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When the Test of Mediation is More Powerful than the Test of the Total Effect

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Abstract

Although previous research has studied power in mediation models, the extent to which the inclusion of a mediator will increase power has not been investigated. First, a study compared analytical power of the mediated effect to the total effect in a single mediator model to identify the situations in which the inclusion of one mediator increased statistical power. Results from the first study indicated that including a mediator increased statistical power in small samples with large coefficients and in large samples with small coefficients, and when coefficients were non-zero and equal across models. Next, a study identified conditions where power was greater for the test of the total mediated effect compared to the test of the total effect in the parallel two mediator model. Results indicated that including two mediators increased power in small samples with large coefficients and in large samples with small coefficients, the same pattern of results found in the first study. Finally, a study assessed analytical power for a sequential (three-path) two mediator model and compared power to detect the three-path mediated effect to power to detect both the test of the total effect and the test of the mediated effect for the single mediator model. Results indicated that the three-path mediated effect had more power than the mediated effect from the single mediator model and the test of the total effect. Practical implications of these results for researchers are then discussed.

Introduction

Most research studies investigate the relation of an independent variable (X) and a dependent variable (Y). The independent variable X represents exposure to a randomized manipulation or an observed variable. An analysis such as linear regression quantifies the relation of X to Y as a regression coefficient and provides a statistical test of whether the coefficient is larger than expected by chance. Prior to such an undertaking, researchers conduct power calculations to assess the probability of observing a statistically significant relation of X to Y given a Type I error rate, sample size, variability, and effect size. Increasing the Type I error rate, increasing the sample size, increasing the effect size, or reducing variability increases power of a statistical test.

As recognition of the importance of mediation analysis has increased, researchers have developed advanced statistical and conceptual models to investigate mediated effects

(MacKinnon, 2008). In the mediation model, there is a hypothesized chain of relations between X and Y such that X first causes a change in a variable M that then causes a change in Y. This model explains how X affects Y by elaborating the relation to be X to M to Y. The mediator M is selected for change based on theory and prior empirical research. In addition to the strong theoretical and empirical rationale behind including the mediator, including a mediator may also increase the power to detect effects in some circumstances. This article describes the reason for this somewhat counterintuitive result, along with the conditions when this result occurs, and then describes the practical applications of these results for study planning.

We present three studies to illustrate when power to detect the mediated effect is greater than power to detect the total effect. Study 1 compares power values for the mediated effect and total effect from a single mediator model to determine when power of the mediated effect is greater than power of the total effect. Study 2 compares power values for the total mediated effect to the total effect from a parallel two mediator model to determine when the test of the parallel two mediator model has more power than the test of the total effect. Study 3 investigates a sequential two mediator model and compares power values for the three-path mediated effect and the total effect to determine when power of the three-path mediated effect is greater than power of the total effect. Study 3 also compares power of the three-path mediated effect to power of the mediated effect from the single mediator model and power of the total effect to see how power increases differentially across models. Results from these studies reveal the nature of how models with one or two mediators increase power over the total effect. Finally, the practical use of these results is described through several research examples.

The Single Mediator Model

Figure 1 shows the relation between an independent variable (X) and a dependent variable (Y), and a three variable model including X and Y when a mediator (M) is added to a model.

The following three regression equations are used to investigate mediation for a single mediator using notation in MacKinnon (2008):

$$Y = i_1 + cX + \varepsilon_1 \quad (1)$$

$$Y = i_2 + c'X + bM + \varepsilon_2 \quad (2)$$

$$M = i_3 + aX + \varepsilon_3 \quad (3)$$

From Equation 1, i_1 is the intercept for the equation, c is the relation between X and Y (also known as the total effect of X on Y), and ε_1 is the variability in Y that is not explained by X. In this article the total effect refers to the c coefficient that represents the relation of X to Y, and applies to all subsequent models described. From Equation 2, b is the relation between M and Y adjusting for X, c' is the relation between X and Y adjusting for M (also known as the direct effect of X on Y that is not through M), and ε_2 is the variability in Y that is not explained by its relations with X and M. From Equation 3, a is the relation between X and

M, and ε_3 is the variability in M that is not explained by X. The coefficients i_2 and i_3 are the intercepts for the equations. The intercepts are not involved in the estimation of mediated effects and could be left out of the equations, but are included here for completeness. Note that both c and c' are coefficients relating X to Y, but c' is a partial effect adjusted for the effects of the mediator. The mediation coefficients from Equations 2 and 3 are estimated using multiple regression or structural equation modeling. A classic example of a single mediator model is the mediation of the relationship between attitudes (X) and behavior (Y) by intentions (M) (Fishbein & Ajzen, 1975).

The Parallel Two Mediator Model

The parallel two mediator model is an extension of the single mediator model, where X is related to Y through a mediator (M_1) and also through an additional mediator (M_2). The mediators each have their own specific effects within this model (as opposed to transmitting the effect of X to M_1 to M_2 to Y), hence the use of the term parallel. The parallel two mediator model is depicted in Figure 2.

The following equations represent the parallel two mediator model using notation from MacKinnon (2008):

$$Y = i_2 + c'X + b_1M_1 + b_2M_2 + \varepsilon_2 \quad (4)$$

$$M_1 = i_3 + a_1X + \varepsilon_3 \quad (5)$$

$$M_2 = i_4 + a_2X + \varepsilon_4 \quad (6)$$

From Equation 4, c' is the relation between X and Y adjusting for M_1 and M_2 , b_1 is the relation between M_1 and Y adjusting for M_2 and X, b_2 is the relation between M_2 and Y adjusting for M_1 and X, and ε_2 is the variability in Y not explained by its relations with X, M_1 , and M_2 . From Equation 5, a_1 is the relation between M_1 and X, and ε_3 is the variability in M_1 that is not explained by X. From Equation 6, a_2 is the relation between M_2 and X, and ε_4 is the variability in M_2 not explained by X. The coefficients i_2 , i_3 , and i_4 are the intercepts for the equations. An example of a parallel two mediator model is Salthouse's (1984) study looking at how the effect of age (X) on typing efficiency (Y) was mediated by reaction time (M_1) and skill (M_2).

The Sequential Two Mediator Model

In the sequential two mediator model, also referred to as the three-path mediator model (Taylor, MacKinnon, & Tein, 2008) or micromediation chain (Cook & Campbell, 1979), two mediators (M_1 and M_2) intervene between X and Y. This model is shown in Figure 3.

