Equivalence of time and aperture domain additive noise in ultrasound coherence

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Ultrasonic echoes backscattered from diffuse media, recorded by an array transducer and appropriately focused, demonstrate coherence predicted by the van Cittert–Zernike theorem. Additive noise signals from off-axis scattering, reverberation, phase aberration, and electronic (thermal) noise can all superimpose incoherent or partially coherent signals onto the recorded echoes, altering the measured coherence. An expression is derived to describe the effect of uncorrelated random channel noise in terms of the noise-to-signal ratio. Equivalent descriptions are made in the aperture dimension to describe uncorrelated magnitude and phase apodizations of the array. Binary apodization is specifically described as an example of magnitude apodization and adjustments are presented to minimize the artifacts caused by finite signal length. The effects of additive noise are explored in short-lag spatial coherence imaging, an image formation technique that integrates the calculated coherence curve of acquired signals up to a small fraction of the array length for each lateral and axial location. A derivation of the expected contrast as a function of noise-to-signal ratio is provided and validation is performed in simulation.

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I. INTRODUCTION

Ultrasound signals received on an array are typically focused and summed coherently in the process of forming a B-mode image, collapsing the channel data into a single signal. Mallart and Fink¹ and Liu and Waag² described the van Cittert–Zernike (VCZ) theorem for ultrasound, predicting a spatial coherence between two array elements for a signal backscattered from randomly positioned, sub-wavelength targets. Rather than removing the aperture domain information, several groups have shown that it is possible to augment the B-mode image using the coherence data.^{3,4}

Short-lag spatial coherence (SLSC) imaging is a technique to create an image solely from the measured coherence, but the results show sensitivity to effects such as channel noise and phase aberration in ways not seen in conventional imaging.⁵ Such effects have been previously exploited to evaluate and improve B-mode image quality. Mallart and Fink described the use of a “coherence factor” to evaluate aberration correction schemes, showing a loss of coherence corresponding to the severity of phase corruption,⁶ while Liu and Waag proposed a “wavefront similarity factor” for their studies.⁷ Lacefield and Waag observed a “spike” in their coherence measurements indicating low signal-to-noise ratio and concluded that with proper normalization, measured coherence could be a useful property of pulse echo signals despite violations of some assumptions made in deriving the VCZ theorem.⁸ Fink *et al.* took advantage of the loss in coherence due to random aberration to smooth speckle texture in B-mode images, similar to spatial compounding.⁹

Pinton *et al.* described three sources of partially correlated clutter that contribute to reduced coherence: multiple reflections returning to the transducer, multiple reflections lengthening the transmit pulse, and phase/amplitude aberration.¹⁰ It has been previously estimated in the blood flow literature that the *in vivo* contribution of these stationary echoes is on the order of 40 dB above blood echo level¹¹ and is often significant with respect to or even exceeds the background tissue level.¹²

We present further study of these effects by describing them as additive noise in the recorded channel data. Section II presents the effect of uncorrelated time domain noise and compares the expected coherence with random magnitude and phase variation. Expressions are developed for effective noise-to-signal ratios in each case, showing their equivalence in the resulting coherence despite differences in signal properties. Section III demonstrates the application of the additive noise expressions to SLSC imaging to describe the previously observed image quality improvement in the presence of channel noise. Section IV discusses some of the implications of the derived noise characteristics for ultrasound imaging.

II. ADDITIVE NOISE

A. Random channel noise

For the purposes of this paper, all signals are transmitted into diffuse media made of random, sub-wavelength scatterers and the backscattered signals are received on an array and properly delayed to align signals from the transmit focal depth. The VCZ theorem states that the mutual coherence of these signals is proportional to the Fourier transform of the transmit intensity field, which is equivalent to the autocorrelation of the transmit aperture function $U_T[x]$ where $\star$ represents cross-correlation normalized by signal length.

