Combining Displacement Field and Grip Force Information to Determine Mechanical Properties of Planar Tissue With Complicated Geometry

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Performing planar biaxial testing and using nominal stress–strain curves for soft-tissue characterization is most suitable when (1) the test produces homogeneous strain fields, (2) fibers are aligned with the coordinate axes, and (3) strains are measured far from boundaries. Some tissue types [such as lamellae of the annulus fibrosus (AF)] may not allow for these conditions to be met due to their natural geometry and constitution. The objective of this work was to develop and test a method utilizing a surface displacement field, grip force–stretch data, and finite-element (FE) modeling to facilitate analysis of such complex samples. We evaluated the method by regressing a simple structural model to simulated and experimental data. Three different tissues with different characteristics were used: Superficial pectoralis major (SPM)

(anisotropic, aligned with axes), facet capsular ligament (FCL) (anisotropic, aligned with axes, bone attached), and a lamella from the AF (anisotropic, aligned off-axis, bone attached). We found that the surface displacement field or the grip force–stretch data information alone is insufficient to determine a unique parameter set. Utilizing both data types provided tight confidence regions (CRs) of the regressed parameters and low parameter sensitivity to initial guess. This combined fitting approach provided robust characterization of tissues with varying fiber orientations and boundaries and is applicable to tissues that are poorly suited to standard biaxial testing. The structural model, a set of C++ finite-element routines, and a MATLAB routine to do the fitting based on a set of force/displacement data is provided in the online supplementary material. [DOI: 10.1115/1.4028193]

Introduction

Soft-tissue characterization using planar biaxial testing and nominal stress–strain curves usually relies on certain conditions¹:

- The sample should have a shape, (e.g., square or cruciform) that tends to produce homogeneous strain fields in the central region.
- If the sample is fibrous, the fiber orientation should be known and aligned with the axes of the test system.
- Strain should be measured far from any rigid boundary, such as a grip or attachment to bone.

These criteria are often met (e.g., Refs. [3–9]), but for some tissue types, meeting one or more criteria is impossible. The tissue may, for example, be too small to allow isolation of a sample that is large enough for biaxial testing and is shaped to create a homogeneous strain region in the center. Aligning the material axes does not allow a biaxial test to generate planar shear strain [10]. Furthermore, if the material axes are improperly aligned, the variation in tissue response will increase, which could hinder elucidation of complex behaviors such as coupling between the two directions [10]. Other groups [e.g., Ref. 10] have used biaxial testing to good effect for off-axis fibrous samples. This is only effective, however, when the fiber orientation is known. When fiber orientation is unknown and/or the objective is to determine the fiber orientation, as in the current study, force–stretch data alone are insufficient. It also may be undesirable or even impossible to remove the tissue from bone, which restricts one's ability to align the tissue fiber direction with the testing apparatus and leads to inhomogeneity of the strain field. An example of these challenges is found in the AF of the intervertebral disk, particularly if one seeks to do single lamella experiments. Although some single lamella experiments have been performed in uniaxial [11–13] and biaxial modes [14–16], testing with intact bone [12–14,16] is attractive both because of direct relevance to in vivo loading and because of minimized tissue damage. Figure 1 shows a lamella of AF attached to axial vertebral bone. This geometry is not conducive to a homogeneous strain field, and the principal fiber direction does not align with the axes of the testing apparatus.

Sample geometries that produce inhomogeneous strain fields pose a considerable challenge to the investigator, but the advent of image-correlation-based methods for tracking motion over the entire tissue during testing [17–24] present new opportunities. The approach of simulating the experiment and then iterating over the model parameters to determine a best fit has been used in elastography [25,26] as well as indentation [27] and can be applied to tissue testing as well. In this study, we demonstrate the use of combined displacement field data, grip force–stretch data, and FE modeling to determine tissue properties from a biaxial test.

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¹For further information on strain uniformity and boundary conditions see Refs. [1] and [2].

Fig. 1 Lamella of the AF. The sample is attached to bone, labeled, and is anisotropic with fibers aligned 30 deg from the horizontal testing axis, along the dotted line. The dissected tissue is too small to be removed from the bone and cut to align the fibers to the direction of pull for biaxial testing.

