On the cyclonic mean circulation of large lakes[†]

(hydrodynamics of lakes/lake currents/dynamics of marginal seas/surface heating and lake circulation/North American Great Lakes)

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ABSTRACT In large lakes, during the heating season, the combination of surface wind drift and solar heating results in the horizontal transport of some of the absorbed heat into the coastal zone. Large-scale vertical movements associated with alternating upwellings and downwellings distribute the heat over a deeper water mass than at mid-lake. As a result, the mean shape of isotherm surfaces is "domed" and, correspondingly, the lake surface is slightly raised near shore and depressed over the center thermocline dome. The mean circulation in geostrophic equilibrium with this surface elevation field is a cyclonic gyre. In Lake Ontario, the amplitude of the mean surface velocity associated with this gyre is of the order of 3 cm sec⁻¹.

Introduction

Four years ago, Emery and Csanady (1) reported observational evidence on the cyclonic mean circulation of many natural closed basins, and proposed an explanation in terms of an air-sea interaction mechanism that depended on the fact that wind blowing over warm water exerts a drag force greater than over cold water. Surface temperature differences were shown by observational evidence to arise from a combination of Ekman drift and surface heating, although the mechanism of this process was not analyzed. The magnitude of the inferred wind-stress curl could not at that time be subjected to direct experimental checks.

Several other possible mechanisms for driving a cyclonic mean circulation have been identified in the interim. Wunsch (2) showed that the Lagrangian drift due to internal Kelvin waves has a cyclonic component at the surface. Bennett (3) suggested that the same surface warming noticed by us leads to greater stratification of the water column at the right-hand shore (looking alongwind) than at the left-hand shore. Increased stratification more effectively insulates the surface layers from the bottom and allows higher velocities. Repetition of this pattern in successive, random wind episodes leads to a cyclonic mean circulation, as would our air-sea coupling mechanism. In a recent paper (4), I argued that the nonlinear (advective) momentum flux in Ekman drift fortifies the right-hand coastal current at the expense of the left-hand one and leads similarly to cyclonic circulation. All three of these alternative mechanisms are effective in a coastal band only: their influence should be negligible beyond about 10 km from shore, and they are relatively weak even within the coastal zone (J. R. Bennett, personal communication).

In another recent contribution (5), the cumulative effects of wind-driven *first-order* flow episodes on a longer-term *mean* circulation were systematically discussed. It was shown that the mean internal pressure field may be influenced by various *pumping* effects, such as a mean flux of lighter fluid in one

direction, and a mean flux of heavier fluid in another. In this conceptual framework, the previously offered explanations of cyclonic mean circulation and some others yet to be referred to may be categorized as shown in Table 1.

Intensive observations of both winds and currents in and over Lake Ontario during the International Field Year on the Great Lakes (IFYGL) and during earlier feasibility studies conducted as IFYGL projects have provided much more detailed evidence bearing on the mean circulation problem than has been available in the past for any natural basin. The wind stress was inferred from 2-m winds by using a constant drag coefficient. One might think that this approach automatically eliminates the Emery-Csanady mechanism. However, the 2-m drag coefficient is little influenced by air stability: under stable conditions low level winds decrease as does the stress. The accuracy of this method of wind-stress determination was sufficient to demonstrate that there was no significant wind-stress curl in July 1972, certainly nothing comparable in amplitude to what was inferred in our 1973 paper and to what would be necessary to explain cyclonic mean circulation. The reason is presumably good horizontal mixing in the air, reducing wind-stress differences. The influence of stability on wind stress is clear on a lakewide basis, the mean stress increasing as the lake warms up, but not on a section by section basis of a lake.

On the other hand, the observations showed conclusively that the mean circulation during summer had two more or less distinct components. In the coastal zone (about 10 km in width), an intense, mean cyclonic current with an amplitude of about 10 cm sec⁻¹ was found in July, extending over most of the lake perimeter although not including its western end which was occupied by a small anticyclonic gyre. The coastal circulation decayed to a negligible amplitude at some 8 km from shore (10). The behavior of the coastal zone is discussed elsewhere (G. T. Csanady and J. T. Scott, unpublished data).