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Goodman presents a more complete discussion of the derivation of the theorem and the required assumptions,¹³

$$\rho[n] = (U_T[x] \star U_T[x])[n]. \quad (1)$$

The VCZ theorem predicts that the spatial covariance function $\rho[n]$ for an array of length N with unity amplitude is proportional to the triangular function $\wedge[n/N]$.¹ Assuming white additive channel noise that is also uncorrelated with the zero-mean signal, this expression has been modified to account for signal power P_S with additive noise power P_N on each channel,¹⁴

$$\rho[n] = \begin{cases} 1, & n = 0 \\ \frac{1 - \frac{n}{N}}{1 + \frac{P_N}{P_S}}, & 1 \leq n < N. \end{cases} \quad (2)$$

As noise power increases, the coherence for $n > 0$ remains linear but with lower amplitude. By adjusting the normalization, Eq. (2) can be written as the sum of the triangular function and a delta function scaled by the noise-to-signal power ratio, for $0 \leq n < N$,

$$\rho[n] \propto \wedge[n/N] + \left(\frac{P_N}{P_S}\right) \delta[n]. \quad (3)$$

Practically, the autocorrelation of the noise across the channel dimension will not produce a true delta function due to finite signal length, adding an inherent source of variance to the coherence curve.

B. Random amplitude apodization

We describe an aperture apodization comprised of a constant term A and an uncorrelated noise term $w[x]$ drawn from a distribution with zero mean and σ^2 variance,

$$U_T[x] = A + w[x]. \quad (4)$$

Substituting Eq. (4) into Eq. (1) and expanding the correlation, the expression can be simplified to the triangular function, a delta function, and a residual noise term $\zeta[n]$. $\mathbf{1}$ is the vector $[1, 1, 1, \dots, 1]$ of the same length as $w[x]$,

$$\rho[n] \propto \wedge[n/N] + \left(\frac{\sigma^2}{A^2}\right) \delta[n] + \zeta[n], \quad (5)$$

$$\zeta[n] = \frac{1}{A} ((\mathbf{1} \star w[x])[n] + (w[x] \star \mathbf{1})[n]). \quad (6)$$

This decomposition of the correlation is shown in Fig. 1 for a sample random apodization. This closely resembles the additive channel noise equation (3) with a scaling term on the delta dependent on both amplitude variance and constant offset, predicting an effective noise-to-signal ratio,

$$\frac{P_N}{P_S} = \frac{\sigma^2}{A^2}. \quad (7)$$

There is also a random component added to the result scaled by A^{-1} , meaning the variance of the coherence curve will increase with the square root of the noise power. This noise level is invariant with the scale of the aperture function, which can be verified by multiplying Eq. (4) by an arbitrary constant.

It is important to note that independent realizations of temporal noise on each channel can actually be viewed as a time-dependent random magnitude apodization and analyzed using the above framework. A random magnitude apodization can also be observed in practice from any variation across transmit elements or related circuitry due to manufacturing or component tolerances.

C. Random binary amplitude apodization

While the theory of the previous section is applicable to any random distribution with zero mean, selecting a binary distribution makes it possible to reduce the number of transmit elements, or transmit events in a synthetic aperture case, without sacrificing beamforming quality.

The element amplitudes can be chosen from a Bernoulli distribution with a probability of taking value 1 given by p . Using the known variance $p(1 - p)$ and mean p , Eq. (7) can be rewritten for this distribution,

$$\frac{P_N}{P_S} = \frac{1 - p}{p}. \quad (8)$$

As expected, the effective noise-to-signal ratio is increased as fewer transmit elements are used. It should also be noted that when Eq. (8) is combined with Eq. (2), the parameter of the Bernoulli distribution p is also the limit of $\rho[n]$ as n approaches 0 (the y-intercept value of the triangular function without the delta function).

D. Random phase apodization

It is also possible to view transmit phase aberration as an apodization, providing a complex aperture description with unity magnitude. Assuming a normal distribution of phase shifts (in units of time) across the array with variance σ^2 , the coherence curve is scaled by a function of $R_{TT}[n]$, the phase autocorrelation,¹⁵

$$\rho[n] = \wedge[n/N] e^{-(2\pi f)^2 (R_{TT}[0] - R_{TT}[n])}. \quad (9)$$

Assuming the phases are uncorrelated, this expression can be reduced to a form similar to Eq. (3) by substituting for the autocorrelation terms,

$$\rho[n] \propto \wedge[n/N] + (e^{(2\pi f)^2 \sigma^2} - 1) \delta[n], \quad (10)$$

$$\frac{P_N}{P_S} = e^{(2\pi f)^2 \sigma^2} - 1. \quad (11)$$

The effective noise-to-signal ratio is a function of the phase variance and transmit pulse frequency, predicting increased sensitivity to phase variations at higher frequencies.

