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Discounting of Monetary Rewards that are Both Delayed and Probabilistic: Delay and Probability Combine Multiplicatively, not Additively

Ariana Vanderveldt, Leonard Green, and Joel Myerson

Washington University in St. Louis

Abstract

The value of an outcome is affected both by the delay until its receipt (delay discounting) and by the likelihood of its receipt (probability discounting). Despite being well-described by the same hyperboloid function, delay and probability discounting involve fundamentally different processes, as revealed, for example, by the differential effects of reward amount. Previous research has focused on the discounting of delayed and probabilistic rewards separately, with little research examining more complex situations in which rewards are both delayed and probabilistic. In two experiments, participants made choices between smaller rewards that were both immediate and certain and larger rewards that were both delayed and probabilistic. Analyses revealed significant interactions between delay and probability factors inconsistent with an additive model. In contrast, a hyperboloid discounting model in which delay and probability were combined multiplicatively provided an excellent fit to the data. These results suggest that the hyperboloid is a good descriptor of decision making in complicated monetary choice situations like those people encounter in everyday life.

Keywords

discounting; delay; probability; hyperboloid function; intertemporal; risk

When alternatives differ on only one dimension, choosing between them can be relatively easy: People almost always prefer a reward that is larger rather than smaller, immediate rather than delayed, and certain rather than probabilistic. Choosing between alternatives can be substantially more difficult, however, when they differ on two dimensions, as they frequently do outside the laboratory. Consider a choice between \$100 now and \$200 in 1 year. Although the larger amount is preferred over the smaller one, getting a reward sooner is preferred over getting one later. Choice may become even more difficult as the number of dimensions increases, for example, when the reward alternatives differ with respect to

Correspondence concerning this article should be addressed to Leonard Green, Department of Psychology, Washington University in St. Louis, St. Louis, MO, 63130. LGreen@wustl.edu.

Ariana Vanderveldt, Department of Psychology, Washington University in St. Louis.

Leonard Green, Department of Psychology, Washington University in St. Louis.

Joel Myerson, Department of Psychology, Washington University in St. Louis.

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amount, delay, and probability of receipt, potentially pitting one's preferences for larger, sooner, and more reliable against each other.

Much progress has been made toward understanding choice between rewards that differ on two dimensions (Green & Myerson, & Vanderveldt, 2014; Madden & Bickel, 2010). Considerable research has examined what economists call intertemporal and risky choice, also termed delay and probability discounting. This research has focused primarily on choices that involve immediate and delayed consequences, in the former case, and on choices that involve certain and probabilistic consequences, in the latter case. These areas are of fundamental concern for psychologists in a variety of areas in part because a tendency to discount the value of delayed outcomes or to ignore the odds against the occurrence of probabilistic outcomes is associated with issues of self-control, impulsivity, and risky behavior.

Research on delay discounting (for a review, see Green & Myerson, 2010) has consistently found that choices between immediate and delayed rewards are well described by a hyperboloid discounting function of the following form:

$$V_d = A / (1 + kD)^{s_d}, \quad (1)$$

where V_d is the subjective value (or present equivalent) of the delayed reward, A is its objective amount, D is the time until receipt of the reward, and k is a parameter reflecting the delay discounting rate, and the entire denominator is raised to the s_d power. Note that when $s_d = 1.0$, this equation reduces to the simple hyperbolic discounting function proposed by Mazur (1987). However, because s_d is typically less than 1.0, the degree of discounting tends to level off as delay increases, indicating that further increases would have relatively little effect on the subjective value of the delayed reward.

Just as delay discounting involves the devaluing of a reward as the time to its receipt increases, probability discounting involves the devaluing of a reward as the odds against its receipt increases (i.e., as the likelihood decreases). As with delay discounting, a hyperboloid function provides an excellent description of probability discounting:

$$V_p = A / (1 + h\theta)^{s_p}, \quad (2)$$

where V_p is the subjective value (or certain equivalent) of the probabilistic reward, A is its objective amount, θ is the odds against its receipt (which, expressed in terms of probability, is $(1-p)/p$), h is a parameter reflecting the probability discounting rate, and the entire denominator is raised to the s_p power. Because s_p , like s_d , is typically less than 1.0, the degree of discounting tends to level off as odds against increases indicating that further increases in the odds against (i.e., further decreases in probability) would have relatively little effect on the subjective value of the probabilistic reward.

As noted in Myerson, Green, and Morris (2011), the preceding equation may be understood in terms of prospect theory (Kahneman & Tversky, 1979), the dominant theoretical perspective in economics on risky choice. Prospect theory posits that the utility of a risky prospect (i.e., a probabilistic outcome) is determined by two underlying mathematical

functions that describe the separate effects of amount and probability. Kahneman and Tversky were agnostic as to the precise mathematical forms of these functions, which they termed a value function and a probability weighting function, respectively. Later, however, Tversky and Kahneman (1992) made working assumptions regarding the forms of these functions in order to better test their theory's predictions. Like prospect theory, Equation 2 may be decomposed into a value function, which (as in Tversky & Kahneman) is assumed to be a power function of amount, and a probability weighting function that has the form: $[1/(1 + h\theta)]^a$, which is simply a biased estimate of the probability, $p = 1/(1 + \theta)$, raised to a power.

For the purpose of comparison with other forms that have been proposed for the probability weighting function, $[1/(1 + h\theta)]^a$ may be rewritten in terms of probability as $\{1/[1 + h(1-p)/p]\}^a$. This probability weighting function has properties similar to those of other proposed forms in that, with appropriate parameter values, it generates an inverse-S-shaped relation between the weight and the probability like that reported in previous studies (e.g., Gonzalez & Wu, 1999; Prelec, 1998; Tversky & Kahneman, 1992).¹

As already noted, both the discounting of delayed rewards and the discounting of probabilistic rewards are well described by hyperboloid functions (Eqs. 1 & 2). Nevertheless, there are significant differences between delay and probability discounting. One important difference is the opposite effect that amount of reward has on the degree of discounting. In delay discounting, smaller amounts are discounted more steeply than larger ones. In probability discounting, the opposite effect is observed: larger amounts are discounted more steeply (Estle, Green, Myerson, & Holt, 2006; Green, Myerson, & Ostaszewski, 1999), contrary to what one might expect based on prospect theory. Moreover, changes in amount affect different parameters of the hyperboloid discounting function depending on whether the reward is delayed or probabilistic (Green, Myerson, Oliveira, & Chang, 2013; Myerson et al., 2011). In the case of delay discounting (Eq. 1), the rate parameter, k , decreases as reward amount increases, while the exponent, s_d , remains relatively constant. In the case of probability discounting (Eq. 2), however, the rate parameter, h , remains relatively constant while the exponent, s_p , increases as reward amount increases.

