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Cite this article: Zhou BZ, Wu GX. 2015 Resonance of a tension leg platform excited by third-harmonic force in nonlinear regular waves. *Phil. Trans. R. Soc. A* **373**: 20140105. <http://dx.doi.org/10.1098/rsta.2014.0105>

One contribution of 12 to a Theme Issue 'Advances in fluid mechanics for offshore engineering: a modelling perspective'.

Subject Areas:

ocean engineering

Keywords:

tension leg platform, third-harmonic resonance, floating body/wave interaction, fully nonlinear theory, higher order boundary element method

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Resonance of a tension leg platform excited by third-harmonic force in nonlinear regular waves

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The resonance of a floating tension leg platform (TLP) excited by the third-harmonic force of a regular wave is investigated based on fully nonlinear theory with a higher order boundary element method (BEM). The total wave elevation and the total velocity potential are separated into two parts, based on the incoming wave from infinity and the disturbed potential by the body. A numerical radiation condition is then applied at the far field to absorb the disturbed potential without affecting the incident potential. The BEM mesh on the free surface is generated only once at the initial time and the element nodes are rearranged subsequently without changing their connectivity by using a spring analysis method. Through some auxiliary functions, the mutual dependence of fluid/structure motions is decoupled, which allows the body acceleration to be obtained without the knowledge of the pressure distribution. Numerical simulation is carried out for the interaction of a floating TLP with waves. The focus is on the motion principally excited by higher harmonic wave forces. In particular, the resonance of the ISSC TLP generated by the third-order force at the triple wave frequency in regular waves is investigated, together with the tensions of the tendons.

1. Introduction

Ringling in the offshore industry usually refers to the transient response of a platform at frequencies much higher than the dominant component of the incoming wave. It is commonly shown by certain kinds of offshore structures such as tension leg platforms (TLPs) and gravity-based structures (GBSs) and has

been a major concern to operators and engineers since it was first observed in a model test of the Hutton TLP in the North Sea [1]. TLPs are generally designed to keep their natural frequencies in heave, pitch and roll degrees of freedom, several times above the dominant wave frequency, whereas natural frequencies in surge, sway and yaw degrees of freedom are designed to be lower than the dominant wave frequency. A report [2] funded by the UK Health and Safety Executive, addressed the ringing of a typical GBS with multiple-columns, which has its natural frequency well above the dominant wave frequency. Although TLPs and GBS with this kind of design avoid the linear resonance, ringing becomes important when the range of natural frequency is several times higher than the dominant wave frequencies. Typically when a transient wave passes through the platform, the structure will continue to oscillate over a period of time at its high natural frequency even when the wave excitation has diminished. This oscillation can be persistent when the damping level is low. Understanding of such ringing behaviour is important because of the high level of stress generated.

A closely related problem is springing, which usually refers to the resonant response of a platform in a stochastic sea state, when the stochastic properties of the motion have become steady. It is highly relevant to the fatigue analysis of the structure. As the natural frequency of a TLP or GBS is high, such resonance is unlikely to be excited by the linear force at the dominant frequency ω of the wave spectrum and it is more likely to be excited by the nonlinear force at $n\omega$, $n = 2, 3, \dots$. It is then evident that both ringing, the transient response, and springing, the steady response in the stochastic sense, are principally due to the higher harmonic components of the nonlinear wave force. Thus in the present work, we will consider the resonant behaviour of a TLP platform excited by the higher harmonic force of a regular wave. Although the regular wave may not truly reflect the stochastic nature of the real sea state, the results obtained can provide some insight into the ringing and springing behaviours of the platform. In particular, we shall use the high-order Stokes wave as the incoming wave together with the fully nonlinear analysis. The main frequency of the Stokes wave may be away from the high natural frequency of the structure, but the higher frequencies in its higher order terms can be close to the natural frequency. However, the significance of the higher modes diminishes gradually in moderately steep waves. As a result, resonant response of a structure typically occurs when its natural frequency is about three to five times the incoming wave frequency, or it is most likely to be excited by the third-order force at the triple wave frequency.

Extensive effort has been made previously to improve the understanding of the ringing and springing related to the high-frequency response and to develop numerical tools to improve the design. The Norwegian Petroleum Directorate and the UK Health and Safety Executive jointly funded a project named 'Higher order wave load effects on large volume structures' in 1993 [3], which focused on a series of field observations, experimental models and numerical simulation research of TLPs and GBS, and produced some very valuable results. Petrauskas & Liu [4] carried out two model tests on springing, one measuring springing forces on a vertical cylinder and the other measuring the response of a TLP. Extensive experiments have also been conducted to study the ringing phenomenon with a circular cylinder in the wave tank [5–7]. Kim *et al.* [8] and Zou *et al.* [9] also carried out the experiment with a model of the ISSC TLP to measure the force and then obtained the response of the platform through equations of motion. In the simulations, while there is work based on empirical equations [10], most research on higher order loads is based on the perturbation theory up to the third order. Typical examples include that by Faltinsen *et al.* [11] for a slender cylinder in long waves and that by Malenica & Molin [12] for a cylinder in finite water depth. Teng & Kato [13] also calculated the third-order wave load at the triple wave frequency on fixed axisymmetric bodies by monochromatic waves. We may note that the perturbation theory is valid only for moderate waves. In deep seas, an offshore structure is designed to operate in a very hostile environment. A more rational approach would be to adopt the fully nonlinear theory.

Much of the work on fully nonlinear wave interactions with three-dimensional structures is based on a numerical wave tank with a wave maker on one side and an absorbing beach on the other side. Wu & Hu [14], Wang *et al.* [15] and Yan & Ma [16] adopted this model with the finite-element method, while Liu *et al.* [17], Bai & Eatock Taylor [18] and Yan & Liu [19]

Figure 1. Sketch of coordinate systems and computation domain.

used the model with the boundary element method (BEM). While this numerical tank reflects experimental practice well, it has similar problems to a physical tank when modelling the true ocean environment in the open sea, because of the way in which the wave is generated and the side wall effect. Ferrant [20] proposed a model for the open sea, in which the total potential is split into the incident potential and the disturbed potential. A model similar to this has been used by Ferrant [21], Ferrant *et al.* [22], Ducroz *et al.* [23] and Shao & Faltinsen [24].

