Published in final edited form as:

Magn Reson Med. 2015 July ; 74(1): 81–92. doi:10.1002/mrm.25374.

Dixon-Type and Subtraction-Type Contrast-Enhanced Magnetic Resonance Angiography: A Theoretical and Experimental Comparison of SNR and CNR

Eric G. Stinson, Joshua D. Trzasko, Paul T. Weavers, and Stephen J. Riederer*

Department of Radiology, Mayo Clinic, MR Research Laboratory, Rochester, Minnesota, USA

Abstract

Purpose—The purpose of this work is to compare the behavior of the signal-to-noise ratio (SNR) and contrast-to-noise ratio (CNR) in contrast-enhanced MR angiography with background suppression performed by either a Dixon-type or subtraction-type method.

Theory and Methods—Theoretical expressions for the SNR and CNR for both background suppression techniques were derived. The theoretical Dixon:subtraction SNR and CNR ratios were compared to empirical ratios measured from phantom and in vivo studies for Dixon techniques utilizing one, two, and three echoes. Specifically, the SNR and CNR ratios were compared as the concentration of contrast material in the blood changed.

Results—Empirical measurements of the SNR and CNR ratios compared favorably with the ratios predicted by theory. As the contrast concentration was reduced, the SNR advantage of the Dixon techniques increased asymptotically. In the ideal case, the SNR improvement over subtraction contrast-enhanced MR angiography was at least twofold for one- and two-echo Dixon techniques and at least a factor of $\sqrt{6}$ for the three-echo Dixon technique.

Conclusion: Expressions showing a contrast concentration-dependent SNR and CNR improvement of at least a factor of two when Dixon-type contrast-enhanced MR angiography is used in place of subtraction-type contrast-enhanced MR angiography were derived and validated with phantom and in vivo experiments.

Keywords

Dixon; fat–water; contrast-enhanced MR angiography; signal-to-noise ratio; contrast-to-noise ratio

INTRODUCTION

Contrast-enhanced magnetic resonance angiography (CE-MRA) typically utilizes a gadolinium-based contrast agent to increase the luminal signal in T_1 -weighted spoiled gradient-recalled echo (SPGRE) images (1). Depending on the concentration, the contrast agent reduces the T_1 of blood from about 2000 ms to between 50 and 300 ms at 3T (2,3). Lipids (fats) have a T_1 of about 250–400 ms at 3T (4), and thus have signal intensity similar

*Correspondence to: Stephen J. Riederer, Ph.D.; 200 First Street SW, Rochester, MN 55905. Riederer@mayo.edu.

to that of contrast-enhanced blood in T_1 -weighted images. As a result, the fat signal can obscure the vascular signal in a contrast-enhanced angiogram.

The luminal signal in CE-MRA is commonly accentuated by suppressing nonvascular signal via subtraction of a precontrast reference image from each contrast-enhanced image (5–8). The result is a difference image in which ideally only the contrast-enhanced vascular signal is present. The subtraction method works well provided there is no motion between the precontrast reference image and the contrast-enhanced image. Otherwise, the method is prone to artifacts from the subtraction of misregistered images. The subtraction method also requires additional scan time to acquire the precontrast reference frame(s).

It is possible to avoid subtraction artifacts and suppress only the troublesome fat signal with methods such as short tau inversion recovery (9) or spectrally selective pulses (10,11). However, these methods are not typically used in CE-MRA because they extend scan time and are sensitive to B_0 inhomogeneities. Dixon-type methods (12) exploit the different resonant frequencies of fats and water to form an image containing only water signal and an image containing only fat signal from images acquired at different echo times. While the source images for the Dixon method are sensitive to B_0 inhomogeneities, B_0 effects can be modeled and removed during the reconstruction.

Recently, dual-echo Dixon-type methods have been proposed to avoid motion artifacts in dynamic contrast-enhanced studies (13,14) and CE-MRA (15,16). Dixon methods have gained interest for CE-MRA because B_0 inhomogeneities can be accounted for in the reconstruction, and scan time is minimally extended when imaging parameters are chosen carefully. In the case of CE-MRA, the water image is used for diagnosis and the fat image is typically discarded. Due to the presence of the background water signal, background suppression for Dixon CE-MRA is not as complete as in subtraction CE-MRA. However, compared to the contrast-enhanced blood signal, the background signal is relatively small due to its longer T_1 . In addition to avoiding subtraction artifacts, dual-echo Dixon CE-MRA has been reported to improve signal-to-noise ratio (SNR) and contrast-to-noise ratio (CNR) compared to subtraction CE-MRA (15–17). Provided B_0 inhomogeneities are known from, e.g., a calibration scan or virtual shimming procedure (18–20), single-echo Dixon-type methods promise the ability to retain short scan times in CE-MRA by utilizing only one echo (21).

Thus, given the considerable contemporary interest in using Dixon CE-MRA for reduced sensitivity to motion artifacts vs. conventional, time-subtracted CE-MRA, as well as the observation that Dixon CE-MRA offers an improvement in SNR and CNR, the motivation for this work is to study these SNR and CNR advantages in more detail. This is an extension of previous work (17).

The purpose of this work is to theoretically analyze and experimentally study the behavior of SNR and CNR in Dixon CE-MRA compared to that of subtraction CE-MRA. In particular, the factors affecting SNR and CNR are investigated and a general framework for calculating the expected SNR and CNR gain from utilizing the Dixon technique in place of a subtraction CE-MRA is provided and validated. To this end, a detailed theoretical analysis is presented

with methodology and experimental results for phantom and in vivo studies using one, two, and three echoes.

THEORY

The SNR of subtraction and Dixon-type CE-MRA are derived from the corresponding forward signal models. The signal model for Dixon imaging with two or more echoes can be described by a linear system of equations by assuming that the water and fat signals are complex. Single-echo Dixon imaging, however, estimates both the water and fat signals from one complex measurement. In general, the solution must then be found with the additional constraint that the water and fat signals are real valued. Because of this difference between single- and multiecho Dixon type imaging, in this work the SNR and CNR analysis will be treated separately for these two cases.

SNR in MRI is typically defined as the magnitude of the signal divided by the standard deviation of the noise in the reconstructed image. Consider, for example, the multicoil image $g = f + \eta$ where the received signal g and the true signal f are complex vectors and η is proper complex (zero-mean) Gaussian noise with covariance Ψ (22). The SNR of a single coil element i is given in Eq. [1].

$$\text{SNR}(g_i) = \frac{|f_i|}{\sqrt{\mathbb{E}[\eta\eta^*]_{i,i}}} = \frac{|f_i|}{\sqrt{\Psi_{i,i}}} \quad [1]$$

In this work, SNR for CE-MRA is defined as the magnitude of the signal within the enhancing vessel (f_v) divided by the standard deviation of the reconstructed noise Ψ_{recon} (Eq. [2]). CNR is similar, but the signal component is the difference between the signal within the vessel and the signal within the background tissue (f_{bkgnd}) as shown in Eq. [3].

$$\text{SNR}_{\text{CEMRA}} = \frac{|f_v|}{\sqrt{\Psi_{\text{recon}}}} \quad [2]$$

$$\text{CNR}_{\text{CEMRA}} = \frac{|f_v - f_{\text{bkgnd}}|}{\sqrt{\Psi_{\text{recon}}}} \quad [3]$$

A number of mathematical conventions are used in this work. $\overline{(\cdot)}$ refers to the complex conjugate of a vector or matrix, $(\cdot)^*$ refers to the conjugate (Hermitian) transpose, and $(\cdot)^T$ refers to the nonconjugate transpose of a matrix. $\mathbb{E}[\cdot]$ denotes the expected value, $\text{Tr}(\cdot)$ denotes the matrix trace, $\det(\cdot)$ denotes the matrix determinant, and $\Re\{\cdot\}$ denotes the real part of a vector or matrix. Additionally, $(\cdot)_{p,q}$ refers to the pq -th element of $(\cdot)$.

