Non-moving Hadamard matrix diffusers for speckle reduction in laser pico-projectors

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Personal electronic devices such as cell phones and tablets continue to decrease in size while the number of features and add-ons keep increasing. One particular feature of great interest is an integrated projector system. Laser pico-projectors have been considered, but the technology has not been developed enough to warrant integration. With new advancements in diode technology and MEMS devices, laser-based projection is currently being advanced for pico-projectors. A primary problem encountered when using a pico-projector is coherent interference known as speckle. Laser speckle can lead to eye irritation and headaches after prolonged viewing. Diffractive optical elements known as diffusers have been examined as a means to lower speckle contrast. This paper presents a binary diffuser known as a Hadamard matrix diffuser. Using two static in-line Hadamard diffusers eliminates the need for rotation or vibration of the diffuser for temporal averaging. Two Hadamard diffusers were fabricated and contrast values measured showing good agreement with theory and simulated values.

Keywords: diffractive diffusers; speckle contrast reduction; laser pico-projectors; Hadamard matrix; Fourier optics

1. Introduction

Due to their power efficiency, brightness, and larger color gamut extremes lasers are ideal light sources for projector applications [1]. A fully coherent source provides an image along the optical axis without the concern of losing focus. Speckle, the granular intensity pattern resulting from constructive and destructive interference, can be observed from laser-illuminated surfaces. The resultant speckle in the projected image strains the eye, resulting in observer's headaches. Speckle contrast associated with the overall intensity pattern is calculated by Equation (1) [2], where σ is the standard deviation and I_{avg} is the average irradiance across the image as described by Equation (2).

$$C = \frac{\sigma}{I_{avg}} \quad (1)$$

$$C = \sqrt{(E[I^2] - E[I]^2)/I_{avg}} \quad (2)$$

Diffusers can be used to offset the original coherence in the laser light prior to reflection from a screen or surface. The degraded coherent optical field results in a change of the image irradiance levels, thus reducing the speckle contrast. A commonly used speckle reduction technique is temporal averaging of the irradiance pattern using a diffractive diffuser [3]. The averaging occurs from rotation or vibration of the diffractive element. A stationary diffuser introduces a single independent phase change. By moving the diffuser either through rotation or vibration, K independent time realizations of the diffuser are

averaged together. Rotation of a pseudo-random pattern diffuser determines the speckle reduction as a function of the number of patterns averaged. The contrast from K patterns can be determined by Equation (3) [4].

$$C = \sqrt{\frac{1}{K}} \quad (3)$$

In order to temporal averaging to completely negate speckle for the human eye, sample rate of 20 Hz, a diffuser would have to rotate a number approaching infinity all within 0.05 s. A rotating or vibrating diffuser would also be susceptible to mechanical failures, reducing product lifespan. Rotational or vibrational methods require additional supporting components, increasing implementation difficulty in pico-projectors which have severely limited spatial allocation. A diffuser that reduces speckle without the need for physical manipulation is ideal for laser pico-projection systems.

Previous work on non-rotating diffusers have had limited success. A more recent article utilized an MEMS device for very small motion of the diffuser [5] where speckle reduction of 0.48 was achieved. Another method used piezoelectric benders to reduce speckle while limiting actual diffuser motion with a contrast of 0.16 [6]. The piezoelectric method, while non-rotational, requires two mechanically vibrating parts making the design susceptible to failure. Various contrast reduction methods [3,7–9] have resulted in significant speckle contrast, but all require motion. Additionally, the majority of speckle reduction methods require large setups or devices that

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would need to be miniaturized for pico-projector application. We present a novel non-rotational, non-moving diffractive diffuser that is easily integrated into laser pico-projectors, significantly reducing speckle contrast.