In this work, we adopt the technique of Ferrant [20] to split the total potential. It attempts to investigate high-frequency resonance during wave interaction with an ISSC TLP by employing the fully nonlinear time domain numerical model in the open sea developed by Zhou *et al.* [25]. The adopted numerical model is verified by comparing the surge added mass of the TLP with the linear frequency-domain results, as well as by comparing amplitudes and phases of the first four harmonic forces on a fixed vertical cylinder with experimental data and other nonlinear numerical results. Numerical simulations are then carried out by setting the triple wave frequency at/close to the natural frequency of the TLP in the pitching mode. Comparison with the results for the body in a single degree of freedom is made to illustrate the coupling effects of the motion in different modes. Numerical results for the motions and tensions of the tendons are presented. In addition, a harmonic analysis of the time history of the fully nonlinear results is performed, from which the importance of higher order effects can be identified. Moreover, the effects of wave frequency on motions in the heave and pitch modes and on the tension of the tendon are analysed.

2. Mathematical model and numerical procedure

(a) Mathematical model

The problem of wave interaction with a TLP in an open sea with water depth d is sketched in figure 1. Two right-handed Cartesian coordinate systems are defined. One is a space-fixed coordinate system $oxyz$ with the oxy plane on the undisturbed free surface and with the z -axis being positive upwards. The other is a body-fixed coordinate system $o'x'y'z'$ with its origin o' placed at the centre of mass of the body. When the body is at its equilibrium position, these two sets of coordinate systems are parallel to each other. The centre of mass is located initially at $\mathbf{X}_{c0}$ in the space-fixed coordinate system, and $\mathbf{X}_c = \mathbf{X}_{c0} + \boldsymbol{\zeta}$ subsequently. Here, $\boldsymbol{\zeta} = (\zeta_1, \zeta_2, \zeta_3)$ is introduced to denote the translational displacements of the mass centre in the x -, y - and z directions, respectively. The rotation of the body is defined through the usual Euler angles $\boldsymbol{\theta} = (\alpha, \beta, \gamma) = (\zeta_4, \zeta_5, \zeta_6)$ to illustrate the displacements in roll, pitch and yaw, the terms commonly used in the naval architecture.

Based on the assumption that the fluid is ideal and incompressible, and flow is irrotational, the velocity potential $\phi(x, y, z, t)$ can be introduced, which satisfies the Laplace equation in the fluid domain R

$$\nabla^2 \phi = 0. \quad (2.1)$$

On the instantaneous free water surface S_F , the fully nonlinear kinematic and dynamic boundary conditions can be given in the following Lagrangian form:

$$\frac{D\mathbf{X}}{Dt} = \nabla \phi \quad (2.2)$$

and

$$\frac{D\phi}{Dt} = -g\eta + \frac{1}{2} \nabla \phi \cdot \nabla \phi, \quad (2.3)$$

where g represents the acceleration due to gravity, $\mathbf{X} = (x, y, z)$ denotes the position vector of a fluid particle on the free surface, η is the elevation of water surface measured from its mean level, $D/Dt = \partial/\partial t + \mathbf{u} \cdot \nabla$ is the total derivative with $\mathbf{u}$ being the velocity of the fluid particle. The boundary condition on the body surface S_B is

$$\frac{\partial \phi}{\partial n} = \mathbf{V} \cdot \mathbf{n}, \quad (2.4)$$

where $\mathbf{V}$ is the velocity of the body surface, $\mathbf{n}$ is the normal of the surface pointing out of the fluid domain, as shown in figure 1. The body surface velocity can be expanded as

$$\mathbf{V} = \mathbf{U} + \boldsymbol{\Omega} \times \mathbf{r}_b, \quad (2.5)$$

where $\mathbf{r}_b = \mathbf{X}'$ is the position vector in the body-fixed coordinate system, $\mathbf{U} = (U_1, U_2, U_3)$ is the translational velocity of the centre of mass, $\boldsymbol{\Omega} = (U_4, U_5, U_6)$ is the rotational velocity, which can be linked to the variation of the Euler angles [26].

Similar to Ferrant [20], the total wave elevation and the total velocity potential on the free surface can be separated arbitrarily into two parts, which can be written as

$$\eta(x, y, t) = \eta_a(x, y, t) + \eta_b(x, y, t) \quad (2.6)$$

and

$$\phi(x, y, \eta(x, y, t), t) = \phi_a(x, y, \eta(x, y, t), t) + \phi_b(x, y, \eta(x, y, t), t). \quad (2.7)$$

The total wave and potential should tend to the incident wave and potential at infinity. This can be achieved if we let

$$\eta_a \rightarrow 0; \quad \phi_a \rightarrow 0 \quad r \rightarrow \infty \quad (2.8)$$

and

$$\eta_b \rightarrow \eta_I; \quad \phi_b \rightarrow \phi_I \quad r \rightarrow \infty, \quad (2.9)$$

where $r = \sqrt{x^2 + y^2}$. Provided equations (2.8) and (2.9) are satisfied, there is no restriction on their individual choice. Thus, when the analytical solution is available for η_I and ϕ_I , such as the higher order Stokes waves, they can be the foundation for the choice of (ϕ_b, η_b) . We write

$$\eta_b(x, y, t) = \eta_I(x, y, t) \quad (2.10)$$

and

$$\phi_b(x, y, \eta(x, y, t), t) = \phi_I(x, y, \eta(x, y, t), t). \quad (2.11)$$

We note that the analytical expression for ϕ_I is for $z \leq \eta_I$. When $z = \eta$ in equation (2.11) is beyond this region, the potential is no longer the originally defined incident potential. In other words, ϕ_b strictly speaking is not the incident potential without the body. It has a contribution from the body due to the change in the wave elevation.