Although much is known about how delay and probability affect discounting in isolation, there have been only a few studies of discounting in which the rewards are both delayed and probabilistic. The paucity of previous research on the discounting of rewards that are both delayed and probabilistic is notable in part because so many everyday choices involve outcomes that are both delayed and uncertain. If one makes a financial investment, for example, there is the possibility, but not the guarantee, that it will pay off in the future. Similarly, if one chooses to smoke cigarettes, there is a chance that one will get cancer later on, but this is not certain. If we want to understand everyday decision making, it is clear that we need to study outcomes that are both delayed and probabilistic. What is unclear is how one should predict people's choices in such situations. For example, it is possible that delay

¹The exponent, s_p , in the probability discounting function is the ratio of the exponent, a , in the probability weighting function to the exponent of the value function (see the Appendix in Myerson et al., 2011).

and probability discounting functions combine in an additive fashion, as Killeen (2009) has suggested; alternatively, they may combine multiplicatively, in a fashion analogous to the multiplicative combination of value and probability weighting in prospect theory (Kahneman & Tversky, 1979). A multiplicative hyperbolic model has been proposed by Ho, Mobini, Chiang, Bradshaw, and Szabadi (1999).

Although the additive model proposed by Killeen (2009) assumed that discounting functions were exponential-power functions, it also is possible to formulate a model in which hyperboloid delay and probability discounting functions combine additively. An additive model of the discounting of delayed and probabilistic rewards assumes that the subjective value of such rewards equals the actual amount of the reward, A , minus both an amount, $f(D)$, which increases as the delay to the reward increases, and an amount, $g(\theta)$, which increases as the odds against increase:

$$V = A - f(D) - g(\theta). \quad (3)$$

According to the hyperboloid discounting model, the amount by which the value of the reward is to be decreased based on the delay is equal to $f(D) = A[1 - 1/(1+kD)^{sd}]$, and the amount by which it is to be decreased based on the odds against is $g(\theta) = A[1 - 1/(1+h\theta)^{sp}]$. Substituting for $f(D)$ and $g(\theta)$ in Equation 3 yields

$$V = A - A[1 - 1/(1+kD)^{sd}] - A[1 - 1/(1+h\theta)^{sp}]. \quad (4)$$

Note that when $\theta = 0$ (i.e., when rewards are certain), Equation 4 reduces to Equation 1, the hyperboloid delay discounting function, and when $D = 0$ (i.e., when rewards are immediate), it reduces to Equation 2, the corresponding probability discounting function.

Alternatively, delay and probability discounting may combine multiplicatively. Although Ho et al. (1999) proposed a multiplicative hyperbolic model, based on Equations 1 and 2 with $s = 1.0$, a multiplicative model of hyperboloid discounting also may be formulated. In this case, the subjective value of a delayed and/or probabilistic reward may be expressed:

$$V = A / [(1+kD)^{sd} * (1+h\theta)^{sp}]. \quad (5)$$

As was the case with the additive model, when rewards are immediate (i.e., $D = 0$), the multiplicative model (Eq. 5) reduces to probability discounting, and when rewards are certain (i.e., $\theta = 0$), the model reduces to delay discounting. Additive and multiplicative discounting models may be compared statistically by testing for an interaction between delay and probability (or odds against). An additive model assumes that the effects of delay and probability are independent: delay has the same effect regardless of probability and probability has the same effect regardless of delay. In contrast, a multiplicative model posits that the effect of delay depends on the probability of reward and vice versa. In particular, the hyperboloid multiplicative model presented above does not just predict that an interaction will be observed – it also predicts the form of that interaction: specifically, the effect of

delay will diminish as the odds against receipt increase, and the effect of probability will diminish as the delay to the possible reward increases.

To date, there has been only one study that examined discounting while systematically varying both delays and probabilities (Yi, de la Piedad, & Bickel, 2006). In this study, participants made choices between a delayed gamble with a specified probability of winning a larger reward and a certain, smaller reward to be received immediately. In their analysis, Yi et al. converted each probability to a supposedly equivalent “delay” based on Rachlin’s argument that delay discounting underlies probability discounting (Rachlin, Logue, Gibbon, & Frankel, 1986; Rachlin, Raineri, & Cross, 1991). They then added these ‘delays’ to the actual delays for each gamble, and fit a delay discounting function to the data. Although they found that a simple hyperbola (Eq. 1, but with s fixed at 1.0) provided a good description of the data, their analysis rests on the debatable assumption that probability discounting can be reduced to delay discounting, despite evidence that this assumption is incorrect (Green & Myerson, 2004; Myerson et al., 2011).

Stevenson (1992) examined the effect of risk on delay discounting of investments, where the risk concerns whether or not the investment will earn a profit (i.e., interest), rather than the risk of whether one will receive a reward or not as in typical probability discounting experiments. Stevenson observed that the effect of delay was significantly less when there was a risk that one would not earn a profit compared to the effect of delay when a profit was certain. A similar result was obtained by Ahlbrecht and Weber (1997), although they did not directly examine discounting functions. The present study tests the generality of Stevenson’s observation, specifically, whether delay discounting is steeper when rewards are certain compared with when there is a possibility that one may not receive a reward at all.

Several other studies have examined the discounting of rewards that are both delayed and probabilistic (Blackburn & El-Deredy, 2013; Reynolds & Schiffbauer, 2004; Weber & Chapman, 2005), but none of them varied both delay and probability parametrically, and thus they do not bear directly on the critical questions asked in the present study: (1) Does the same hyperboloid function that describes delay and probability discounting when they are examined separately also describe discounting when both delay and probability are varied simultaneously? (2) Do delay and probability discounting combine additively or multiplicatively in determining the subjective value of a delayed, probabilistic reward? Importantly, the present study addresses these critical questions in the context of a new mathematical model (Eq. 5) that not only describes the separate effects of delay and probability on the subjective value of an outcome, but also describes the effects of these attributes on subjective value when outcomes are both delayed and probabilistic, as they frequently are in everyday choice situations.