Farther offshore another component of the mean circulation was in evidence: a cyclonic gyre was in geostrophic balance with the mean internal-mass distribution. Pickett and Richards (11) demonstrated this for July 1972: their mean circulation pattern and dynamic height distribution is reproduced here in Fig. 1. Note that the current observations shown relate to 15-m depth, which is just below the layer occupied by wind drift. The amplitude of this mean circulation component is about an order of magnitude less than the coastal circulation, but its scale is larger, with the most intense flow coinciding approximately with the 60- to 120-m isobaths. None of the mechanisms identified so far offers a satisfactory explanation of this mean circulation component.

The purpose of the present paper is to show that the geostrophic lakewide summer circulation component may be explained by invoking a pumping mechanism affecting heated surface water. Emery and I (1) did not analyze the occurrence of surface warming in the direction of Ekman drift, nor did we elaborate on the important implication that this signifies

Abbreviations: CBL, coastal boundary layer; IFYGL, International Field Year on the Great Lakes.

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Table 1. Schematic matrix of circulation mechanisms

Causes of cyclonic circulation	Mechanism	Refs.
Internal pressure field (mass pumping)	Boundary diffusion	6
	Early mid-lake circulation	Present paper
	Finite amplitude upwelling	7
	Spring thermocline	8, 9
Internal stress distribution	Differential bottom friction	3
Distributed momentum sources	Momentum pumping	2, 4
Direct forcing	Wind stress curl	1

shoreward transport of heat. It is now argued that this mechanism is alone capable of establishing a domed mean thermocline structure during the heating season, which implies by geostrophic balance a cyclonic mean circulation of an amplitude about that observed.

A simple conceptual model

As an idealization of a long and narrow lake consider an infinitely long channel, at the surface of which a longshore wind stress acts and solar heat is absorbed. The channel is supposedly wide enough for a cross-channel surface drift to be established due to the earth's rotation. Far from the shores, an Ekman layer is supposed to form at the surface, in which a quantity of water U_t is transported to the right of the wind:

$$U_t = \int_{-h_t}^0 u dz = \frac{F}{f} \quad [1]$$

in which h_t is a top layer depth, F is the (kinematic) wind stress along positive y , the x axis pointing across the channel, and f is the Coriolis parameter. The idealization is intended to simulate the effects of a single wind-stress episode and F will be regarded constant, as will be the surface heat flux.

In simple Ekman models, a two-dimensional flow pattern is often assumed in which the surface drift is returned in the same cross section at greater depths. However, as has been argued by Pedlosky (12), Birchfield (13, 14), and others, in a natural basin of irregular depth distribution, vorticity constraints due to the earth's rotation at least partially prevent such simple recirculation. In a fluid of very low viscosity, the return flow takes place in a narrow coastal boundary layer (CBL). The exact nature of the return flow is immaterial for the arguments which follow: Eq. 1 will be supposed to hold true at sufficiently large distances from the two shores. The region close to the shores is supposedly occupied CBLs of unspecified and possibly complex structure. The left-hand CBL (looking along the wind) delivers fluid at the rate U_t to the center region top layer, while the right-hand CBL absorbs it. Outside the CBLs, the cross-channel velocity is a function of the vertical coordinate alone, $u = u(z)$ and vertical motions are absent.

The heat balance of the fluid outside the CBLs is described by the equation:

$$\frac{\partial T}{\partial t} + \frac{\partial(uT)}{\partial x} = \frac{\partial}{\partial z} \left(K_z \frac{\partial T}{\partial z} \right) + \frac{\partial}{\partial x} \left(K_x \frac{\partial T}{\partial x} \right) \quad [2]$$

in which K_z and K_x are eddy thermometric conductivities along z and x , respectively, which may be functions of location.

During the heating season, major temperature changes in natural basins are confined to the surface mixed layer, with vertical heat transfer across the thermocline being slow enough to permit neglecting it in a single wind-stress episode. Boundary conditions to be imposed on the idealized model are, correspondingly, a constant heat flux Q at the surface and a zero heat flux at the thermocline:

$$\begin{aligned} K_z \frac{\partial T}{\partial z} &= \frac{Q}{\rho c_p} \quad (z = 0) \\ K_z \frac{\partial T}{\partial z} &= 0 \quad (z = -h_t). \end{aligned} \quad [3]$$

The second condition insulates the bottom layer from any temperature changes, so that the calculations below refer to the mixed layer only. However, heat transfer to the deeper waters is not excluded within the CBLs.

FIG. 1. Dynamic height distribution and currents 15-m deep in Lake Ontario, averaged over July 1972 (11). Dynamic heights are in millimeters; current barbs represent 1 cm sec^{-1} .