Substituting equations (2.6), (2.7), (2.10) and (2.11) into the governing equation and boundary conditions for ϕ , we obtain the following governing equation and the free surface boundary conditions of the boundary value problem for (ϕ_a, η_a) :

$$\nabla^2 \phi_a = 0 \quad \text{in } R, \quad (2.12)$$

$$\frac{Dx}{Dt} = \frac{\partial \phi_a}{\partial x} + \frac{\partial \phi_b}{\partial x} \quad \text{on } S_F, \quad (2.13)$$

$$\frac{Dy}{Dt} = \frac{\partial \phi_a}{\partial y} + \frac{\partial \phi_b}{\partial y} \quad \text{on } S_F, \quad (2.14)$$

$$\frac{D\eta_a}{Dt} = \frac{\partial \phi_a}{\partial z} + \frac{\partial \phi_b}{\partial z} - \frac{\partial \eta_b}{\partial t} - \nabla \phi_b \cdot \nabla \eta_b - \nabla \phi_a \cdot \nabla \eta_b \quad \text{on } S_F \quad (2.15)$$

and
$$\frac{D\phi_a}{Dt} = -g(\eta_b + \eta_a) - \frac{\partial \phi_a}{\partial t} + \frac{1}{2} |\nabla \phi_a|^2 - \frac{1}{2} |\nabla \phi_b|^2 \quad \text{on } S_F. \quad (2.16)$$

An artificial damping zone is applied on the outer annulus of a circular computational domain to absorb the scattered wave. In this study, both ϕ - and η -type damping terms are added to the free surface conditions in equations (2.13)–(2.16) [25].

On the body surface S_B , the boundary condition for ϕ_a is

$$\frac{\partial \phi_a}{\partial n} = \mathbf{V} \cdot \mathbf{n} - \frac{\partial \phi_b}{\partial n}. \quad (2.17)$$

To complete the boundary value problem, the initial conditions on the free surface can be given as

$$\begin{cases} \phi_a = 0 & z = \eta_b \\ \eta_a = 0 \end{cases} \quad t = 0. \quad (2.18)$$

In order to avoid an abrupt start and allow a gradual development of the body motion and disturbed potential, all the terms involving η_b and ϕ_b are multiplied by a modulation function.

In this study, the fifth-order Stokes wave [27] is used as the incoming wave, with the wave amplitude A being defined as half of the wave height or distance between wave peak and trough. Although the fifth-order incident wave does not satisfy the fully nonlinear free surface condition exactly, it is expected to be a good approximation for the fully nonlinear theory and can provide accurate results for the resonant behaviour driven principally by the third-harmonic force.

(b) Solution procedure

The higher order BEM is used to solve the mixed boundary value problem described above at each time step. The boundary surface is discretized by the quadratic isoparametric elements. In particular, the body surface is divided into several parts, and a mesh with eight-node structured quadrilateral elements or six-node structured triangle elements is generated on each part and these meshes are then unified. On the free surface, unstructured meshes are generated by the Gambit software [28] at the beginning of the calculation. In the Lagrangian form of the free surface boundary condition, the nodes of elements move both horizontally and vertically. It is possible that elements become distorted as time progresses. When the overall fluid domain does not change significantly, mesh rearrangement on the free surface is implemented by adopting the spring analogy method to obtain the horizontal coordinates of new nodes on the free surface. Those nodes on the intersections with the body surface and the far-field boundary are treated separately. Interpolation is then used to update the vertical coordinates and the potential at each new node [25]. To improve the efficiency of mesh regeneration on the body surface, elements between the upper surfaces of the pontoons at the bottom of the platform remain unchanged and the remeshing is applied only to the rest for the platform surface.

(c) Hydrodynamic forces and body motions

The equation of motions for a rigid body can be written as [26]

$$[M]\dot{\mathbf{U}} = \mathbf{F}_h + \mathbf{F}_e \quad (2.19)$$

and

$$[I]\dot{\boldsymbol{\Omega}} + \boldsymbol{\Omega} \times [I]\boldsymbol{\Omega} = \mathbf{N}_h + \mathbf{N}_e, \quad (2.20)$$

where $\mathbf{U}$ and $\boldsymbol{\Omega}$ are defined after equation (2.6), and $\dot{\mathbf{U}} = [a_1, a_2, a_3]^T$ and $\dot{\boldsymbol{\Omega}} = [a_4, a_5, a_6]^T$ indicate the translational acceleration and the angular acceleration, respectively. $[M]$ and $[I]$ in the equations are mass and rotational inertia matrixes, and $\mathbf{F}_e = [f_{e1}, f_{e2}, f_{e3}]^T$ and $\mathbf{N}_e = [f_{e4}, f_{e5}, f_{e6}]^T$ are the external force and moment about the centre of mass. As the rotational centre is at the centre of the body mass, there is no coupling between translational and rotational motions on the left-hand sides of equations (2.19) and (2.20). However, the coupling does exist implicitly on the right-hand sides. The hydrodynamic force $\mathbf{F}_h = (f_1, f_2, f_3)$ and moment $\mathbf{N}_h = (f_4, f_5, f_6)$ on the body can be obtained by integrating the pressure over its wetted surface

$$f_i = -\rho \iint_{S_B} \left(\frac{\partial \phi}{\partial t} + \frac{1}{2} |\nabla \phi|^2 + gz \right) n_i \, ds, \quad (2.21)$$

where ρ is the fluid density, $(n_1, n_2, n_3) = \mathbf{n}$, and $(n_4, n_5, n_6) = \mathbf{r}_b \times \mathbf{n}$. If the incoming wave is only in the x -direction and the body is symmetric about the x - z plane, $\boldsymbol{\Omega} \times [I]\boldsymbol{\Omega} = 0$, $\alpha = 0$, $\gamma = 0$.

An effective method for dealing with the temporal derivative in the hydrodynamic force is that proposed by Wu & Eatock Taylor [29]. In this approach, some auxiliary functions $\psi_i (i = 1, \dots, 6)$ are introduced. These functions satisfy the Laplace equation in the fluid domain, are zero on the free surface and $\partial \psi_i / \partial n = n_i$ on the body surface. Through the use of these functions, the equation of motion can be written as [29]

$$\sum_{j=1}^6 (m_{ij} + c_{ij}) a_j = Q_i - mg \delta_{i,3} + f_{ei}, \quad (2.22)$$

where

$$c_{ij} = \rho \iint_{S_B} \psi_i n_j \, ds \quad (2.23)$$

and

$$\begin{aligned} Q_i = & \rho \iint_{S_B} [\nabla \psi_i [(\mathbf{U} + \boldsymbol{\Omega} \times \mathbf{r}_b) \cdot \mathbf{n}] \cdot [\nabla \phi - (\mathbf{U} + \boldsymbol{\Omega} \times \mathbf{r}_b)] + \psi_i (\boldsymbol{\Omega} \times \mathbf{U}) \cdot \mathbf{n}] \, ds \\ & - \rho \iint_{S_B} \left(gz + \frac{\partial \phi_b}{\partial t} + \frac{1}{2} |\nabla \phi|^2 \right) n_i \, ds + \rho \iint_{S_B} \psi_i \frac{\partial (\phi_b)_t}{\partial n} n_i \, ds \\ & - \rho \iint_{S_F} \left[g(\eta_a + \eta_b) + \frac{\partial \phi_b}{\partial t} + \frac{1}{2} |\nabla \phi|^2 \right] \frac{\partial \psi_i}{\partial n} \, ds, \end{aligned} \quad (2.24)$$