The sequential two mediator model is represented by Equations 7, 8, and 9, using notation from MacKinnon (2008) and Taylor et al. (2008):

$$Y = i_2 + b_4X + b_3M_2 + b_6M_1 + \varepsilon_4 \quad (7)$$

$$M_1 = i_3 + b_1X + \varepsilon_2 \quad (8)$$

$$M_2 = i_4 + b_2M_1 + b_5X + \varepsilon_3 \quad (9)$$

From Equation 7, b_4 is the relation between X and Y adjusting for M_1 and M_2 , b_3 is the relation between M_2 and Y adjusting for X and M_1 , b_6 is the relation between M_1 and Y adjusting for M_2 and X, and ε_4 is the variability in Y not explained by its relations with X, M_1 , and M_2 . From Equation 8, b_1 is the relation between M_1 and X, and ε_2 is the variability in M_1 not explained by X. And from Equation 9, b_2 is the relation between M_2 and M_1 adjusting for X, b_5 is the relation between M_2 and X adjusting for M_1 , and ε_3 is the variability in M_2 not explained by its relations with M_1 and X. The coefficients i_2 , i_3 , and i_4 are the intercepts for the equations.

The sequential two mediator model contains several different effects of X on Y. In this model, b_4 represents the direct effect of X on Y instead of c' as for the other models. The three effects that form the total mediated effect are the three-path mediated effect ($b_1b_2b_3$), the two-path mediated effect passing through M_1 (b_1b_6), and the two-path mediated effect passing through M_2 (b_5b_3). The total mediated effect of X on Y passing through either mediator is the sum of the three mediated effects, $b_1b_2b_3 + b_1b_6 + b_5b_3$. As in the two previous models, the total mediated effect is equal to the difference between the total and direct effects shown here in Equation 10:

$$b_1b_2b_3 + b_1b_6 + b_5b_3 = c - b_4 \quad (10)$$

Taylor et al. (2008) describe a sequential two mediator model from Tekleab, Bartol, and Liu (2005) where the effect of pay (X) on turnover (Y) was mediated by perceived distributive justice (M_1) and pay raise satisfaction (M_2).

Statistical Power

The power of a statistical test is that test's ability to detect an effect when an effect is truly present in the population (Neyman & Pearson, 1933). Power depends on several key parameters involved in hypothesis testing, namely the Type I error rate, sample size, and effect size. The four parameters Type I error, sample size, effect size, and power are interdependent, and given any three, the fourth may be calculated. A power value of .80 is the standard for adequate power of a hypothesis test in the social sciences (Cohen, 1988). The power to detect the total effect of X on Y, c , is the probability of observing a statistically significant t value given the noncentral distribution for the true total effect of X on Y. The probability that both a and b are statistically significant is the product of the probability to detect each effect separately given that power to detect a and b are independent (Tofighi, MacKinnon, & Yoon, 2009). For the sequential two mediator model, the probability that b_1 , b_2 , and b_3 are statistically significant is the product of the probability to detect each effect separately given that all paths are uncorrelated. The mediated effect ab is the product of two random variables and the total mediated effect $a_1b_1 + a_2b_2$ is the sum of two products of two random variables. The power to detect these effects is not equal to the probability of observing a statistically significant t value given the noncentral

distribution (as it would be for one random variable), because the distribution of the product of two random variables is non-normal (Aroian, 1944; Craig, 1936). Appendix A contains additional details on power computations.

Tests of Mediation

There are different approaches to estimating mediated effects from the regression models (MacKinnon et al., 2002). For the single mediator model, the product of the $\hat{a}$ and $\hat{b}$ coefficients ($\hat{a}\hat{b}$) is an estimator of the mediated effect. The effect of X on Y after adjustment for M ($\hat{c}'$) is an estimator of the direct effect. If a mediated effect is present and $\hat{c}'$ is zero then there is complete mediation. If a mediated effect is present and $\hat{c}'$ is not zero then there is partial mediation. The mediated effect is also equal to the difference between the $\hat{c}$ and $\hat{c}'$ coefficients, $\hat{c} - \hat{c}'$. As a result, the total effect $\hat{c}$ breaks down into a direct effect $\hat{c}'$ and a mediated effect $\hat{a}\hat{b}$. For the multiple regression equations described above, $\hat{a}\hat{b} = \hat{c} - \hat{c}'$ and therefore $\hat{c} = \hat{c}' + \hat{a}\hat{b}$. The rationale behind the $\hat{a}\hat{b}$ mediation measure is that mediation depends on the extent to which the independent variable affects the mediator (coefficient $\hat{a}$) and the extent to which the mediator affects the dependent variable (coefficient $\hat{b}$). The $\hat{a}\hat{b}$ measure reflects how much a one unit change in X affects Y indirectly through M, and the $\hat{c} - \hat{c}'$ measure reflects how much M explains the relation between X and Y. The joint significance test of mediation (MacKinnon, Lockwood, Hoffman, West, & Sheets, 2002) tests whether the $\hat{a}$ path is statistically significant and whether the $\hat{b}$ path is statistically significant, and rejects the hypothesis of no mediation if both paths are statistically significant. In addition to the joint significance test, a commonly implemented test of mediation assesses significance of the mediated effect $\hat{a}\hat{b}$ by dividing the effect by its multivariate delta standard error based on first derivatives, $s_{ab} = \sqrt{a^2 s_b^2 + b^2 s_a^2}$ (Sobel, 1982).

For the parallel two mediator model, the estimator of the mediated effect of M_1 is the product of $\hat{a}_1$ and $\hat{b}_1$, and the mediated effect of M_2 is the product of $\hat{a}_2$ and $\hat{b}_2$. Individually, these are the specific mediated effects ($\hat{a}_1\hat{b}_1$ and $\hat{a}_2\hat{b}_2$) and together, they are the total mediated effect ($\hat{a}_1\hat{b}_1 + \hat{a}_2\hat{b}_2$). In this model, as $\hat{c}$ is the total effect and $\hat{c}'$ is the direct effect of X on Y adjusting for M_1 and M_2 , $\hat{a}_1\hat{b}_1 + \hat{a}_2\hat{b}_2 = \hat{c} - \hat{c}'$. As there is no test of joint significance for the total mediated effect $\hat{a}_1\hat{b}_1 + \hat{a}_2\hat{b}_2$, the product of coefficients test assesses significance of the total mediated effect by dividing the total mediated effect by its multivariate delta standard error based on first derivatives,

$$s_{a_1b_1+a_2b_2} = \sqrt{s_{a_1}^2 b_1^2 + s_{b_1}^2 a_1^2 + s_{a_2}^2 b_2^2 + s_{b_2}^2 a_2^2 + 2a_1a_2s_{b_1b_2}} \text{ (MacKinnon, 2008).}$$