Under a steady wind and given steady surface heat flux, a form of quasi equilibrium may be envisaged in which the water is heating up at a constant rate r in time, while there is also a constant cross-channel temperature gradient s , in the uniform central portion of the channel, outside the CBLs. For this portion of the channel, therefore, one may try to find a solution of Eq. 2 of the following form:

$$T = rt + sx + \theta(z). \quad [4]$$

On the reasonable assumption that K_x is not a function of the x coordinate in mid-channel, Eq. 2 now yields

$$r + us = \frac{\partial}{\partial z} \left(K_z \frac{\partial \theta}{\partial z} \right). \quad [5]$$

After one integration this becomes

$$K_z \frac{\partial \theta}{\partial z} = \int_{-h_t}^z (r + us) dz \quad [6]$$

in which the boundary condition at $z = -h_t$ has already been applied. The top boundary condition results in the relationship:

$$\frac{Q}{\rho c_p} = rh_t + sU_t \quad [7]$$

which is a bulk heat balance, apportioning the total heat gain between local heating and transport to the right.

If no heat at all were communicated to the deep water, the total heat income of the basin QA , in which A is the surface area, would be distributed over the top-layer volume only. If a narrow nearshore band in which the total depth is less than h_t is neglected, the top-layer volume is $h_t A$, so that the rate of heating would be

$$r = \frac{Q}{\rho c_p h_t}. \quad [8]$$

Eq. 7 shows now that, in this case, the horizontal temperature gradient is zero in the center portion of the lake, and the whole top layer is heating up uniformly everywhere.

It is known, however, that heat exchange between surface and bottom waters is appreciable in CBLs. Specifically, the water supplied by the left-hand CBL is partly derived by entrainment from the bottom layer, because here the thermocline is close to the surface, this being an upwelling zone. The entrained water cools the surface layer and keeps the heating rate r below the basin average value given by Eq. 8. Alternatively, one may express this result by saying that the heat income of the left-hand CBL is distributed over a water volume greater than its surface area times the thermocline depth h_t . We write therefore:

$$r = (1 - \phi) \frac{Q}{\rho c_p h_t} \quad [8a]$$

in which ϕ is between zero and unity, and is determined by the effective volume over which the incoming heat is distributed in the left-hand CBL. Eq. 7 now supplies the horizontal temperature gradient in the center region

$$s = \phi \frac{Q}{\rho c_p U_t}. \quad [9]$$

A second integration of Eq. 6 yields the temperature distribution in the vertical:

$$\theta(z) = \int_{-h_t}^z \frac{dz}{K_z} \int_{-h_t}^z (r + us) dz. \quad [10]$$

Without loss of generality, θ may be taken to be zero at $z = -h_t$, as implied by the integration limits. It is now evident that

Eq. 4 satisfies Eq. 2 and the boundary conditions, if $\theta(z)$ is as given by Eq. 10. For any prescribed distribution of $u(z)$ and $K_z(z)$ the temperature field may therefore be calculated by simple quadrature. This temperature field characterizes the quasi equilibrium situation envisaged in the above simple model.

For the arguments of this paper the details of the vertical temperature distribution are immaterial. From Eq. 7 it is easily shown that the right-hand CBL gains more heat than the left-hand CBL loses, by the amount

$$\frac{Q_H}{\rho c_p} = sU_t l = \phi \frac{Q}{\rho c_p} l.$$

Here, Q_H is horizontal heat flux in energy per unit length of shoreline and unit time, and l is channel width outside CBLs.

The excess heat flowing into the right-hand CBL is equivalent to the heat income over a fraction ϕ of the exposed channel surface. If ϕ is a relatively large fraction, heat advection into the right-hand CBL is substantial.

The above argument can be summarized as follows: the rate of heating following the fluid in the center portion of the channel is, by Eq. 7, $Q/\rho c_p h_t = r + su_t$, in which $u_t = U_t/h_t$. The left-hand CBL delivers to the center portion a fluid warming at the rate r . If r is less than the center rate of heating, horizontal differences in temperature must arise, and the fluid arriving in the right-hand CBL is warmer than it was when it left the left-hand CBL. The argument can be turned around as follows: observed horizontal temperature gradients across wind under steady conditions imply entrainment of bottom water from the left-hand CBL and horizontal flux of heat into the right-hand CBL. Basin-wide this implies enhanced heat transfer to deep water within coastal boundary layers. An interesting point to note is that the horizontal heat flux from the center portion of the basin to the right-hand CBL proceeds "up" the temperature gradient, being an advective rather than a diffusive flux.