where m_{ij} , $m_{i+3,j+3}$, $i, j = 1, 2, 3$ in equation (2.22) correspond, respectively, to M_{ij} of $[M]$ in equation (2.19) and I_{ij} of $[I]$ in equation (2.20), and $\delta_{i,j} = 1 (i = j)$ and $\delta_{i,j} = 0 (i \neq j)$. In this way, the acceleration of the body can be obtained directly once the potential has been found without knowledge of the pressure. By using these auxiliary functions, the fluid–structure interaction problem is decoupled, and can be solved more easily.

When the results from the simulations become periodic at t_0 with period T , the amplitude of each harmonic component of the force $F(t)$ can be obtained by Fourier analysis

$$f^{(m)} = \frac{2}{N_T \times T} \left| \int_{t_0}^{t_0 + N_T \times T} F(t) e^{im\omega t} \, dt \right|, \quad m = 1, 2, \dots \quad (2.25)$$

where N_T is an integer.

Figure 2. Configuration of the ISSC TLP: (a) plane view and (b) side view (unit: m).

3. The ISSC TLP

(a) Specification of the ISSC TLP prototype

The ISSC TLP [30] is chosen as the case study. Figure 2 shows the configuration of the structure, which consists of four circular cylindrical columns, four rectangular pontoons and four vertical tendons. One end of each tendon is tied at the centre of the column bottom, and the other end is fixed on the sea bottom. Its geometrical and dynamic data are summarized in table 1. As the incident wave considered here propagates only in the x -direction and the structure is symmetric, it responds only with motions in 3 d.f., namely surge, heave and pitch motions. In this case, the tensions of No. 1 tendon and No. 4 tendon are identical, and those of No. 2 tendon and No. 3 tendon are also the same. Thus, only the tensions of No. 1 and No. 2 will be provided.

(b) Forces on the tendon

The tendon is treated as a spring with no mass, and only the effect of the axial stiffness is considered. The stiffness of the tension leg system is assumed to be large enough that it keeps tendons in the tight state throughout the whole process of the movement. The force on the tendon is calculated as follows according to the location of the tendon:

$$F_c = k\Delta l + T_0, \quad (3.1)$$

where k is the stiffness coefficient of the tendon, Δl is the variation of the length due to the platform motions from all modes and T_0 is the pre-tension force of the tendon, which satisfies $4T_0 + mg = F_B$ to guarantee that it is in balance initially, where m is the body mass and F_B is the buoyancy. The direction of F_c from each tendon will be along its line. Δl is obtained based on all the translational motions of the platform as well as its rotational motions through the Euler angles. F_c will then lead to forces and moments on the platform in all modes and will be incorporated into f_{ei} in equation (2.22).

In the space-fixed system, let $O_1(x_1, y_1, z_1)$ and $O_2(x_2, y_2, z_2)$ be the coordinates of the two ends of the tendon tied at the pontoon and the sea bed, respectively. The latter will be $O_2(x'_2, y'_2, z'_2)$ in the body-fixed system. As $\zeta_2 = \zeta_4 = \zeta_6 = 0$, the variation of the length can be obtained as

$$\Delta l = \sqrt{P^2 + (Q + l_0)^2} - l_0, \quad (3.2)$$

where

$$P = \zeta_1 \pm a(\cos \zeta_5 - 1) - \bar{h} \sin \zeta_5; \quad Q = \zeta_3 - (\pm a) \sin \zeta_5 + \bar{h}(1 - \cos \zeta_5), \quad (3.3)$$

Table 1. Main particulars of the ISSC TLP.

parameter	value
spacing between column centres	86.25 m
column radius	8.435 m
total height	67.5 m
draught	35.0 m
water depth	450.0 m
pontoon width	7.5 m
pontoon height	10.5 m
displacement	5.455×10^7 kg
weight	4.05×10^7 kg
vertical position of mass centre above keel	38.0 m
vertical position of rotation centre above keel	38.0 m
roll moment of inertia	8.237×10^{10} kg m ²
pitch moment of inertia	8.237×10^{10} kg m ²
yaw moment of inertia	9.807×10^{10} kg m ²
length of tendon	415.0 m
vertical stiffness of combined tendons	8.13×10^8 N m ⁻¹
roll and pitch effective stiffness	1.512×10^{12} N m rad ⁻¹

and the $\pm$ sign corresponds to tendons one and two, and a and $\bar{h}$ are the distances from the upper end of the tendon to the centre of mass in the x' - and z' directions, respectively.

(c) Natural frequencies of the platform

The natural frequencies are calculated according to the un-damped linear equation for free motion in frequency domain, or

$$\sum_{j=1}^6 \left[-\omega_n^2 (m_{i,j} + a_{m(i,j)}) + (K_{i,j} + C_{i,j}) \right] \zeta_j = 0, \quad (3.4)$$

where, $m_{i,j}$ are defined after equation (2.22), $a_{m(i,j)}$, $K_{i,j}$ and $C_{i,j}$ are coefficients of the added mass matrix, the stiffness matrix due to tendons and the hydrostatic restoring matrix, respectively. The coefficients $K_{i,j}$ of the stiffness matrix of TLP means the force or moment in mode i due to the unit displacement plus the pre-tension force in mode j . In the linear case, the equivalent stiffness matrix is obtained when only the first-order terms are retained

$$[K] = \begin{bmatrix} 4T_0/l_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4T_0 l_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4k & 0 & 0 & 0 \\ 0 & 4T_0 \bar{h}/l_0 & 0 & 4ka^2 & 0 & 0 \\ -4T_0 \bar{h}/l_0 & 0 & 0 & 0 & 4ka^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 8T_0 a^2/l_0 \end{bmatrix}.$$

The hydrostatic restoring coefficients of heave, roll and pitch in matrix $[C]$ are, respectively, $C_{33} = 8.97 \times 10^6$ N m⁻¹, $C_{44} = 3.165 \times 10^9$ N m · rad⁻¹ and $C_{55} = 3.165 \times 10^9$ N m · rad⁻¹. Other terms in matrix $[C]$ are zero.

