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The Distribution of the Product Explains Normal Theory Mediation Confidence Interval Estimation

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Abstract

The distribution of the product has several useful applications. One of these applications is its use to form confidence intervals for the indirect effect as the product of 2 regression coefficients. The purpose of this article is to investigate how the moments of the distribution of the product explain normal theory mediation confidence interval coverage and imbalance. Values of the critical ratio for each random variable are used to demonstrate how the moments of the distribution of the product change across values of the critical ratio observed in research studies. Results of the simulation study showed that as skewness in absolute value increases, coverage decreases. And as skewness in absolute value and kurtosis increases, imbalance increases. The difference between testing the significance of the indirect effect using the normal theory versus the asymmetric distribution of the product is further illustrated with a real data example. This article is the first study to show the direct link between the distribution of the product and indirect effect confidence intervals and clarifies the results of previous simulation studies by showing why normal theory confidence intervals for indirect effects are often less accurate than those obtained from the asymmetric distribution of the product or from resampling methods.

An indirect effect (mediated effect, intermediate end point effect) occurs when an independent variable (X) causes a mediator variable (M), which in turn causes the dependent variable (Y; MacKinnon, 2008; Sobel, 1990). Mediation analysis is used in different disciplines such as psychology, sociology, and epidemiology to understand the causal mechanisms underlying an effect. An example of a single mediator model in prevention research is that a workplace health promotion intervention may increase healthy eating habits by improving dietary coworker norms (Ranby et al., 2011). The indirect effect is the product of the path from the intervention program to the dietary coworker norms mediator and the path from the mediator to the outcome healthy eating habits (see Figure 1).

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Using the product of path coefficients method in mediation analysis, a researcher can estimate confidence limits and test the significance of the indirect effect. The rise in the use of mediation analysis has led to the investigation of the statistical performance of various tests of significance testing and confidence interval construction for the indirect effect (MacKinnon, Lockwood, Hoffman, West, & Sheets, 2002; MacKinnon, Lockwood, & Williams, 2004; Preacher & Hayes, 2008; Preacher & Selig, 2012; Shrout & Bolger, 2002; Taylor & MacKinnon, 2012). Methods for the confidence limit construction for the product of coefficients approach to test mediation (e.g., bootstrapping methods and Monte Carlo confidence intervals) have been compared with the standard z test method confidence limits (also known as normal theory confidence limits), which assume a normal distribution for the indirect effect. Recent research showed that the use of normal theory confidence limits for significance testing can lead to incorrect conclusions because the indirect effect is the product of two coefficients (MacKinnon et al., 2002; MacKinnon, Warsi, & Dwyer, 1995) and the product of two normally distributed coefficients may not follow a normal distribution (Lomnicki, 1967; Springer & Thompson, 1966). However, until now no studies have shown how and when this situation occurs. This study aims to demonstrate how the distribution of the product explains the results of previous statistical simulation studies of the indirect effect. Prior simulation studies have concluded that the reason methods such as the bootstrap are more accurate than normal theory methods is that they more accurately follow the distribution of the indirect effect. But these earlier studies did not explain empirically that the distribution of the product was the reason earlier simulation studies found that normal theory confidence limits were inaccurate. In this article we show that the moments of the underlying distribution of the product explain when normal theory methods are inaccurate. We specifically investigate for which values of a and b ab is most nonnormal and how the moments of the indirect effect influence the coverage and imbalance of the normal theory confidence limits. We also use a real data example to demonstrate the effect of ignoring the distribution of the product and suggest programs to compute confidence limits for the indirect effect.

ESTIMATING THE INDIRECT EFFECT

The basic mediation model can be summarized in three equations including three variables: X , the independent variable; Y , the dependent variable; and M , the mediator (MacKinnon, 2008; MacKinnon & Dwyer, 1993):

$$Y = i_1 + cX + e_1 \quad (1)$$

$$M = i_2 + aX + e_2 \quad (2)$$

and

$$Y = i_3 + c'X + bM + e_3. \quad (3)$$

Equation 1 estimates the overall effect of the predictor X on the outcome, Y (the c regression coefficient). Equation 2 estimates the effect of X on the mediator (the a path). Equation 3

estimates the effect of the independent predictor X on the outcome Y adjusting for the effect of the mediating variable M (the c' path) and the effect of M on Y adjusted for the independent variable X (the b path). The intercepts are i_1 , i_2 , and i_3 and residual errors are e_1 , e_2 , and e_3 .

