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PATH OPTIMIZATION AND CONTROL OF A SHAPE MEMORY ALLOY ACTUATED CATHETER FOR ENDOCARDIAL RADIOFREQUENCY ABLATION

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Abstract

This paper introduces a real-time path optimization and control strategy for shape memory alloy (SMA) actuated cardiac ablation catheters, potentially enabling the creation of more precise lesions with reduced procedure times and improved patient outcomes. Catheter tip locations and orientations are optimized using parallel genetic algorithms to produce continuous ablation paths with near normal tissue contact through physician-specified points. A nonlinear multivariable control strategy is presented to compensate for SMA hysteresis, bandwidth limitations, and coupling between system inputs. Simulated and experimental results demonstrate efficient generation of ablation paths and optimal reference trajectories. Closed-loop control of the SMA-actuated catheter along optimized ablation paths is validated experimentally.

1. Introduction

Atrial Fibrillation (AF) is the most common cardiac arrhythmia, affecting over 2.7 million Americans, and is a leading cause of thrombus and stroke [1], [2]. AF occurs when the heart's normal electrical impulses are disrupted by random electrical activity, preventing organized contraction of the heart's upper chambers, causing only partial blood evacuation to the lower chambers. Treatment typically requires isolating these abnormal electrical impulses by forming scars in the atrial tissue. Published success rates of open-heart procedures (i.e. Cox Maze [3]) exceed 90%, but the surgeries are highly invasive and complex, encompassing the risks and long recovery associated with open-heart surgery [4]. Alternatively, minimally-invasive catheter ablation can be used to create lesions by burning or freezing the atrial tissue. Current catheter ablation procedures are tedious, time-consuming, and result in significant X-ray exposure to the patient and medical team [5], [6]. Success requires highly skilled catheter manipulation in the beating heart, and constant surface contact for transmural and continuous lesions [7], [8].

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Teleoperated (i.e. robotic) catheters have the potential to enhance catheter maneuverability in the heart, reduce procedure times, and increase the range of motion and contact stability during ablation. Previous work has focused on the design, optimization, and fabrication of shape memory alloy (SMA) actuated robotic catheters for endocardial ablation [9], [10]. The SMA-actuated catheter used here (Figure 1) consists of two bending segments: the distal segment containing the catheter tip, and the adjacent proximal segment. Each segment is actuated by four offset SMA tendons, which produce bending in the catheter. Pulse-width modulation (PWM) is used regulate electrical power in each tendon based on commands sent from a custom C++ control application. The ability to create continuous, transmural lesions is the fundamental performance objective of this SMA-actuated catheter. This requires not only precise position control of the catheter tip, but also sufficient surface contact, most easily achieved through maintaining the catheter tip perpendicular to the atrial tissue.

Catheter tip position (x_D, y_D, z_D) (Figure 2) is kinematically dependent on the proximal (P) and distal (D) bending (θ_P, θ_D) and orientation (ϕ_P, ϕ_D) angles. Multiple combinations of these angles can achieve a given tip position (Figure 3); this redundancy enables limited control of catheter tip orientation. Optimization can be used to determine the most advantageous tip position and orientation to create transmural lesions in minimal time.

Widespread clinical adoption of this robotic catheter ablation technology requires robust, real-time control of the bending segments, which is complicated by the highly nonlinear, hysteretic behavior of the thermally-activated SMA tendons. Hysteresis compensation of SMA actuators has been investigated extensively [11], [12], [13], [14]. Additionally, the literature is replete with control algorithms developed to address SMA bandwidth limitations [15], [16], [17], [18], [19], [20], [21]. However, the multivariable control problem presented here introduces additional challenges: axial compression, coupling between tendons, tendon slack, and redundancy in catheter tip positioning.

This paper presents a trajectory optimization and control strategy for the dual segment SMA-actuated catheter, with emphasis on the efficient creation of continuous, transmural lesions in atrial tissue during endocardial radiofrequency ablation. The proposed clinical procedure for creating lesions is as follows:

- I.** Using the catheter's joystick, the physician navigates the catheter tip and specifies three critical locations on the endoatrial surface.
- II.** An algorithm computes an ablation path through these physician-specified locations.
- III.** Optimal bending and orientation angles are generated for several locations along the path.
- IV.** Real-time control algorithms ensure adequate tracking of the reference trajectory during ablation.

Steps I and II are discussed in Section 2.1. For Step III, the optimization problem is presented in Section 2.3 and the solution method is detailed in Section 2.4. The controller

for Step IV is described in Section 2.5. Simulated and experimental results are presented in Section 3, with concluding remarks presented in Section 4.

The method presented here serves as a first step toward developing an optimization and control strategy for this sophisticated clinical procedure. Two key assumptions are made in the derivations and analyses presented here. First, the desired ablation paths are assumed to be planar. While this assumption might be valid for short lesions along the atrial wall, its validity is questionable for longer lesions along concave surfaces of the heart. In such cases, step II can be modified to account for the more complex ablation path; all other steps will remain valid. The second assumption is that the optimal bending and orientation angles are obtainable within the confined spaces of the heart. A special term was included in the cost function derived in Section 2.3 to address this, however future work must focus on refining this term and possibly adding tissue contact constraints to the optimization.