2. Hadamard diffuser

A binary diffuser commonly used in speckle reduction techniques has an associated phase shift of either 0 or π for each individual phase element of the structure [10,11]. This corresponds to a common binary pattern of 0 or 1. A Hadamard matrix is a square matrix used for performing Hadamard transforms (HT) that is composed entirely of +1 and -1 values [10]. A special property of the Hadamard matrix is the relationship between the matrix and its transpose as described in Equation (4). This definition requires all Hadamard matrices to be square with orders $n = 2, 4, 8, 16, \dots$ where $H_1 = [1]$ [10]. Multiple versions of the Hadamard matrix exist, each with a unique binary pattern.

$$H_n H_n^T = H_n^T H_n = n I_n \quad (4)$$

The first type of Hadamard matrix is the Sylvester matrix and is described in Equation (5) with the condition that $H_1 = [1]$. An order 16 Sylvester matrix is shown in Figure 1 where the dark area denotes +1 and the white space denotes -1.

$$H_{2^n} = \begin{pmatrix} H_{2^{n-1}} & H_{2^{n-1}} \\ H_{2^{n-1}} & -H_{2^{n-1}} \end{pmatrix} \quad (5)$$

The other Hadamard matrix types are known as the Walsh-Paley and the Walsh matrix, and can be constructed through recursion [10]. While each matrix still follows the basic definition, each has an exclusive pattern creating different propagated irradiance patterns when observed. The -1 elements of the matrix can be replaced with 0 and a binary diffuser can be developed from the pattern.

Figure 1. Order 16 Sylvester matrix.

The measured speckle contrast for an initial *rotating* Hadamard diffuser design was 0.12 [11]. The original design requires 150–200 rpms for rotating diffusers in order to achieve speckle contrast at or below 0.20 as discussed previously regarding temporal averaging of a diffuser. Variations in the Hadamard matrix pattern types will reduce speckle contrast without the need for temporal averaging. Using two stationary statistically correlated Hadamard diffusers with varying phase patterns in tangent can reduce the speckle more than two generic uncorrelated binary diffusers. A similar experiment was conducted in 2012 in which a single optimized binary diffuser was manipulated into two separate diffuser structures [12]. While stationary diffuser speckle was reduced, the method did not lower the contrast more than previously demonstrated using single rotating diffuser contrast values.

3. Theoretical analysis

A single scattering, Fourier model is used for initial analysis of the proposed dual Hadamard diffuser reduction technique and proceeds with a full analysis of optical system simulation. A 16-order matrix is created as an individual cell and duplicated to fill the total size of the array dependent on the analysis of the entire propagation system. For any speckle reduction technique, Hadamard matrices included, a decrease by $(M)^{-1/2}$ can be achieved where M is the degree of freedom [4,13]. The degree of freedom represents the number of independent diffraction patterns that are created by a stationary diffuser [3,8,9]. A 16-order Hadamard matrix pattern can be expected to have a contrast measurement of 0.707. Placing two differing Hadamard patterns in series along the optical path, within the Fresnel zone of the primary diffuser allows for a speckle value of 0.343. Due to the correlation between the two patterns, one-half of the contrast reduction associated with the fully developed speckle can be assumed [4,14].

Calculation of the expected contrast value requires an extension of Equation (2), which can be done by assuming N independent pixels for the image irradiance. The standard deviation of the irradiance can now be expanded given by Equation (6).

$$C = \frac{\sqrt{\sum_{n=1}^N (I_i - I_{\text{Avg}})^2}}{I_{\text{Avg}}} \quad (6)$$

The image irradiance level can be found using Fourier propagation with the assumption that the illuminated diffuser speckle pattern will propagate onto a projection lens prior to observation onto a screen. This layout is illustrated in Figure 2 where z_1 and z_2 represent the propagation distances between the diffuser, lens, and

Figure 2. Current diffuser layout using projection lens.

observation screen and f represents the focal length of the lens. Initial analysis will involve a single diffuser to verify the approach to satisfy the previously expected contrast values.