The bootstrap test also assesses significance of the total mediated effect for the parallel two mediator model by bootstrapping confidence intervals around $\hat{a}_1\hat{b}_1 + \hat{a}_2\hat{b}_2$. The bootstrap test generates asymmetric confidence intervals based on the distribution of the mediated effect instead of assuming a normal distribution (MacKinnon et al., 2002; MacKinnon, Lockwood, & Williams, 2004). To obtain a bootstrap estimate of confidence limits for the mediated effect for a sample of size N , the bootstrap procedure takes a bootstrap sample of size N with replacement from the original sample and estimates a mediated effect. This process is

repeated a large number of times to create a set of bootstrapped estimates that form a distribution, and that distribution is used to calculate confidence limits for the mediated effect (MacKinnon, 2008). MacKinnon (2008) and MacKinnon et al. (2002) contain additional information on significance tests for these models.

For the sequential two mediator model, a joint significance test evaluates the significance of the $\hat{b}_1$, $\hat{b}_2$, and $\hat{b}_3$ coefficients separately (MacKinnon et al., 2002; Taylor et al., 2008). If all paths are statistically significant, mediation is present. For information on other tests of mediation for the sequential two mediator model, see Taylor et al. (2008).

Causal assumptions of mediation—This article aims to illustrate when the tests of mediation have more power than the test of the total effect. For the single mediator model, we make several assumptions regarding the causal relations among variables for identification of effects following VanderWeele and Vansteelandt (2009): (1) no confounders of the X to M relation, (2) no confounders of the M to Y relation, (3) no confounders of the X to Y relation, and (4) no effects of X on confounders that then affect the relation of X to Y. Randomization of X satisfies assumptions 1 and 3 which allows for estimators $\hat{a}$ and $\hat{c}$ to be treated as causal effects. Randomization of X does not satisfy assumptions 2 and 4 which are required for interpretation of $\hat{b}$ and $\hat{c}'$ as causal effects. Statistical methods that accommodate confounders as well as methods that assess sensitivity may address the violation of these assumptions (Imai, Keele, & Tingley, 2010). Satisfying causal assumptions of mediation becomes more complex with multiple mediators, and is an active area of research (Albert & Nelson, 2011; Avin, Shpitser, & Pearl, 2005; Imai & Yamamoto, 2013; Lange, Rasmussen, & Thygesen, 2013; Robins, 2003).

Increased Power Through Inclusion of Additional Variables

The literature on how including additional variables in statistical analyses can be beneficial is extensive. This idea can be traced back to the foundations of path analysis when it was suggested that researchers should analyze complex or indirect relations among variables, rather than analyzing only direct influences (Wright, 1921; Wright, 1934). R. A. Fisher also summarized the concept: "...When asked in a meeting what can be done in observational studies to clarify the step from association to causation, Sir Ronald Fisher replied: 'Make your theories elaborate'" (R. A. Fisher, cited by Cochran, 1965, p. 252). Literature in epidemiology also discussed the idea of making theories elaborate, where researchers proposed that it may be necessary to study the effects of a factor on a disease outcome through multiple channels, requiring more complex theories (Hill, 1965; Yerushalmy & Palmer, 1959). Research on causal inference argues that planned statistical analyses of observational studies should be based on elaborate theories (Rosenbaum, 2010), and that certain definitions of causality are inadequate because they do not address underlying causal mechanisms (Cox, 1992).

The increase in power resulting from the inclusion of covariates is particularly well-documented. One of the primary uses of a covariate in analysis of covariance (ANCOVA) is to increase the precision of a randomized experiment by reducing error variability (Cochran, 1957; Cox, 1960; Huck, 1972). Miller and Chapman (2001) agree that the main goal of

using ANCOVA should be to increase power to detect effects instead of using it to control for group differences, as is often done in practice. Although theory must determine whether inclusion of a mediator is appropriate and inclusion of a covariate may be data-driven, both serve to increase statistical power to detect effects in some situations.

Surrogate endpoints are also frequently used in medical research as a means of increasing power. Researchers may wish to study the effect of X on an outcome Y which is difficult or costly to measure, so they will find an intermediate endpoint Z that is a surrogate for Y such that X affects Z as it would Y , and Z is a predictor of Y (Prentice, 1989, p. 432). For example, the presence of polyps has been used as a surrogate endpoint for the outcome of colon cancer (Freedman & Schatzkin, 1992). Sample size or effect size increases through the use of surrogate endpoints (thereby increasing power).

Related to these concepts is the idea of an intensive design: including intermediate points of measurement and using the weighted average of those responses for each subject as an outcome instead of just one response (Kraemer & Thieman, 1989). The intensive design increases power without requiring an increased sample size over a posttest-only randomized experimental design (Kraemer & Thieman, 1989), and also increases power over a pretest-posttest design given certain conditions, although this design requires more measurement points (Maxwell, 1998; Venter, Maxwell, & Bolig, 2002).

Increased Power Through Inclusion of Mediators

In light of this research, it follows that including a mediator would increase power in some situations. Cox (1960) showed that including intermediate variables in the relation between X and Y would increase precision. MacKinnon et al. (2002) identified the tests of mediation with the best power and Type I error rates, and found that due to the requirement of a significant X to Y relation, the Baron and Kenny (1986) and Judd and Kenny (1981) causal steps tests for mediation are underpowered. Fritz and MacKinnon (2007) replicated this result, finding that when a and b paths were small and c' was zero, the sample size required to detect the mediated effect at .80 power for the Baron and Kenny test was 20886. Shrout and Bolger (2002) also stated that power to detect mediation is greater than power to detect a total effect of X on Y in some situations, and gave an example of a situation where power to detect the a and b effects would be greater than power to detect the c total effect. This means that some tests of mediation detect significant mediated effects when the relation between X and Y is nonsignificant (Fritz, Cox, & MacKinnon, 2013; MacKinnon, 2008, 394–395; Shrout & Bolger, 2002; Taylor et al., 2008). It follows that including a mediator, or mediators, results in increased power over the total effect in some situations. The purpose of this article is to describe when these situations occur.

Study 1

As described above, there is evidence that the single mediator model will have more power to detect effects than the total effect under certain circumstances. Study 1 compared power values of the mediated effect ab and the total effect c for the single mediator model to determine when the mediated effect has more power.

