936	0.3686	0.1795	0.7434	0.5131	0.3112	0.1398
Qiu [15]	0.9075	0.6665	0.4619	0.2930	0.8084	0.5969	0.4151	0.2618
Rakkiyappan [16]	1.3881	1.0391	0.6803	0.3940	1.2324	0.8885	0.5822	0.3393
Lakshmanan [17]	1.5088	1.1251	0.8070	0.5419	1.3390	1.0111	0.7323	0.4955
Cheng [18]	1.5924	1.1582	0.8247	0.5645	1.3723	1.0379	0.7385	0.5513
Theorem 1 (N=3)	2.1540	1.7919	1.4319	1.0743	2.1198	1.7583	1.3992	1.0422
Theorem 1 (N=4)	2.1745	1.8121	1.4520	1.0943	2.1395	1.7677	1.4088	1.0616

Table 3 Maximum upper bound of $d = h$ with different values of c

c	0	0.1	0.2	0.3	0.4	0.42	0.43
Cheng [18]	1.2585	1.0464	0.8630	0.7079	0.578	0.5550	0.541
Theorem 1 (N=3)	1.9494	1.6209	1.2924	0.9642	0.6366	0.5712	0.5385

proposed method in this work provides a less conservative delay bound in comparison to the one in [18]. But, when $c \geq 0.43$, the proposed stability criterion in [18] gives a much less conservative result than those in our paper.

Conclusion

This paper has discussed robust stability problem for neutral systems with mixed time-varying delays and nonlinear perturbations. Based on a novel Lyapunov functional method and linear matrix inequality technology, delay-dependent stability conditions are derived by using a delay-dividing approach. The proposed criteria are both delay-dependent and rate-dependent. Two numerical examples are given to demonstrate the effectiveness of the proposed method.

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