SNR and CNR for Multiecho Dixon CE-MRA

The signal model for each voxel in an n -echo, c -coil Dixon acquisition can be described in matrix form as

$$G_m = A_m X S + N_m \quad [4]$$

where the subscript “m” refers to the multiecho Dixon case, G_m is an $n \times c$ (echoes $\times$ coils) matrix whose columns represent the multiecho signal from each of the coils. A matrix of Dixon system parameters $A_m = B_m C_m$ contains terms for the phase shifts from B_0 inhomogeneities (B_m) and chemical shift (C_m)

$$B_m = \begin{bmatrix} e^{i\phi_1} & & 0 \\ & \ddots & \\ 0 & & e^{i\phi_n} \end{bmatrix} \quad [5a]$$

$$C_m = \begin{bmatrix} 1 & e^{i\theta_1} \\ \vdots & \vdots \\ 1 & e^{i\theta_n} \end{bmatrix} \quad [5b]$$

where $\phi_t = \gamma B_0(\text{TE}_t)$, $\theta_t = 2\pi f(\text{TE}_t)$, γ is the gyromagnetic ratio, f is the chemical shift in Hz, and TE is the echo time of the acquired image. $X = \begin{bmatrix} W \\ F \end{bmatrix}$ is a 2×1 column vector containing the true water and fat signals (W and F). S is a $1 \times c$ matrix of the sensitivities of the c coil elements, and N_m is an $n \times c$ matrix of (zero-mean) proper complex Gaussian noise whose rows have covariance Ψ . That is, $\mathbb{E}[N_m^T \delta_p \delta_p^T \overline{N_m}] = \Psi$, $\forall p$, where p is an echo (row) index and δ_p is the vector which selects the p th row. This definition is equivalent to the traditional noise covariance matrix for vector signals (i.e., $\mathbb{E}[\eta \eta^*] = \Psi$), but accounts for the transposition of the coils dimension. Because (i) the coils dimension is transposed and (ii) multiple echoes are included in N_m , the noise covariance of our multiecho signal model is $\Lambda = \mathbb{E}[N_m^* N_m] = n\Psi^T$.

The target quantity, X , can be estimated from G_m via ordinary least squares minimization, which corresponds to the maximum likelihood estimate (23).

$$\hat{X}_m = \arg \min_{X \in \mathbb{C}} [\text{Tr}((A_m X S - G_m) \Lambda^{-1} (A_m X S - G_m)^*)] \quad [6]$$

It is assumed that in the coils dimension the solution is weighted by Λ^{-1} to account for noise covariance between coils, while the echoes dimension is unweighted under the assumption that the noise is uncorrelated between echoes.

The solution in the echoes dimension is the left-side pseudoinverse of A_m , and reconstruction comprises of the application of the linear filter $H_L = (A_m^* A_m)^{-1} A_m^*$. Similarly, the solution in the coils dimension is the right-side noise-weighted pseudoinverse of S , defined as $H_R = \Lambda^{-1} S^* (S \Lambda^{-1} S^*)^{-1}$. Thus,

$$\hat{X}_m = H_L G_m H_R = (A_m^* A_m)^{-1} A_m^* G_m \Lambda^{-1} S^* (S \Lambda^{-1} S^*)^{-1} \quad [7]$$

Note that H_L has dimensions $2 \times n$ and H_R has dimensions $c \times 1$. A Kronecker delta function ($\delta^T = [1 \ 0]$) is used to select from $\hat{X}_m$ the water signal $\hat{W}_m$ which is relevant in CE-MRA. If the fat signal were of interest, the delta function $\delta^T = [0 \ 1]$ could be used. Thus, the maximum likelihood estimate $\hat{W}_m$ of the water signal is found from measurements G_m via Eq. [8].

$$\begin{aligned} \hat{W}_m &= \delta^T \hat{X}_m = \delta^T H_L A_m X S H_R + \delta^T H_L N_m H_R \\ &= W + \delta^T H_L N_m H_R \end{aligned} \quad [8]$$

The SNR of the water signal is defined as:

$$\text{SNR}_m(\hat{W}_m) = \frac{|W|}{\sqrt{\delta^T \mathbb{E}[H_L N_m H_R H_R^* N_m^* H_L^*] \delta}} \quad [9]$$

From the derivation shown in Appendix A and signal assumptions from Eqs. [2] and [3], the resultant SNR and CNR expressions for multiecho, multicoil Dixon CE-MRA are found.

$$\text{SNR}_m(\text{vessel}) = \frac{|f_{\text{CEbl}}| \sqrt{\det(C_m^* C_m)}}{\sqrt{n(S \Lambda^{-1} S^*)^{-1}}} \quad [10a]$$

$$\text{CNR}_m(\text{vessel:background}) = \frac{|f_{\text{CEbl}} - f_{\text{bkgrnd}}| \sqrt{\det(C_m^* C_m)}}{\sqrt{n(S \Lambda^{-1} S^*)^{-1}}} \quad [10b]$$

where $|f_{\text{CEbl}}|$ is the water signal $|W|$ from contrast-enhanced blood, n is the number of echoes, $\det(C_m^* C_m) = n^2 - \sum_{k=1}^n e^{i\theta_k} \sum_{j=1}^n e^{-i\theta_j}$, and the scalar $\sqrt{(S\Lambda^{-1}S^*)^{-1}}$ is a weighted root sum of squares aggregate.

SNR and CNR for Single-Echo Dixon CE-MRA

Mirroring Eq. [4], the signal equation for single-echo, multicoil Dixon acquisition can also be described in matrix form as

$$G_s = A_s X S + N_s \quad [11]$$

where the subscript “s” refers to the single-echo Dixon case, and G_s is an $1 \times c$ matrix containing the signal from each coil of a single-echo acquisition. The matrix of Dixon parameters is now a 1×2 matrix

$$A_s = e^{i(\phi(t) + \phi_0)} [1 \quad e^{i\theta}] \quad [12]$$

where $\phi(t) = \gamma B_0(\text{TE})$, $\theta = 2\pi f(\text{TE})$, and ϕ_0 is the shared initial phase of W and F . N_s is a $1 \times c$ matrix containing zero-mean, proper complex Gaussian noise with covariance Λ . Because X is assumed to be real valued in the single-echo case, the water and fat signals are constrained to have equal phase ϕ_0 at time $t=0$. Similar to the multiecho case, X can be estimated from G_s via least squares minimization:

$$\hat{X}_s = \arg \min_{X \in \mathbb{R}} [\Re\{\text{Tr}((A_s X S - G_s)\Lambda^{-1}(A_s X S - G_s)^*)\}] \quad [13]$$

where the restriction that X be real is part of the minimization process. The solution for the single-echo, c -coil case is derived in Appendix C and given by:

$$\hat{X}_s = \mathcal{H}\{G_s\} = [\Re\{A_s^* A_s\}]^{-1} \Re\{A_s^* G_s \Lambda^{-1} S^*\} (S \Lambda^{-1} S^*)^{-1} \quad [14]$$

which is similar to Eq. [4] in Ref. (24). As in the multiecho case, the water signal is the first component from this result, $\hat{W}_s = \delta^T \hat{X}_s = W + \delta^T \mathcal{H}\{N_s\}$. The SNR of the water image for single-echo imaging is defined as:

$$\text{SNR}_s(\hat{W}_s) = \frac{|W|}{\sqrt{\delta^T \mathbb{E}[\mathcal{H}\{N_s\} \mathcal{H}\{N_s\}^*] \delta}} \quad [15]$$