2. Methods

2.1 Ablation Path

The methods described here begin with three physician-specified points on the atrial wall (A , B , and C), expressed in global XYZ coordinates (the global coordinate system is located at the base of the proximal segment, Figure 4a). An ablation path is constructed on the plane containing these points. This plane is defined using the surface normal

$$\begin{aligned} \mathbf{n} &= \mathbf{r}_{AB} \times \mathbf{r}_{AC} \\ &= [(y_B - y_A)(z_C - z_A) - (y_C - y_A)(z_B - z_A)] \mathbf{i} \\ &\quad - [(x_B - x_A)(z_C - z_A) - (x_C - x_A)(z_B - z_A)] \mathbf{j} \\ &\quad + [(x_B - x_A)(y_C - y_A) - (x_C - x_A)(y_B - y_A)] \mathbf{k} \\ &= n_x \mathbf{i} + n_y \mathbf{j} + n_z \mathbf{k} \end{aligned} \quad (1)$$

where $\mathbf{r}_{AB}$ and $\mathbf{r}_{AC}$ are vectors directed from point A to points B and C , respectively. The ablation plane is constructed such that:

$$n_x(x - x_A) + n_y(y - y_A) + n_z(z - z_A) = 0 \quad (2)$$

This ablation plane defines a new local $\hat{X}\hat{Y}\hat{Z}$ coordinate system, shown in Figure 4. The following three rotation matrices describe the transformation from $\hat{X}\hat{Y}\hat{Z}$ to XYZ coordinates:

$$\begin{aligned} \mathbf{R}_Z(\alpha) &= \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ \mathbf{R}_Y(\beta) &= \begin{bmatrix} \cos \beta & 0 & \sin \beta \\ 0 & 1 & 0 \\ -\sin \beta & 0 & \cos \beta \end{bmatrix} \\ \mathbf{R}_X(\psi) &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \psi & -\sin \psi \\ 0 & \sin \psi & \cos \psi \end{bmatrix} \end{aligned} \quad (3)$$

where

$$\begin{aligned}
\alpha &= \tan^{-1} \left(\frac{y_C - y_A}{x_C - x_A} \right) \\
\beta &= -\tan^{-1} \left(\frac{z_C - z_A}{\sqrt{(x_C - x_A)^2 + (y_C - y_A)^2}} \right) \quad (4) \\
|\psi| &= \cos^{-1} \left(\frac{\mathbf{z}' \cdot \mathbf{n}}{\|\mathbf{n}\|} \right)
\end{aligned}$$

and where $\mathbf{z}' = \cos(\alpha) \sin(\beta) \mathbf{i} + \sin(\alpha) \sin(\beta) \mathbf{j} + \cos(\beta) \mathbf{k}$. Defining the total rotation matrix $^{XYZ}\mathbf{R}_{\hat{x}\hat{y}\hat{z}} = \mathbf{R}_Z(\alpha) \mathbf{R}_Y(\beta) \mathbf{R}_X(\psi)$ gives the transformation matrix

$$^{XYZ}\mathbf{T}_{\hat{x}\hat{y}\hat{z}} = \begin{bmatrix} ^{XYZ}\mathbf{R}_{\hat{x}\hat{y}\hat{z}} & x_A \\ & y_A \\ & z_A \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_\alpha c_\beta & -s_\alpha c_\beta + c_\alpha s_\beta s_\psi & s_\alpha s_\beta + c_\alpha s_\beta c_\psi & x_A \\ s_\alpha c_\beta & c_\alpha c_\beta + s_\alpha s_\beta s_\psi & -c_\alpha s_\beta + s_\alpha s_\beta c_\psi & y_A \\ -s_\beta & c_\beta s_\psi & c_\beta c_\psi & z_A \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (5)$$

The inverse transformation is then used to compute the local coordinates:

$$\begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \\ 1 \end{bmatrix} = ^{XYZ}\mathbf{T}_{\hat{x}\hat{y}\hat{z}}^{-1} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \quad (6)$$

where

$$^{XYZ}\mathbf{T}_{\hat{x}\hat{y}\hat{z}}^{-1} = \begin{bmatrix} c_\alpha c_\beta & s_\alpha c_\beta & -s_\beta & -c_\alpha c_\beta x_A - s_\alpha c_\beta y_A + s_\beta z_A \\ -s_\alpha c_\beta + c_\alpha s_\beta s_\psi & c_\alpha c_\beta + s_\alpha s_\beta s_\psi & c_\beta s_\psi & -(-s_\alpha c_\beta + c_\alpha s_\beta s_\psi) x_A - (c_\alpha c_\beta + s_\alpha s_\beta s_\psi) y_A - c_\beta s_\psi z_A \\ s_\alpha s_\beta + c_\alpha s_\beta c_\psi & -c_\alpha s_\beta + s_\alpha s_\beta c_\psi & c_\beta c_\psi & -(s_\alpha s_\beta + c_\alpha s_\beta c_\psi) x_A - (-c_\alpha s_\beta + s_\alpha s_\beta c_\psi) y_A - c_\beta c_\psi z_A \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (7)$$

Two ablation paths, from A to B ($q=1$) and from B to C ($q=2$), are created using cubic splines based on the distance $r_q(k)$ from the midpoint M of physician-specified points A and C (Figure 4b) to the k^{th} path point:

$$\begin{aligned}
r_q(k) &= a_q \varphi_q(k)^3 + b_q \varphi_q(k)^2 + c_q \varphi_q(k) + d_q \\
\hat{x}(k) &= \hat{x}_M - r_q(k) \cos(\varphi_q(k)) \\
\hat{y}(k) &= r_q(k) \sin(\varphi_q(k)) \quad (8)
\end{aligned}$$

where $\varphi_A = \varphi_1(k)$, $\varphi_B = \varphi_2(k)$, and $\varphi_C = \varphi_C$. The physician-specified points expressed in transformed coordinates $(\hat{x}_A, \hat{y}_A, \hat{z}_A)$, $(\hat{x}_B, \hat{y}_B, \hat{z}_B)$, $(\hat{x}_C, \hat{y}_C, \hat{z}_C)$ are used to define boundary conditions