Experiment 1

Method

Participants—Fifty-four participants (34 females, 20 males; mean age = 21.4) were recruited from the Washington University Department of Psychology Human Subjects Pool. Participants were tested individually in a small room with a computer and received either

course credit or payment for their participation. Data from four participants (3 females) were excluded from the analyses because of their near-exclusive choice of immediate, certain rewards across conditions, but inclusion of their data did not affect the significance of any of the main effects or interactions in any of the ANOVAs reported below.

Materials and Procedure—Participants were instructed that they would be asked to make a series of choices between two hypothetical amounts of money shown on the computer screen. One amount could be received immediately and was guaranteed whereas the other amount was larger, but might be both delayed in time and probabilistic in its receipt. For example, a participant might be asked to choose between receiving \$300 right now for sure and an 80% chance of receiving \$800 in 6 months. Participants indicated which they preferred by pressing one of two keys. Before the experimental session began, participants were given four practice trials involving one amount and two delays crossed with two probabilities. The values used for the practice trials were similar, but not identical to those used in the experimental trials.

Five delays (0 [immediate], 1 month, 6 months, 2 years, and 5 years) were crossed with five probabilities (which were presented to participants as percent chance of reward: 10%, 25%, 40%, 80%, and 100% [certain]), at each of two amounts (\$800 and \$40,000). Each condition involved a unique combination of a delay, a probability, and an amount of delayed/probabilistic reward. When the delay was 0, the procedure reduced to a standard probability discounting task; when the probability was 100%, the procedure reduced to a standard delay discounting task.

The reward amount conditions were counter-balanced: Half of the participants experienced all of the conditions with the \$800 amount first, followed by all of the conditions with the \$40,000 amount; the other half of the participants experienced the \$40,000 amount conditions before the \$800 amount conditions. For each reward amount, the various delay and probability combinations were administered in a random order.

For each delay/probability-amount combination, participants made five choices. On the first choice, the smaller amount was half of the larger, delayed/probabilistic amount (i.e., \$400 in the \$800 conditions and \$20,000 in the \$40,000 conditions). For each subsequent choice in a condition, the amount of the smaller reward was adjusted based on the participant's previous choice (Du, Green, & Myerson, 2002). The size of each adjustment was half that of the preceding adjustment. For example, when given a choice between \$400 right now for sure and an 80% chance of \$800 in 6 months, if the participant chose the \$400, the amount of the smaller reward was decreased to \$200 on the next trial. If the participant instead had chosen the larger amount, the smaller reward was increased to \$600 on the next trial. If the participant then chose the \$600, the smaller amount on the third trial would be reduced to \$500 (half of the previous adjustment).

This titrating procedure converged on an indifference point (i.e., an immediate, certain amount that was approximately subjectively equivalent to the larger, delayed and probabilistic reward), estimated as the amount of smaller reward that would have been presented had there been an additional (sixth) trial (Du et al., 2002).

Results

Figure 1 presents the group mean indifference points from all of the conditions plotted first as a function of delay (left panels) and then again, this time as a function of odds against (right panels). The curved lines represent the fits of the hyperboloid discounting functions, with Equation 1 used to fit the data when it was plotted as a function of delay, and Equation 2 used to fit the data when it was plotted as a function of odds against.

Note that each of the five curves in the left panels corresponds to delay discounting at a different probability, and each of the five curves in the right panels corresponds to probability discounting at a different delay. Each curve in the left panels was obtained by fitting Equation 1 but with the value of A equal to the certain equivalent of the reward at the specified probability, so that the curve describes the effect of delay on rewards with that probability. For example, when fitting a delay discounting function to the indifference points for rewards that had a 40% chance of being received after different delays, A was equal to the subjective value of an immediate 40% chance of a reward (\$220, see Figure 1). Similarly, each curve in the right panels was obtained by fitting Equation 2 but with the value of A equal to the present equivalent of the reward at the specified delay, so that the curve describes the effect of probability on rewards at that delay. For example, when fitting a probability discounting function to the indifference points for rewards that were delayed by 2 years and had different probabilities, A was equal to the subjective value of a certain reward that would be received after a 2 year delay (\$610; see Figure 1).

As may be seen, delay discounting was observed when probabilities were high, but there was little or no effect of delay when probabilities were lower (see the left panels of Figure 1). In contrast, steep probability discounting was observed at all delays (see the right panels of Figure 1). However, the size of the decrease in subjective value with increases in the odds against receiving a reward was greater at shorter delays, as indicated by the fact that although the subjective values were similar at all delays when the odds against were high, the subjective values were lower at longer delays when the odds against were low.

Equation 1 provided good fits to the delay discounting data in five of the six cases where the likelihood of reward was 40% or higher, but failed to converge or provided parameter estimates that did not differ significantly from zero in the cases where the likelihood of reward was 25% or lower (see Table 1). In the five cases where good fits were not obtained, this reflected the fact that as probability decreased, subjective values showed little or no change with increases in delay (see the left panel of Fig. 1). In contrast, Equation 2 provided excellent fits to the probability discounting data in all 10 cases in Figure 1 (2 amounts $\times$ 5 delays); all R^2 s $> .99$ (see Table 1). In all the cases where the parameters of Equations 1 and 2 could be reliably estimated, s was less than 1.0, and significantly so in all but one case. This is consistent with previous findings showing that the hyperboloid typically provides a better description of delay and probability discounting than a simple hyperbola (McKerchar et al., 2009; Ostaszewski, Green, & Myerson, 1998).

In order to examine the effects of amount on delay and probability discounting when rewards are both delayed and probabilistic, we examined the areas under the delay and probability discounting curves. The area under the curve (AuC) provides a theoretically

neutral measure of the degree of discounting because it does not require assuming a particular mathematical form for the discounting function (Myerson, Green, & Warusawitharana, 2001). The AuCs for each probability condition and for each delay condition were normalized by converting indifference points into proportions of the objective, larger amounts (\$800 or \$40,000) and delays and odds against into proportions of the longest delay and highest odds against examined in the present study. As a result of this normalization, an AuC could range from 1.0 (indicating no discounting relative to the actual amount) to 0.0 (indicating complete discounting).