Method

Analytical power for the single mediator model was calculated based on derivations in Appendix A. Both SAS and R programs to compute analytical power for the single mediator model may be downloaded at <http://ripl.faculty.asu.edu/downloadable-programs/others/>. For the single mediator model, population variances and covariances for X, M, and Y were calculated based on different combinations of population path parameters and sample sizes. The covariance matrix for the single mediator model is given in MacKinnon (2008). Sample sizes of 50, 100, 200, 500, 1000, and 5000 were chosen based on those commonly found in the social sciences and in mediation research (Fairchild, MacKinnon, Taborga, & Taylor, 2009; MacKinnon et al., 2002; MacKinnon et al., 2004). Population path parameters were chosen in accordance with those used in prior research on mediation models for the single mediator model (0, .14, .39, and .59) for each of the a , b , and c' paths (Fritz & MacKinnon, 2007; MacKinnon et al., 2002). This $4 \times 4 \times 4 \times 6$ factorial design resulted in 384 combinations of path parameters and sample size.

The predicted variances and covariances were then used to calculate zero-order and first-order partial correlation effect sizes corresponding to the population a and b paths as found in MacKinnon (2008). The variances and covariances for the single mediator model were also used to calculate the true a and b coefficients and their true standard errors. Power was then calculated in two ways. First the correlations corresponding to a and b coefficients were used to calculate a z test, which was then used to compute analytical power. Then the path coefficients a and b and their true standard errors were used to calculate a t test, which was used to compute analytical power. The z and t tests yielded the same power values. Analytical formulas for power of the joint significance test of mediation for the single mediator model are also found in Wang and Xue (2012). For both methods of computing analytical power, the individual power values for a and b were multiplied to calculate analytical power, analogous to the joint significance test.

Results

Single mediator model analytical results—Analytical power of the test of the total effect and the joint significance test for the mediated effect from the single mediator model where c' is zero or small are shown in Table 1 (for all cases where power of the test of the mediated effect was greater than power of the test of the total effect, the direct effect c' was always zero or small; when c' is equal to zero this indicated full mediation). Only power of the models is considered in this table (combinations where a or b is zero are measures of Type I error rate).

Combinations of parameters where the test of the mediated effect had more power than the test of the total effect are underlined. When a and b were small ($a = b = .14$), the test of the mediated effect had more power than the test of the total effect at larger sample sizes. The test of the total effect had more power than the test of the mediated effect at smaller sample sizes when a and b were small. For example, in Table 1 for the case where a and b were small ($a = b = .14$), the test of the total effect had more power than the test of the mediated effect for a sample size of 50. The test of the mediated effect had more power than the test of the total effect in every other sample size. However, when a and b were large ($a = b = .$

59), the test of the mediated effect had more power than the test of the total effect at smaller sample sizes. For $a = b = .59$, the test of the mediated effect had more power than the test of the total effect in sample sizes up to 200. At sample sizes larger than 200, the test of the total effect and the joint significance and product of coefficients tests all had power of approximately one for large a and b . In summary, two patterns of results emerged for the single mediator model. The test of the mediated effect had more power than the test of the total effect when sample size was large and coefficients were small, and when sample size was small and coefficients were large. At large sample sizes with large coefficients, power of both tests approached one. Additionally, the test of the mediated effect had more power than the test of the total effect only in cases where c' was zero or small.

The analytical power results were replicated in an empirical simulation study that is not reported here. Results from the empirical simulation are available at <http://ripl.faculty.asu.edu/publications/computer-programs/>.

Comparison of standard errors of total and mediated effects—For the majority of the combinations of parameters and sample sizes studied, the test of mediation was more powerful than the test of the total effect when c' was equal to zero, and therefore when the total and mediated effects were equal (that is, $ab = c$). For the test of the mediated effect to be more powerful than the test of the total effect, the t test of significance for ab must be larger than the t test of significance for c . Because the tests of significance are computed by dividing the effects by their standard errors, it follows that if the test of the mediated effect is more powerful than the test of the total effect, the standard error of the total effect must be larger than the standard error of the mediated effect.

Table 2 shows a comparison of the analytical standard errors of c and ab for the single mediator model next to power of the tests of c and ab , for combinations where a and b are greater than zero and c' was equal to zero at $N = 100$ (a subsample of the combinations used in the results above).

When ab was equal to c and c' was equal to zero, the standard error of c was larger than the standard error of ab . Furthermore, when the standard error of c was larger than the standard error of ab , power of the test of ab was larger than power of the test of c .

A further study of situations where the test of the mediated effect had more power than the test of the total effect at smaller sample sizes and smaller coefficients showed that although the standard error of c was always larger than the standard error of ab when ab and c are equal, when a or b approached zero the power of the test of ab became smaller than power of the test of c . This pattern occurred for cases where a or b was very small but not zero. For example at $N = 100$, when a was equal to .14 and b was equal to .001, c was equal to .0014. The standard error of ab was .01422 and the standard error of c was .10102 so the standard error of ab was smaller than the standard error of c , but power of the test of ab was equal to .00714 and power of the test of c was equal to .02508. Although power of the test of c approached 0.025 for very small coefficients, power of the test of ab decreased to below 0.025.

Type I error rates—For all sample sizes, when $a = b = 0$ the test of joint significance had Type I error rates of .001. This result is a replication of previous literature on Type I error rates for tests of mediation (MacKinnon et al., 2002).

Discussion

A comparison of analytical power of the mediated and total effects shows that the mediated effect has increased power with small parameter sizes and large sample size, and with large parameter sizes and small sample size. Furthermore, the conditions for when power of the test of the mediated effect will be greater than power of the test of the total effect are also found in the comparison of standard errors of c and ab , and therefore in terms of variances and covariances among variables. This means that given an expected covariance matrix among variables based on previous literature, it would be possible to use those variances and covariances to determine if the standard error of c will be larger than the standard error of ab . If it is and ab and c are expected to be equal, the test of mediation will have more power than the test of the total effect.

Proximal Versus Distal Mediators: Effects on Power—When b is larger than a (that is, M is closer in time or more highly related to Y than to X), M may be considered a distal mediator. When a is larger than b (that is, M is closer in time or more highly related to X than to Y), M may be considered a proximal mediator. According to previous research, the test of ab will be more powerful for models with distal mediators than for models with proximal mediators due to collinearity between X and M (Hoyle & Kenny, 1999; Kenny & Judd, 2013). When collinearity between X and M is high, the standard error of the b path is increased, leading to a less powerful test of significance. Results from Fritz and MacKinnon (2007) support this point, demonstrating that required sample size is larger for conditions where a is larger than b .