$$\begin{aligned}
r_A &= \hat{x}_M - \frac{\hat{x}_C}{2}, r'_A = 0, \varphi_A = 0 \\
r_B &= \sqrt{(\hat{x}_M - \hat{x}_B)^2 + (-\hat{y}_B)^2}, r'_B = 0, \varphi_B = \tan^{-1} \left(\frac{\hat{y}_B}{\hat{x}_M - \hat{x}_B} \right) \\
r_C &= \hat{x}_C - \hat{x}_M, r'_C = 0, \varphi_C = \pi \cdot \text{sign}(\varphi_B) \quad (9)
\end{aligned}$$

which are used to calculate the coefficients of (8):

$$\begin{aligned}
a_1 &= \frac{2(r_A - r_B)}{\varphi_B^3} \\
b_1 &= -\frac{3}{2}\varphi_B a_1 \\
c_1 &= 0 \\
d_1 &= r_A \\
a_2 &= \frac{c_2}{2\varphi_B \varphi_C} \\
b_2 &= -\frac{(\varphi_B + \varphi_C)}{2\varphi_B \varphi_C} c_2 \\
c_2 &= \frac{r_B - r_C}{-\frac{\varphi_B^2}{6\varphi_C} + \frac{\varphi_C^2}{6\varphi_B} + \frac{\varphi_B}{2} - \frac{\varphi_C}{2}} \\
d_2 &= r_C + \left(\frac{\varphi_C}{6\varphi_B} - \frac{\varphi_C}{2} \right) c_2
\end{aligned} \tag{10}$$

The reference ablation path is specified by transforming from $(x(\hat{k}), \hat{y}(k), \hat{z}(k))$ to $(x(k), y(k), z(k))$ coordinates using (5).

2.2 Catheter Kinematics

The catheter tip measurements and the reference ablation path are specified in global Cartesian coordinates (x, y, z) , while control is based on the system's generalized coordinates $(\theta_P, \varphi_P, \theta_D, \varphi_D)$, requiring conversion between the two coordinate systems. First, inverse kinematics between the tip coordinates of the proximal segment (x_P, y_P, z_P) and the generalized coordinates (θ_P, φ_P) are derived. Because the proximal segment's base establishes the origin of the global coordinate system, and because proximal segment bending does not exceed 90° (due to geometric constraints within the heart), a straightforward trigonometric relationship exists (Figure 5):

$$\theta_P = \tan^{-1} \left(\frac{2z_P \sqrt{x_P^2 + y_P^2}}{z_P^2 - x_P^2 - y_P^2} \right) \tag{11}$$

$$\phi_P = \tan^{-1} \left(\frac{y_P}{x_P} \right) \tag{12}$$

For the distal segment, the global coordinates (x_D, y_D, z_D) must be transformed to local coordinates $(\tilde{x}_D, \tilde{y}_D, \tilde{z}_D)$, with origin at the base of the distal segment (the tip of the proximal segment, Figure 6):

$$\begin{bmatrix} \tilde{x}_D \\ \tilde{y}_D \\ \tilde{z}_D \\ 1 \end{bmatrix} = {}^{XYZ}\mathbf{T}_{\tilde{x}\tilde{y}\tilde{z}}^{-1} \begin{bmatrix} x_D \\ y_D \\ z_D \\ 1 \end{bmatrix} \tag{13}$$

The local coordinate system is aligned such that the $\tilde{Z}$ axis points in the direction of the proximal segment tip and the $\tilde{X}$ axis is aligned with the SMA tendon of the distal segment nearest to the SMA tendon of the proximal segment aligned with the global X axis. The rotation matrices required for the transformation matrix ${}^{XYZ}\mathbf{T}_{\tilde{x}\tilde{y}\tilde{z}}$ are

$$\begin{aligned}
\mathbf{R}_z(\phi_P) &= \begin{bmatrix} \cos \phi_P & -\sin \phi_P & 0 \\ \sin \phi_P & \cos \phi_P & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
\mathbf{R}_Y(\theta_P) &= \begin{bmatrix} \cos \theta_P & 0 & \sin \theta_P \\ 0 & 1 & 0 \\ -\sin \theta_P & 0 & \cos \theta_P \end{bmatrix} \\
\mathbf{R}_Z(\xi - \phi_P) &= \begin{bmatrix} \cos(\xi - \phi_P) & -\sin(\xi - \phi_P) & 0 \\ \sin(\xi - \phi_P) & \cos(\xi - \phi_P) & 0 \\ 0 & 0 & 1 \end{bmatrix}
\end{aligned} \quad (14)$$

where ξ is the offset angle between the proximal and distal tendons. Letting $\eta = \xi - \phi_P$, the distal transformation matrix becomes

$${}^{XYZ}\mathbf{T}_{\tilde{x}\tilde{y}\tilde{z}} = \begin{bmatrix} {}^{XYZ}\mathbf{R}_{\tilde{x}\tilde{y}\tilde{z}} & x_P \\ y_P \\ z_P \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_{\phi_P} c_{\theta_P} c_{\eta} - s_{\phi_P} s_{\eta} & -c_{\phi_P} c_{\theta_P} s_{\eta} - s_{\phi_P} c_{\eta} & c_{\phi_P} s_{\theta_P} & x_P \\ s_{\phi_P} c_{\theta_P} c_{\eta} + c_{\phi_P} s_{\eta} & -s_{\phi_P} c_{\theta_P} s_{\eta} + c_{\phi_P} c_{\eta} & s_{\phi_P} s_{\theta_P} & y_P \\ -s_{\theta_P} c_{\eta} & s_{\theta_P} s_{\eta} & c_{\theta_P} & z_P \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (15)$$