As may be seen in the left panel of Figure 2, which plots the normalized areas under the delay discounting curves shown in Figure 1, the degree of discounting increased (i.e., the AuCs decreased) as the probability of reward decreased. An ANOVA confirmed that the effect of probability was significant, $F(4,45) = 228.08, p < .001, \eta_p^2 = .95$. Although the main effect of amount was not significant, $F(1,48) = 2.20, p = .145$, there was an amount $\times$ probability interaction $F(4,45) = 4.62, p = .003, \eta_p^2 = .29$. Planned comparisons revealed that there was a significant effect of amount when the probability of reward was 1.00, reflecting the fact that discounting was steeper when the amount of reward was smaller, ($F(1,48) = 16.81, p < .001, \eta_p^2 = .26$) but there was no effect of amount at lower probabilities, all F s < 2.22 , all p s $> .142$.

As may be seen in the right panel of Figure 2, which plots the normalized areas under the probability discounting curves shown in Figure 1, the degree of discounting tended to increase slightly as delay increased. An ANOVA revealed that the effect of delay was significant, $F(4,45) = 10.23, p < .001, \eta_p^2 = .48$. The main effect of amount approached significance, $F(1,48) = 3.63, p = .063$, and there was no significant amount $\times$ delay interaction, $F(4,45) = 1.66, p = .176$. Planned comparisons revealed significant effects of amount when the delay was 1 month and 6 months, reflecting the fact that discounting was steeper when the amount of reward was larger, $F(1,48) = 6.68, p = .013, \eta_p^2 = .12$, and $F(1,48) = 6.91, p = .011, \eta_p^2 = .13$, respectively, but there was no effect of amount at the other delays, all F s < 1.44 , all p s $> .237$.

The subjective values (i.e., indifference points) from all conditions were submitted to two repeated-measures ANOVAs, one for each reward amount, in order to determine whether, as predicted by a multiplicative model but not an additive model, there was an interaction between delay and probability. A significant interaction was observed at each amount: $F(16,34) = 3.42, p = .001, \eta_p^2 = .62$ for the \$800 amount; $F(16,34) = 2.58, p = .010, \eta_p^2 = .55$ for the \$40,000 amount.

As may be seen in Figure 3, which plots the group mean subjective values as a function of both delay and probability, the multiplicative model (Eq. 5) provided excellent fits at both amounts (both R^2 s = .997; Table 2 shows the values of the parameters for fits of Equation 5 to the data at each amount). Fits also were generally good at the individual level, with median R^2 s of .934 and .938 for the \$800 and \$40,000 amounts, respectively. As may be seen in Figure 4, which presents data from individuals for whom the R^2 s for fits of Equation

5 were at the 25th, 50th, and 75th percentiles, the multiplicative interaction is clearly apparent in individual data.

To determine whether simpler versions of the multiplicative model would provide equally good fits, the group data were fit with reduced models that had two rate parameters but no exponent (i.e., models based on simple hyperbolic discounting), as well as with a three-parameter hyperboloid in which the entire denominator is raised to the same power (rather than having separate exponents for the delay and probability discounting components as in the full model). The reduced two-parameter version of the multiplicative model is similar to that proposed by Ho et al. (1999), and may be written as

$$V = A / [(1 + kD) * (1 + h\theta)], \quad (6)$$

and the three-parameter version may be written as

$$V = A / [(1 + kD) * (1 + h\theta)]^S. \quad (7)$$

Because adding free parameters to a model increases the variance accounted for, differences between nested models like those just described often are compared using incremental-*F* tests to assess whether the increase in the explained variance is significantly greater than what would be expected simply as a result of the additional free parameters. When we did so, we found that the full four-parameter multiplicative model (Eq. 5) provided a significantly better fit to the data for both the \$800 and \$40,000 amounts two-parameter version (Eq. 6), both incremental-*F*s(2,21) > 30.75, *ps* < .001. The four-parameter multiplicative model also provided a significantly better fit than the three-parameter version (Eq. 7) for both amounts: *F*(1,21) = 18.53, *p* < .001, for the \$800 amount; *F*(1,21) = 4.50, *p* < .05, for the \$40,000 amount. These results strongly suggest that when outcomes are both delayed and probabilistic, discounting is best described by the multiplicative combination of separate hyperboloid delay and probability discounting functions, each with its own rate parameter and exponent (Eq. 5).

Discussion

The results of Experiment 1 demonstrate that the hyperboloid function, which describes the discounting of rewards that are either delayed or probabilistic when the effects of delay and probability are studied separately (Green & Myerson, 2004), also provides an excellent description of choices that are more complex and involve rewards that are both delayed and probabilistic. As for the question of whether, in such more complex situations, delay and probability discounting combine in an additive or a multiplicative fashion, the finding of a significant interaction between delay and probability at each reward amount is consistent with a multiplicative model.

One interesting aspect of the present data that deserves further investigation is the fact that the effect of probability on the degree of delay discounting appeared to be greater than the effect of delay on the degree of probability discounting. Indeed, increases in the odds against receiving a reward virtually eliminated the effect of delay on subjective value, whereas

increases in the delay until receiving a reward merely attenuated the effect of probability (e.g., in Figure 1, note the relatively shallow discounting when the probability of reward was 1.0, in which case the task was a typical delay discounting task). One aspect of the experimental procedure that might have contributed to this result is that the conditions were presented in a random order, and delay and/or probability changed with every new condition. That is, too many things might have been changing across questions, leading participants to focus primarily on one dimension. This interpretation implies that participants tended to focus more on the probability dimension, and as a consequence, their choices were less affected by delay. Experiment 2 was designed to address this issue. In addition, because this is the first study to compare additive and multiplicative models of discounting, it is important to assess the robustness of the finding that delay and probability hyperboloid discounting combine multiplicatively to determine the subjective value of rewards.