As the test of ab is more powerful when b is larger than a , it follows that when comparing power of the test of ab to power of the test of c , the gain in power over the test of c would be greater in conditions where b is larger than a . This gain in power is seen in the results from this study. For example, in Table 1 at $N = 200$, $a = .39$, $b = .59$, $c' = 0$, adding a mediator increased power to detect effects by .211, and at $N = 200$, $a = .59$, $b = .39$, $c' = 0$, adding a mediator increased power to detect effects by 0.152. The increase in power to detect effects was larger for the condition where b was larger than a .

Study 2

Study 1 demonstrated that power to detect the mediated effect was greater than power to detect the total effect for the single mediator model. Study 2 aims to demonstrate that having two mediators also increases statistical power to detect effects, and identifies when the test of the total mediated effect $a_1b_1 + a_2b_2$ will have more power than the test of the total effect c for the parallel two mediator model.

Method

Determination of path parameters for the parallel two mediator model—The parallel two mediator model covariance matrix is shown in Table 3. Covariance algebra was used to derive formulas for the variance of each variable in the parallel two mediator model, and the covariances between each pair of variables.

The variances and covariances were then used to derive full and partial correlation effect sizes for the parallel two mediator model where ρ is the correlation. Formulas for the effect sizes are as follows:

$$\rho_{XM_1} = \frac{\sigma_{XM_1}}{\sqrt{\sigma_X^2} \sqrt{\sigma_{M_1}^2}} \quad (11)$$

$$\rho_{M_1Y.XM_2} = \frac{\rho_{M_1Y.X} - \rho_{M_1M_2.X}\rho_{M_2Y.X}}{\sqrt{1 - \rho_{M_1M_2.X}^2} \sqrt{1 - \rho_{M_2Y.X}^2}} \quad (12)$$

$$\rho_{XM_2} = \frac{\sigma_{XM_2}}{\sqrt{\sigma_X^2} \sqrt{\sigma_{M_2}^2}} \quad (13)$$

$$\rho_{M_2Y.XM_1} = \frac{\rho_{M_2Y.X} - \rho_{M_1M_2.X}\rho_{M_1Y.X}}{\sqrt{1 - \rho_{M_1M_2.X}^2} \sqrt{1 - \rho_{M_1Y.X}^2}} \quad (14)$$

$$\rho_{XY.M_1M_2} = \frac{\rho_{XY.M_1} - \rho_{XM_2.M_1}\rho_{YM_2.M_1}}{\sqrt{1 - \rho_{XM_2.M_1}^2} \sqrt{1 - \rho_{YM_2.M_1}^2}} \quad (15)$$

Where Equation 11 is the zero-order correlation effect size for a_1 , Equation 12 is the second-order partial correlation effect size for b_1 , Equation 13 is the zero-order correlation effect size for a_2 , Equation 14 is the second-order partial correlation effect size for b_2 , and Equation 15 is the second-order partial correlation effect size for c' . Appendix B shows how the effect size formulas were produced from the variances and covariances between variables.

Regression coefficients for this model were derived based on the above formulas that correspond approximately to Cohen's (1988) small, medium, and large correlation effect sizes. Parameters were identified based on the correlation results that would be as close to Cohen's guidelines for small, medium, and large effect sizes (.1, .3, and .5) as possible. It was determined that for a_1 , b_1 , a_2 , and b_2 , path coefficients of .101, .314, and .577 corresponded to small, medium, and large effects. For c' , path coefficients of .131, .400, and .740 corresponded to small, medium, and large effects. These path coefficients were necessary for generating data to calculate empirical power for the parallel two mediator model.

Empirical power for the parallel two mediator model—Empirical power was calculated by generating data with a known mediated effect and computing the proportion of times the test for mediation was statistically significant. Sample sizes chosen for the parallel two mediator model were 50, 100, 200, 500, and 1000. Parameter values (described above) were 0, .101, .314, and .577 for the a_1 , a_2 , b_1 , and b_2 coefficients, and 0, .131, .400, and .577 for the c' coefficient. This 5×4^5 design resulted in 5120 combinations of parameters and sample size, and 500 datasets were simulated for each combination. All variables were simulated to be continuous for both models, and all tests of significance were two-tailed. The product of coefficients test was used to assess significance for the parallel two mediator model. In this study, power was assessed for testing two null hypotheses: $H_0: c = 0$ for the test of the total effect and $H_0: a_1b_1 + a_2b_2 = 0$ for the product of coefficients test of significance. For each combination of parameters and sample size, the proportion of times out of that the test was significant out of 500 generated datasets was the empirical power for combinations with non-zero path parameters (the Type I error rate for combinations with path parameters equal to zero).

Bootstrap—Bootstrap estimation was used to calculate power for the total mediated effect because there is not a joint significance test for the parallel two mediator model, and the z test using the multivariate delta method standard error for the total mediated effect is known to be inaccurate. Bootstrap estimation was conducted for 10 combinations of parameters and sample size to compare to power of the total effect and total mediated effect. The combinations were randomly selected from the combinations where the total mediated effect was more powerful than the total effect based on the empirical power results. One sample of data was simulated for X , M_1 , M_2 , and Y based on the mediation equations, and then bootstrap estimation was conducted for that single dataset by sampling with replacement from that sample of data to get N observations, forming the first bootstrap sample. In each bootstrap sample, the values of a_1 , b_1 , a_2 , b_2 , and c were generated and saved. This process of randomly sampling with replacement and saving values of the coefficients was repeated 1000 times to create 1000 bootstrap samples for the first simulated data set, and the saved bootstrapped values of a_1 , b_1 , a_2 , b_2 , and c were used to create confidence intervals for the total mediated effect $a_1b_1 + a_2b_2$ and the total effect c . This process of simulating a dataset and generating a bootstrap confidence interval was repeated 1000 times, simulating 1000 datasets. The means of the number of times the confidence intervals did not include zero from the 1000 simulated datasets were the bootstrapped power values of the total and total mediated effects.

Results

Parallel two mediator model empirical results—Table 4 contains empirical power values for both the product of coefficients test of the total mediated effect for the parallel two mediator model and the test of the total effect where $c' = 0$ or .131 (corresponding to zero or small effect sizes for c' , which were the only values of c' at which the test of the total mediated effect was found to have more power than the test of the total effect). To reduce the amount of information presented, only combinations where $a_1 = b_1$ and $a_2 = b_2$ are included. Combinations of parameters and sample size where the test of the total mediated effect was more powerful than the test of the total effect are underlined, and combinations of

parameters and sample size where the test of the total mediated effect exceeded .80 when the test of the total effect did not are underlined and italicized.