where ${}^{XYZ}\mathbf{R}_{\tilde{x}\tilde{y}\tilde{z}} = \mathbf{R}_Z(\phi_P) \mathbf{R}_Y(\theta_P) \mathbf{R}_Z(\xi - \phi_P)$. The inverse transformation matrix is therefore

$$\begin{aligned}
& {}^{XYZ}\mathbf{T}_{\tilde{x}\tilde{y}\tilde{z}}^{-1} \\
&= \begin{bmatrix} c_{\phi_P} c_{\theta_P} c_{\eta} - s_{\phi_P} s_{\eta} & s_{\phi_P} c_{\theta_P} c_{\eta} + c_{\phi_P} s_{\eta} & -s_{\theta_P} c_{\eta} & -(c_{\phi_P} c_{\theta_P} c_{\eta} - s_{\phi_P} s_{\theta_P}) x_P - (s_{\phi_P} c_{\theta_P} c_{\eta} + c_{\phi_P} s_{\eta}) y_P + s_{\theta_P} z_P \\ -c_{\phi_P} c_{\theta_P} s_{\eta} - s_{\phi_P} c_{\eta} & -s_{\phi_P} c_{\theta_P} s_{\eta} + c_{\phi_P} c_{\eta} & s_{\theta_P} s_{\eta} & -(c_{\phi_P} c_{\theta_P} s_{\eta} - s_{\phi_P} c_{\eta}) x_P - (-s_{\phi_P} c_{\theta_P} s_{\eta} + c_{\phi_P} c_{\eta}) y_P - s_{\theta_P} z_P \\ c_{\phi_P} s_{\theta_P} & s_{\phi_P} s_{\theta_P} & c_{\theta_P} & -c_{\phi_P} s_{\theta_P} x_P - s_{\phi_P} s_{\theta_P} y_P - c_{\theta_P} z_P \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (16)
\end{aligned}$$

Because the distal segment can bend up to 180° , two distinct kinematic cases must be considered. For $\theta_D < 90^\circ$, after conversion to local coordinates, the inverse kinematics are equivalent to the proximal segment case (11)-(12). However, for $90^\circ \leq \theta_D \leq 180^\circ$, a slight modification is required for the bending angle calculation (see Figure 7):

$$\theta_D = \pi - \tan^{-1} \left(\frac{2\tilde{z}_D \sqrt{\tilde{x}_D^2 + \tilde{y}_D^2}}{\tilde{x}_D^2 + \tilde{y}_D^2 - \tilde{z}_D^2} \right) \quad (17)$$

Because θ_D is unknown, however, the choice of algorithm must be based on the Cartesian coordinates:

$$\theta_D = \begin{cases} \tan^{-1} \left(\frac{2\tilde{z}_D \sqrt{\tilde{x}_D^2 + \tilde{y}_D^2}}{\tilde{z}_D^2 - \tilde{x}_D^2 - \tilde{y}_D^2} \right) & \text{if } \tilde{z}_D^2 \geq \tilde{x}_D^2 + \tilde{y}_D^2 \\ \pi - \tan^{-1} \left(\frac{2\tilde{z}_D \sqrt{\tilde{x}_D^2 + \tilde{y}_D^2}}{\tilde{x}_D^2 + \tilde{y}_D^2 - \tilde{z}_D^2} \right) & \text{else} \end{cases} \quad (18)$$

$$\phi_D = \tan^{-1} \left(\frac{\tilde{y}_D}{\tilde{x}_D} \right) \quad (19)$$

The forward kinematics, illustrated in Figure 8 for the proximal segment, are:

$$\begin{aligned} x_p &= \frac{L_P}{\theta_P} (1 - \cos \theta_P) \cos \phi_P \\ y_p &= \frac{L_P}{\theta_P} (1 - \cos \theta_P) \sin \phi_P \\ z_p &= \frac{L_P}{\theta_P} \sin \theta_P \end{aligned} \quad (20)$$

where L_P is the length of the proximal segment.

As noted previously, the forward kinematics for the distal segment must consider two cases based on distal bending angle. The conversion for $\theta_D < 90^\circ$ is

$$\begin{aligned} \tilde{x}_D &= \frac{L_D}{\theta_D} (1 - \cos \theta_D) \cos \phi_D \\ \tilde{y}_D &= \frac{L_D}{\theta_D} (1 - \cos \theta_D) \sin \phi_D \\ \tilde{z}_D &= \frac{L_D}{\theta_D} \sin \theta_D \end{aligned} \quad (21)$$

and for $90^\circ < \theta_D < 180^\circ$ is

$$\begin{aligned} \tilde{x}_D &= \frac{L_D}{\theta_D} (1 - \cos (\pi - \theta_D)) \cos \phi_D \\ \tilde{y}_D &= \frac{L_D}{\theta_D} (1 - \cos (\pi - \theta_D)) \sin \phi_D \\ \tilde{z}_D &= \frac{L_D}{\theta_D} \sin (\pi - \theta_D) \end{aligned} \quad (22)$$

where L_D is the length of the distal segment. These local coordinates are then transformed to global coordinates:

$$\begin{bmatrix} x_D \\ y_D \\ z_D \\ 1 \end{bmatrix} = {}^{XYZ}\mathbf{T}_{\tilde{x}\tilde{y}\tilde{z}} \begin{bmatrix} \tilde{x}_D \\ \tilde{y}_D \\ \tilde{z}_D \\ 1 \end{bmatrix} \quad (23)$$

where ${}^{XYZ}\mathbf{T}_{\tilde{x}\tilde{y}\tilde{z}}$ is given in (15).