Methods

Fitting Procedure

Model. The model of choice for the current study is a simple structural model for fibrous tissue [28] whose parameters correspond roughly to fiber stiffness, A , fiber nonlinearity, B , preferred fiber orientation direction, μ , and fiber spread, κ (note that these parameters are representative of the fiber architecture but do not necessarily correlate with a specific fiber measurement, and, particularly in complex, multicomponent tissues, the meaning of the parameters cannot be related directly to the tissue architecture or properties of constituents). The fiber population is taken to be distributed according to a bidirectional von Mises distribution (1), and the tension in a fiber is taken to be exponential in its Green strain (2). The model equations are as follows:

$$f(\theta; \kappa, \mu) = \frac{1}{\pi I_0(\kappa)} \exp(\kappa \cos[2(\theta - \mu)]), \quad \theta \in [0, \pi] \quad (1)$$

$$S_f = A(\exp[B(\lambda_f^2 - 1)] - 1) \quad (2)$$

$$S_{ij} = \int_0^\pi S_f(\theta) n_i(\theta) n_j(\theta) f(\theta) d\theta \quad (3)$$

In Eqs. (1)–(3), $f(\theta)$ is the probability of a fiber oriented in direction θ , I_0 is the modified Bessel function of the first kind and order 0, and λ_f^2 is the squared fiber stretch given by $C_{ij} n_i(\theta) n_j(\theta)$, where C_{ij} is the right Cauchy–Green tensor and $n_i(\theta)$ is the unit vector in direction θ . S_{ij} is the tension in the tissue. This model was chosen for convenience and familiarity, and it was treated as a representative anisotropic model.

Algorithm. The four parameters in the model (A , B , μ , κ) were fit to experimental results by minimizing the error in the two-term objective function, φ

$$\varphi = \text{SSE}_{\text{Grip Forces}}(A, B, \mu, \kappa) + w * \text{SSE}_{\text{Nodal Displacements}}(A, B, \mu, \kappa) \quad (4)$$

The first term in the objective function, $\text{SSE}_{\text{Grip Forces}}$, represents the sum of squared errors between the model and experimental grip force-stretch data at incremental extensions. A FE simulation (written in C++) was used to solve for the stress and displacement fields over the planar, two-dimensional domain of the sample using Eqs. (1)–(3) as previously described in Ref. [28], at incremental grip extensions. For the FE model, the domain of each experimental sample was taken from a video frame and meshed using ABAQUS (Simulia, Providence, RI) with an average of 306 linear quadrilateral elements per sample. The second term

in the function, $w * \text{SSE}_{\text{Nodal Displacements}}$, represents the sum of squared errors between the FE model and experimental nodal displacement fields for each sample mesh, scaled by weighting factor, w . The weighting factor was calculated based on the errors from the initial guess to balance the grip force-stretch and nodal displacement field errors. Model displacement fields were taken from the FE nodes, excluding grip nodes. Experimental displacements were calculated for each sample using its FE mesh and digital image correlation (DIC) as previously described [21].

The two-term objective function (4) was minimized using Newton–Raphson iteration (with trust region control) using an analytical Jacobian [29,30]. Analytical expressions were provided for the Jacobian and numerical approximations for the Hessian. Further explanation of the parameter determination scheme is given in the flow chart provided in the supplemental material [31]. Confidence regions for each parameter were calculated using a linearized form of the model with the simplification that the other three parameters were assumed known, shown in Eqs. (5) and (6) [32].

$$(\psi_i - \hat{\psi}_i)^T \hat{Z}_{ij}^T \hat{Z}_{ij} (\psi_i - \hat{\psi}_i) \leq 4(\varphi/[n-4])F(4, n-4, 1-\alpha) \quad (5)$$

$$\text{where } \hat{Z}_{ij} = \frac{\partial \varepsilon_j}{\partial \psi_i} \quad (6)$$

ψ_i is the set of parameters, where all terms in the expression with a hat represent those related to the fitted parameter set. Z_{ij} represents the linear approximation of the error of the predictor variables (grip force-stretch data, nodal displacement field data) while incrementing one parameter and holding the remaining three parameters constant for each grip stretch increment, j . n represent the total number of observations from the predictor variables at each grip stretch increment. F represents the F -statistic, where α is the 0.05 confidence level. All computations were carried out on a single core at the University of Minnesota Supercomputing Institute.

Simulated Experiments. Simulated data sets were created using a forward simulation with previously fitted parameters [33]² to assess the performance of the method.

- Parameter values were varied $\pm 15\%$ to assess sensitivity of φ to the different parameters. Parameter sensitivity was defined as

$$\text{Parameter Sensitivity} = \text{Param} \cdot (d\varphi/d\text{Param}) \quad (7)$$

- The initial guess of each parameter was varied $\pm 15\%$ to assess the robustness of the fitted parameters.
- The effect of noise was assessed by fitting parameters to data perturbed with white Gaussian noise, varying signal-to-noise ratio between 2.5 and 100.