Figure 2 shows the aperture and time domain descriptions for each additive noise condition and the resulting

FIG. 1. Individual terms in the cross-correlation of a random amplitude aperture modeled as signal and noise and the resulting sum. Notice the delta function present in the noise term that adds to the lag-zero position of the sum. The rescaling of the total curve would normalize the delta function value to 1 and reduce the amplitude of the higher-lag values.

coherence curves. All cases use parameters selected to produce an effective noise-to-signal ratio of 0.3, resulting in the roughly the same linear function offset by a delta function despite the different conditions used to create each.

E. Maximum length sequences

Each of the described apodizations produces a coherence curve that shows deviations from the ideal linear function of Eq. (2). In order to produce a smoother curve, it is possible to optimize the applied apodization. One potential solution is to use a “maximum length sequence,” a pseudo-random binary sequence with circular autocorrelation that well approximates a train of delta functions.¹⁶ The sequence can be produced using feedback from a shift register and contains all binary sequences of the length of the register except for the zero sequence. The length of the sequence is fixed at multiples of $2^n - 1$, with delta functions in the autocorrelation appearing at every $2^n - 1$ lags. This approach also fixes the number of ones and zeros in the sequence and therefore fixes the effective noise-to-signal ratio at around 0.5. A modification of this technique or other optimization techniques would be required to achieve other noise-to-signal ratios.

One choice, given an array with 2^n elements, is to use the longest possible sequence and to pad the result with either a 1 or 0. The autocorrelation in this case will be dependent on the particular shift of the sequence and will not show the smallest possible variance because the assumption of periodicity is violated. It is also possible to create a shorter sequence and repeat the pattern as necessary, although padding with a partial sequence will still be necessary. This case better approximates a periodic sequence and in cases such as SLSC imaging, delta functions present in the correlation curve outside the region of interest do not affect the result. The effect of truncating or otherwise altering a maximum length sequence has been previously studied,¹⁷ but in practice the deviation from the delta function is

still less than is expected from a random binary sequence. The results for both choices of maximum length sequence are shown in Fig. 3 and compared to a random binary sequence, all showing an effective noise-to-signal ratio of 1. The root-mean-square error of this random binary sequence compared to the linear function of Eq. (2) over the first 20 lags is 0.058, while the optimized maximum length sequences show errors of 0.014 and 0.012 for the longer and shorter sequences, respectively.

III. SHORT-LAG SPATIAL COHERENCE IMAGING

A. Imaging technique

Short-lag spatial coherence imaging uses an alternative method of image formation from conventional B-mode imaging in an effort to suppress clutter observed *in vivo*. Both techniques begin by delaying the echoes received on each array channel to focus the radio frequency (RF) signals. Each line of a B-mode image is then produced by summing the RF data and performing envelope detection, followed by log compression. Instead, SLSC imaging uses a measurement of $\rho[n]$ by the normalized cross-correlation of axial kernels between each pair of delayed echo signals,

$$\rho[n] = \frac{1}{N-n} \sum_{i=1}^{N-n} \frac{\sum_{k=k_1}^{k_2} s_i[k] s_{i+n}[k]}{\sqrt{\sum_{k=k_1}^{k_2} s_i^2[k] \sum_{k=k_1}^{k_2} s_{i+n}^2[k]}}. \quad (12)$$

Each pixel R_{sl} in the SLSC image is created by summing the corresponding coherence curve up to M lags and normalizing by the maximum pixel brightness in the field of view,⁵

$$R_{sl} \propto \sum_{n=1}^M \rho[n]. \quad (13)$$

FIG. 2. Comparison of additive noise models calibrated to the same effective noise-to-signal ratio of 0.30. Aperture domain magnitude and phase, time domain sample signals, and calculated coherence curves are shown for each case. For temporal noise, one time sample is shown in the aperture domain and appears as a random magnitude apodization. For the aperture domain noise sources, three sample channels are shown in the time domain. Temporal and random phase noise are normally distributed. Random magnitude is drawn from a standard uniform distribution.