2.3 Optimization Problem

Accurate ablation path tracking requires determination of reference angles ($\theta_{P,r}$, $\phi_{P,r}$, $\theta_{D,r}$, $\phi_{D,r}$) which achieve desired catheter tip positions ($x_{D,r}$, $y_{D,r}$, $z_{D,r}$). By combining (15) and (23), the relationship between these generalized and Cartesian coordinates is:

$$\begin{aligned} x_{D,r} &= \begin{pmatrix} c_{\phi_{P,r}} c_{\theta_{P,r}} c_{\eta,r} - s_{\phi_{P,r}} s_{\eta,r} \end{pmatrix} \tilde{x}_{D,r} + \begin{pmatrix} -c_{\phi_{P,r}} c_{\theta_{P,r}} s_{\eta,r} - s_{\phi_{P,r}} c_{\eta,r} \end{pmatrix} \tilde{y}_{D,r} + c_{\phi_{P,r}} s_{\theta_{P,r}} \tilde{z}_{D,r} + x_{P,r} \\ y_{D,r} &= \begin{pmatrix} s_{\phi_{P,r}} c_{\theta_{P,r}} c_{\eta,r} - c_{\phi_{P,r}} s_{\eta,r} \end{pmatrix} \tilde{x}_{D,r} + \begin{pmatrix} -s_{\phi_{P,r}} c_{\theta_{P,r}} s_{\eta,r} + c_{\phi_{P,r}} c_{\eta,r} \end{pmatrix} \tilde{y}_{D,r} + s_{\phi_{P,r}} s_{\theta_{P,r}} \tilde{z}_{D,r} + y_{P,r} \\ z_{D,r} &= -s_{\theta_{P,r}} c_{\eta,r} \tilde{x}_{D,r} + s_{\theta_{P,r}} s_{\eta,r} \tilde{y}_{D,r} + c_{\theta_{P,r}} \tilde{z}_{D,r} + z_{P,r} \end{aligned} \quad (24)$$

where $(x_{P,r}, y_{P,r}, z_{P,r})$ and $(x_{D,r}, y_{D,r}, z_{D,r})$ are functions of $(\theta_{P,r}, \phi_{P,r}, \theta_{D,r}, \phi_{D,r})$ given by (20) and (21)-(22), respectively.

Because (24) is three equations and four unknowns, $(\theta_{P,r}, \phi_{P,r}, \theta_{D,r}, \phi_{D,r})$, additional performance criteria can be considered, including catheter tip orientation and path smoothness. An optimization problem is constructed to compute reference angles at each point k along the path. The cost function contains two key terms that penalize deviation from the path generated in Section 2.1:

$$J_1(k) = \sqrt{(\hat{x}_{D,r}(k) - \hat{x}(k))^2 + (\hat{y}_{D,r}(k) - \hat{y}(k))^2} \quad (25)$$

$$J_2(k) = \sqrt{(\hat{z}_{D,r}(k) - \hat{z}(k))^2} \quad (26)$$

where $J_1(k)$ ensures planar ablation path adherence and $J_2(k)$ ensures tissue contact via normal penetration.

A third cost function term can be added to penalize deviation of catheter tip orientation (represented by unit vector $\mathbf{u}(k) = u_x(k)\mathbf{i} + u_y(k)\mathbf{j} + u_z(k)\mathbf{k}$) from surface normal $\mathbf{n}$ (1)

$$J_3(k) = \sqrt{\left(\frac{\mathbf{n}}{\|\mathbf{n}\|} - \mathbf{u}(k)\right)^T \left(\frac{\mathbf{n}}{\|\mathbf{n}\|} - \mathbf{u}(k)\right)} \quad (27)$$

where, in distal coordinates, the unit vector components are

$$\begin{aligned} \tilde{u}_x(k) &= \sin(\theta_{D,r}(k)) \cos(\phi_{D,r}(k)) \\ \tilde{u}_y(k) &= \sin(\theta_{D,r}(k)) \sin(\phi_{D,r}(k)) \\ \tilde{u}_z(k) &= \cos(\theta_{D,r}(k)) \end{aligned} \quad (28)$$

which are transformed to the global system using the rotation matrix ${}^{XYZ}\mathbf{R}_{\tilde{x}\tilde{y}\tilde{z}}$ given in (15)

$$\begin{bmatrix} u_x(k) \\ u_y(k) \\ u_z(k) \end{bmatrix} = {}^{XYZ}\mathbf{R}_{\tilde{x}\tilde{y}\tilde{z}} \begin{bmatrix} \tilde{u}_x(k) \\ \tilde{u}_y(k) \\ \tilde{u}_z(k) \end{bmatrix} \quad (29)$$

A fourth cost function can be used to penalize large variations in reference angles between adjacent path points

$$\begin{aligned} & + \left(\phi_{P,r}(k) - \phi_{P,r}(k-1)\right)^2 \\ & + \left(\theta_{D,r}(k) - \theta_{D,r}(k-1)\right)^2 \\ & + \left(\phi_{D,r}(k) - \phi_{D,r}(k-1)\right)^2 \sqrt{\left(\theta_{P,r}(k) - \theta_{P,r}(k-1)\right)^2 + \left(\phi_{P,r}(k) - \phi_{P,r}(k-1)\right)^2 + \left(\theta_{D,r}(k) - \theta_{D,r}(k-1)\right)^2 + \left(\phi_{D,r}(k) - \phi_{D,r}(k-1)\right)^2} \\ J_4(k) &= \sqrt{\left(\theta_{P,r}(k) - \theta_{P,r}(k-1)\right)^2 + \left(\phi_{P,r}(k) - \phi_{P,r}(k-1)\right)^2 + \left(\theta_{D,r}(k) - \theta_{D,r}(k-1)\right)^2 + \left(\phi_{D,r}(k) - \phi_{D,r}(k-1)\right)^2} \end{aligned}$$

This function helps ensure that reference trajectories remain within the system bandwidth, and reduces the range of angles the catheter is required to track. The latter is critical during endocardial ablation, where contact with tissue limits the catheter's range of motion.