The initial propagation prior to incidence on the diffuser as coherent monochromatic Gaussian light is defined by Equation (7).

$$U_G(x, y) = A_G e^{\frac{x^2 + y^2}{w_0^2}} \quad (7)$$

A_G represents the amplitude of the beam and w_0 is the initial waist of the beam. The diffuser can be modeled as a single scattering phase screen with the Hadamard matrix structure defined in Equation (6). This matrix can be replicated to represent the whole of the diffuser as described in Equation (8).

$$t_A(x, y) = e^{j\pi/2} \begin{bmatrix} H_{16} & \dots & H_{16} \\ \dots & \dots & \dots \\ H_{16} & \dots & H_{16} \end{bmatrix} \quad (8)$$

The $\frac{\pi}{2}$ in the exponential is the phase shift that separates the two levels of the diffusers. The incident illumination can be multiplied by the diffuser within the Fourier domain since it is modeled as a phase screen. It is assumed that the Gaussian illumination is a negligibly small distance from the incident diffuser which allows direct convolution of the incident beam and the diffuser. The initial optical field immediately after the diffuser is defined in Equation (9).

$$U_{Inc}(\xi_{Inc}, \eta_{Inc}) = \mathfrak{S}\{U_G(x_{Inc}, y_{Inc}) \times t_A(x_{Inc}, y_{Inc})\} \quad (9)$$

Propagation after the diffuser can be accomplished through the transfer matrix approach [13], taking into account the distance of propagation is still within the near-field region. Equation (10) shows the result of propagation after the diffuser.

$$U_D(x_D, y_D) = \frac{e^{jkz}}{j\lambda z} \iint U_{Inc}(\xi, \eta) e^{j\frac{k}{2z}[(x-\xi)^2 + (y-\eta)^2]} d\xi d\eta \quad (10)$$

Due to the very small size of the diffuser elements in x and y , the phase propagated pattern size will increase rapidly along the z axis causing the lens to be well within the focal length and thus, within the Fresnel region of the diffuser output. This satisfies the condition for Fresnel approximation given by Equation (11) [15].

$$z^3 \gg \frac{\pi}{4\lambda} [(x - \xi)^2 + (y - \eta)^2]_{\max}^2 \quad (11)$$

An important element in measuring the speckle is to ensure that the lens does not decrease the size of the speckle spots so much that the individual pixels of the camera used for measurement do not cause averaging, independent of the rest of the system [16]. This “matching” of lens to speckle screen was used extensively for measuring laser beam size effects and diffuser speed while maintaining the individual speckle spot size, effectively comparing the speckle contrast across the surface [3]. A lens, L , can be described as a phase function applied to the incoming wave. The transmittance function of the projection lens is described by Equation (12) [15].

$$t_L(x_L, y_L) = P(x_L, y_L) e^{-j\frac{k}{2f}(x_L^2 + y_L^2)} \quad (12)$$

For a lens function, it can be shown that the distance d is replaced by the focal length of the lens f . The aperture function P can be simply described as a circle [15, 17]. Propagation of the lens can occur using the Fraunhofer propagation equation and replacing the distance parameter z with the focal length f of the lens [17]. The generic Fraunhofer equation is described in Equation (13). Substituting z for the focal length f and inserting Equations (10) and (12) results in Equation (14).

$$U_f(x_f, y_f) = \frac{e^{jkz}}{j\lambda z} e^{j\frac{k}{2z}(x^2 + y^2)} \iint U_1(\xi, \eta) e^{-j\frac{k}{2z}(x\xi + y\eta)} d\xi d\eta \quad (13)$$

$$U_f(x_f, y_f) = \frac{e^{jkf}}{j\lambda f} e^{j\frac{k}{2f}(x_f^2 + y_f^2)} \times \iint U_D(x_D, y_D) P(x_f, y_f) e^{-j\frac{k}{2f}(x_f x_D + y_f y_D)} dx_D dy_D \quad (14)$$

U_f represents the amplitude of the wave of the resultant propagation at the focal plane of the lens L .