Representative Experiments. Three representative tissue types were tested. SPM represents highly aligned tissue with fibers oriented with the test axes. FCL represents highly aligned tissue with fibers oriented with the test axes and a small sample with rigid boundaries. A lamella from the AF represents highly aligned tissue with fibers oriented off-axis and a small sample with rigid boundaries. All samples were obtained from the University of Minnesota Anatomy Bequest Program and approved by institutional review. The tissues used and testing protocol are detailed in Table 1. The SPM was dissected to a cruciform shape. The FCL was dissected from a right L3–L4 motion segment. The thin membrane covering the FCL was removed, and the facet capsule was isolated from the motion segments with ligament attachments to the superior and inferior articular facets intact. The ligamentum flavum and trabecular bone within the facet joints were removed

²This parameter set is from a previous test of a different annulus fibrosus lamella sample, fitted prior to improving the technique as shown in the current study.

Table 1 Tissue characteristics and testing protocols

Alignment	SPM On axis	FCL On axis	AF Lamella Off axis
Donor	76F	60M	65M
Preload (N)	0.2	1	2
Preconditioning (grip stretch)	1.50	1.14	1.15
Grip strain rate (%/s)	1	1	1
Maximum grip stretch	1.50	1.14	1.15

Fig. 2 Sensitivity of force (F) and displacement (U) error to model parameters. The parameter κ has been multiplied by 10 for visual clarity. Sensitivities of nodal measurements in the one and two directions are different due to the anisotropy of the model. A and B affect grip force but have less influence on the displacements, whereas displacements are more sensitive to μ and κ .

to create a planar bone–ligament–bone configuration. AF, from the anterior region of a L3–L4 intervertebral disk, was dissected to a single lamella with axial vertebral attachments, using a technique similar to that of Bass et al. [14]. Sandpaper was attached to sample arms with cyanoacrylate glue.

To obtain the displacement field, each sample was speckled with Verhoeff's stain (Sigma-Aldrich, St. Louis, MO) to provide image texture over the sample domain while it was filmed (Canon Rebel T2i, Melville, NY) during the experiment. Still images taken from the filmed experiments were used to calculate the displacement field of the sample (using a custom DIC code³) at incremental extensions. Further detail to obtain the displacement field by this method has been described in literature (i.e., Ref. [34]).

Each sample was attached to an Instron–Sacks biaxial tester, and a preload was applied. Samples were preconditioned for eight cycles and tested in equibiaxial extension, a common loading configuration. Images and grip force–stretch data were acquired for each test, and the grip force–stretch data were zeroed with respect to the preload. Levels of preload and maximum grip stretch were chosen to stretch each tissue outside of its toe region and thus produce a significant nonlinear response. The structural model was used to fit parameters to each data set, and parameter CRs were calculated as described in Algorithm section.

Results

Simulated Experiments. Sensitivities of the error in simulated force–stretch data and displacement field to the parameter values are shown in Fig. 2. Arm force–stretch data (F_1 , F_2) and nodal displacement field data (U_1 , U_2) were taken from a single node, showing how their values changed by adjusting the parameter

³This code is available for licensing at http://license.umn.edu/technologies/20130022_robust-image-correlation-based-strain-calculator-for-tissue-systems. There is no charge for an academic license.

Fig. 3 Parameter error using simulated data perturbed with White Gaussian noise. Relative error for the fitted parameter is shown for each parameter: κ ($\bullet$), μ ($\square$), A ($\blacktriangle$), and B (∇). Parameters, κ , μ , and B are plotted on the left axis and parameter A is plotted on the right. Parameters κ , μ , and B are largely unchanged by the noise, and the relative error for A was less than 5% for signal to noise ratios greater than ten.

Fig. 4 Representative experiment grip force data fitted with the simple structural model based on grip force and nodal displacement data. A single arm along or near the fiber axis is shown for each tissue type: SPM ($\diamond$), FCL ($\square$), and AF lamella ($\triangle$). Data at low Green strains are inset to better visualize the FCL and AF lamella data. Each data set is fitted well considering all four grip forces as well as nodal displacements are fitted simultaneously.

values. The subscript refers to the direction of the measurement. The arm force–stretch data are more sensitive to the parameters μ , A , and B than to κ . Horizontal nodal displacement field data (U_1) were more sensitive to the parameter κ , and vertical nodal displacement field data (U_2) were more sensitive to the parameters κ and μ , where greater sensitivity to the parameters leads to more accurate parameter estimation.

Parameters κ , μ , and B were largely unaffected by noise, and the error in A was less than 5% for signal to noise ratios greater than ten (Fig. 3). For all initial guesses studied, the fitting based on both grip force–stretch data and nodal displacement field data converged to the correct result (within 8%).

Representative Experiments. Parameters were fit to experimental data using three approaches: grip force–stretch data alone (FORCE), nodal displacement field data alone (DISP), and grip force–stretch and nodal displacement field data simultaneously (BOTH). Figure 4 shows grip force–stretch data on the arm along or closest in alignment to the fiber axis of each tissue type, for the experiments (points) and the BOTH fit (lines). The four-parameter model fits the grip force–stretch data well. Parameter values for the BOTH approach of each tissue are shown in Table 2. The FCL and AF lamella parameters denote strong alignment and a highly nonlinear response.