Table 2. Natural frequencies of the platform.

	surge	sway	heave	roll	pitch	yaw
ω_n (rad s ⁻¹)	0.0612	0.0612	3.488	3.401	3.401	0.0764
$\omega_n \sqrt{R/g}$	0.0567	0.0567	3.235	3.154	3.154	0.0708

In order to get a non-zero solution of ζ_j from equation (3.1), the determinant of coefficient matrix has to be zero, from which the following four independent equations can be obtained:

$$-\omega^2(m + a_{m(33)}) + C_{33} + K_{33} = 0, \quad (3.5)$$

$$-\omega^2(m_{66} + a_{m(66)}) + K_{66} = 0, \quad (3.6)$$

$$(-\omega^2(m + a_{m(11)}) + K_{11})(-\omega^2(m_{55} + a_{m(55)}) + C_{55} + K_{55}) + \omega^2 a_{m(15)}(-\omega^2 a_{m(51)} + K_{51}) = 0 \quad (3.7)$$

$$\text{and } (-\omega^2(m + a_{m(22)}) + K_{22})(-\omega^2(m_{44} + a_{m(44)}) + C_{44} + K_{44}) + \omega^2 a_{m(24)}(-\omega^2 a_{m(42)} + K_{42}) = 0. \quad (3.8)$$

The solutions of these equations are obtained by iteration as $a_{m(i,j)}$ are a function of ω . It can be seen that from equations (3.5) and (3.6), the heave motion and the yaw motion are both fully independent of other modes and their natural frequencies can be obtained from these equations, respectively. The surge motion and the pitch motion are coupled, as illustrated in equation (3.7). Two positive roots are 0.0612 and 3.401 rad s⁻¹. If only a single degree of motion is allowed, equations for surge and pitch natural frequencies will be $(-\omega^2(m + a_{m(11)}) + K_{11}) = 0$ and $(-\omega^2(m_{55} + a_{m(55)}) + C_{55} + K_{55}) = 0$, respectively. This gives the natural frequencies of the independent surge and pitch motions as $\omega'_{n1} = 0.0612$ rad s⁻¹ and $\omega'_{n5} = 3.254$ rad s⁻¹, respectively. This indicates that the surge natural frequency is hardly influenced by the pitch motion, while the pitch natural frequency becomes slightly larger when the motion is coupled with surge. Similar analysis can be made for sway and roll in equation (3.8). Table 2 lists the solutions of equations (3.5)–(3.8), where R is the radius of the column. It can be seen from the table that the natural frequencies in surge, sway and yaw are very small, whereas those in heave, roll and pitch are very large. It ought to point out that the natural frequencies in surge and pitch are from their coupled motions, and the same applies to sway and roll. For simplicity, they are named as the natural frequencies for the individual modes.

(d) Damping matrix

When a system is excited from the rest near its natural frequency, it will take many periods before the motion becomes periodic if the damping of the system is small. Here, a large part of the platform is far below the free surface and its water plane is wall sided. Therefore, the damping through wave radiation, in heave for example, in the potential theory is relatively small. Thus, extensive simulation over many cycles of a periodic wave would be required for the body motion to become periodic. Also the potential theory has ignored the viscous damping which can be important relative to radiation damping in such a case. Thus, additional damping $B_{i,i}$ is introduced in the calculation based on the empirical equation $B_{i,i} = \alpha_i m_{i,i}$, where $\alpha_i = 2d_i(2\pi/T_{0i})$ is the coefficient and the d_i is usually chosen empirically. Here, we take $d_1 = 0.6$, $d_3 = 0.01$ and $d_5 = 0.01$, which have been chosen as small as possible on the basis that the motions have reached a periodic state after 30 periods through numerical tests. Thus, the damping coefficients of surge, heave and pitch in matrix $[B]$ are, respectively, $B_{11} = 3.14 \times 10^6$ N · s m⁻¹, $B_{33} = 2.83 \times 10^6$ N · s m⁻¹ and $B_{55} = 5.6 \times 10^9$ N · m · s rad⁻¹. Other terms in matrix $[B]$ are zero. The damping force, that is, $f_{ei} = -B_{i,i}U_i$, $i = 1, \dots, 6$, is then added to the external force f_{ei} in equation (2.22).

Figure 3. Comparison of the surge added mass. (Online version in colour.)

4. Numerical results

(a) Verification through comparison with linear frequency-domain results

The ISSC TLP has been used extensively as a benchmark for case studies. A well-known example is the exercise in 1987 [30]. It was found at the time that the first-order surge added mass from the calculations of 17 organizations differed from each other significantly. Newman & Scлавounos [31] subsequently showed that the difference was very much due to the insufficient number of elements used because of computer power. They then provided updated results. Here, we use the same case for comparison. We impose a surge displacement $\zeta_1 = S_1 \sin(\omega t)$ for the structure and obtain the surge force $F(t)$ when it has become periodic at $t > t_0$. For the linear case, the force can be decomposed into a term of added mass with acceleration and another term of damping with velocity, from which the surge added mass can be found from Fourier analysis.

Figure 3 presents the comparison between the present numerical results and the linear frequency-domain solutions for $S_1/R = 0.01$, where R is the radius of the platform column. The results in [31] are recalculated by the demonstration program of the WAMIT code [32]. Within each quadrant of the platform, 1145 higher order elements are adopted for TEST14 in the WAMT demonstration program, and 630 higher order elements (eight nodes for quadrilateral elements and six for triangle elements) are used in the recalculation based on the code developed by Teng & Eatock Taylor [33]. Elements of similar size to the latter are used on the body surface in the present calculation. The time step is selected as $\Delta t = T/40$, where $T = 2\pi/\omega$. Elements of similar size to that on the body surface are used on the free surface near the body and they increase gradually away from the body; the increase ratio depends on the wavelength. It can be seen that the present results are in good agreement with the solutions from the code of Teng & Eatock Taylor [33]. This may be expected as the same mesh is used on the body surface in these two calculations. However, the present calculation is made in the time domain with elements on both the body surface and the free surface, while the solutions using Teng and Eatock Taylor's code in the frequency domain are made with elements on the body surface only. Results from 630 elements using Teng and Eatock Taylor's code are also provided in the figure and good agreement between results from different meshes can be seen. This suggests that 630 elements in the present calculation can provide sufficiently accurate results. Comparison with the results calculated using the WAMIT code shows no visible graphical difference.