The test of the total mediated effect was more powerful than the test of the total effect when effect sizes were large at smaller sample sizes, and when effect sizes were small at larger sample sizes, as was true for the single mediator model. For example, the test of the total mediated effect was more powerful than the test of the total effect when the a_1 , a_2 , b_1 , and b_2 coefficients were all large for $N = 50$, but both power values were equal to one for all larger sample sizes. However, the test of the total mediated effect was more powerful than the test of the total effect when the a_1 , a_2 , b_1 , and b_2 coefficients were small for $N = 200$ and above. In addition, there were more combinations of parameters and sample size where the test of the total mediated effect was more powerful than the test of the total effect at lower sample sizes because as sample size increased, the power of both tests approached one for the larger effect sizes.

Beyond looking at combinations of parameters and sample size where the test of the total mediated effect was more powerful than the test of the total effect, it is of interest to look at combinations where the test of the total mediated effect exceeded adequate power of .80 (Cohen, 1988) and the test of the total effect did not. There are 204 total combinations where this occurs. Table 4 shows that the test of the total mediated effect exceeded power of .80 and the test of the total effect did not in large samples with small coefficients and in small samples with large coefficients (the same trend found above) for a smaller portion of cases.

Parallel two mediator model bootstrap results—Comparisons of bootstrapped and empirical power values for the total effect and the total mediated effect for the parallel two mediator model are given in Table 5.

Power values were very similar for the bootstrap and empirical methods for the same test. The largest discrepancy between empirical and bootstrapped power values for the total mediated effect was a discrepancy of .038, which occurred at $N = 50$, $a_1 = .101$, $b_1 = .314$, $a_2 = .577$, and $b_2 = .577$. The largest discrepancy between empirical and bootstrapped power values for the total effect was a discrepancy of .045, which occurred at $N = 50$, $a_1 = .577$, $b_1 = .577$, $a_2 = .314$, and $b_2 = .101$. These bootstrap results confirm that the empirical power comparisons for the total and total mediated effects for the parallel two mediator model are accurate. Most importantly, the bootstrap results confirm the cases where power to detect the total mediated effect is greater than power to detect the total effect.

Type I error rates—For the parallel two mediator model, when either a_1 or b_1 was zero the a_1b_1 term was zero, and when either a_2 or b_2 was zero the a_2b_2 term was zero. Therefore, results from the empirical simulation with combinations where both the a_1b_1 and a_2b_2 terms are zero provide empirical Type I error rates. Combinations where $a_1 = b_1 = a_2 = b_2 = 0$ are provided here as measures of empirical Type I error for the product of coefficients test in Table 6.

As for the single mediator model, the product of coefficients test of the parallel two mediator model had low Type I error rates.

Discussion

Results show that when c' is zero or small, empirical power for the test of the total mediated effect is greater than power for the test of the total effect in two cases: (1) when effect sizes are small and sample size is large, and (2) when effect sizes are large and sample size is small, similar to the pattern of results found in Study 1. Looking at the combinations where empirical power exceeds .80 for the test of the total mediated effect, and not for the test of the total effect, reveals that these combinations follow the same general trend that all combinations did, but a smaller amount of combinations meet these criteria. In addition, power values from bootstrap sampling for the parallel two mediator model show the same discrepancy between power for the test of the total effect and power for the test of the total mediated effect. The differential effect of proximal and distal mediators is also shown for the parallel two mediator model results, where when b_1 or b_2 is larger than a_1 or a_2 , the increase in power is greater.

Study 3

Studies 1 and 2 demonstrated that power to detect the mediated effect was greater than power to detect the total effect for the single and parallel two mediator models in some situations. Drawing on results from these studies, Study 3 is designed to demonstrate that power to detect the three-path mediated effect $b_1b_2b_3$ has greater than power to detect the total effect c for the sequential two mediator model as well. Furthermore, Study 3 compares power of the single mediator model to power of the sequential two mediator model to demonstrate how power might change by adding an additional mediator in the sequential model.

Method

Analytical power for the sequential two mediator model—Analytical power of the three-path mediated effect for the sequential two mediator model was calculated using the same method as the single mediator model. A SAS program to compute analytical power for the sequential two mediator model is available at <http://ripl.faculty.asu.edu/downloadable-programs/others/>. Path coefficient values were set to .14, .39, and .59 to correspond to small, medium, and large effect sizes as used for the single mediator model. Power was calculated for sample sizes of 50, 100, 200, 500, 1000, and 5000, corresponding to sample sizes commonly used in the social sciences and the sample sizes used for the single mediator model. Population variances and covariances for X , M_1 , M_2 , and Y were calculated based on the different combinations of population path parameters and sample sizes, and were then used to calculate both correlation effect sizes corresponding to the population b_1 , b_2 , and b_3 paths and the true b_1 , b_2 , and b_3 coefficients and their standard errors. As with the single mediator model, analytical power was computed using the true coefficients and their standard errors to calculate a t test. To calculate power of the joint significance test for the three-path mediated effect, the power values for b_1 , b_2 , and b_3 were multiplied.

Comparison of single and sequential two mediator models—It is of particular interest to compare power of the single and sequential two mediator models because in a theoretical mediational chain, the sequential two mediator model would be the result of

adding a second mediator to a single mediator model. Comparing the single mediator and sequential two mediator models will answer the question of whether adding multiple mediators will increase power to detect effects of X on Y over a single mediator.

Analytical power was calculated for the total effect c and the mediated effects ab (in the single mediator model) and $b_1b_2b_3$ (in the sequential two mediator model), and all effects were set to be equal. A c coefficient value of .05 for the single mediator model corresponded to a model with one coefficient at a value of .1 and one coefficient at a value of .5; for the sequential two mediator model this corresponded to a model with two coefficients at values of .3 and one coefficient at a value of .555. At a c coefficient value of .09, for the single mediator model this corresponded to a model with both coefficients at a value of .3; for the sequential two mediator model this corresponded to a model with two coefficients at values of .5 and one coefficient at a value of .36. At a c coefficient value of .15, for the single mediator model this corresponded to a model with one coefficient at a value of .3 and one coefficient at a value of .5; for the sequential two mediator model this corresponded to a model with two coefficients at values of .5 and one coefficient at a value of .6. Power was calculated for these models at a sample size of $N = 100$ and the direct effect was set to zero for both mediation models. The b_5 and b_6 coefficients were set to zero as well for the sequential two mediator model.