Combining (25), (26), (27), and (30), the cost function becomes

$$J(k) = pJ_1(k) + qJ_2(k) + rJ_3(k) + sJ_4(k) \quad (31)$$

where p , q , r , and s are weighting constants for each individual cost function. The optimization problem is thus to minimize $J(k)$ subject to constraints on the reference angles:

Minimize:

$$J(k) = f(\theta_{P,r}(k), \phi_{P,r}(k), \theta_{D,r}(k), \phi_{D,r}(k)) \quad (32)$$

Subject To:

$$\begin{aligned} 0 &\leq \theta_{P,r}(k) \leq \frac{\pi}{2} \\ 0 &\leq \phi_{P,r}(k) \leq 2\pi \\ 0 &\leq \theta_{D,r}(k) \leq \pi \\ 0 &\leq \phi_{D,r}(k) \leq 2\pi \end{aligned} \quad (33)$$

$$\theta_{P,r}(k) \leq \max \left\{ \frac{\pi}{2} + \frac{r_{stiff}}{\cos \phi_{P,r}(k)} \theta_{D,r}(k) \cos(\phi_{D,r}(k) + \xi), \right. \\ \left. \frac{\pi}{2} + \frac{r_{stiff}}{\sin \phi_{P,r}(k)} \theta_{D,r}(k) \sin(\phi_{D,r}(k) + \xi) \right\}$$

The first four constraints ensure the reference angles are within the catheter's range of motion: 90° and 180° of omnidirectional bending for the proximal and distal segments, respectively. The final constraint accounts for coupling between the distal and proximal segments. Because the distal pull wires run through the proximal segment, actuation of the distal SMA tendons causes significant bending in the proximal segment:

$$\theta_P^{(D)} = r_{stiff} \theta_D \quad (34)$$

where r_{stiff} is the bending stiffness ratio between the distal and proximal segments. The actual bending in the proximal segment is assumed to be a linear combination of the bending caused by the proximal SMA tendons $\theta_P^{(P)}$ and the distal SMA tendons $\theta_P^{(D)}$:

$$\theta_P = \theta_P^{(P)} + \theta_P^{(D)} \quad (35)$$

Accounting for the direction of bending (ϕ_P , ϕ_D), the final constraint is derived to ensure the proximal SMA tendons can meet the actuation requirements of the reference angles.

2.4 Parallel Genetic Algorithms

Because genetic algorithms [22] are well-established methods for solving multimodal, nonlinear, and multivariate optimization problems, they are used here to minimize (32) subject to the constraints in (33). To increase computational efficiency and ensure that a global minimum is found, a parallel approach [23] is employed by subdividing the design space. Partitioning the proximal and distal orientation angle space (ϕ_P , ϕ_D) into four

quadrants, (0-90°, 90°-180°, 180°-270°, and 270°-360°) gives 16 subpopulations. Genetic algorithms are run in parallel to find the optimal design for each subpopulation, from which the global optimum is selected.

Subpopulations of size 40 (10 times the number of design variables) are used to balance the tradeoff between computational efficiency and global optimization. Each subpopulation is randomly initialized within the appropriate regional design variable bounds. Each generation, candidate designs are sorted by fitness and the top 20% are automatically advanced to the next generation. Crossover is performed on the top 80% of candidate designs using fitness proportionate selection to pair parent designs and the BLX- α [24] method to generate children designs. A mutation rate of 5% is applied to the subpopulation. The process is repeated until the minimum subpopulation fitness converges.

Candidate designs are evaluated using a fitness function $F(k)$ which combines the cost function $J(k)$ (31) with a penalty term $P(k)$ on constraint violations

$$F(k) = J(k) + r_p P(k) \quad (36)$$

$$P(k) = \sum_{j=1}^9 \max[g_j(k), 0]^2 \quad (37)$$

where $g_j(k) = 0$ are the eight regional design variable bound constraints and the nonlinear constraint (33). The penalty weight increases linearly with generation number: $r_p = 0.01 * n_{gen}$. This ensures the entire design space is searched by allowing some constraint violations initially, but also ensures that the final solution is feasible by later rejecting constraint violations.

2.5 Controller

A closed-loop controller was developed and implemented to track the optimal reference angles generated in Section 2.4. The bending and orientation angles of each segment (θ, ϕ) are decoupled into two planar bending angles ($\theta_X = \theta \cos \phi$, $\theta_Y = \theta \sin \phi$), each controlled by an antagonistic SMA tendon pair: (SMA_{X+} , SMA_{X-}) and (SMA_{Y+} , SMA_{Y-}). The complete system block diagram is shown in Figure 9. The remainder of this section describes controller synthesis for each antagonistic controller block.