The commonly used approach to calculate the point estimate of the indirect effect is the product of coefficients, ab , assuming that e_2 , and e_3 are independent. The *standard z method* for testing significance of the indirect effect uses the point estimate of the effect, ab , divided by its standard error and compares it with the normal distribution. The most commonly used standard error (σ) of ab derived by Sobel (1982) based on first derivatives using the multivariate delta method is shown in Equation 4:

$$\sigma_{ab} = \sqrt{a^2\sigma_b^2 + b^2\sigma_a^2}. \quad (4)$$

Then, the normal theory symmetrical confidence limits for the indirect effect are constructed using the following formula:

$$ab \pm z_{1-\omega/2} * \sigma_{ab} \quad (5)$$

where z is the value on the z distribution for the Type I error rate ω .

Confidence limits of the indirect effect ab indicate the upper and lower boundaries of the confidence interval that represents the range of values which have a specified probability of including the true value of ab given an infinite number of samples. Several other methods to construct confidence limits for the indirect effect have been investigated. Those methods obtain sampling distributions of ab in different ways. For example, the percentile bootstrapping method draws random samples with replacement from the original sample. The product ab for each resample is then computed resulting in an empirical sampling distribution of ab . Then the observations corresponding to the $\omega/2$ and $1 - \omega/2$ percentiles define the confidence limits. The bias-corrected bootstrap corrects potential bias in ab , and the accelerated bias-corrected bootstrap corrects bias in both ab and the changes in the standard deviation of ab as ab varies. Another method to compute the confidence limits for the indirect effect uses the standardized a and b to compute ab and then uses the critical values for the distribution of the product (Meeker, Cornwell, & Aroian, 1981) to create asymmetric confidence limits (MacKinnon et al., 2002).

Confidence Limits Bias

Although the standard error estimator of the indirect effect may be unbiased, the confidence limits using the standard error estimator can be inaccurate (MacKinnon, Fritz, Williams, & Lockwood, 2007; MacKinnon et al., 2004). Even though it is expected that for a 5% Type I error rate, 2.5% of the confidence intervals will be to the left of the true value of the indirect effect and 2.5% will be to the right of the true value, several simulation studies showed that there is imbalance in the number of times a true value falls to the left or right of the confidence limits of an indirect effect (MacKinnon et al., 2004; MacKinnon et al., 1995; Preacher & Selig, 2012; Stone & Sobel, 1990). MacKinnon et al. (1995) showed that under

different variance estimators (i.e., multivariate delta method, Sobel, 1982; second-order Taylor series, Goodman, 1960; and unbiased variance estimator, Goodman, 1960), for positive values of the indirect effect, the true value was more often to the right than to the left of the normal theory confidence limits. The asymmetry decreased as the sample size increased. Bollen and Stine (1990) used a bootstrapping approach to correct for the inaccuracy of the normal theory confidence limits. Yet, simulation studies showed that bootstrap estimation of the indirect effect also leads to asymmetry of percentage of confidence intervals to the left or right of the true value of the indirect effect (Bollen & Stine, 1990; Lockwood & MacKinnon, 1998). Similarly, the standard z test confidence limits were found to have higher imbalance than the asymmetric distribution of the product method that incorporates skewness and kurtosis of ab (MacKinnon et al., 2004). Among the methods to compute confidence limits for the indirect effect, the standard z test (normal theory method) is the most commonly used approach in mediation analysis and had the highest confidence limit imbalance (MacKinnon, 2008).

Furthermore, the standard z test has low coverage due to its assumption of normality. It was found that when values of parameters a and b were small, the coverage for the indirect effect estimated with the standard z test was low, and the coverage rate converged around .95 when a and b increased (MacKinnon et al., 2004; Preacher & Selig, 2012). Yet, coverage was equal to 1.0 when a and b were zero. Similar results were also found for the coverage rates of the standardized indirect effect (Cheung, 2009).

This study aims to demonstrate how the *distribution* of the product of coefficients explains the results of previous simulation studies on mediation confidence interval estimation performance. There are numerous methods to compute confidence intervals of ab . The goal of this study is not comparing those methods as this has been studied in the past. Instead, this study focuses on the normal theory confidence limits and aims to demonstrate that the inaccuracy in the standard z test for mediation confidence interval estimation occurs when the nonnormal distribution of the product is most nonnormal.