B. Effects of noise on contrast

Contrast is calculated as the difference between the SLSC value inside and outside a lesion, normalized by the value outside the lesion,

$$\text{contrast} = \frac{R_{\text{out}} - R_{\text{in}}}{R_{\text{out}}}. \quad (14)$$

Previous work described one source of contrast in SLSC imaging as the non-uniform scattering function present at boundaries.^{5,18} The difference in scattering amplitude across a boundary provides decorrelation that enhances contrast, even though normalized cross correlation would otherwise remove the dependence on signal amplitude. However, contrast can also be observed away from a boundary where a uniform scattering function is expected, such as at the center of a large hypoechoic lesion. This work proposes additive

noise in both the time and aperture domains as the source of this contrast, present throughout the entire SLSC image.

The contrast due to additive noise can be expressed as a ratio of signal powers and a signal-to-noise ratio by substituting Eqs. (13) and (2) into Eq. (14) for both the signal inside and outside the lesion, P_{Si} and P_{So} , respectively. Noise power P_N is assumed to be constant across both regions,

$$R_{\text{out}} = \sum_{n=1}^M \left(\frac{1 - \frac{n}{N}}{1 + \frac{P_N}{P_{So}}} \right), \quad (15)$$

$$R_{\text{in}} = \sum_{n=1}^M \left(\frac{1 - \frac{n}{N}}{1 + \frac{P_N}{P_{Si}}} \right), \quad (16)$$

FIG. 3. Comparison of coherence curves produced by binary random apodizations. The solid curve shows one realization of a binary random sequence and the large resulting variance from a linear function. Both the maximum length sequences, from seven shift registers (MLS-7) and from five (MLS-5), show a more linear coherence curve (dashed lines) compared to the random sequence. The sequence produced by five shift registers shows periodicity, producing delta functions in the autocorrelation every $2^5 - 1 = 31$ lags in exchange for smaller variations from the linear function.

$$\text{contrast} = \frac{1 - \frac{P_{Si}}{P_{So}}}{1 + \frac{P_{Si}}{P_N}}. \quad (17)$$

The contrast predicted by Eq. (17) is particularly interesting in the limits of the noise power. As noise power

approaches zero, the contrast also goes to zero, making the hypoechoic region indistinguishable from the background. As noise power approaches infinity, the image reaches the intrinsic contrast of the region as would be seen in a B-mode image. Although the delta function dominates both coherence curves in this case, the SLSC integrals begin at the first lag and allow for comparison of only the linear regions such that high contrast is observed. Intermediate noise powers produce a monotonically increasing curve for increasing channel noise up to the maximum contrast value,

$$\lim_{P_N \rightarrow 0} \frac{1 - \frac{P_{Si}}{P_{So}}}{1 + \frac{P_{Si}}{P_N}} = 0, \quad (18)$$

$$\lim_{P_N \rightarrow \infty} \frac{1 - \frac{P_{Si}}{P_{So}}}{1 + \frac{P_{Si}}{P_N}} = 1 - \frac{P_{Si}}{P_{So}}. \quad (19)$$

This result suggests that given data with a high signal-to-noise ratio, a large amount of channel noise should be added to SLSC data in either the time or aperture domain to produce the best contrast possible. Field II (Refs. 19 and 20) was used to simulate layers with -3 , -6 , -12 , and -20 dB scatterer amplitude compared to the background layer. A simulated 5 MHz transducer with 128 elements was used to perform a linear scan with a focal point at 4 cm. Figure 4(a) shows the results for 100 realizations of additive temporal noise at a range of noise powers compared to the model given in Eq. (17), showing good agreement over the full range of noise powers.

FIG. 4. Field II simulated image metrics for SLSC images of layers of varying intrinsic contrast (-20 , -12 , -6 , -3 dB relative to the background) using 100 realizations of random additive channel noise. (a) Contrast from simulation and the model given in Eq. (17), showing good agreement. Contrast increases from zero to a maximum as noise power increases. (b) Speckle SNR decreases as noise power increases, as previously reported experimentally. (c) Contrast-to-noise ratio shows a peak, representing an optimum noise power for imaging performance. The peak shifts with intrinsic layer contrast, determined primarily by the SLSC contrast.

FIG. 5. Sample Field II simulated SLSC images for a single speckle and noise realization over a range of intrinsic contrasts (-20 , -12 , -6 , -3 dB relative to the background) and noise powers (-40 to 20 dB relative to the signal). Boxes indicate regions away from the boundary with a uniform scattering function where contrast was measured. All images are normalized by the same value and the dynamic range displayed is from 0 to 1. Axes are proportional and the displayed axial range is 10 mm.