Experiment 2

To test the hypothesis that the difficulty of dealing with the frequent interleaved changes in both reward dimensions (i.e., delay and probability) associated with randomization was the source of the finding that probability had greater effects than delay on discounting in Experiment 1, we revisited the discounting of rewards that are both delayed and probabilistic, examining discounting within blocks of conditions in which only one dimension was varied. For half of the participants (Group 2A), all of the delays were studied within a block and probability was varied across blocks. For the other half of the participants (Group 2B), all of the probabilities were studied within a block and delay was varied across blocks. In addition to simplifying the structure of the task, this procedure emphasized one dimension of the choices with which participants were presented: the delay dimension in the case of Group 2A and the probability dimension in the case of Group 2B. If blocking has the effect of making one dimension more salient, then participants in Group 2A should show steeper delay discounting than participants in Group 2B, and participants in Group 2B should show steeper probability discounting than participants in Group 2A.

Method

Participants—One hundred and two participants (61 females, 41 males; mean age = 20.9) recruited from the Washington University Department of Psychology Humans Subjects Pool received either partial course credit or payment for their participation. Data from one male participant in Group 2A and one female participant in Group 2B were excluded from the analyses because of their near-exclusive choice of immediate, certain rewards across conditions.

Materials and Procedure—The procedure was similar to that used in Experiment 1, with the primary difference being the order in which the conditions were presented to participants to potentially emphasize either the delay to or the probability of receiving the reward. For half of the participants (Group 2A), the conditions were arranged in five blocks, one for each probability, with delay varied within each block; for the other half (Group 2B), the conditions also were arranged in five blocks, but for this group there was one block for each

delay, with probability varied within each block. As in Experiment 1, two amounts of delayed/probabilistic reward (\$800 and \$40,000) were studied in counter-balanced order, and for each amount, five delays (0 [immediate], 1 month, 6 months, 2 years, and 5 years) were crossed with five probabilities (presented as the percent chance of reward: 10%, 25%, 40%, 80%, and 100% [certain]). Importantly, the same delays, probabilities, and amounts used in Experiment 1 also were used in Experiment 2 to facilitate comparison between the experiments.

For Group 2A, the first block of conditions involved either an \$800 or a \$40,000 reward available with a likelihood of 100% (as in the typical delay discounting task), and the delays within that block were presented in random order. The four subsequent blocks, each involving one of the four other probabilities, were presented in random order, and within each block the delays were presented in random order. The next five blocks of conditions involved the other reward amount, and again, the initial block consisted of the 100% probability conditions, followed by four blocks, each involving one of the four other probabilities, presented in random order.

For Group 2B, the first block of conditions involved either an \$800 or a \$40,000 reward available immediately (as in the typical probability discounting task), and the probabilities within that block were presented in random order. The four subsequent blocks, each involving one of the four other delays, were presented in random order, and within each block the probabilities were presented in random order. The next five blocks of conditions involved the other reward amount, and again, the initial block consisted of the immediate reward conditions, followed by four blocks, each involving one of the four delays, presented in random order.

Results

Figures 5 and 6 present the group mean indifference points for Groups 2A and 2B, respectively, from all of the conditions plotted first as a function of delay (left panels) and then again, this time as a function of odds against (right panels). The curved lines represent the fits of the hyperboloid discounting functions, with Equation 1 used to fit the data when it was plotted as a function of delay and Equation 2 used to fit the data when it was plotted as a function of odds against. Each of the five curves in the left panels corresponds to delay discounting at a different probability, and each of the five curves in the right panels corresponds to probability discounting at a different delay. For both groups, delay discounting was observed when probabilities were high, but there was little or no effect of delay when probabilities were lower, whereas steep probability discounting was observed at all delays, although it was steepest at shorter delays. This pattern is similar to that observed in Experiment 1.

For both Groups 2A and 2B, hyperboloid discounting functions provided very good descriptions of delay discounting in most cases and excellent descriptions of probability discounting in all cases (see Table 3). The parameters of the delay discounting function could not be estimated reliably (i.e., estimated values were not significantly different from zero) in two cases, both of which were associated with low probabilities and relatively little effect of delay on subjective value. In all but one of the cases where the parameters of

Equations 1 and 2 could be reliably estimated, s was less than 1.0, and of these 37 cases, s was significantly less than 1.0 in 32 cases. The finding that s was significantly less than 1.0 in so many cases indicates that the hyperboloid provides a better description of delayed and probabilistic choice than the simple hyperbola.

AuCs were calculated for each probability condition and for each delay condition depicted in Figures 5 and 6. As may be seen in the left panels of Figure 7, which plots the normalized areas under the delay discounting curves shown in the left panels of Figures 5 and 6, for both groups the degree of discounting increased (i.e., the AuCs decreased) as the probability of reward decreased. Consistent with this increase in discounting with decreases in probability, separate ANOVAs on the normalized AuCs for each group revealed that the effect of probability was significant in both cases (both $F(4,45) > 104.41$, both $ps < .001$, both $\eta_p^2 > .90$). The main effect of amount was significant for Group 2A, $F(1,48) = 9.63$, $p = .003$, $\eta_p^2 = .17$, but not for Group 2B, $F(1,48) < 1.0$, although the amount $\times$ probability interaction was significant in both cases (both $F(4,45) > 6.37$, both $ps < .001$, both $\eta_p^2 > .36$) reflecting the fact that the difference between the AuCs for the two amounts was greatest when the probability of a delayed reward was 1.0 (see the left panels of Fig. 7).

In addition, as may be seen in the right panels of Figure 7, which plots the normalized areas under the probability discounting curves shown in the right panels of Figures 5 and 6, the degree of discounting tended to increase as delay increased. Separate ANOVAs confirmed that the effect of delay was significant for both Groups 2A and 2B (both $F(4,45) > 14.56$, both $ps < .001$, both $\eta_p^2 = .56$). The main effect of amount was not significant for Group 2A, $F(1,48) < 1.0$, but it was for Group 2B, $F(1,48) = 24.68$, $p < .001$, $\eta_p^2 = .34$, and the amount $\times$ delay interaction was significant in both cases, $F(4,45) = 2.70$, $p < .05$, $\eta_p^2 = .19$, and $F(4,45) = 5.33$, $p = .001$, $\eta_p^2 = .32$, respectively, reflecting the fact that the difference between the AuCs for the two amounts was greatest when there was no delay to the probabilistic reward or when the delay was brief (see right panels of Fig. 7).