(b) Verification through comparison with nonlinear results

We can further check the accuracy of the present results through comparison with other fully nonlinear results. The wave diffraction around a bottom-mounted free-surface piercing vertical

Figure 4. The initial meshes on the free surface and the cylinder for Mesh b: (a) free surface and (b) cylinder surface.

Figure 5. Comparison of the first-harmonic horizontal force: (a) amplitude of the force and (b) phase of the force. (Online version in colour.)

circular cylinder with radius of R and water depth of $d/R = 20$ at wavenumber $kR = 0.245$ is considered. This is the same case which has been studied experimentally by Huseby & Grue [34] and numerically by Ferrant [21] and Shao & Faltinsen [24] using a fully nonlinear theory.

Two different meshes are used in the present calculation. Mesh a has 664 elements and 2209 nodes on the full boundary, and Mesh b has 1320 and 4255, respectively. The initial meshes on the free surface and the cylinder for Mesh b are shown in figure 4. The ratio between the radius of the outer truncated cylinder radius and R is 46 and the radial length of the damping zone is set to be one wavelength. Figures 5–8 show the results for the first-, second-, third- and fourth-harmonic forces, respectively. It can be seen from these figures that the present results from the two meshes are almost graphically identical. To check whether the fifth-order Stokes incident wave is a good approximation for the present case, we have run the simulation with Mesh b using the twentieth-order Stokes wave [35], and it makes hardly any visible difference to the results. Comparisons with the experimental results of Huseby & Grue [34], and fully nonlinear numerical results of Ferrant [21] and Shao & Faltinsen [24] show that they are in fairly good agreement. There is some discrepancy between all the numerical results and the experimental data in the second-harmonic forces. As commented by Shao & Faltinsen [24], the free second-harmonic waves which originate at the wave maker in the experiment may be the reason.

Figure 6. Comparison of the second-harmonic horizontal force: (a) amplitude of the phase and (b) phase of the force. (Online version in colour.)

Figure 7. Comparison of the third-harmonic horizontal force: (a) amplitude of the force and (b) phase of the force. (Online version in colour.)

(c) Analysis of platform motion near the pitch natural frequency

To undertake the convergence study for the higher order force on the ISSC platform, we run a simulation for uncoupled pitch motion at $A/R = 0.12$ and $\omega\sqrt{R/g} = \omega'_{n5}\sqrt{R/g}/3 = 1.006$. The damping zone starts from a horizontal distance at $D_{dc} = 15R$ to the centre of the space-fixed system and the radial length of the damping zone is set to be one linear wavelength $\lambda = 5.7R$. Two different meshes are chosen. There are 1775 elements and 11 395 nodes on the full boundary in Mesh a, as shown in figure 9, and the number of elements and nodes of Mesh b is 3486 and

Figure 8. Comparison of the fourth-harmonic horizontal force: (a) amplitude of the force and (b) phase of the force. (Online version in colour.)

Figure 9. The initial meshes on the free surface and the TLP for Mesh a: (a) free surface and (b) TLP surface.

Table 3. Error analysis of first three harmonic pitch amplitudes at different meshes and time steps.

	Mesh a ($\Delta t_1 = T/40$)	Mesh b ($\Delta t_1 = T/40$)	Mesh a ($\Delta t_2 = T/80$)	$ f_{ia} - f_{ib} /f_{ib}$ (%)	$ f_{i\Delta t_1} - f_{i\Delta t_2} /f_{i\Delta t_2}$ (%)
$\zeta_5^{(1)} R/A$	0.0002111	0.0002106	0.0002110	0.24	0.05
$\zeta_5^{(2)} R/A$	0.00003965	0.00003936	0.00003980	0.74	0.38
$\zeta_5^{(3)} R/A$	0.0002738	0.0002628	0.0002756	4.18	0.65

21 993. It can be seen from figure 10 that the curves from two meshes at $\Delta t = T/40$ and those from Mesh a at $\Delta t = T/40$ and for $\Delta t = T/80$ are graphically identical. The error analysis is presented in table 3. $|f_{ia} - f_{ib}|/f_{ib}$ in the table denotes the relative difference of results from two different meshes with $\Delta t = T/40$. $|f_{i\Delta t_1} - f_{i\Delta t_2}|/f_{i\Delta t_2}$ in the table denotes the relative difference between results from two different time steps with Mesh a. They are less than 5%. This shows that the result with Mesh a and time-interval $\Delta t = T/40$ is sufficiently accurate for these results.

Figure 10. Convergence study with mesh and time step for pitch motion at $\omega\sqrt{R/g} = 1.006$, $A/R = 0.12$. (Online version in colour.)

Based on table 2, we consider the case in which the frequency of the incident wave is set as $\omega\sqrt{R/g} = \omega_{n5}\sqrt{R/g}/3 = 1.051$, to highlight the third-harmonic resonance in pitch. Three cases with Stokes wave of amplitudes $A/R = 0.06, 0.12$ and 0.18 are calculated, corresponding to $A = 0.5, 1.0$ and 1.5 m defined as half of the wave height. The initial meshes on the free surface and the TLP shown in figure 9 are adopted. The time step is selected as $\Delta t = T/40$, where $T = 2\pi/\omega$.

Motions of the TLP at three different wave amplitudes are given in figure 11. Based on the linear theory, results should be independent to the wave amplitude when it is normalized by A . Thus, any difference in these cases is due to the nonlinear effect. Figure 11*a* shows that the drift motion in surge is large, while the drift motions in heave and pitch in figure 11*b,c* are very small. This is because the restoring force in surge is small, while large in heave and pitch. Table 2 shows that the surge natural frequency is much lower than that in pitch. When the wave frequency is near the pitch natural frequency, no resonance is expected to be excited by ω in surge and it is even far less likely by 2ω or 3ω . Figure 11*a* shows that the oscillatory motion of surge is dominated by the component of ω , while motions at 2ω and 3ω are small. The figure also shows that the drift motion of surge ζ_1/A is very much affected by A , as ζ_1 is dominated by the term proportional A^2 , in this case based on the perturbation theory. The amplitude of the oscillatory motion of ζ_1/A at ω is not very much affected by A , as it is dominated by the linear term. Figure 11*b,c* shows that heave and pitch motions are not dominated by a single frequency, even at a smaller wave amplitude. In fact, as the triple wave frequency is set at the natural frequency of pitch, which is also close to that of heave (table 2), the nonlinear components in the motions will be relatively more significant. Because of this, as A increases, the heave and pitch motions at 2ω and 3ω become more and more pronounced. However, compared with the surge motion, the overall motion amplitudes of heave or pitch are still much smaller. Thus, the motion of the platform has a component with lower frequency and larger amplitude, as well as larger drift, and a component with higher frequency and smaller amplitude.