Figure 5 shows the ratio of the parameter magnitude to width of its 95% CR for each fitting approach. A high ratio indicates a precise estimate, and a low ratio indicates a high degree of uncertainty. The FCL and AF lamella data show three of the four

Table 2 Representative experiment parameter values

Tissue type	Parameters			
	κ	μ (deg)	A (N/mm)	B
SPM	0.135	124.812	4.683	1.616
FCL	2.777	0.144	0.225	9.751
AF lamella	4.000	22.500	0.144	9.750

Fig. 5 Representative experiment parameter estimates with each fitting approach based on the ratio of parameter value to the width of the CR. The fitting approaches shown refer to the data used to inform the model: grip forces and nodal displacements (BOTH), grip forces alone (FORCE), and nodal displacements alone (DISP). The tissue data fitted are (a) SPM, (b) FCL, and (c) AF lamella. The horizontal line represents the location where the fitted parameter value is equivalent to the width of the CR. Fitted parameters above the line are a more precise estimate than those below. The BOTH approach consistently fits three of the four parameters more precisely than the other approaches in the FCL and the AF lamella. The BOTH approach has a ratio larger than one for each parameter in each tissue type as opposed to the FORCE and DISP approaches.

parameters are described more precisely using the BOTH approach rather than FORCE or DISP. For all parameters, the ratio is consistently greater than one using the BOTH approach, whereas the FORCE and DISP approaches were less precise. In all tissues evaluated, DISP described κ , μ , and B poorly.

Discussion

Simulated and experimental data were used to evaluate a regression scheme using grip force-stretch and nodal displacement fields, which allowed robust fitting with low sensitivity to noise and initial guess and tight CRs. This combined approach produced unique parameter sets that accurately described the tissue tested. In general, the BOTH approach generated tighter CRs than the magnitude of the parameter, whereas the FORCE and DISP approaches often produced broad CRs, making the parameter less precisely determined in each tissue type and parameter. Although one direction of the nodal displacement data (U_2) is more sensitive to κ , it was not sensitive enough to present in the CRs from the tissues tested.

The fitted representative experimental data were consistent with previous studies. Moduli and stiffness were calculated from the fitted data for direct comparison to published data. While the lack of published data on biaxial passive mechanics of SPM prevents direct comparison, passive inflation testing has been performed on rat diaphragm, which is also a large, wide-span skeletal muscle [35]. Moduli along the fiber direction in the SPM of the current study (0.77 MPa) were comparable to those in Ref. [35] (0.92 MPa). Few tensile tests have been performed on isolated FCLs, but one comparable study [20] of preconditioning in uniaxial tension measured the elastic modulus along the fibers (1.33 ± 0.49 MPa). In the current study, the average tangent elastic modulus along the fibers (calculating the stiffness from both arms along the fibers) is within range (1.08 MPa). Additionally, some tensile tests of single AF lamella have been performed in biaxial tension. One such study [14] reports an axial tangent modulus (16 MPa) about twice that calculated in the current study (9.88 MPa).

In the current study, we used our previous structural model for convenience and familiarity, but any anisotropic model would be suitable to apply this method. For example, Flynn and Rubin have proposed a three-parameter strain energy density model based on total dilatation, an orthotropic invariant, and a measure of the elastic distortional deformation [36]. Sun and Sacks have published a planar soft-tissue seven parameter generalized Fung-elastic constitutive model with two restrictions for numerical stability [37]. Other constitutive equations have been published with application to tissue from, e.g., anterior cruciate ligament [38], abdominal aortic aneurysm tissue [39], and healthy renal artery [40].

This evaluation has some limitations. The approach described herein assumes the tissue properties are homogeneous. We have shown previously [41] that inhomogeneous tissue properties can be determined through inverse modeling through partitioning. Performance of the current model could be modified following [41] to account for tissue property inhomogeneity, but that might require significantly more complex experiments. It is possible that in the simulated experiments the range of initial guesses and noise used were not large enough to evaluate the capabilities of the method. Also, fitted representative experimental data were obtained from a single experiment of each tissue type. The robustness of fitting would be evaluated further by fitting repeated experiments of a tissue or experiments using different loading configurations.

For an isotropic material, grip force-stretch data could be sufficient to specify the model, and the techniques described herein would be unnecessary. For more complex geometries and orientations, however, more information is needed to define the model parameters robustly. This comes at a moderate computational cost to generate the Jacobian and Hessian. The combination of force-stretch and displacement field data for an equibiaxial test is a good option, especially when fiber direction is not known or controlled.

Supplemental Material

The fitting code and instructions for use are available in the supplemental material, available under the “[Supplemental Data](#)” tab on the ASME Digital Collection. The code is specific to our model [28] but could be adapted to other models as needed.

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