MOMENTS OF THE DISTRIBUTION OF THE PRODUCT

When computing the product of two normally distributed variables, if both variables have means equal to zero leading to a product equal to zero, the distribution of the product is symmetric with a kurtosis of six (Craig, 1936). In the case of nonzero means, the distribution of the product is skewed and also has excess kurtosis. The estimator for the indirect effect corresponds to the product of two regression coefficients often assumed to have a normal distribution. The ratios of the means to standard errors of each random variable are $z_a = a/s_a$ and $z_b = b/s_b$. The four central moments of the product of correlated (ρ) z_a and z_b are then as follows (Cornwell, Aroian, & Taneja, 1978, p. 127; Craig, 1936; Meeker et al., 1981):

$$\text{Mean} = \mu = z_a z_b + \rho \quad (6)$$

$$\text{Variance} = \sigma_y^2 = z_a^2 + z_b^2 + 2z_a z_b \rho + 1 + \rho^2 \quad (7)$$

$$\text{Skewness} = v_{3,y} = \{6[\rho(z_a^2 + z_b^2) + z_a z_b(1 + \rho^2)] + 2\rho(3 + \rho^2)\} / \sigma_y^3 \quad (8)$$

$$\text{Kurtosis} = v_{4,y} - 3 = \{12(z_a^2 + z_b^2)(1 + 3\rho^2)\} + 24z_a z_b \rho(3 + \rho^2) + 6(1 + 6\rho^2 + \rho^4) / \sigma_y^4 \quad (9)$$

where ρ is the correlation between z_a and z_b .

Single mediator model indirect effects have uncorrelated a and b coefficients except for latent variable mediation models (MacKinnon, 2008). For uncorrelated z_a and z_b (i.e., $\rho = 0$) the four moments reduce to (Cornwell et al., 1978; Craig, 1936)

$$\text{Mean} = \mu = z_a z_b \quad (10)$$

$$\text{Variance} = \sigma^2 = z_a^2 + z_b^2 + 1 \quad (11)$$

$$\text{Skewness} = v_3 = 6(z_a z_b) / (z_a^2 + z_b^2 + 1)^{3/2} \quad (12)$$

$$\text{Kurtosis} = v_4 = \{12(z_a^2 + z_b^2) + 6\} / (z_a^2 + z_b^2 + 1)^2 \quad (13)$$

Following the aforementioned formulas for the two uncorrelated variables, when the product of the means of the two random variables is nonzero, the distribution is skewed (up to a maximum value of ± 1.5) and has kurtosis (up to a maximum value of 6); the product approaches a normal distribution as one of or both z_a and z_b get large in absolute value (Aroian, 1947; Aroian, Taneja, & Cornwell, 1978). Aroian (1947) showed that the distribution of the product approaches the gamma distribution. The analytical solution for the distribution of the product is provided by the Bessel function used for the solution of differential equations. The distribution of the product is approximately proportional to the Bessel function of the second kind with a purely imaginary argument (Aroian, 1947; Craig, 1936). Figure 2 shows the plots of true moments for independent z_a and z_b ranging between -5 and 5 that include the common values in social sciences. As can be seen from Figure 2, kurtosis and skewness of the distribution of the product increase as the values of z_a and z_b get closer to zero.

In this study, we investigated how sample size; effect sizes of a , b , and c' ; and the four moments of ab influenced confidence interval estimation performance of the indirect effect in a single mediator model using a Monte Carlo simulation.

METHOD

The SAS 9.2 programming software was used for the simulation. X , M , and Y were continuous variables. X was generated from the normal distribution using the RANNOR function. M was produced from X and RANNOR function to add random error. Y was produced from X and M and again the RANNOR function to add random error. The

regression parameters (a , b , and c') reflecting the relation between the variables X , M , and Y varied as $-.59$, $-.39$, $-.14$, 0 , $.14$, $.39$, and $.59$ corresponding to positive and negative values of Cohen's criteria for zero, small (2% of the variance), medium (13% of the variance), and large (26% of the variance) effect sizes, respectively (Cohen, 1988, pp. 412–414; corresponding to partial correlations of 0, 0.14, 0.36, and 0.51, respectively). The simulation was conducted for sample sizes of 50, 200, 500, 1,000, and 5,000 per condition. A total of 1,000 replications of each condition were simulated. To summarize, a 5 (sample size) $\times$ 7 (a path) $\times$ 7 (b path) $\times$ 7 (c' path) factorial design with a total of 1711 different conditions was used in the simulation study.