Simplification (Appendix D) and application of CE-MRA signal assumptions lead to the SNR and CNR expressions for single-echo, multicoil Dixon CE-MRA:

$$\text{SNR}_s(\text{vessel}) = \frac{|f_{\text{CEbl}}| |\sin \theta| \sqrt{2}}{\sqrt{(S\Lambda^{-1}S^*)^{-1}}} \quad [16a]$$

$$\text{CNR}_s(\text{vessel:background}) = \frac{|f_{\text{CEbl}} - f_{\text{bkgrnd}}| |\sin \theta| \sqrt{2}}{\sqrt{(S\Lambda^{-1}S^*)^{-1}}} \quad [16b]$$

SNR and CNR for Subtraction CE-MRA

The signal equation for c -coil subtraction CE-MRA is shown in Eq. [17].

$$G_{\Delta} = G_2 - G_1 = X_{\Delta}S + N_{\Delta} \quad [17]$$

where $G_1 = X_1S + N_1$ and $G_2 = X_2S + N_2$ are the precontrast and postcontrast images, respectively, G is a $1 \times c$ matrix containing the complex difference between the images G_1 and G_2 , X is a 1×1 matrix containing the true subtracted signal, and N is a $1 \times c$ matrix containing (zero-mean) proper complex noise with covariance $\Lambda = 2\Lambda$. The multiplier of 2 is a result of the subtraction operation on the noise from the individual images (with covariance Λ). The reconstruction can again be solved using least squares minimization:

$$\hat{X}_{\Delta} = \arg \min_{X_{\Delta} \in \mathbb{C}} [Tr((X_{\Delta}S - G_{\Delta})\Lambda_{\Delta}^{-1}(X_{\Delta}S - G_{\Delta})^*)] \quad [18]$$

Reconstruction comprises the application of a linear filter H to the signal G

$$\hat{X}_{\Delta} = G_{\Delta}SH_{\Delta} + N_{\Delta}H_{\Delta} = X_{\Delta} + N_{\Delta}X_{\Delta} \quad [19]$$

where $H_{\Delta} = \Lambda_{\Delta}^{-1}S^*(S\Lambda_{\Delta}^{-1}S^*)^{-1}$. The SNR of the subtraction image is defined as:

$$\text{SNR}(\hat{X}_{\Delta}) = \frac{|X_{\Delta}|}{\sqrt{E[(N_{\Delta}H_{\Delta})(N_{\Delta}H_{\Delta})^*]}} = \frac{|X_{\Delta}|}{\sqrt{H_{\Delta}^*(\Lambda_{\Delta})H_{\Delta}}} \quad [20]$$

where we have leveraged that $(N H)$ is a scalar. Application of CE-MRA signal assumptions, substitution of $\Lambda = 2\Lambda$, and simplification gives:

$$\text{SNR}_{\Delta}(\text{vessel}) = \frac{|f_{\text{CEbl}} - f_{\text{bl}}|}{\sqrt{2(S\Lambda^{-1}S^*)^{-1}}} \quad [21a]$$

The CNR equation can similarly be formed with the signal difference. Assuming there was no motion between the unenhanced and enhanced image acquisitions, the true subtracted background signal is zero, and for the subtraction case, SNR and CNR are equal.

$$\text{CNR}_{\Delta}(\text{vessel:background}) = \frac{|f_{\text{CEbl}} - f_{\text{bl}}|}{\sqrt{2(S\Lambda^{-1}S^*)^{-1}}} \quad [21b]$$

SNR and CNR Ratios

To compare subtraction and Dixon-type CE-MRA, the differences in SNR are evaluated using the ratio of $\text{SNR}_{\text{Dixon}}$ to SNR_{Δ} . The same is done for CNR. While each metric has already been derived separately, use of the ratio of SNR or CNR normalizes the comparison with respect to equal imaging parameters. This is a similar approach as taken in (25) when calculating the g -factor which is based on the SNR ratio of an undersampled image to a fully-sampled image.

For the case of multiecho Dixon vs. subtraction CE-MRA, the SNR and CNR ratios are given in Eqs. [22a] and [22b], respectively.

$$\frac{\text{SNR}_m}{\text{SNR}_{\Delta}} = \frac{|f_{\text{CEbl}}|}{|f_{\text{CEbl}} - f_{\text{bl}}|} \sqrt{\frac{2\det(C_m^* C_m)}{n}} \quad [22a]$$

$$\frac{\text{CNR}_m}{\text{CNR}_{\Delta}} = \frac{|f_{\text{CEbl}} - f_{\text{bkgrnd}}|}{|f_{\text{CEbl}} - f_{\text{bl}}|} \sqrt{\frac{2\det(C_m^* C_m)}{n}} \quad [22b]$$

where n is the number of echoes and $\det(C_m^* C_m) = n^2 - \sum_{k=1}^n e^{i\theta_k} \sum_{j=1}^n e^{-i\theta_j}$. In the case of two echoes, Eq. [22a] reduces to:

$$\frac{\text{SNR}_m}{\text{SNR}_{\Delta}} = \frac{|f_{\text{CEbl}}|}{|f_{\text{CEbl}} - f_{\text{bl}}|} \sqrt{2\sqrt{1 - \cos \Delta\theta}} \quad [23]$$

where $\Delta\theta = \theta_2 - \theta_1$, the difference between the phase angles at TE_1 and TE_2 . Additionally, if n echoes are evenly spaced around the unit circle, Eq. [22a] reduces to:

$$\frac{\text{SNR}_m}{\text{SNR}_\Delta} = \frac{|f_{\text{CEbl}}|}{|f_{\text{CEbl}} - f_{\text{bl}}|} \sqrt{2n} \quad [24]$$

For the case of single-echo Dixon vs. subtraction CE-MRA, the SNR and CNR ratios are given in Eqs. [25a] and [25b], respectively.

$$\frac{\text{SNR}_s}{\text{SNR}_\Delta} = \frac{|f_{\text{CEbl}}|}{|f_{\text{CEbl}} - f_{\text{bl}}|} 2 |\sin \theta| \quad [25a]$$

$$\frac{\text{CNR}_s}{\text{CNR}_\Delta} = \frac{|f_{\text{CEbl}} - f_{\text{bkgrnd}}|}{|f_{\text{CEbl}} - f_{\text{bl}}|} 2 |\sin \theta| \quad [25b]$$

Note that each of the SNR and CNR ratios contains signal-dependent terms ($|F_{\text{Dixon}}(f)|/F(f)$) and noise-dependent terms (other terms). The signal components can be estimated from the signal equation for the pulse sequence of interest, while the noise terms can be calculated from the phase angles between water and fat at the acquired echo times. For CE-MRA, a SPGRE pulse sequence is often used. From the corresponding signal equation (1,26), a T_1 (and, therefore, contrast-material concentration) dependent change in the SNR and CNR ratios is expected. Specifically, at least a twofold SNR improvement is expected when optimal echo times are chosen. Plots of the theoretical improvement of Dixon over Subtraction CE-MRA vs. contrast-material concentration given a typical SPGRE pulse sequence are shown in Figure 1a–c. Additionally, the effect of the choice of θ or θ (which depend on TE) on the SNR and CNR is shown in Figure 1d–f. The number of echoes, n , used in the Dixon acquisition affects the SNR and CNR ratios in the multiecho Dixon vs. subtraction case (Fig. 1d–f). For the Dixon case, depending on the type of acquisition (e.g., multishot vs. multiecho, unipolar vs. bipolar readout, single-phase vs. time-resolved, etc.), there may be a scan time or temporal resolution tradeoff associated with obtaining additional echoes.

METHODS

Overview

Experimental performance in comparison with theory of one-, two-, and three-echo Dixon vs. subtraction CE-MRA was evaluated through phantom and in vivo measurements. Images were acquired in a multishot fashion; each echo was acquired in a separate repetition. This allowed echo times to be chosen independently of other scan parameters such as number of readout samples or receiver bandwidth. The echo times and resultant phase angles between the water and fat signals used in this work are given in Table 1.