It may also be observed that the average surface temperature of the two CBLs in the above simple model is the same as the average surface temperature of the mid-lake portion. The left-hand CBL is cold, the right-hand CBL is warm, and over the center portion there is a linear gradient between the two CBL temperatures.

Mean circulation

Consider now the average effect of many wind-stress episodes combined with surface heating. The wind varies randomly in direction as well as in magnitude, whereas the quasi equilibrium situation envisaged in the simple model above may only be expected to be established in relatively long periods of steady winds. Nevertheless, a tendency toward those conditions should be present all the time, and one might perhaps model actual conditions by supposing that something like the quasi equilibrium heat transport to the right-hand shore exists for a certain fraction, ϕ_t , of the time. However, the location of the "right-hand" shore is different from storm to storm, so that the results of the above analysis are best formulated in terms of the average heat flux into the two CBLs, which is half the value given by Eq. 11.

The net result then is that a fraction $\phi^* = \frac{1}{2}\phi_t$ of the surface heat income of the basin outside coastal boundary layers is transported into those same boundary layers by a succession of storms, as a result of the combined effect of surface Ekman drift and surface heating. The average heat income of the CBLs is greater than the local surface heat flux, whereas the heat income

FIG. 2. Temperature of Lake Ontario averaged for 20- to 40-m depths, as observed on a ship cruise between 26th and 28th June 1972 (courtesy of the staff of Canada Centre for Inland Waters). There had been some strong east and west winds in the weeks prior to the survey, including Hurricane Agnes on 22–23 June.

of the mid-lake portion is less. If ϕ_t and ϕ are both about 0.5, then ϕ^* should be about 0.1.

Because the mean surface temperature of the CBLs is no different from the mean temperature at mid-lake, the extra long-term mean heat income of the CBLs must all be supposedly utilized in warming a larger volume of water per unit surface area than at mid-lake. In order to accommodate the larger volume of warm water near the shore, the mean thermocline has to slope downward as one approaches the shore, and has to intersect the bottom at some distance offshore. The precise geometry of the shore zone and the quantity of warm water to be accommodated determine this distance. The slope of the mean thermocline lakeward of its locus of intersection with the bottom is determined by the slope in first-order downwelling episodes (when the horizontal scale is the internal radius of deformation) together with various dissipative effects which act to make the thermocline relax toward a horizontal equilibrium position. The details of such dissipative processes are little known. It is not possible to say how large the actual horizontal scale of the mean downwelled thermocline should be, except that it should presumably be larger than the internal radius of deformation (which is typically 5 km).

A “domed” mean thermocline implies a mean dynamic height distribution characterized by a low in the center and a high near the shores. If, as is reasonable to suppose, the mean motion of the deep, cold layer is negligible, the dynamic height distribution implies cyclonic mean flow in the upper layers of the lake.

The above inferences apply to average conditions within the entire CBL surrounding a lake, or to the effect of varying winds with negligible mean stress. When the mean wind stress is appreciable, the CBL to its right should be warmer than that to its left, on the average, and there should be a corresponding mean surface temperature gradient over the center part. The mean circulation in the center part then also contains Ekman drift close to the surface, but there should be little effect at greater depth.

Comparison with observation

The approximate character of the above conclusions needs little further emphasis. The discussion of the observational evidence

will be focused on the degree to which the phenomena revealed by the simple model are quantitatively significant.

The average heat absorption by Lake Ontario is given by Rodgers and Anderson (15) as about $400 \text{ cal cm}^{-2} \text{ day}^{-1}$ in June and $256 \text{ cal cm}^{-2} \text{ day}^{-1}$ in July. During the same period, the typical wind stress in the course of a sustained storm is about 0.5 dyne cm^{-2} . Putting $\phi = 0.5$ in Eq. 9, one calculates a crosswind temperature gradient of $3 \cdot 10^{-7} \text{ cm}^{-1}$, or about a 2° temperature contrast from the north to the south shore of Lake Ontario during a sustained west or east wind.

Surface temperature differences of this magnitude have frequently been observed in aerial surveys conducted during the heating season (1, 16). In some instances, the north–south temperature difference *outside* CBLs was as high as $4\text{--}5^\circ$ which suggests that ϕ is close to unity and r is close to zero. Pronounced upwelling on the left-hand shore was often noticeable. The sign of the temperature gradient reversed with changes in wind direction, higher temperatures always occurring to the right of the wind. Later in the season, as surface heating became weak, the temperature differences also became smaller. Generally, however, the observed magnitude of the surface temperature differences was large enough to require ϕ in the above theory to be a substantial fraction.