In order to determine whether blocking so as to emphasize one or the other dimension (i.e., delay or probability) affects the degree of delay or probability discounting, planned comparisons were conducted comparing the indifference points for Groups 2A and 2B. Results of these comparisons failed to reveal an interaction either between group and delay or between group and probability (both $F(4,95) < 1.24$, both $ps > .30$), consistent with the similar patterns of results obtained when the delay and probability AuCs for the two groups were analyzed separately.

As noted previously, a multiplicative model predicts an interaction between delay and probability (i.e., the degree of delay discounting changes depending on the probability of the delayed reward, and the degree of probability discounting changes depending on the delay until the probabilistic reward). An additive model predicts no interaction. Accordingly, the subjective values (i.e., indifference points) from all conditions were submitted to repeated-measures ANOVAs, one for each reward amount. For both Group 2A and Group 2B,

planned comparisons revealed significant interactions at both amounts (all $F_s(16,34) > 4.61$, all $p_s < .001$, all $\eta_p^2 > .68$), consistent with a multiplicative model.

As was the case with the data from Experiment 1, the multiplicative model (Eq. 5) provided excellent fits to the mean subjective values for all delay/probability conditions, with R^2 s greater than .99 for both groups (see Table 4, which also presents the values of the parameters for fits of Equation 5 to the data for each amount). The excellent fits of the multiplicative model may be seen in Figure 8, which plots the group mean subjective values as a function of both delay and probability. To determine whether simpler versions of the multiplicative model would provide equally good fits, the data were fit with a reduced model that had two rate parameters but no exponent (i.e., a model based on simple hyperbolic discounting; see Eq. 6), as well as with a three-parameter hyperboloid in which the entire denominator is raised to the same power (rather than having separate exponents for the delay and probability discounting components as in the full model; see Eq. 7).

As was observed in Experiment 1, the full four-parameter multiplicative model (Eq. 5) provided significantly better fits to the data from both groups than the two-parameter version (Eq. 6) for both the \$800 and \$40,000 amounts (all incremental- $F_s(2,21) > 39.3$, $p_s < .001$). The four-parameter multiplicative model also provided significantly better fits than the three-parameter version (Eq. 7) (all incremental- $F_s(1,21) > 13.8$, $p_s < .002$). These results argue that delay and probability discounting are best described by a model in which separate hyperboloid discounting functions, each with its own rate parameter and exponent, are combined multiplicatively.

Discussion

The results of Experiment 2 again demonstrated that the hyperboloid function provides an excellent description of choices when rewards are both delayed and probabilistic. Moreover, significant interactions between delay and probability were observed, consistent with a multiplicative (rather than an additive) model of discounting. In addition, we compared a four-parameter multiplicative model based on the assumption that separate two-parameter hyperboloid functions describe delay and probability discounting to a reduced multiplicative model based on simple hyperbolic discounting functions like that proposed by Ho et al. (1999). The results showed that the full, four-parameter model produced significantly better fits, consistent with previous findings showing that hyperboloid discounting functions provide better descriptions of discounting data than simple hyperbolas when delay and probability discounting are examined separately (for a review, see Green & Myerson, 2004).

As may be recalled, in Experiment 1 the effect of probability on the degree of delay discounting appeared to be greater than the effect of delay on the degree of probability discounting. To determine whether this was a consequence of the random interleaving of all delay/probability conditions, conditions in Experiment 2 were blocked so as to emphasize either the delay dimension (Group 2A) or the probability dimension (Group 2B). However, despite the blocking, the effect of probability on delay discounting was again greater than the effect of delay on probability discounting, and no significant difference was observed between the two blocking procedures.

Some previous studies (Robles & Vargas, 2008; Robles, Vargas, & Bejarano, 2009) have reported that the order in which choices are presented (e.g., varying the amount of immediate reward in ascending or descending order) affects participants' indifference points, whereas others have failed to observe such effects (Green, Myerson, & McFadden, 1997; Holt, Green, & Myerson, 2012; Robles & Vargas, 2007; Rodzon, Berry, & Odum, 2011). The present results are consistent with the latter group of studies and suggest that although order may sometimes influence discounting, it is not a very robust effect.

As for the fact that probability had greater effects than delay on the subjective value of rewards that were both delayed and probabilistic, it is possible that probability is simply the more salient dimension, although interpretation is complicated by the fact that as in the previous experiment, the degree of delay discounting observed in the present experiment was less than the degree of probability discounting when each was examined separately. This difference, in turn, may be a consequence of the particular amounts, delays, and probabilities studied. It is to be noted, however, that the delays ranged up to 5 years and the probabilities were as low as .10, and the same results were obtained with both relatively small and large amounts of reward.

General Discussion

Many everyday decisions involve outcomes that are both delayed and probabilistic, yet most research on discounting has focused on one dimension or the other (Green, Myerson, & Vanderveldt, 2014; Madden & Bickel, 2010). In two experiments, we examined the discounting of rewards that were both delayed and probabilistic, systematically varying both dimensions in a combined discounting procedure. Mathematical functions with the same hyperboloid form that describes the effect of delay on the subjective value of a certain reward and the effect of probability on an immediate reward were shown to also describe the effect of delay on the subjective value of a probabilistic reward as well as the effect of probability on the subjective value of a delayed reward. Notably, the effect of delay varied systematically with the probability of reward and the effect of probability varied with delay, a result inconsistent with a model in which dimensions combine in an additive fashion, as implied by the additive utility model proposed by Killeen (2009).

A model in which hyperboloid discounting functions combine multiplicatively (Eq. 5) provided excellent fits to the data from both experiments (see Tables 2 and 4, and Figs. 3 and 8). As predicted by the multiplicative hyperboloid model, the decrease in subjective value as the odds against increase was greater when the delay was brief, and the decrease in subjective value as delay increases was greater when the probability of reward was high (i.e., when the odds against are low). Importantly, the multiplicative hyperboloid model provided significantly better fits than a multiplicative model based on simple hyperbolic discounting functions (Ho et al., 1999). These results are consistent with past research showing that a hyperboloid provides a better description of simple delay and probability discounting than a simple hyperbola (for a review, see Green & Myerson, 2004), and show that the superiority of the hyperboloid form extends to more complicated situations like those in the present study in which rewards are both delayed and probabilistic.