After the focal plane of the lens, propagation to the observation plane can be evaluated using the Fresnel propagation equation described previously by Equation (11). The final amplitude output of the system U_{Obs} can be described

by Equation (15) where Equation (14) can be viewed as the initial amplitude field and is propagated using the Fresnel approximation to the final observation screen.

$$U_{Obs}(x_{Obs}, y_{Obs}) = \frac{e^{jkz_2}}{j\lambda z_2} \iint \frac{e^{jkf}}{j\lambda f} e^{\left[j\frac{k}{2f}(x_f^2 + y_f^2) \right]} \iint U_D(x_D, y_D) P(x_f, y_f) e^{\left[-j\frac{2\pi}{\lambda f}(x_f x_D + y_f y_D) \right]} x_D dy_D e^{j\frac{k}{2z}[(x - \xi)^2 + (y - \eta)^2]} d\xi d\eta \quad (15)$$

The distance from the focal plane of the lens to the final observation plane is greater than that previously described for the diffuser to the lens. This distance z_2 is still well within the near-field distance requirement and satisfies Equation (12). Far-field propagation will need to be considered for projection onto surfaces for image viewing (actual projector system), but is not applicable for the detection measurement [4,16].

Contrast measurements for the output speckle images can now be found from the previously discussed contrast equation involving speckle image intensity as described in Equation (7). The intensity of the observation measurement from the optical field in the observation plane can be broken into pixel elements as shown in Equation (16).

$$I_{Obs}(x_{Obs}, y_{Obs}) = |U_{Obs}(x_{Obs}, y_{Obs})|^2 \quad (16)$$

Thus, the overall contrast created by the Hadamard diffuser can be attributed to taking multiple Fourier transforms of the initial phase screen t_A . A second phase screen can now also be included by evaluating Equation (8) again with all elements the same except for the phase screen. The analysis for both diffusers will need to maintain the same z value associated with the diffusers prior to the lens to maintain Fresnel region propagation. A simulation will be used further to the previous analysis and provide an accurate accounting of the contrast reduction provided by the diffuser. Modeling of the z value to match the phase elements of both diffusers will be discussed in Section 3.

4. Simulating diffusers

Simulation of the primary diffuser is accomplished first to ensure the proper contrast evaluation prior to a two-diffuser simulation [18]. Both contrast values will be compared with the expected theoretical values for accuracy. Fourier analysis requires that the discrete values associated with the sampling of the arrays be properly related to the linear size of each array. In this situation, the two primary physical objects that require extended examination are the diffuser, or diffusers, and the lens.

A positive lens is used in imaging the aperture of the lens and will determine the final speckle spot size in the observation plane [4]. This configuration is known as subjective speckle. An important part of measuring the speckle contrast is ensuring that the speckle spots will not be averaged together in a single pixel of the observation screen [16]. This will be fixed by ensuring a large array for the lens that will project the pattern propagated from the diffuser. An 8000×8000 array group will be used for each screen, i.e. Gaussian input, diffuser, and lens. This means a sampling size of 3.89μ per pixel given a 54.8 cm diameter lens and a 25.4 cm linear, square diffuser. The most advanced photolithography machines are capable of incrementing diffuser size of around 1μ . This physical size limitation is well within the current standards for producing an actual diffuser and our simulated sampling size stands. The final output for the single Hadamard diffuser propagated through a projection lens is shown below in Figure 3. The output directly after the diffuser and a smaller cropped version of the final output image are shown as well.

The contrast provided by the final image is 0.67 ± 0.02 comparable to the theoretical value of 0.707 for a single Hadamard diffuser [11]. A two-diffuser output image is shown next in Figure 4. The distance between the two diffusers was optimized for lowest possible speckle and is currently set at 8 mm. The speckle contrasted provided by the final image is 0.43 ± 0.02 . The theoretical limit was calculated to be 0.343.