For each simulated sample, the indirect effect was computed as the product of a and b . The four moments of ab were calculated using the aforementioned formulas 10–13 for the case of uncorrelated a and b . Ninety-five percent confidence limits for ab were obtained by using Equation 5 with the z value of 1.96. Two dependent variables were computed: confidence interval coverage and confidence interval imbalance. Confidence interval imbalance was computed by subtracting the proportion of times the true value of ab was above the upper confidence limit from the proportion of times the true value of ab was below the lower confidence limit. Coverage was computed as the proportion of 95% confidence intervals that contained the true value of ab .

RESULTS

Figure 3 shows the relationship between the moments of the product of coefficients and coverage and imbalance. There is a curvilinear relationship between skewness of the product of coefficients and coverage. As skewness gets closer to 0, coverage gets closer to .95; however when skewness is equal 0, coverage is 1. As skewness increases in absolute value, coverage decreases. The data points in the graph where the skewness is equal to its maximum value (i.e., ± 1.15) correspond to z_a and z_b with small effect size values (those are also the values where kurtosis is equal to 3.5). Similarly, as kurtosis gets closer to 0, coverage gets closer to .95. When kurtosis reaches its maximum value of 6.0, coverage becomes equal to 1. Also, as skewness in absolute value and kurtosis increase, imbalance increases.

We conducted hierarchical regression analyses to predict the coverage and imbalance of the product of coefficients (i.e., ab) by the four moments (see Table 1). At the first step, sample size; critical ratios for a , b , and c' ; and mean and variance of ab were entered into the regression equation. At the second step, skewness, kurtosis, and their interaction were added, and at the third step the square of skewness and its interaction with kurtosis were added into the equation to investigate the curvilinear relationship of skewness with the coverage and imbalance as shown by Figure 3. As shown in Table 1, imbalance increases as sample size decreases. For imbalance, the explained variance (R^2) significantly increases from .13 to .86 when the square of skewness and its interaction with kurtosis are added into the equation. Imbalance has a significant curvilinear relationship with skewness and also interacts with kurtosis. Moreover, results show that as kurtosis increases coverage increases significantly. Coverage also has a significant curvilinear relationship with skewness and its interaction with kurtosis that leads to a significant increase in the variance explained (R^2

increases from .27 to .76). Furthermore, we also added the first and second derivatives of the skewness and kurtosis to the regression equations to see if the slope and acceleration of the moments of the product of coefficients predict coverage and imbalance. We did not report those results for the derivatives in Table 1 because their effects were close to zero and nonsignificant. Figure 3 and the regression analyses demonstrate that the failure of the normal theory approach to confidence interval construction for mediation can be explained by the moments of the underlying distribution of the product of coefficients. Results explain how and why the imbalance and coverage issues for the normal theory confidence limits occur.

EMPIRICAL EXAMPLE

The following example illustrates the disparity between the normal theory confidence limits and the asymmetric distribution of the product confidence limits. The Adolescent Training and Learning to Avoid Steroids (ATLAS) program was designed to prevent the use of anabolic androgenic steroids among high school athletes (Goldberg et al., 1996; MacKinnon et al., 2004). The data for this example come from a total of 861 athletes. The data were also used for illustration in MacKinnon et al. (2004). One of the hypotheses of the study was that X-exposure to the program would increase M-the perceived severity of steroid use, which would in turn increase Y-proper nutrition behaviors.

Ninety-five percent confidence limits were computed using the observed estimates of a , b , σ_a , and σ_b (0.27309, 0.07357, 0.08942, and 0.03004, respectively). The standard error of the indirect effect for the normal theory confidence limits was calculated using the multivariate delta method (Sobel, 1982). The four moments for ab in this example using z ratios and assuming independent a and b were mean $\mu = 7.4795$, variance $\sigma^2 = 16.3249$, skewness $\nu_3 = 0.6804$, and kurtosis $\nu_4 = 0.7126$.

The normal theory produced a 95% confidence interval between -0.001 and 0.041 , whereas the asymmetric distribution of the product confidence limits were 0.003 and 0.045 (MacKinnon et al., 2007; Tofighi & MacKinnon, 2011; see Figure 4). The former confidence interval includes the value of zero, thus leading to the conclusion that the indirect effect was nonsignificant, whereas the latter indicated there was a significant indirect effect in the data. From the RMediation plot, one can see that the distribution of the indirect effect for the ATLAS data has a slight positive skew and kurtosis greater than zero, thus making the normal theory confidence limits less accurate than the asymmetric distribution of the product confidence limits.