Similar patterns of discounting were observed at both the smaller (\$800) and the larger (\$40,000) amounts of delayed/probabilistic reward when the data for each of the two amounts were examined separately. When discounting of the two amounts was compared, typical delay and probability magnitude effects were observed when the rewards were certain but delayed and when they were immediate but probabilistic: When the rewards were certain but delayed, the larger (delayed) amount was discounted less steeply, whereas when they were immediate but probabilistic, the larger (probabilistic) amount was discounted more steeply. However, magnitude effects tended to be minimal when rewards were both delayed and probabilistic, suggesting that in such cases, the opposite magnitude effects observed in simple delay and probability discounting tend to cancel each other out.

The data reported by Yi et al. (2006) and by Blackburn and El-Deredy (2013) both show effects of reward amount similar to those observed in the present study. As in the present study, Yi et al. examined the discounting of two reward amounts (\$10 and \$1,000) and systematically varied both delay and probability, although for purposes of analysis, they converted all probabilities to “delays” by multiplying the odds against by 35.3 (based on Rachlin et al., 1991) and adding this value to the programmed delay to obtain what they termed “composite delays.” Analyses based on the composite delays revealed significant effects of amount consistent with previous findings for delay discounting (Green & Myerson, 2004). However, inspection of the median indifference points in their Figure 2 (top panel) reveals that, as in the present study, there were differences in the degree to which the two amounts were discounted when delays were brief and probabilities of reward were high (as indicated by low composite delay values) but not when delays were long and probabilities were low (as indicated by high composite delay values).

A similar pattern may be seen in the data reported by Blackburn and El-Deredy (2013), who examined delay discounting when the probability of reward was .50, and their data also show greater discounting of a smaller reward amount (£100) than of a larger amount (£1,000) when the delay is brief but not when it is long. Interestingly, when obtaining a reward was guaranteed but the amount was uncertain, a smaller uncertain amount was discounted more steeply than a larger uncertain amount even at the longest delay (10 years) examined. This difference between the discounting of an uncertain reward amount and of a reward whose very delivery is uncertain is potentially a very important finding and deserves further systematic investigation.

A number of studies have examined the effects of adding a common delay to certain and probabilistic choices (Abdellaoui, Diecidue, & Onculer, 2011; Baucells & Heukamp, 2010; Keren & Roelofsma, 1995; Noussair & Wu, 2006; Sagristano, Trope, & Liberman, 2002; Weber & Chapman, 2005) or adding a common probability to immediate and delayed choices (Keren & Roelofsma, 1995; Weber & Chapman, 2005). The general consensus from these experiments is that people become more risk tolerant (i.e., less sensitive to probability) when rewards are further in the future. Despite the many procedural differences between these experiments and the current study, the present results also show decreases in the effect of probability as delay increases. The present results also show decreases in the effect of delay as probability decreases. Both of these findings, the effect of delay on probability

discounting and the effect of probability on delay discounting, are predicted by the multiplicative discounting model (but not by an additive model).

As shown in Figures 3 and 8, however, increasing the odds against a delayed reward virtually eliminated the effect of delay whereas increasing the delay to a probabilistic reward only attenuated the effect of probability. There are at least two possible reasons why the effect of probability on the value of delayed/probabilistic rewards was greater than the effect of delay: One possibility is because the odds against receiving a reward receives greater weight than the delay to reward when individuals evaluate delayed/probabilistic rewards – probability is simply the more important factor.

Alternatively, the greater effect of probability in the present study may merely be an artifact of the amounts and the range of delays and probabilities examined in the present study. More specifically, over the range of delays and probabilities examined in the present study, increasing the odds against resulted in a greater decrease in subjective value even when delay was not a factor than did increasing the delay when probability was not a factor. One way to determine whether the greater effect of probability on the subjective value of delayed/probabilistic rewards is an artifact or whether probability is actually more salient than delay would be to repeat the present study using a reward amount and a range of delays and odds against such that the decrease in subjective value of an immediate reward at the lowest probability is equivalent to the decrease in subject value of a certain reward at the longest delay. If the greater effect of probability were to be observed under these conditions, such a finding would indicate that the greater effect of probability observed here is more than an artifact.

Regardless of whether or not probability is more salient than delay, the results of the current study demonstrate that the effects of delay and probability combine multiplicatively rather than additively. Moreover, a multiplicative model based on hyperboloid discounting functions provides a better account of behavior when rewards are both delayed and probabilistic than one based on simple hyperbolic discounting. Of course, it remains to be seen if this finding will hold across different contexts and commodities. What is clear from the current results, however, is that a multiplicative hyperboloid discounting model provides an excellent description of behavior in situations more complex than those typically studied in discounting experiments.

Although focusing on simple situations in which only one factor is varied experimentally makes analytic sense, it is also important to take the results of studies of such simple situations and see how they play out in more complex situations. The present findings show that a model that assumes the multiplicative interaction of the principles that operate in simple discounting situations can explain decisions in more complicated situations like those that individuals encounter every day, situations that frequently involve monetary outcomes that are both delayed and probabilistic.

It has proven very informative to examine decision making by special populations that are described as impulsive or risk-taking (e.g., substance abusers, pathological gamblers) using delayed outcomes as well as probabilistic outcomes (e.g., Petry & Madden, 2010). However,

the choices that they actually confront, and have problems dealing with, typically involve outcomes that are both delayed and probabilistic. In the case of substance abusers, for example, both incarceration and many of the health consequences tend to be both uncertain and delayed, as are the outcome of psychological interventions or treatment. Accordingly, psychological tests designed like the present experiments, that examine choices involving outcomes that are delayed, uncertain, and both uncertain and delayed, may prove useful in understanding problem behavior in the real world.

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Figure 1.

Mean indifference points and best fitting curves as predicted by Equation 1 (delay discounting) and Equation 2 (probability discounting). The left panels show delay discounting at each probability, and the right panels show probability discounting at each delay. The top panels correspond to the \$800 reward conditions, and the bottom panels correspond to the \$40,000 reward conditions.