Complex images were formed via discrete inverse Fourier transform of fully-sampled 3D Cartesian k-space data. The data were fully sampled in that no acceleration such as parallel acquisition or partial Fourier was performed. Dixon-based fat/water separation was then performed using a technique appropriate for the number of echoes. In single-echo Dixon imaging, $\phi(t)$ and ϕ_0 were estimated from a dual-echo calibration scan using a phase-constrained reconstruction (24). The single-echo Dixon separation was then performed using Eq. [14]. Dual-echo Dixon separation was performed using a generalized dual-echo separation (27). Three-echo separation used a voxel independent IDEAL method included in the ISMRM Fat-Water Toolbox (28,29). Subtraction images were formed via complex image-space subtraction of a precontrast image from a postcontrast image both acquired at the shortest acquired echo time for the technique.

Region of interest (ROI)-based SNR and CNR measurements were performed on each reconstructed image. A debiased estimate of the signal magnitude was obtained using the method from Ref. (30), i.e., $|f|_{\text{debiased}} = \sqrt{\max(|f|^2 - \sigma^2, 0)}$ where σ^2 is the measured variance in the noise. No noise correction was necessary, as the complex (dual- and three-echo) or real (single-echo) image was used for noise estimation.

Ratios of measured Dixon SNR to measured subtraction SNR were computed and compared against the ratio predicted by theory. In this study, consistent with our experimental method, the signal equation for an SPGRE sequence was used to calculate the signal terms of the theoretical SNR ratio.

Phantom Experiments

An anthropomorphic fat–water phantom was constructed to simulate an abdomen with subcutaneous fat by layering solid vegetable shortening around a bovine gelatin (b-gel) center (Fig. 2a). A cylindrical void in the center of the phantom allowed vials of gadopentetate dimeglumine (Magnevist; Bayer HealthCare Pharmaceuticals, Wayne, NJ, USA)-doped (Gd-doped) b-gel to be inserted to simulate an enhancing abdominal aorta. The phantom measured $18 \times 18 \times 12.5$ cm (length $\times$ width $\times$ height), and the cylindrical vials were 1.6 cm in diameter and 6 cm long.

For accurate comparison with the theoretical SNR and CNR ratios, the T_1 relaxation times of the Gd-doped and un-doped b-gel were experimentally determined (31). The contrast-material concentration in each Gd-doped vial was then calculated from the measured T_1 s and the published relaxivity of the contrast agent (3).

The phantom was imaged at 3T (Signa; GE Healthcare, Waukesha, WI) in a birdcage head coil. Scan parameters are shown in Table 1. In the phantom study, data from a dual-echo experiment acquired in the same session were used to estimate ϕ and ϕ_0 for the single-echo reconstruction. To avoid adverse effects from any subtle changes in the main magnetic field, the single-echo scans were performed immediately following the completion of the dual-echo scans. The start times of the scans were approximately 3 min apart. Additionally, the same prescan values (center frequency, transmit and receive gains, and shims) were used for all experiments.

All ROIs contained measurements from 10 consecutive axial slices. f_v was estimated as the mean signal within the Gd-doped vial, f_{bgnd} was estimated as the mean signal within the undoped b-gel background, and the reconstructed noise was estimated as the standard deviation in the (nonsignal producing) air surrounding the object. For all experiments, the ROIs for estimating f_v , f_{bgnd} , and the standard deviation of the noise contained about 80, 16,000, and 7000 voxels per slice, respectively.

In Vivo Experiments

CE-MRA studies of the thighs were used to examine the signal behavior of fully-sampled data in vivo. The thighs were chosen because they typically contain fat and can be imaged relatively free of motion. The goal of these experiments was to verify a contrast-material concentration dependent advantage of Dixon vs. subtraction CE-MRA as seen in theory. To that end, gadobenate dimeglumine (MultiHance; Bracco Diagnostics, Princeton, NJ) was injected with a stepped protocol during IRB-approved volunteer imaging sessions. Fully-sampled, dual-echo, or three-echo images were acquired for each injection volume: 0 mL (precontrast), 1, 2, 4, and 12 mL. Images were acquired with 10 or 12 circumferential elements of a custom-built phased array receiver coil for the single-/three-echo experiment and the dual-echo experiment, respectively (32). The single-echo data were taken as the center echo (e.g. TE=2.80 ms) of the three-echo data. Additional scan parameters are shown in Table 1.

The vessel ROIs were placed in the femoral arteries and contained approximately 85 voxels. The background signal ROIs were placed in the muscle surrounding the vessel and contained about 130 voxels per slice. Noise was measured in the air surrounding the signal producing object and contained about 3000 voxels per slice. All ROIs spanned 10 consecutive slices.

Because the in vivo contrast concentration is unknown, the trends of SNR and CNR were investigated. Empirical results were compared to theoretical results calculated from published T_1 values of unenhanced blood and muscle (2) and r_1 values of contrast material (3). Measured SNR ratio values were plotted against the injection volume while the theoretical values were plotted against an arbitrary contrast concentration that was determined to show the same trend. Note that the purpose of the theoretical curve is to provide a reference of the asymptotic behavior expected in the SNR ratio, and not to accurately predict the concentration of contrast agent in the blood.

RESULTS

Images from the dual-echo phantom experiment are shown in Figure 2. The water image depicts the Gd-doped vial against the undoped b-gel with good suppression of the fat signal. Compared to the Gd-doped vial, the background water signal in the b-gel is relatively benign but still nonzero.

Figure 3 shows the resultant SNR and CNR ratios compared with the theoretical prediction for the phantom scanned with one, two, and three echoes. A good agreement between theoretical and measured SNR and CNR is seen for all three Dixon methods. In all cases, Dixon CE-MRA shows an SNR and CNR advantage of approximately twofold or more over

subtraction CE-MRA. As in theory, the magnitude of the advantage varies according to the number of echoes used in the reconstruction and the contrast-material concentration. That is, as the concentration of contrast in the Gd-doped vials is reduced, the relative SNR and CNR improvements in Dixon CE-MRA are increased.

Comparisons between the SNR and CNR ratios predicted by theory and those measured in vivo are shown in Figure 4. In all cases, the measured SNR and CNR ratios follow the trends predicted by theory. A Dixon CE-MRA SNR advantage of at least a factor of two is observed for all tested contrast doses with a larger advantage at lower contrast concentrations. Dixon CE-MRA shows a CNR advantage for most contrast concentrations, but at low contrast concentrations the advantage decreases. The 1 mL injection volume shows a CNR ratio of ~1, indicating that the CNRs of Dixon and subtraction CE-MRA are approximately equal.

Images and signal profiles from the in vivo thigh studies are shown in Figure 5. For display, these images were normalized to have an equal standard deviation within the noise ROIs used for the SNR measurements. Therefore, because the noise level is equal, the signal seen in the images is indicative of the SNR, and the difference between the vessel signal and background signal shows the CNR. The Dixon images show excellent fat suppression when any number of echoes are used in the reconstruction.

For all results comparing a specific Dixon technique to subtraction, the signal was measured across a line in a representative axial plane. The specific line profile used for the three-echo Dixon vs. subtraction comparison is identified in the 12 mL result in Figure 5b.

Measurements across this profile for the Dixon (blue) and subtraction (red) results are shown in Figure 5c and illustrate the SNR advantage of Dixon CE-MRA. This is similarly shown for the two-echo (Fig. 5e) and one-echo (Fig. 5g) cases.

DISCUSSION

A general theoretical expression has been derived for the relative improvement in SNR and CNR when a Dixon-type single- or multiecho acquisition is used in place of a subtraction-type single-echo acquisition for CE-MRA. The expression was validated with phantom and in vivo data. The improvement, when expressed as a ratio, has terms that depend on signal and terms that depend on noise. While previous work in Dixon imaging has explored image quality improvement dependent on the noise terms through analysis of the equivalent number of signal averages (NSA) (33), to our knowledge there has been no comparison of the signal terms or in the context of CE-MRA. The signal terms are an important component of this analysis as their inclusion shows the contrast concentration-dependent change in SNR and CNR. That is, the noise terms indicate a baseline shift in SNR and CNR but the signal terms indicate the asymptotic trend observed in phantom and in vivo experimental studies. The high SNR ratio at low contrast concentrations suggests that Dixon CE-MRA may be useful in situations where low contrast dose is desired.