Each antagonistic SMA pair is controlled using a hybrid approach combining three terms: a feedforward (FF) term for hysteresis compensation, a proportional+integral+derivative (PID) term for robustness, and a proportional cubed (P3) term to decrease response time:

$$V = V_{FF} + V_{PID} + V_{P3} \quad (38)$$

The feedforward term V_{FF} is an estimate of the steady-state voltage required to maintain the reference angle. Considering only outer-loop hysteresis data, Figure 10, two analytical functions can be derived, one for heating and one for cooling

$$\begin{aligned} V_{ss,h} &= a_{h0} \operatorname{atan} [a_{h1} (\theta_r - a_{h2})] + a_{h3} (\theta_r - a_{h2}) + a_{h4} \\ V_{ss,c} &= a_{c0} \operatorname{atan} [a_{c1} (\theta_r - a_{c2})] + a_{c3} (\theta_r - a_{c2}) + a_{c4} \end{aligned} \quad (39)$$

where the coefficients a_{hi} and a_{ci} are optimized to fit open-loop experimental data. For simplicity, minor hysteresis loops are ignored. Alternatively, the feedforward voltage is computed using the outer-loop relationships (39) and tracking error $e = \theta_r - \theta_m$

$$V_{FF} = V_{ss,c} + \frac{V_{ss,h} - V_{FF,prev}}{1 + \exp[\chi(\varepsilon - e)]} - \frac{V_{ss,c} - V_{FF,prev}}{1 + \exp[\chi(-\varepsilon - e)]} \quad (40)$$

Parameters χ and ε establish smooth transitions between the outer-loop heating and cooling curves. Parameter ε represents the tracking error at which the control will transition between curves and parameter χ sets the rate of transition. These parameters are chosen through experimental iterations to balance the tradeoff between tracking accuracy and stability.

Due to the presence of unmodeled dynamics, parametric uncertainties, external disturbances and measurement errors, feedforward control alone cannot provide the required tracking performance. For robustness, a PID term is added

$$V_{PID} = k_p e + k_i \int_0^t e d\tau + k_d \frac{de}{dt} \quad (41)$$

Additionally, a proportional cubed term [19] is incorporated to decrease response times by rapidly heating tendons when tracking error is large

$$V_{P3} = k_3 e^3 \quad (42)$$

Because the control law is synthesized for an antagonistic SMA tendon pair, voltage is applied to either the SMA_+ or SMA_- tendon, depending on the sign of $V(t)$.

As noted previously, bending in the distal segment results in additional deflection in the proximal segment. To account for this, proximal segment model-based control terms (39) utilize $\theta_r^{(P)}$ (35) instead of θ_r .

2.6 Simulation and Experimental Methods

Simulations were conducted in MATLAB (The MathWorks, Inc., Natick, MA) to optimize genetic algorithm parameters (number of subpopulations, population size, crossover rate, and mutation rate) and cost function weighting factors (31). Real-time optimization and control algorithms were programmed in Visual Studio C++ (Microsoft Corporation, Redmond, WA) for experimental evaluation on the SMA-actuated robotic catheter.

An EnSite NavX System (St. Jude Medical, St. Paul, MN), used clinically for visualization and navigation during cardiac ablation procedures, was modified to provide real-time position measurement of catheter electrodes. For laboratory testing, the catheter was submerged in a saline-filled custom tank fabricated with EnSite-compatible patch electrodes (Figure 11a). During operation, a pulse width modulation (PWM) amplifier received real-

time control commands via serial communication to regulate SMA tendon power, Figure 11b.

Although the EnSite NavX was not developed to provide real-time measurements for catheter control algorithms, experiments were conducted to establish its potential for this purpose. Specifically, the system's measurement accuracy, precision, bandwidth and linearity were quantified by placing a plastic cube (8.5 cm sides) in the saline-filled tank and acquiring steady-state position data (15 Hz sample rate) corresponding to the cube's corners and midpoints. Nonlinear measurement errors (Figure 12a) and low-frequency positional variations (Figure 12b) were observed even after implementing a 3rd-order Butterworth filter (0.75 Hz cutoff frequency). Collectively, these measurement limitations resulted in 7.1 ± 3.5 mm of position uncertainty and ± 0.8 mm imprecision for the 2.7 mm diameter catheter. Despite these limitations, the EnSite system is presently the best option available for endocardial sensing, and was therefore chosen for real-time control of the SMA-actuated robotic catheter.

3. Results

The three specified points and the computed path used for optimization simulations are shown in Figure 13. The cost function weighting factors (31) were initially chosen as $p_0 = 1$, $q_0 = 1$, $r_0 = 10$, and $s_0 = 10$, which specifies approximately equal emphasis on each term. By independently amplifying each weighting factor, individual cost function contributions were quantified and compared (Table 1). First, planar path adherence was emphasized, followed by normal penetration (a measure of tissue contact force). Next catheter tip normality was emphasized. Finally, large variations in adjacent reference angles were heavily penalized. The final weighting factors $p = 2$, $q = 2$, $r = 5$, and $s = 5$ were determined by iterative simulation to optimize the tradeoff between path adherence and lesion quality, as shown in Figure 14. These weighting factors allow small deviations from the desired path (evident in the 1 mm tracking error peak of Figure 14b) to put more emphasis on tip normality and to reduce variations between adjacent reference angles. If necessary, these factors could be modified by the physician to better address the procedural requirements.

Experimental performance of this path optimization strategy was quantified using the SMA-actuated catheter in a saline-filled tank (Figure 11a). The catheter was navigated to three arbitrary points, which were stored and used to compute the optimal reference path. Path optimization took approximately 20 seconds to complete. This optimization process was repeated four times (using different specified points); optimal reference trajectories are shown in Figure 15. These results demonstrate efficient real-time generation of continuous ablation paths, and performance indices J_1 and J_2 emphasize path adherence to produce relatively arc-like trajectories. However, contributions from J_3 and J_4 (which emphasize tip normality and limit large variations in reference angles, respectively) result in optimal ablation trajectories that are not perfectly smooth arcs.

Next, closed-loop control of the SMA-actuated catheter was demonstrated for one optimized ablation path. Because clinical ablation procedures require a series of discrete, overlapping lesions, the optimized path was discretized into 16 specific ablation locations, which were

implemented as step references every 40 seconds. Experimental results reveal an average response time of approximately 2 seconds with less than 10° overshoot. However, average steady-state tip tracking error (Figure 16) was 5.5 mm (approximately twice the diameter of the ablation catheter tip). Although this tracking error was larger than expected, and unacceptably large for clinical use, much of it can be traced back to the sensing limitations discussed in Section 2.6.