DISCUSSION

In this article, we investigated how the moments of the distribution of the product contribute to confidence interval coverage and imbalance for the indirect effect. Results of the current simulation study showed that as skewness in absolute value increases, coverage decreases. And as skewness in absolute value and kurtosis increase, imbalance increases. Our results explain the reason for the cases when imbalance and low coverage of the estimated indirect

effect occur in previous simulation studies (MacKinnon et al., 2004; MacKinnon et al., 1995; Preacher & Selig, 2012).

We furthermore illustrated the difference between testing the significance of the indirect effect using normal theory versus asymmetric distribution of the product. These results are important because they explain the observed improvement in confidence interval coverage when the distribution of the product or the bootstrap is used as the product ab does not have a normal distribution but follows the distribution of the product. In this way, it explains the results of statistical simulation studies of indirect effects.

This study emphasizes that the distribution of the product of coefficients should be taken into account when testing indirect effects in order to reach accurate conclusions. Normal theory confidence limits assume a normal distribution for the indirect effect although that may not be true. Ignoring the distribution of the product may alter Type I error rates and statistical power. Currently there are a few methods to improve confidence limits for the indirect effect. PROD-CLIN computes asymmetric confidence intervals on the basis of the distribution of the product (MacKinnon et al., 2007; also available in R software, Tofighi & MacKinnon, 2011). However, the current version of PRODCLIN only computes confidence limits for the indirect effects consisting of two regression coefficients. There is need for the computation of asymmetrical confidence limits on the basis of the distribution of the product of three or more paths. Bootstrap methods to estimate confidence limits for the indirect effect in both moderated and unmoderated multiple mediator models are available (PROCESS macro available in SAS and SPSS; Hayes, 2013). Furthermore, a Monte Carlo method for k paths is available in SAS (Wurpts & MacKinnon, 2013) and is being added to the RMediation program (Tofighi & MacKinnon, 2013). Methods such as the Monte Carlo method, which does not have parametric assumptions for ab , and various bootstrapping methods have also produced better results, especially for small sample sizes (MacKinnon et al., 2004). Yet, bootstrapping methods require raw data that may not always be available. Also, some resampling methods for the indirect effect have been shown to have inflated Type I error rates (Fritz, Taylor, & MacKinnon, 2012; MacKinnon et al., 2004). Another alternative to compute confidence limits for the indirect effect is the permutation test method that requires extensive computation but also generates more accurate confidence intervals and significance testing because it does not require that the indirect effect divided by its standard error has a normal distribution (Taylor & MacKinnon, 2012).

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FIGURE 1.
Single mediation model.

(a) Distribution of the product plot of kurtosis by z_a and z_b

(b) Distribution of the product plot of skewness by z_a and z_b

FIGURE 2.

Distribution of the product of coefficients by z_a and z_b (color figure available online).

FIGURE 3.

The relationship between the moments of the distribution of the product and confidence interval coverage and imbalance.

FIGURE 4.
RMeditation plot for the ATLAS (The Adolescent Training and Learning to Avoid Steroids)
data distribution of the product.

TABLE 1
Summary of Regression Analyses Predicting Confidence Interval Coverage and Imbalance

Variable	Imbalance			Coverage		
	β	t	R^2	β	t	R^2
Step 1						
N	-.31**	-8.53		.05	1.39	
a	.01	.33		.00	.06	
b	-.00	-.04		.01	.57	
c'	.00	.06		-.01	-.36	
Mean (ab)	.00	.05		-.01	-.36	
Variance (ab)	-.05	-1.50	.12	-.1**	-2.67	.01
Step 2						
Skewness (ab)	.02	.60		.01	.33	
Kurtosis (ab)	-.04	-1.81		.54**	25.01	
Skewness (ab)	-.01	-.35	.13	-.00	-.12	.27
* Kurtosis (ab)						
Step 3						
Skewness ² (ab)	1.72**	86.53		-1.34**	-51.13	
Skewness ² (ab)	-1.12**	-56.48	.86	.81**	30.96	.76
* Kurtosis (ab)						
Step 4						
Skewness ³ (ab)	.04	.68		-.06	-.87	
Skewness ³ (ab)	-.02	-.26	.86	.09	1.17	.76
* Kurtosis (ab)						

Note. a , b , and c' are critical ratios (e.g., z_a).

* $p < .05$.

** $p < .001$.