When the phase angles between the water signal and the fat signal are chosen optimally, Dixon CE-MRA should theoretically provide at least twofold improvement in SNR at any

contrast concentration. The baseline factor of 2 is due to the noise terms in the SNR ratio and is highly dependent on the choice of θ or $-\theta$ (and thus TE). While a near optimal (in the SNR sense) θ was chosen here, it may be advantageous to trade SNR for a shorter TR while optimizing a pulse sequence for fast imaging (16). The signal terms give rise to the contrast concentration-dependent behavior of the SNR and CNR ratios (Fig. 1b) while the θ -dependent terms provide a baseline shift.

The SNR and CNR ratios presented here describe the improvement when the Dixon method is used in place of a subtraction method for background suppression. It should be noted that when multiple echoes are used for the Dixon acquisition, the SNR and CNR improvements may come at the expense of scan time or temporal resolution. For CE-MRA, however, the goal is simply to suppress the background fat signal and not quantify it. Therefore, the fewest number of echoes to accomplish background suppression should be used for CE-MRA, as extended scan time could affect the capture of a first-pass contrast agent bolus. Further, scan time can be minimized by utilizing a multiecho sequence (as opposed to the multishot sequence used here), bipolar readout gradients, and careful choice of echo times. Additionally, the results presented here show the difference in SNR and CNR between Dixon- and subtraction-type acquisitions with all other parameters equal. Parameter alteration of one acquisition vs. the other would require adjustment of the ratios.

Previous work in single-echo Dixon has indicated an NSA improvement over the source image of $|\sin \theta|$ (19), while we derived and observed an number of signal averages improvement of $\sqrt{2} |\sin \theta|$. This difference can be resolved by clarifying what is considered the “source” image. In this work, the source image is defined as the acquired complex image. Previous work [e.g. Ref. (19)] defined the source image as the magnitude of the measured complex data. When these differences are resolved, the NSA results are equivalent.

The theoretical expression for the CNR ratio with either measured T_1 values (phantom) or T_1 values from literature (in vivo) match experimental results well. However, as contrast concentration changes, the CNR ratio behaves differently in the two cases due to the difference in the T_1 values of the background phantom materials and in vivo tissues. In the phantom, $T_{1,\text{bkgnd}} > T_{1,\text{bl}}$, while in vivo $T_{1,\text{bkgnd}} < T_{1,\text{bl}}$ giving rise to the different trends that predict the theoretical CNR results shown in Figures 3 and 4. Note that while the in vivo signal terms of the CNR expression will typically be less than or equal to 1 due to the residual water signal in the background, it is the baseline shift from reduced noise that gives Dixon CE-MRA a higher CNR than subtraction CE-MRA.

This work uses a relatively simple signal model that does not include multippeak fat models, T_2^* decay effects, or other effects. Multiple fat peaks can be incorporated by including a weighted sum of chemical shift terms in place of the single phase term included here. Accounting for other effects requires modifications to the model itself in addition to requiring more time samples to estimate the additional unknown parameters. In CE-MRA, time is often at a premium, so we consider the model presented here (that accounts for B_0 and chemical shift) to be a good tradeoff between accuracy and acquisition time. Further reduction in acquisition time could be realized with accelerated imaging techniques [e.g.,

SENSE (25)]. Analysis of the SNR and CNR of accelerated Dixon and subtraction images can be performed in a similar way as presented here, but with the addition of an undersampling matrix.

The theoretical expressions for the SNR and CNR of the Dixon- and subtraction-type background suppression methods were derived assuming known and ideal coil sensitivity matrices S and main magnetic field inhomogeneities, B_0 . In reality, these quantities contain noise that could influence the comparison between the theoretical values and the experimental results. While the effect of the propagation of uncertainty in these variables is beyond the scope of this work, both quantities are assumed smooth and noise can be limited by using low-pass filtered approximations of S and B_0 .

In conclusion, we have derived and validated a theoretical expression for the improvement in SNR and CNR when a Dixon-type single- or multiecho acquisition is used in place of a subtraction-type single-echo acquisition. The improvement is dependent on contrast concentration. For commonly used dosages of contrast material, single-, dual-, and three-echo Dixon CE-MRA can consistently provide an approximately twofold improvement in blood-to-background CNR vs. conventional subtraction CE-MRA.

Acknowledgments

Grant sponsor: NIH; Grant numbers: HL070620, EB000212, and RR018898.

The authors thank Tom Hulshizer for assistance in the design of the fat-water phantom. We also acknowledge the use of the ISMRM Fat-Water Toolbox (<http://ismrm.org/workshops/FatWater12/data.htm>) for some of the results shown in this article.

References

1. Prince MR, Yucel EK, Kaufman JA, Harrison DC, Geller SC. Dynamic gadolinium-enhanced three-dimensional abdominal MR arteriography. *J Magn Reson Imaging*. 1993; 3:877–881. [PubMed: 8280977]
2. Stanisz GJ, Odobina EE, Pun J, Escaravage M, Graham SJ, Bronskill MJ, Henkelman RM. T1, T2 relaxation and magnetization transfer in tissue at 3T. *Magn Reson Med*. 2005; 54:507–512. [PubMed: 16086319]
3. Pintaske J, Martirosian P, Graf H, Erb G, Lodemann KP, Claussen CD, Schick F. Relaxivity of gadopentetate dimeglumine (Magnevist), gadobutrol (Gadovist), and gadobenate dimeglumine (MultiHance) in human blood plasma at 0.2, 1.5, and 3 tesla. *Invest Radiol*. 2006; 41:213–221. [PubMed: 16481903]
4. de Bazelaire CMJ, Duhamel GD, Rofsky NM, Alsop DC. MR imaging relaxation times of abdominal and pelvic tissues measured in vivo at 3.0 T: preliminary results. *Radiology*. 2004; 230:652–659. [PubMed: 14990831]
5. Douek PC, Revel D, Chazel S, Falise B, Villard J, Amiel M. Fast MR angiography of the aortoiliac arteries and arteries of the lower extremity: value of bolus-enhanced, whole-volume subtraction technique. *AJR Am J Roentgenol*. 1995; 165:431–437. [PubMed: 7618572]
6. Adamis MK, Li W, Wielopolski PA, Kim D, Sax EJ, Kent KC, Edelman RR. Dynamic contrast-enhanced subtraction MR angiography of the lower extremities: Initial evaluation with a multisection two-dimensional time-of-flight sequence. *Radiology*. 1995; 196:689–695. [PubMed: 7644630]
7. Wang Y, Johnston DL, Breen JF, Huston J 3rd, Jack CR, Julsrud PR, Kiely MJ, King BF, Riederer SJ, Ehman RL. Dynamic MR digital subtraction angiography using contrast enhancement, fast data acquisition, and complex subtraction. *Magn Reson Med*. 1996; 36:551–556. [PubMed: 8892206]