4. Conclusions

Simulated and experimental results validate the potential of the proposed ablation path optimization approach. This method proved to be computationally efficient (requiring only 20 seconds to optimize), producing continuous ablation paths and optimal reference trajectories through any physician-specified points. Additionally, the physician can easily modify the conditions for optimality by adjusting weights associated with each cost function.

Closed-loop control of the SMA-actuated catheter along each optimized path was efficient; step responses to several discrete path locations demonstrated acceptable response times with low overshoot. However, steady-state tracking error was larger than expected. The 5.5 mm average tracking error can be attributed to real-time sensing limitations of the EnSite NavX system. Individual catheter electrode measurements lack precision, varying on average 1.6 mm peak-to-peak, even when the catheter is held steady. Because catheter tip position is calculated from the four generalized angles, which depend on three different electrode measurements, this error has a tendency to propagate through the forward kinematic equations (23). Future work will focus on modifying the controller to compensate for measurement variation and ensure low tracking error and stability.

This research showed promising results towards the ultimate goal of fully-automated catheter control during endocardial ablation procedures. However, before clinical use can be considered, the methods presented here must be tested in a more realistic clinical environment. The confined spaces of the heart and interactions with elastic tissues introduce additional challenges that must be fully considered and experimentally tested. The optimization problem presented here includes a cost term meant to ensure adequate contact force during ablation based on position alone. Future work could focus on enhancing this term and incorporating force feedback into the control algorithms. Another cost term penalized large variation between adjacent angles. This term was intended to improve the likelihood that bending angles are obtainable within the confines of the heart (by penalizing large variations from the physician-demanded angles). A successful clinical procedure would require more assurance that the angles are obtainable. This cost function term should therefore be modified to incorporate the boundaries of the heart chambers (as mapped via the EnSite system).

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Research Highlights

- A path optimization and control strategy for shape memory alloy actuated cardiac ablation catheters is presented
- Catheter tip locations and orientations are optimized using parallel genetic algorithms
- Closed-loop control of the SMA-actuated catheter along optimized paths is validated experimentally

Figure 1.
SMA-actuated catheter showing proximal and distal bending segments and catheter handle with joystick for user input.

Figure 2.

Coordinate systems of the SMA-actuated catheter: proximal segment bending and orientation angles (θ_p, ϕ_p) and catheter tip location (x_D, y_D, z_D) relative to the global XYZ coordinate system, and distal segment bending and orientation angles (θ_D, ϕ_D) relative to the local $\tilde{X}\tilde{Y}\tilde{Z}$ coordinate system.

Figure 3.
Kinematic redundancy of the two segment catheter showing identical tip position achieved with different catheter orientations.

Figure 4.

Ablation path generation: (a) three physician-specified points, with rendering of catheter orientation for each, and the vectors used to create the ablation plane they define; (b) planar ablation path showing the k^{th} set of discrete coordinates associated with the cubic spline calculation.

Figure 5. Conversion from proximal tip global Cartesian coordinates to proximal segment generalized coordinates.

Figure 6. Global XYZ and local $\tilde{X}\tilde{Y}\tilde{Z}$ coordinate systems and coordinates for the distal segment, related by the proximal segment angles θ_p and ϕ_p .

Figure 7. Conversion from distal tip local Cartesian coordinates to distal segment generalized coordinates for $\theta_D > 90^\circ$

Figure 8.
Conversion from proximal segment generalized coordinates to proximal tip global Cartesian coordinates

Figure 9.

Catheter closed-loop controller showing decoupling to regulate voltage in antagonistic SMA actuator pairs.

Figure 10.

Outer-loop hysteresis curve relating steady-state voltage to bending angle for an antagonistic SMA tendon pair

Figure 11.

Experimental closed-loop control setup: (a) catheter submerged in saline-filled custom tank fabricated with EnSite-compatible patch electrodes; (b) control loop showing PWM amplifier for powering SMA tendons and EnSite NavX System for real-time catheter position feedback.

Figure 12.

EnSite NavX accuracy and precision: (a) comparison between known cubic geometry and measured tip locations (shown as 300 measured data points at each cube corner and midpoint); (b) close-up view of one 300 data point measurement showing variation within a 1.6 mm diameter sphere.

Figure 13.
Selected points and computed path used for optimization simulations

Figure 14.

Simulated optimal ablation trajectory: (a) generated ablation path and catheter orientation rendering at each optimized path point, (b) performance factors including planar tracking error, normal penetration, and tip normality, and (c) optimal reference angles.

Figure 15.
Optimal ablation paths (solid curves) generated from experimentally recorded catheter tip locations (dots)

Figure 16.

Experimental results showing reference and average catheter orientations and tip locations.

Table 1

Simulated cost functions associated with independently magnifying corresponding weights

Weights	Planar Deviation (J_1) [mm]	Normal Penetration (J_2) [mm]	Deviation from Normal (J_3) [deg]	Average Change in Angles (J_4) [deg]
p_0, q_0, r_0, s_0	0.01	4.09	42.5	10.6
$10 p_0, q_0, r_0, s_0$	0.01	5.43	40.6	23.7
$p_0, 10q_0, r_0, s_0$	0.61	0.00	54.3	14.5
$p_0, q_0, 10r_0, s_0$	2.94	9.85	1.0	34.5
$p_0, q_0, r_0, 10s_0$	38.20	0.41	13.6	5.7
$\frac{1}{2}p_0, \frac{1}{2}q_0, \frac{1}{2}r_0, \frac{1}{2}s_0$	0.00	0.08	55.1	20.1