To analyse the tensions in tendons caused by the platform motions, equation (3.2) can be written as

$$\Delta l = \frac{(P^2 + Q^2)x + 2Q}{\sqrt{(Px)^2 + (Qx + 1)^2 + 1}}, \quad (4.1)$$

where $x = 1/l_0$. As l_0 is much larger than the motion amplitude, the right-hand side of equation (4.1) can be expanded into the Taylor series of x and only the first two terms will be kept. We obtain

$$\Delta l \approx \frac{P^2}{2l_0} + Q = \frac{\zeta_1^2}{2l_0} + \frac{[\pm a(\cos \zeta_5 - 1)]^2}{2l_0} + \frac{(\bar{h} \sin \zeta_5)^2}{2l_0} + \zeta_3 - (\pm a) \sin \zeta_5 + \bar{h}(1 - \cos \zeta_5). \quad (4.2)$$

From figure 11, it can be seen that the maximum amplitudes of surge, heave and pitch are $|\zeta_1| \approx 1.0$, $|\zeta_3| \approx 1.9 \times 10^{-3}$ and $|\zeta_5| \approx 8.9 \times 10^{-5}$, respectively. Substituting them into equation (4.2), we can find that the magnitudes of the first, the fourth and the fifth terms are of order 10^{-3} , while the

Figure 11. Motions of the TLP in Stokes wave with $\omega\sqrt{R/g} = 1.051$ at different wave amplitudes: (a) surge, (b) heave and (c) pitch. (Online version in colour.)

remaining terms are of order 10^{-7} . As a result, equation (4.2) becomes

$$\Delta l \approx \frac{\zeta_1^2}{2l_0} + \zeta_3 - (\pm a)\zeta_5. \quad (4.3)$$

The three terms above correspond to the contributions from surge, heave and pitch, respectively, and the tether tension contributions can be denoted as $F_{c1} = k\zeta_1^2/2l_0$, $F_{c2} = k\zeta_3$ and $F_{c3} = ka\zeta_5$.

Equation (4.3) shows that Δl is proportional to heave and pitch motions linearly. Thus, their individual contributions to the tensions can be obtained by multiplying the results in figure 11*b,c* by k , respectively. The dominant term from surge in this particular example is of second order. It has been seen in figure 11*a* that the surge is dominated by the mean drift and the first-order oscillatory motions. Thus, it will contribute to the tensions in tendons in terms of a steady force, and oscillatory forces at the wave frequency and the double frequency. Equation (4.3) also shows that the surge and heave will have the same effect on tendons one and two. The effect of pitch has the opposite phase in terms of sign. To show the behaviour of the tensions of the tendons at the higher frequency resonance, figure 12*a,b* provides the tensions in tendons one and two, respectively, excluding T_0 , together with individual tension components in figure 13. It can be seen from figure 13 that the heave motion leads to a steady downward force. This is partly cancelled by the steady upward force caused by the surge. Thus, the mean force in the tendons shown in figure 12*a,b* is much smaller. The oscillatory components at single, double and triple frequencies at large wave amplitude are obvious in both figure 12*a,b*. However, due to different signs before the component due to the pitch in equation (4.3), the triple frequency component

Figure 12. Tensions of tendons one and two with $\omega\sqrt{R/g} = 1.051$ at different wave amplitudes: (a) one and (b) two. (Online version in colour.)

Figure 13. Tension components due to the different motions at $\omega\sqrt{R/g} = 1.051$, $A/R = 0.18$. (Online version in colour.)

has opposite phase in figure 12*a,b*, as the pitch motion is at near resonance excited by the triple frequency component in the wave. We note that the steady drift motion of surge is much larger than the oscillatory component. This leads to the observation that the oscillatory component of $F_{c1} = k\zeta_1^2/2l_0$ is dominated by the first-harmonic component and the second one is hardly visible in figure 13.

To show the coupling effects, results of coupled motion and uncoupled motion in each mode are compared, where the latter means that only a single degree of motion is allowed. The comparison in figure 14*a* at $\omega\sqrt{R/g} = 1.051$ and $A/R = 0.18$ shows that there is little difference between coupled and uncoupled surge motions. This means that the surge is hardly affected by the heave and pitch. This is expected as their motions are very small. Figure 14*c* shows that the uncoupled pitch motion is significantly different from the coupled one. In particular, the triple frequency component becomes far less obvious. The main reason for this is the change of the natural frequency. As discussed in §3, the coupled pitch natural frequency is $\omega_{n5}\sqrt{R/g} = 3.154$ which is different from the uncoupled pitch natural frequency $\omega'_{n5}\sqrt{R/g} = 3.018$. Thus the uncoupled motion in figure 14*c* is no longer at triple frequency resonance and it is therefore not

Figure 14. Coupled and uncoupled motions of the TLP at $\omega\sqrt{R/g} = 1.051$, $A/R = 0.18$: (a) surge, (b) heave and (c) pitch. (Online version in colour.)

surprising to see its triple frequency component of motion disappears. However, it can be seen from figure 14 that the pitch motions at 2ω and 3ω are very pronounced when the triple frequency is at its own natural frequency $\omega'_{n5}\sqrt{R/g} = 3\omega\sqrt{R/g} = 3.018$. Retuning to figure 14b, it can be seen that the uncoupled heave motion is also very much different from the coupled one. Part of reason is that the effect by the resonance of pitch motion at the triple wave frequency is no longer there. Another reason is related to what is shown by Wu [36]. For this TLP which is symmetric about the plane $x' = 0$, periodic symmetric pure surge motion will lead to vertical force with components of frequencies $2n\omega$, $n = 0, 1, 2, \dots$. Thus, the surge motion can significantly change the steady and double frequency vertical force components.