5. Experimental results and discussion

Hadamard diffuser designs were manufactured by the Microoptics division of Jenoptik Optical Systems, Inc (Huntsville)¹. Speckle contrast results were measured for the diffusers using a 532.8 nm DPSS laser with pseudo-random linear polarization. The laser was spatial filtered prior to incidence on the diffuser. A single diffuser was measured with a measured speckle contrast minimum of $0.62 (\pm 0.02)$ using a 25 mm focal length projection lens. The image was captured using a CCD camera with an array size of 480×640 pixels and a pixel size of $7 \mu\text{m}^2$ giving a total detector size of 3.36×4.48 mm. Figure 5 illustrates the output results that were used for contrast measurement. The image results from the single physical Hadamard diffuser matches the output results of a zoomed-in version of the simulation output in Figure 3(c).

Two diffusers were aligned along a common optical axis and the image contrast was measured. The projection lens was again a 25 mm focal length positive lens. Final contrast results for dual Hadamard diffusers were slightly more difficult to compare with simulated values due to mitigating factors such as spatial alignment, separation distance between diffusers, and speckle spot size. Each of these factors can arguably effects the speckle

(a)

(b)

(c)

Figure 3. Single diffuser simulation results. (a) Intermediate image screen; (b) final image screen; (c) final image normalized.

output, so all conditions of testing have been fully analyzed for each contrast result. A standard separation distance between the diffuser of approximately 5 mm (± 0.5 mm) was set up. A minimum speckle contrast of 0.40 (± 0.08) was achieved. It is noted that a significant variance in the speckle measurements was observed with

(a)

(b)

(c)

Figure 4. Dual diffuser simulation results. (a) Image directly after second diffuser; (b) final image output from dual diffuser system; (c) close up image output from dual diffuser system.

an 8–12% change in the contrast resulting from the lack of optimization with the separation distance between the two diffusers. Simulation results confirm that separation distance between the diffusers can have a significant impact on the resulting speckle contrast value which was

Figure 5. Physical Hadamard diffuser – image capture.

confirmed with the physical measurements fluctuations. Further research is underway to optimize the separation distance to enhance the speckle reduction capacity of the Hadamard matrix diffuser set. Figure 6 illustrates the output from a dual diffuser setup.

The minimum value achieved was within 2–3% of expected simulation results although still greater than the 6% that was theoretically predicted. It is hypothesized that a possible difference in contrast is due to larger than anticipated Gaussian beam effects and pixel to speckle spot size matching [16]. While the simulation attempts to overcome pixel size mismatch by utilizing large array

Figure 6. Dual Hadamard diffusers – image capture.

sizes, the camera array size, and pixel size are fixed. This is still of minor importance as differences in theoretical vs. physical measurements are within the variance caused by the distance discrepancies of the two diffusers.

The effects of linear polarization were evaluated with the physical diffuser set. A previous study showed that polarization has little to no impact on rotating diffusers [3]. A polarizer was implemented prior to the laser's incidence on the diffuser. Speckle contrast was measured for various linear polarization positions. The contrast measured from polarized images was $0.46 (\pm 0.02)$. This is a slightly higher speckle contrast than with random polarization. However, this change is within error parameters caused by the physical setup of the diffuser so it is of interest only in situations where the laser is completely polarized.

6. Conclusion

A static, dual in-line Hadamard diffuser optical system has been demonstrated for speckle reduction in pico-projection applications. Hadamard diffusers were fabricated and measured with good agreement between experimental contrast measurements and simulated values. A minimum contrast of $0.40 (\pm 0.08)$ was achieved without using rotation or vibrational movements. Using this type of structured binary matrix maintains partial coherence of the laser source, unlike static pseudo-random grayscale-style diffusers, necessary to preserve the perceived brightness of the laser. This static design ensures no mechanical failure modes and no vibration that could result in optical component alignment issues within a small enclosed system. The in-line Hadamard diffuser notion allows for a greater reduction in speckle than a single diffuser while still maintaining partial coherence desired for integrated laser pico-projector sources.

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Note

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