8. Leiner T, de Weert TT, Nijenhuis RJ, Vastbinder GBC, Kessels AG, Ho KYJ, van Engelshoven JM. Need for background suppression in contrast-enhanced peripheral magnetic resonance angiography. *J Magn Reson Imaging*. 2001; 14:724–733. [PubMed: 11747029]
9. Bydder GM, Steiner RE, Blumgart LH, Khenia S, Young IR. MR imaging of the liver using short TI inversion recovery sequences. *J Comput Assist Tomogr*. 1985; 9:1084–1089. [PubMed: 4056142]
10. Haase A, Frahm J, Hanicke W, Matthaei D. 1H NMR chemical shift selective (CHESS) imaging. *Phys Med Biol*. 1985; 30:341–344. [PubMed: 4001160]
11. Meyer CH, Pauly JM, Macovski A, Nishimura DG. Simultaneous spatial and spectral selective excitation. *Magn Reson Med*. 1990; 15:287–304. [PubMed: 2392053]
12. Dixon WT. Simple proton spectroscopic imaging. *Radiology*. 1984; 153:189–194. [PubMed: 6089263]
13. Saranathan M, Rettmann D, Bayram E, Lee C, Glockner J. Multiecho time-resolved acquisition (META): A high spatiotemporal resolution Dixon imaging sequence for dynamic contrast-enhanced MRI. *J Magn Reson Imaging*. 2009; 29:1406–1413. [PubMed: 19425056]
14. Michaely HJ, Morelli JN, Budjan J, Riffel P, Nickel D, Kroeker R, Schoenberg SO, Attenberger UI. CAIPIRINHA-Dixon-TWIST (CDT)-volume-interpolated breath-hold examination (VIBE): a new technique for fast time-resolved dynamic 3-dimensional imaging of the abdomen with high spatial resolution. *Invest Radiol*. 2013; 48:590–597. [PubMed: 23538886]
15. Leiner, T., Versluis, B., Geerts, L., Alberts, E., Hendrikse, J., Vonken, EJ., Eggers, H. Subtractionless first-pass single-dose peripheral MRA using two-point Dixon fat-saturation. Proceedings of the 20th Annual Meeting of ISMRM; Melbourne, Australia. 2012; p. 525
16. Leiner T, Habets J, Versluis B, Geerts L, Alberts E, Blanken N, Hendrikse J, Vonken EJ, Eggers H. Subtractionless first-pass single contrast medium dose peripheral MR angiography using two-point Dixon fat suppression. *Eur Radiol*. 2013; 23:2228–2235. [PubMed: 23591617]
17. Stinson, EG., Trzasko, JD., Riederer, SJ. SNR and CNR performance comparison in Dixon- vs. subtraction-type contrast-enhanced MR angiography. Proceedings of the 21st Annual Meeting of ISMRM; Salt Lake City, Utah, USA. 2013; p. 4458
18. Xiang, QS. Improved single point water-fat imaging with virtual shimming. Proceedings of the 9th Annual Meeting of ISMRM; Glasgow, Scotland, UK. 2001; p. 789
19. Yu H, Reeder SB, McKenzie CA, Brau ACS, Shimakawa A, Brittain JH, Pelc NJ. Single acquisition water-fat separation: Feasibility study for dynamic imaging. *Magn Reson Med*. 2006; 55:413–422. [PubMed: 16372279]
20. Ma J. A single-point Dixon technique for fat-suppressed fast 3D gradient-echo imaging with a flexible echo time. *J Magn Reson Imaging*. 2008; 27:881–890. [PubMed: 18302201]
21. Eggers, H., Geerts-Ossevoort, L., Vonken, EJ., Leiner, T. Single-echo Dixon imaging of first-pass contrast-enhanced peripheral angiography without subtraction. Proceedings of the 20th Annual Meeting of ISMRM; Melbourne, Australia. 2012; p. 1199
22. Neeser F, Massey J. Proper complex random processes with applications to information theory. *IEEE Trans Inf Theory*. 1993; 39:1293–1302.
23. Kay, SM. Fundamentals of statistical signal processing, Vol. I: estimation theory. Upper Saddle River, NJ: Prentice Hall PTR; 1993. p. 595
24. Bydder M, Yokoo T, Yu H, Carl M, Reeder SB, Sirlin CB. Constraining the initial phase in water-fat separation. *Magn Reson Imaging*. 2011; 29:216–221. [PubMed: 21159457]
25. Pruessmann KP, Weiger M, Scheidegger MB, Boesiger P. SENSE: sensitivity encoding for fast MRI. *Magn Reson Med*. 1999; 42:952–962. [PubMed: 10542355]
26. Haase A, Frahm J, Matthaei D, Hänicke W, Merboldt KD. FLASH imaging. Rapid NMR imaging using low flip-angle pulses. *J Magn Reson*. 1986; 67:258–266.
27. Eggers H, Brendel B, Duijndam A, Herigault G. Dual-echo Dixon imaging with flexible choice of echo times. *Magn Reson Med*. 2011; 65:96–107. [PubMed: 20860006]
28. Reeder SB, Wen Z, Yu H, Pineda AR, Gold GE, Markl M, Pelc NJ. Multicoil Dixon chemical species separation with an iterative least-squares estimation method. *Magn Reson Med*. 2004; 51:35–45. [PubMed: 14705043]

29. Hu HH, Börnert P, Hernando D, Kellman P, Ma J, Reeder S, Sirlin C. ISMRM workshop on fat-water separation: Insights, applications and progress in MRI. *Magn Reson Med*. 2012; 68:378–388. [PubMed: 22693111]
30. McGibney G, Smith MR. An unbiased signal-to-noise ratio measure for magnetic resonance images. *Med Phys*. 1993; 20:1077–1078. [PubMed: 8413015]
31. Barral JK, Gudmundson E, Stikov N, Etezadi-Amoli M, Stoica P, Nishimura DG. A robust methodology for in vivo T1 mapping. *Magn Reson Med*. 2010; 64:1057–1067. [PubMed: 20564597]
32. Johnson CP, Weavers PT, Borisch EA, Grimm RC, Hulshizer TC, LaPlante CC, Rossman PJ, Glockner JF, Young PM, Riederer SJ. Three-station three-dimensional bolus-chase MR angiography with real-time fluoroscopic tracking. *Radiology*. 2014; 272:241–251. [PubMed: 24635676]
33. Glover GH. Multipoint Dixon technique for water and fat proton and susceptibility imaging. *J Magn Reson Imaging*. 1991; 1:521–530. [PubMed: 1790376]
34. Meyer, CD. Matrix analysis and applied linear algebra. Philadelphia, PA: Society for Industrial and Applied Mathematics; 2000. p. 718

APPENDIX A: DERIVATION OF MULTIECHO DIXON SNR

Starting with Eq. [9],

$$\text{SNR}_m(\hat{W}_m) = \frac{|W|}{\sqrt{\delta^T \mathbb{E}[H_L N_m H_R H_R^* N_m^* H_L^*] \delta}} \quad [9 \text{ revisited}]$$

we need to simplify $\mathbb{E}[\cdot]$ which is a 2×2 matrix. Multiply each side of the expected value by a 2×2 identity matrix formulated as a sum of delta functions where

$$\begin{aligned} \delta_1 &= \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{ and } \delta_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \\ I &= \sum_{a=1}^2 \delta_a \delta_a^T = \sum_{b=1}^2 \delta_b \delta_b^T \\ \mathbb{E}[(H_L N_m H_R)(H_L N_m H_R)^*] & \quad [A1] \\ &= \mathbb{E}\left[\left(\sum_{a=1}^2 \delta_a \delta_a^T\right) H_L N_m H_R H_R^* N_m^* H_L^* \left(\sum_{b=1}^2 \delta_b \delta_b^T\right)\right] \\ &= \sum_{a=1}^2 \sum_{b=1}^2 \delta_a H_R^* \mathbb{E}[(N_m^* H_L^* \delta_b)(\delta_a^T H_L N_m)] H_R \delta_b^T \end{aligned}$$

where we have leveraged that $(\delta_a^T H_L N_m H_R)$ is scalar. Let $Q = H_L^* \delta_b \delta_a^T H_L$ and evaluate $\mathbb{E}[N_m^* Q N_m]$ element by element to gain some understanding of its behavior.