Results

Sequential two mediator model analytical results—Table 7 shows analytical power values for the joint significance test of the three-path mediated effect. Power values did not differ across values of the direct effect b_4 . Only power values for the three-path mediated effect are considered in this table (no combinations with zero paths for b_1 , b_2 , or b_3), and the b_5 and b_6 coefficients were set to zero (meaning no two-path mediated effects were present in the model). Combinations of parameters where the joint significance test of the three-path mediated effect is more powerful than the test of the total effect are underlined. The test of the total effect had more power than the test of the three-path mediated effect only at $N = 50$ when two of the three coefficients were small and at $N = 100$ when all three of the coefficients were small. For some combinations of large effects and large samples, power values for both tests approached one and were approximately equal. For all other combinations of parameters and sample sizes, the test of the three-path mediated effect had more power than the test of the total effect.

Comparison of standard errors of total and three-path mediated effects—

Because the b_4 , b_5 , and b_6 coefficients were set to zero when computing power for the sequential two mediator model, the total mediated effect was equal to $b_1b_2b_3$ and $b_1b_2b_3 = c$. As with the single mediator model, for the test of the total mediated effect to be more powerful, the t value from the test of significance for $b_1b_2b_3$ must be larger than the t value from the test of significance for c , and so the standard error of the total effect must be larger than the standard error of the total mediated effect.

Analytical standard errors of $b_1b_2b_3$ and c for the sequential two mediator model are shown in Table 8 for combinations where none of the b_1 , b_2 , or b_3 coefficients equal zero and $N = 100$.

As with the single mediator model, when the total effect was equal to the total mediated effect and the direct effect was equal to zero, the standard error of c was larger than the standard error of $b_1b_2b_3$ and power of the test of $b_1b_2b_3$ was greater than power of the test of c . The exceptions to this pattern were the cases where power of the test of $b_1b_2b_3$ was less than power of the test of c as shown in Table 7 even though the standard error of $b_1b_2b_3$ was smaller than the standard error of c . Power of the test of c was greater than power of the test of $b_1b_2b_3$ only when two of the three coefficients were small at the smallest sample size studied, and in these conditions, the power value dropped below .025 (the same pattern of results produced in Study 1).

Results for comparison of models—The comparison of power for the single and sequential two mediator models are shown in Table 9. Power of the test of the total effect is given as well for comparison. The total mediated effects for both models were equal to each other, and were also equal to c .

For each of the conditions, the test of the total effect had the least power and the test of the three-path mediated effect for the sequential two mediator model had the most power. The mediated effect for the single mediator model had power values that fell between the test of the total effect and the test of the sequential two mediator model. These results indicate that in some situations not only did the test of the single mediator model have more power to detect effects than the test of the total effect, the test of the sequential two mediator model had more power to detect effects than either the test of the single mediator model or the test of the total effect.

Discussion

Power of the three-path mediated effect from the sequential two mediator model is greater than power of the test of the total effect for almost all combinations where the total mediated effect $b_1b_2b_3$ is non-zero. As with the single mediator model, the standard error of the total mediated effect is always smaller than the standard error of the total effect, leading to a more powerful test of mediation. Furthermore, a comparison of the sequential two mediator model to the single mediator model and the total effect shows that the sequential two mediator model has more power than either the single mediator model or the test of the total effect. The finding that the test of the three-path mediated effect from the sequential two mediator model has more power than the mediated effect from the single mediator model and the test of the total effect indicates that including multiple mediators may be more beneficial to power than including a single mediator or testing only the total effect for significance.

Examples

To provide more concrete examples of how testing the mediation hypothesis provides greater statistical power, we describe several hypothetical studies. Suppose a researcher is planning an intervention study to increase safe sex practices. The study is limited to 100

participants, and past research suggests that the researcher would find a correlation of about .2 for the total effect of X on Y. A correlation of .2 corresponding to a total effect of X on Y with $N = 100$ would have power of .517. If there is theoretical evidence for a mediator (for example condom use self-efficacy) that has a correlation of .387 between the intervention and the mediator and .371 between the mediator and the outcome, then the power to detect the mediated effect of X to M to Y is equal to .815.

For a second example, imagine a media intervention to increase exercise among elderly patients with a correlation of .1 and a maximum of 400 elderly persons available for the study. The power to detect a correlation of .1 with 400 participants is .515. If the researcher has a mediator (such as awareness of places to exercise) with a .148 correlation with X and .669 with Y, then the power to detect the mediated effect is .847. For this same example, if the researcher has a mediator (such as exercise self-efficacy) with a .33 correlation with X and .287 with Y, then the power to detect the mediated effect is .999.

In a third example, a researcher expects a correlation between X and Y of .05 for an intervention to promote vegetable consumption. The power of detecting that correlation is equal to .197 with $N = 500$. However, if the researcher has a mediator (such as social norms about healthy eating) with a correlation of .447 for X to M and .100 for M to Y, then the power to detect the mediated effect of .05 is equal to .605. If the sample size was $N = 1000$, the power to detect the relation of X to Y would increase from .348 to .884 for the same size correlations between X and M and M and Y.

For a final example, say a researcher expects a correlation between X and Y of .07 based on prior research for an intervention to increase social competence. A correlation of .07 with $N = 500$ would have power of .36. However, if the researcher adds a mediator (such as skills in social settings) with a correlation of .14 for X to M and .52 for M to Y, then the power to detect the mediated effect of .07 is equal to .86.

General Discussion

The purpose of this article was to identify situations where the test of the mediated effect is more powerful than the test of the total effect for both single and two mediator models. The inclusion of mediators increases power when coefficients are small and sample size is large, and when coefficients are large and sample size is small. These results extend from the mediated effect for the single mediator model to the total mediated effect for the parallel two mediator model, and to the three-path mediated effect for the sequential two mediator model. These are the most important findings from these studies, as they can inform a researcher with a fixed sample size and fixed effect sizes of the magnitude of power increase that will result from including one or two mediators. Furthermore, a comparison of the standard errors of c and ab for the single mediator model when c and ab were equal determined that when $ab = c$, the standard error of c is always larger than the standard error of ab and therefore the single mediator model is always more powerful than the test of the total effect. This result is also found for the sequential two mediator model, where the standard error of c is always larger than the standard error of $b_1b_2b_3$ for the sequential two mediator model. These findings will be useful for researchers working with known complete

mediation models, as this means that complete mediation models with one or two mediators are always more powerful than the test of the total effect except when coefficients approach zero.