(d) Motions of the tension leg platform at the different wave frequencies

Table 2 has shown that the un-damped and linear heave and pitch natural frequencies are $\omega_{n3}\sqrt{R/g} = 3.235$ and $\omega_{n5}\sqrt{R/g} = 3.154$, respectively. Extensive simulations near these frequencies are made for the cases of $A/R = 0.18$. Results of motions are decomposed based on equation (2.25), after they have become periodic. The amplitudes of the first-, second- and third-harmonic motions are presented in figure 15a, b and c, respectively. It can be seen from figure 15a, that the surge motion is mainly from the dominant wave frequency as ω , 2ω and 3ω are all far away from its

Figure 15. Amplitudes of the first-, second- and third-harmonic motions versus wave frequency at $A/R = 0.18$: (a) surge, (b) heave and (c) pitch. (Online version in colour.)

Figure 16. Amplitudes of the first- and second-harmonic tensions of tendons one and two versus wave frequency at $A/R = 0.18$: (a) one and (b) two. (Online version in colour.)

natural frequency. It is most interesting to see from figure 15*b,c* that the maxima of the third-harmonic heave and pitch motions occur at $\omega\sqrt{R/g} \approx 1.081$ and $\omega\sqrt{R/g} \approx 1.055$, respectively, which implies the natural frequencies at 3.243 and 3.165, respectively. They are slightly different from the un-damped linear results in table 2. Figure 15*b* shows that the away from the resonant region the third-harmonic heave motion amplitude is significantly reduced and it is smaller than the first- and second-harmonic ones. Similar results for the pitch can be seen in figure 15*c*. All these show the importance of avoiding the triple wave frequency getting near the natural frequency of heave or pitch.

The amplitudes of the first-, second- and third-harmonic tensions of tendons one and two are shown in figure 16*a,b*, respectively. It can be seen from the figures that the variations of the first- and second-harmonic tensions of the tendons are smooth, while there are several peaks and troughs in the third-harmonic ones. From equation (4.3), it can be seen that the third-harmonic tensions of the tendons are due to the combined effect of the third-harmonic heave and pitch motions, as the contribution of the third-harmonic surge motion is negligible. From figure 15*b,c*, we can see that the natural pitch and heave frequencies are $3 \times \omega\sqrt{R/g} \approx 3 \times 1.055 = 3.165$ and $3 \times \omega\sqrt{R/g} \approx 3 \times 1.081 = 3.243$, respectively, which may be slightly different from the undamped natural frequencies in table 2. At $\omega\sqrt{R/g} \approx 1.055$, the maximum amplitude of $k\zeta_5^{(3)}/\rho gAR^2$ is about 0.5, while that of $k\zeta_3^{(3)}/\rho gAR^2$ is about 0.05. Thus the contribution to the tensions of the tendons at this frequency mainly comes from the pitch motion. As a result, the third-harmonic tensions of tendons one and two both show a maximum around $\omega\sqrt{R/g} \approx 1.055$. At $\omega\sqrt{R/g} \approx 1.081$, which is related to the heave resonance, the maximum amplitude of $k\zeta_3^{(3)}/\rho gAR^2$ is about 0.19, while that of $k\zeta_5^{(3)}/\rho gAR^2$ is about 0.14. Thus, the contributions to the tensions of the tendons from the heave and pitch motions are comparable. This means that the peak in heave does not automatically lead to a peak in the tensions. In fact, due to the combined effect of heave and pitch, the second local peak of the tension occurs at $\omega\sqrt{R/g} \approx 1.078$ for tendon one and $\omega\sqrt{R/g} \approx 1.085$ for tendon two, slightly away from the heave resonant frequency at $\omega\sqrt{R/g} \approx 1.081$. We may also note from figure 16*b* that there is a local trough at $\omega\sqrt{R/g} \approx 1.074$ for tendon two. This is because the amplitudes of $k\zeta_3^{(3)}/\rho gAR^2$ and $k\zeta_5^{(3)}/\rho gAR^2$ are comparable but their phases are almost opposite one another for tendon two, which leads to the partial cancellation in the tension.

5. Conclusion

A higher order BEM has been used to investigate the resonance of a floating TLP generated by the third-harmonic force in nonlinear regular waves. The comparison of the surge added mass at small amplitude motion, together with the comparison for the higher order force on a vertical cylinder suggest that the present model provides accurate results. The resonant behaviour generated by the third-order force at the triple wave frequency is well captured and investigated. It becomes more pronounced when the incoming wave amplitude increases as the triple frequency force is dominated by the term of the cube of the wave amplitude. When the triple frequency of the incoming wave is close to the higher natural frequency of the platform, its surge motion with its lower natural frequency will have large amplitude at wave frequency, as well as a large drift. Its heave and pitch motions with higher natural frequencies will be dominated by responses at the triple wave frequency, even though the amplitudes may be much smaller than that of the surge. The comparison between the results from the coupled motion and the uncoupled motion around this wave frequency shows that the surge motion is hardly affected by heave and pitch motions, while heave and pitch motions, as well as the pitch natural frequency, are very much affected by the surge. Extensive simulations have been made when the triple wave frequency is near the natural frequencies of the heave and pitch. It is noted that these natural frequencies are affected by the wave amplitude. The third-harmonic heave and pitch motion amplitudes are significantly reduced when the triple wave frequency is away from their corresponding natural frequencies. The effect of the heave and pitch on the tensions of the tendons is dominated by their linear terms, while the effect of the surge is dominated by the second-order term. The different phases of these motions can reinforce the tensions in some tendons while cause partial cancellation in others. In the design and operation of this type of offshore platform, it is important to avoid the high-frequency resonance and to adjust the phase difference of each motion, or to control their adverse effects.

Acknowledgements. The authors have greatly benefited from the most inspiring and helpful discussions with Profs R. Eatock Taylor and J. N. Newman. The valuable contribution from Prof. B. Teng and Dr Y. L. Shao is also gratefully acknowledged.

Funding statement. The authors gratefully acknowledge financial support from the Lloyd's Register Foundation (LRF) through the joint centre involving University College London, Shanghai Jiaotong University and Harbin Engineering University. The LRF helps to protect life and property by supporting engineering-related education, public engagement and the application of research. This work is also supported by the National Natural Science Foundation of China (51409066), and the China Postdoctoral Science Foundation (2014M550183).

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