Once ϕ is a substantial fraction, one expects the warm layer to be deeper in the mean near shore than at mid-lake. That such was the case in July 1972 has already been pointed out (cf. Fig. 1). A more specific illustration of this effect is shown in Fig. 2, which gives the mean temperature distribution between 20- and 40-m depths in late June 1972. There was a succession of storms in early June, followed by Hurricane Agnes on June 22–23. Their net result was deep heat penetration at both north and south shores, but not at the center. Surface heating, of course, is intense in June. The depth of heat penetration is greater in the coastal bands than in the center by about a factor of two. The width of the warm band is greater along the north shore because of shallower water: the 5° isotherm in this figure coincides approximately with the 120-m isobath along both shores. Comparison with Fig. 1 shows that the main cyclonic geostrophic current is located just shoreward of this isobath and has a width of about 10 km.

Approximately the same mean thermal structure in early

summer in Lake Ontario was described in greater detail by Sweers (17), some years prior to IFYGL. Further illustrations by Pickett and Richards (11) for July 1972 also show that (i) the mean wind stress was appreciable and eastward, (ii) the mean surface temperature increased from north to south by about 2°, and (iii) the coastal boundary layer at the south shore was warmer to a greater mean depth than at the north shore, while the mean thermocline at the center part of the lake was shallower than in either boundary layer. The difference in dynamic height between center and shore, corresponding to the mean early-summer thermal structure, is of the order of 0.6 cm and implies a geostrophic surface layer velocity (relative to a supposedly quiet bottom layer) of the order of 2 cm sec⁻¹. This mean velocity was in fact observed by current meters moored 15-m deep, i.e., below the layer affected by Ekman drift (barbs in Fig. 1).

All this observational evidence is consistent with the simple conceptual model developed above and shows that the phenomena modelled are quantitatively significant. Specifically, the fraction ϕ^* of the basin surface, the heat income of which is transported into the coastal zone over a longer period, may be estimated to be about 0.1, judging by the observed extra heat content of the coastal zone.

The above ideas can also be applied to the explanation of the central cold pool observed in Lake Superior by Ragotzkie and Bratnick (18) in late July. Ragotzkie and Bratnick postulated upwelling of cold water in the center portion of the lake associated with divergence of surface Ekman transport. This would require considerable cyclonic curl in the wind stress which was almost certainly not present. The above discussion has shown that it is not necessary to postulate divergent *volume* transport of water; divergent *heat* transport to the right of the wind in successive storms is sufficient to explain preferential heating of the nearshore zone. There must be, of course, a source region for the Ekman drift, an analog of the left-hand CBL in the above simple model. This may be somewhat offshore in the case of Lake Superior, which has a different summer thermal structure than Lake Ontario. At the present time it is not profitable to take these speculations further.

Conclusion

While there may be a number of different mechanisms implicated in the cyclonic circulation within the coastal zone or in mid-lake in other seasons, the IFYGL evidence on Lake Ontario gives strong support to the above conceptual model of mid-lake circulation in the early heating season. This model should be distinguished from (i) a boundary-diffusion mechanism [see, e.g., Wunsch (6)] and (ii) spring thermocline models of the type discussed by Bennett (8), Huang (9), or Csanady (19). In a stratified fluid, mixing at the boundaries generates a dynamic height distribution giving rise to geostrophic currents [mechanism (i), above]. Mixing, alone, reduces surface temperatures and increases those at depth: this clearly differs from the situation discussed above. In early spring, shallow waters heat up first, so that horizontal density gradients appear giving rise to cyclonic mean flow near shore [mechanism (ii), above]. Whereas this effect is akin to the one discussed here, it is much weaker than the massive shoreward heat transport by Ekman drift in late spring and early summer. Also, the spring thermocline is a shallow near-surface phenomenon unlike the massive later penetration of heat near shore.

It should also be pointed out that the mid-lake circulation is

not the only mechanism to produce a large-scale cyclonic gyre in geostrophic equilibrium with an appropriate density field. As demonstrated in a discussion of finite amplitude upwelling (7), the asymmetry of the upwelling versus the downwelling zone coupled with a Kelvin wave-like behavior of the downwelling zone, can generate a domed thermocline well after the heating season (as it did in October, 1972).

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