Sample SLSC images are shown for a range of intrinsic contrast and noise power values in Fig. 5. The images demonstrate contrast due to both the non-uniform scattering function, visible only around the boundary, and due to the additive noise, visible throughout the image but especially away from the boundary where the scattering function is uniform. While it is difficult to visually appreciate the improved contrast at high noise powers, the contrast is driven by the relative decorrelation rate between the two layers, which is noticeably faster for the layers with high intrinsic contrast.

Despite the increase in contrast, it has been shown experimentally that speckle signal-to-noise ratio falls as channel noise is added, resulting in some optimum noise power to maximize the contrast-to-noise ratio.¹⁸ Figures 4(b) and 4(c) show the simulated speckle SNR and CNR for the same layer simulation, showing the role that contrast plays in shifting the peak of the CNR curve as a function of noise power. Increasing noise also affects the observed image texture. In experimental environments with low channel noise, the parameters described in the previous sections can be tuned to reach this peak and optimize image quality.

Increasing noise power also affects the texture observed in the SLSC image. A given noise power preferentially degrades coherence in low-amplitude sections of the echo signals that would form dark speckle regions in B-mode imaging. High and low coherence regions in the presence of moderate noise therefore somewhat resemble the corresponding B-mode speckle texture. Although the pattern is stationary as a function of noise, the difference between localized light and dark regions vary with noise power in the same way as bulk regions of varying scattering amplitudes.

IV. CONCLUSION

While additive noise is conventionally a time domain signal, we have presented an aperture dimension equivalent and described the effect on the normalized coherence curve. Random sequences applied as magnitude or phase on transmit produce a delta function at the lag-zero position of the coherence curve plus some residual noise at higher lag values. When normalized, this additive term affects the shape of the entire coherence curve and suppresses coherence from the signal of interest. Modifications to either the aperture magnitude or phase will produce changes in the coherence curve and can be used to intentionally reshape the curve as in the case of maximum length binary sequences. Coherence imaging is particularly sensitive to noise conditions, which inherently makes them sensitive to the choice of aperture as well. In the case of SLSC imaging, contrast can be modified using the aperture to optimize image quality. This can be performed in transmit for conventional imaging or in post-processing for synthetic aperture imaging.

The presented aperture domain framework also holds true for clutter that appears in ultrasound imaging as partially correlated and uncorrelated signals from off-axis scattering, phase aberration, and reverberation. The use of coherence in weighting or producing images allows for reduction of the artifacts associated with such phenomena because the original coherence signal is still present. If an appropriate model of the coherence of the artifact is used, the coherence of the original signal may be recovered to improve image quality. SLSC imaging already takes advantage of this to some degree by integrating the coherence curve beginning at the lag-1 position, avoiding the delta function if it is present. In the case of partially correlated noise, such as slow-varying

aberration, the delta function will be blurred and extend into the short-lag region, so further consideration is necessary to completely remove the artifact such as the use of a “middle-lag” region. Assuming knowledge of the cause of the artifact, because the coherence is the autocorrelation of a single aperture function it would also be possible to pose phase or magnitude corruption as a deconvolution problem.

It is important to note that the equivalence established here can only be applied in the context of spatial coherence. A similar analysis of aperture noise could be performed for conventional B-mode imaging and speckle correlation for the purpose of spatial compounding. Following the derivation illustrated in Fig. 3 of Wagner *et al.*,²¹ the autocorrelations of the transmit aperture and the receive aperture each produce the triangular function plus delta function of Eq. (3). The speckle correlation curve is given by the square of the convolution of these two functions, reducing the magnitude of the delta function relative to the other components. The resulting speckle correlation is robust to large amounts of aperture domain noise, suggesting that spatial compounding would be largely unaffected.

Further understanding of the sources of clutter *in vivo* is essential to improving the quality of clinical results. Simulation and experimental work performed in the absence of noise and clutter often presents an unrealistic picture of the image quality, overlooking the difficulties faced in many patients. Improvements in suppressing or leveraging clutter to produce clearer images should be pursued to increase the success rate of ultrasound in challenging imaging environments.

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