$$\mathbb{E}[N_m^* Q N_m]_{p,q} = \sum_{k=1}^n \sum_{l=1}^n \mathbb{E}[(N_m^*)_{p,k} Q_{k,l} (N_m)_{l,q}] \quad [\text{A2a}]$$

$$= \sum_{k=l=1}^n Q_{k,k} \mathbb{E}[(N_m^*)_{p,k} (N_m)_{k,q}] + \sum_{k(\neq l)=1}^n \sum_{l(\neq k)=1}^n Q_{k,l} \mathbb{E}[(N_m^*)_{p,k} (N_m)_{l,q}] \quad [\text{A2b}]$$

The first sum contains information solely from the same echo and is allowed to have correlated noise along the coils dimension (i.e., $\mathbb{E}[(N_m^*)_{p,k} (N_m)_{k,q}] = \Lambda_{p,q}$). The second term is from different echoes, and with the assumption that noise vectors from different echoes are independent, $\mathbb{E}[(N_m^*)_{p,k} (N_m)_{l,q}] = 0$ for all $k \neq l$ and all terms vanish. Thus,

$$\mathbb{E}[N_m^* Q N_m]_{p,q} = \sum_{k=1}^n Q_{k,k} \Lambda_{p,q} = \text{Tr}(Q) \Lambda_{p,q} \quad [\text{A3}]$$

The product of the matrices within the trace operator can be permuted cyclically without changing the trace of the product (34), so:

$$\text{Tr}(Q) = \text{Tr}(H_L^* \delta_b^T H_L) = \text{Tr}(\delta_a^T H_L H_L^* \delta_b) = \delta_a^T (A_m^* A_m)^{-1} \delta_b \quad [\text{A4}]$$

where we have used the definition of H_L , and noted that $(A_m^* A_m)^{-1}$ is Hermitian. Substituting the results of Eqs. [A3] and [A4] into Eq. [A1] results in

$$\mathbb{E}[(H_L N H_R)(H_L N H_R)^*] = \sum_{a=1}^2 \sum_{b=1}^2 \delta_a \delta_a^T (A_m^* A_m)^{-1} \delta_b H_R^* \Lambda H_R \delta_b^T \quad [\text{A5}]$$

$$= (S \Lambda^{-1} S^*)^{-1} (A_m^* A_m)^{-1} \quad [\text{A6}]$$

where $H_R^* \Lambda H_R = (S \Lambda^{-1} S^*)^{-1}$ and is scalar and the sums have been resolved. Expanding and simplifying $(A_m^* A_m)^{-1}$ (Appendix B), substituting into Eq. [9], and applying the water delta functions gives the SNR of the water signal in multiecho, multicoil Dixon imaging:

$$\text{SNR}_m(\hat{W}) = \frac{|W| \sqrt{\det(C_m^* C_m)}}{\sqrt{n(S \Lambda^{-1} S^*)^{-1}}} \quad [\text{A7}]$$

where $\det(C_m^* C_m) = n^2 - \sum_{k=1}^n e^{i\theta_k} \sum_{j=1}^n e^{-i\theta_j}$ and n is the number of echoes.

APPENDIX B: SIMPLIFICATION OF $(A^* A)^{-1}$

Using Eqs. [5a] and [5b], with the knowledge that B_m is a unitary diagonal matrix, $B_m^* B_m = I$ and

$$(A_m^* A_m)^{-1} = (C_m^* B_m^* B_m C_m)^{-1} = (C_m^* C_m)^{-1} \quad [\text{B1}]$$

$$= \begin{bmatrix} n & \sum_{k=1}^n e^{i\theta_k} \\ \sum_{j=1}^n e^{-i\theta_j} & n \end{bmatrix}^{-1} \quad [\text{B2}]$$

$$= \frac{1}{\det(C_m^* C_m)} \begin{bmatrix} n & -\sum_{k=1}^n e^{i\theta_k} \\ -\sum_{j=1}^n e^{-i\theta_j} & n \end{bmatrix} \quad [\text{B3}]$$

where n is the number of echoes, and $\det(C_m^* C_m) = n^2 - \sum_{k=1}^n e^{i\theta_k} \sum_{j=1}^n e^{-i\theta_j}$.

In the single-echo case, $n = 1$ and

$$[\Re\{A_s^* A_s\}]^{-1} = \begin{bmatrix} 1 & \cos \theta \\ \cos \theta & 1 \end{bmatrix}^{-1} = \frac{1}{\sin^2 \theta} \begin{bmatrix} 1 & -\cos \theta \\ -\cos \theta & 1 \end{bmatrix} \quad [\text{B4}]$$

APPENDIX C: SINGLE-ECHO MULTICOIL RECONSTRUCTION

To find a solution to $\hat{X}_s = \arg \min_{X \in \mathbb{R}} [\Re\{\text{Tr}((A_s X S - G_s) \Lambda^{-1} (A_s X S - G_s)^*)\}]$ it is useful to first expand the expression to be minimized.

$$\mathcal{L} = \Re\{\text{Tr}(A_s X S \Lambda^{-1} S^* X^T A_s^* - (A_s X S \Lambda^{-1} G_s^* + G_s \Lambda^{-1} S^* X^T A_s^*) + G_s \Lambda^{-1} G_s^*)\} \quad [\text{C1a}]$$

$$= \Re\{\text{Tr}(A_s X S \Lambda^{-1} S^* X^T A_s^*)\} - \Re\{\text{Tr}(2\Re\{G_s \Lambda^{-1} S^* X^T A_s^*\})\} + \Re\{\text{Tr}(G_s \Lambda^{-1} G_s^*)\} \quad [\text{C1b}]$$

$$\begin{aligned} &= \Re\{\text{Tr}(X^T A_s^* A_s X S \Lambda^{-1} S^*)\} - \Re\{\text{Tr}(2\Re\{X^T A_s^* G_s \Lambda^{-1} S^*\})\} + \Re\{\text{Tr}(G_s \Lambda^{-1} G_s^*)\} \\ &= X^T \Re\{A_s^* A_s\} X S \Lambda^{-1} S^* - 2X^T \Re\{A_s^* G_s \Lambda^{-1} S^*\} + \Re\{G_s \Lambda^{-1} G_s^*\} \end{aligned} \quad [\text{C1c}]$$

where we have noted that the center two terms in Eq. [C1a] are scalar conjugates and we have again invoked the circular permutation property of the trace (34) in Eq. [C1c]. Differentiating $\mathcal{L}$ with respect to X and setting it equal to zero allows the least squares solution of X to be found.

$$\nabla_X \mathcal{L} = 2\Re\{A_s^* A_s\} X S \Lambda^{-1} S^* - 2\Re\{A_s^* G_s \Lambda^{-1} S^*\} = 0 \quad [\text{C2}]$$

The estimate for X , $\hat{X}_s$, is then given by

$$X \approx \hat{X}_s = [\Re\{A_s^* A_s\}]^{-1} \Re\{A_s^* G_s \Lambda^{-1} S^*\} (S \Lambda^{-1} S^*)^{-1} \quad [\text{C3}]$$

APPENDIX D: SIMPLIFICATION OF SINGLE-ECHO SNR

Beginning with Eq. [15],

$$\text{SNR}_s(\hat{W}_s) = \frac{|W|}{\sqrt{\delta^T \mathbb{E}[\mathcal{H}\{N_s\} \mathcal{H}\{N_s\}^*] \delta}}$$

it is useful to deal only with the denominator, $\mathcal{D}$. Expanding $\mathcal{H}\{N_s\}$ and $\mathcal{H}\{N_s\}^*$ where $\mathcal{H}\{\cdot\}$ is defined in Eq. [14], it is seen that $(S \Lambda^{-1} S^*)^{-2}$ is a scalar and can be removed from the radical.