Figure 2.

Mean area under the curve (AuC) for delay discounting data at each probability (left panel) and probability discounting data at each delay (right panel). Note the different scales on the y-axes.

Figure 3.

Three-dimensional plots of the mean indifference points and best-fitting curves as predicted by the multiplicative discounting model (Eq. 5) for the \$800 (left panel) and \$40,000 (right panel) reward conditions.

Figure 4.

Three-dimensional plots of individual participants' indifference points and best-fitting curves as predicted by the multiplicative discounting model (Eq. 5) for the \$800 reward (left panels) and the \$40,000 condition (right panels). Plots represent the individuals whose fits (R^2) of the multiplicative discounting model (Eq. 5) were at the 25th, 50th, and the 75th percentiles.

Figure 5.

Mean indifference points and best fitting curves as predicted by Equation 1 (delay discounting) and Equation 2 (probability discounting) for Group 2A. The left panels show delay discounting at each probability, and the right panels show probability discounting at each delay. The top panels correspond to the \$800 reward conditions, and the bottom panels correspond to the \$40,000 reward conditions.

Figure 6.

Mean indifference points and best fitting curves as predicted by Equation 1 (delay discounting) and Equation 2 (probability discounting) for Group 2B. The left panels show delay discounting at each probability, and the right panels show probability discounting at each delay. The top panels correspond to the \$800 reward conditions, and the bottom panels correspond to the \$40,000 reward conditions.

Figure 7.

Mean area under the curve (AuC) for delay discounting data at each probability (left panels) and probability discounting data at each delay (right panels). Note the different scales on the y-axes in the left and right panels. Data from Groups 2A and 2B are shown in the top and bottom panels, respectively.

Figure 8.

Three-dimensional plots of the mean indifference points and best-fitting curves as predicted by the multiplicative discounting model (Eq. 5). The left panels show the \$800 conditions and the right panels show the \$40,000 conditions. Data from Group 2A and 2B are shown in the top and bottom panels, respectively.

Table 1

Parameter estimates and proportions of variance (R^2) accounted for by Equation 1 (delay discounting) and Equation 2 (probability discounting) in Experiment 1.

	Delay Discounting					Probability Discounting				
	100%	80%	40%	25%	10%	Now	1 Month	6 Months	2 Years	5 Years
\$800										
h/k	0.218	0.213	0.032	---	---	2.731	3.553	3.859	3.451	5.069
s	0.141	0.154	0.334	---	---	0.767	0.653	0.624	0.626	0.557
R^2	0.991	0.977	0.987	---	---	0.999	0.997	0.998	0.998	0.997
\$40,000										
h/k	0.603	---	0.387	---	---	5.293	2.815	4.165	3.065	6.736
s	0.047	---	0.075	---	---	0.622	0.798	0.703	0.779*	0.570
R^2	0.904	---	0.875	---	---	0.999	0.999	0.999	0.998	0.999

Note. Equation 1 failed to converge or provided parameter estimates that did not differ significantly from zero for the 25% and 10% delay discounting conditions at the \$800 reward amount and the 80% delay discounting condition at the \$40,000 reward amount.

* Not significantly different from 1.0, $p > 0.05$. All other values of s are significantly less than 1.0.

Table 2

Parameter estimates and proportions of variance (R^2) accounted for by the multiplicative discounting model (Eq. 5) in Experiment 1.

	\$800	\$40,000
k	0.167	0.140
h	3.637	4.278
s_d	0.155	0.083
s_p	0.650	0.683
R^2	0.997	0.997

Table 3

Parameter estimates and proportion of variance (R^2) accounted for by Equation 1 (delay discounting) and Equation 2 (probability discounting) for Group 2A and Group 2B in Experiment 2.

		Delay Discounting				Probability Discounting					
		100%	80%	40%	25%	10%	Now	1 Month	6 Months	2 Years	5 Years
Group 2A	\$800										
	<i>h/k</i>	0.219	0.707	0.476	1.889	38.449	3.736	3.675	3.071	2.089	1.180
	<i>s</i>	0.315	0.157	0.182	0.101	0.060	0.560	0.543	0.631	0.665	0.648*
	<i>R</i> ²	0.987	0.985	0.962	0.901	0.990	0.999	0.997	0.999	0.997	0.996
	\$40,000										
	<i>h/k</i>	0.241	0.082	0.024	0.174	0.020	2.806	3.979	2.567	3.286	4.897
	<i>s</i>	0.146	0.285	0.449*	0.211	0.507*	0.721	0.577	0.723*	0.698	0.583
	<i>R</i> ²	0.977	0.943	0.929	0.906	0.789	0.999	0.999	0.997	0.999	0.999
Group 2B	\$800										
	<i>h/k</i>	0.368	0.366	0.389	0.001	0.043	3.561	3.352	8.078	3.457	9.151
	<i>s</i>	0.164	0.183	0.133	6.461*	0.212*	0.574	0.541	0.413	0.519	0.388
	<i>R</i> ²	0.960	0.987	0.991	0.984	0.652	0.999	0.999	0.999	0.999	0.998
	\$40,000										
	<i>h/k</i>	0.723	3.227	2.397	---	---	3.683	7.333	4.058	8.284	5.932
	<i>s</i>	0.085	0.060	0.069	---	---	0.716	0.537	0.636	0.474	0.547
	<i>R</i> ²	0.975	0.858	0.898	---	---	0.999	0.999	0.999	0.999	0.999

Note. Equation 1 failed to converge or provided parameter estimates that did not differ significantly from zero for the 25% and 10% delay discounting conditions at the \$40,000 reward amount for Group 2B.

* Not significantly different from 1.0, $p > 0.05$. All other values of s are significantly less than 1.0.

Table 4

Parameter estimates and proportions of variance (R^2) accounted for by the multiplicative discounting model (Eq. 5) in Experiment 2.

	Group 2A		Group 2B	
	\$800	\$40,000	\$800	\$40,000
k	0.436	0.122	0.323	0.752
h	3.368	3.425	4.736	5.474
s_d	0.220	0.203	0.172	0.082
s_p	0.570	0.654	0.493	0.583
R^2	0.991	0.997	0.995	0.996