$$\mathcal{D} = (S\Lambda^{-1}S^*)^{-1} \sqrt{\delta^T Z Q Z^T \delta} \quad [\text{D1}]$$

where $Z = [\Re\{A_s^* A_s\}]^{-1}$ and $Q = \mathbb{E}[\Re\{A_s^* N_s \Lambda^{-1} S^*\} (\Re\{A_s^* N_s \Lambda^{-1} S^*\})^T]$. Note that $N_s \Lambda^{-1} S^*$ is a scalar, and $N_s \Lambda^{-1} S^* = (N_s \Lambda^{-1} S^*)^T = \overline{S \Lambda^{-1} N_s^T}$. Thus,

$$Q = \mathbb{E} \left[\Re\{A_s^* \overline{S \Lambda^{-1} N_s^T}\} \left(\Re\{A_s^* \overline{S \Lambda^{-1} N_s^T}\} \right)^T \right] \quad [\text{D2}]$$

Noting that $\Re\{(\cdot)\} = \frac{1}{2}[(\cdot) + \overline{(\cdot)}]$, the expression is expanded further:

$$Q = \frac{1}{4} \mathbb{E} [A_s^* \overline{S \Lambda^{-1} N_s^T} N_s \Lambda^{-1} S^* \overline{A_s} + A_s^* \overline{S \Lambda^{-1} N_s^T} \overline{N_s} \Lambda^{-1} S^T A_s + A_s^T S \Lambda^{-1} N_s^* N_s \Lambda^{-1} S^* \overline{A_s} + A_s^T S \Lambda^{-1} N_s^* \overline{N_s} \Lambda^{-1} S^T A_s] \quad [\text{D3}]$$

As N_s is a proper complex Gaussian noise matrix, the pseudocovariance $\Omega = \mathbb{E}[N_s^T N_s] = \mathbb{E}[N_s^* \overline{N_s}]$ is zero (22) and

$$Q = \frac{1}{4} \mathbb{E} \left[A_s^* \overline{S \Lambda^{-1} N_s^T} \overline{N_s} \Lambda^{-1} S^T A_s + A_s^T S \Lambda^{-1} N_s^* N_s \Lambda^{-1} S^* \overline{A_s} \right] \quad [\text{D4}]$$

$$= \frac{1}{2} \Re\{A_s^T S \Lambda^{-1} \mathbb{E}[N_s^* N_s] \Lambda^{-1} S^* \overline{A_s}\} \quad [\text{D5}]$$

$$= \frac{1}{2} \Re\{A_s^T S \Lambda^{-1} S^* \overline{A_s}\} \quad [\text{D6}]$$

Noting that $S \Lambda^{-1} S^*$ is purely real and scalar, and that $\Re\{A_s^T \overline{A_s}\} = \Re\{A_s^* A_s\}$

$$Q = \frac{1}{2} (S \Lambda^{-1} S^*)^{-1} \Re\{A_s^* A_s\} \quad [\text{D7}]$$

Substituting this result into Eq. [D1]:

$$\mathcal{D} = (S\Lambda^{-1}S^*)^{-1} \sqrt{\delta^T [\Re\{A_s^* A_s\}]^{-1} \frac{1}{2} (S\Lambda^{-1}S^*) \Re\{A_s^* A_s\} ([\Re\{A_s^* A_s\}]^{-1})^T \delta} \quad [\text{D8}]$$

Expansion of $([\Re\{A_s^* A_s\}]^{-1})^T$ (Appendix B) and applying the delta functions results in

$$\mathcal{D} = \sqrt{\frac{(S\Lambda^{-1}S^*)^{-1}}{2}} \frac{1}{|\sin \theta|} \quad [\text{D9}]$$

Substitution of Eq. [D9] into Eq. [15] gives the SNR of the water signal in single-echo, multicoil Dixon imaging.

$$\text{SNR}_s(\widehat{W}_s) = \frac{|W| |\sin \theta| \sqrt{2}}{\sqrt{(S\Lambda^{-1}S^*)^{-1}}} \quad [\text{D10}]$$

FIG. 1.

Theoretical SNR and CNR improvement of dual-echo Dixon vs. subtraction CE-MRA over a range of contrast material concentrations. **a:** As contrast concentration approaches 0 mM, the SNR ratio approaches infinity. **b:** The noise component of the SNR ratio is constant over contrast concentrations, while the signal component varies. **c:** The behavior of the CNR ratio at low contrast concentrations is dependent on the T_1 relaxation times of the background tissue and unenhanced blood. When $T_{1,\text{bkgrnd}} < T_{1,\text{bl}}$ (as is expected in vivo) the CNR ratio tends toward zero at very low contrast concentrations. SNR ratio dependence on θ (for 1 echo) or θ (for >1 echo) at 0.5 mM contrast concentration (**d**), and 8 mM contrast concentration (**e**). In (**f**), the maxima of (**d**) and (**e**) are shown in the context of the SNR ratio plots varying over contrast concentration.

FIG. 2.

The phantom and contrast-enhanced vials used in the phantom experiments (a). Images from the dual-echo Dixon experiment comparing the SNR and CNR in pre-contrast subtraction (b) and Dixon water (d) images. An image of the fat signal is shown in (c).

FIG. 3.

Ratios of measured and theoretical Dixon:subtraction SNR and CNR in a phantom for single-echo (a, b), dual-echo (c, d), and three-echo (e, f) Dixon imaging. At all contrast concentrations, Dixon CE-MRA shows an SNR and CNR advantage over subtraction CE-MRA. At low contrast concentrations the advantage increases.

FIG. 4.

Ratios of in vivo measured and theoretical Dixon:subtraction SNR and CNR for single-echo (a, b), dual-echo (c, d), and three-echo (e, f) Dixon imaging. At all contrast concentrations, Dixon CE-MRA shows an SNR advantage over subtraction CE-MRA. At low contrast concentrations the SNR advantage increases, while the CNR advantage decreases due to the relationship between the T_1 relaxation times of unenhanced blood and the background signal (see Figure 1c).

FIG. 5.

In vivo images from single-, dual-, and three-echo Dixon CE-MRA compared to subtraction CE-MRA. Profiles through vessels and background (dotted lines) are also included to show the signal difference between the Dixon images and subtraction images. Note that these images have been normalized to have an equal noise level in the background, thus the relative signal intensity in the profiles approximates the relative SNR between Dixon and subtraction CE-MRA.

Table 1

Imaging Parameters for Phantom and In Vivo Studies Performed at 3T

	One echo	Two echo	Three echo
TE(s) (ms)	2.80	2.30/3.50	2.05/2.80/3.55
Phase offset between water and fat	450°	370°/563°	330°/450°/570°
Phantom			
Field of view (cm ³)	9.0 × 22.0 × 22.0	6.0 × 22.0 × 22.0	6.0 × 22.0 × 22.0
Sampling matrix	60 × 224 × 224	30 × 224 × 224	30 × 224 × 224
Resolution (mm ³)	1.50 × 0.98 × 0.98	2.00 × 0.98 × 0.98	2.00 × 0.98 × 0.98
TR (ms)/FA/BW (kHz)	6.5/18°/±62.5	6.5/18°/±62.5	6.5/18°/±62.5
TEs used for estimation of ϕ (ms)	2.30/3.50	–	–
Number and arrangement of receive coil elements	1 birdcage	1 birdcage	1 birdcage
In vivo			
Field of view (cm ³)	36.0 × 36.0 × 19.2	40.0 × 40.0 × 22.4	36.0 × 36.0 × 19.2
Sampling matrix	220 × 220 × 120	250 × 250 × 140	220 × 220 × 120
Resolution (mm ³)	1.64 × 1.64 × 1.60	1.60 × 1.60 × 1.60	1.64 × 1.64 × 1.60
TR (ms)/FA/BW (kHz)	5.56/18°/±62.5	5.90/30°/±62.5	5.56/18°/±62.5
TEs used for estimation of ϕ (ms)	2.05/3.55	–	–
Number and arrangement of receive coil elements	10 circumferential	12 circumferential	10 circumferential

Field of view, Sampling matrix, and Resolution are reported as S/I × L/R × A/P. All imaging was performed using a 3DFT acquisition.