Fit with Earlier Literature

Previous research on significance tests of mediation has shown that some tests are more powerful because they do not require the total effect to be significant. The results of these studies confirm that in some situations when the mediated effect and the total effect are equal, the test of the mediated effect will be significant but the test of the total effect will not. This replicates findings in existing literature, and extends this concept to models with multiple mediators as well. The results here also confirm that the joint significance test of mediation for the single mediator model is more powerful than the product of coefficients test using the multivariate delta standard error, which replicates findings in MacKinnon et al. (2002).

Limitations

These studies show that mediators increase statistical power given certain circumstances. However, in order to take advantage of the increase in power that a mediation model may provide, there must be theoretical support for inclusion of a mediator in the planning stages of a study. The inclusion of mediators must be theory-driven and not data-driven; it is inappropriate to decide to include a mediator post hoc simply because one fails to find a significant relationship between X and Y. A mediation hypothesis must be specified in the design prior to randomization or data collection, based on theory or evidence from prior research.

It is also important to realize that including a mediator will not always increase power. In some situations, including a hypothesized mediator may fail to change the relation between X and Y or decrease the power to detect the relation between X and Y. The current studies show that including a mediator will increase power when the standard error of c is greater than the standard error of ab and ab is equal to c . However, ab and c will not always be equal; in fact, ab will typically be less than or equal to c (for consistent mediation models, but not for inconsistent mediation models). When the ratio of ab over its standard error is equal to the ratio of c over its standard error, there will be no difference in power to detect effects. When the ratio of ab over its standard error becomes smaller than the ratio of c over its standard error, the test of c will be more powerful than the test of ab . And for the parallel two mediator model, when the specific mediated effects are in opposite directions, the total mediated effect would be smaller and therefore less powerful compared to the test of the single mediator model.

Future Directions

Although the current studies examine single and two mediator models, the gain in power achieved from adding mediators could be assessed for more complex models with multiple mediators as well, such as structural equation mediation models, longitudinal mediation models, and mediation models with binary variables. Power of complex mediation models has been assessed through empirical simulation (Thoemmes, MacKinnon, & Reiser, 2010),

but analytical power has not been computed for these models. It would also be of interest to compare power of the parallel and sequential two mediator models. The power gained from including more than two mediators in a model could be studied as well. Additionally, the results presented here could be used in the planning stages of a study as a method of increasing power to detect effects. Future research could focus on how these results can be used to choose mediators in the planning stage of research to increase power.

Sufficient power is a key component in the design and implementation of any research study. This article demonstrates that in some situations, inclusion of mediators increases power to detect effects. An important finding of this article is that including multiple mediators in a model will increase power over and above the test of the total effect. Another finding is the specific conditions under which including one or more mediators will increase power. This finding is important for planning in research design, and provides an analytical formula for calculating when the inclusion of a mediator will result in a more powerful statistical test as well as guidelines for researchers with an expected effect size and sample size who wish to use mediators to increase power. These findings will be of use for all researchers who are interested in mediation as a method for increasing statistical power.

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Appendix A

For the single mediator model described in Equations 2 and 3, and assuming uncorrelated errors across equations, normally distributed variables and a linear system of relations (McDonald, 1997), the true covariances among X, M, and Y for the single mediator model can be used to obtain expected values of regression coefficients and the standard errors of those coefficients to compute predicted power for the test of a , b , and c at any combination of effect size and sample size. The expected $\hat{a}_T$ coefficient is given in Equation A.1. The subscript T is used to indicate that this value is the true or theoretical value given the covariance matrix and sample size.

$$\hat{a}_T = \frac{Cov(X, M)}{Var(X)} \quad (A.1)$$

The true or theoretical (true is represented by the T in the subscript) variance of the estimator $\hat{a}_T$, $\sigma_{\hat{a}_T}^2$ (standard error is the square root of the variance), is equal to:

$$\sigma_{\hat{a}_T}^2 = \frac{\sigma_{e3}^2}{(N - 2) Var[X]} \quad (A.2)$$

In Equation A.2, σ_{e3}^2 is the true error variance from Equation 3 which predicts M from X. An estimator of the true error variance represented in two ways is shown in equations A.3 and A.4.

$$\hat{\sigma}_{e3T}^2 = \frac{\sum e^2}{N - p - 1} \quad (\text{A.3})$$

$$\hat{\sigma}_{e3T}^2 = \hat{\sigma}_M^2 - a^2 \hat{\sigma}_X^2 \quad (\text{A.4})$$

In Equation A.3, p is the number of X variables (which is equal to 1 here for the one predictor case), and $\sum e^2$ is the sum of the squared differences between the predicted and observed M scores. Equation A.4 involves the variances of X and M and the $\hat{a}$ coefficient. Similar formulas are used to determine the expected values of $\hat{b}$ and $\hat{c}'$ as shown in equations A.5 and A.6.

$$\hat{b}_T = \frac{\text{Var}[X] \text{Cov}[M, Y] - \text{Cov}[X, M] \text{Cov}[X, Y]}{\text{Var}[X] \text{Var}[M] - \text{Cov}[X, M]^2} \quad (\text{A.5})$$

$$\hat{c}'_T = \frac{\text{Var}[M] \text{Cov}[X, Y] - \text{Cov}[X, M] \text{Cov}[M, Y]}{\text{Var}[X] \text{Var}[M] - \text{Cov}[X, M]^2} \quad (\text{A.6})$$

$$\sigma_{bT}^2 = \frac{\sigma_{e2}^2}{N - 3} \frac{\frac{1}{\text{Var}[M]}}{1 - r_{XM}^2} \quad (\text{A.7})$$

$$\sigma_{c'T}^2 = \frac{\sigma_{e2}^2}{N - 3} \frac{\frac{1}{\text{Var}[X]}}{1 - r_{XM}^2} \quad (\text{A.8})$$

$$\hat{\sigma}_{e2T}^2 = \hat{\sigma}_Y^2 - b^2 \hat{\sigma}_M^2 - c'^2 \hat{\sigma}_X^2 + bc' \hat{\sigma}_{MX}^2 \quad (\text{A.9})$$

Equations A.7 and A.8 show the true variance of $\hat{b}_T$ and $\hat{c}'_T$ respectively, where N is the number of participants and $\hat{\sigma}_{e2T}^2$ is the expected true error variance from Equation 2. Equation A.9 shows an estimator of the true error variance for Equation 2 predicting Y from X and M and is analogous to the error variance formula in Equation A.4, but with $p = 2$ predictors (X and M).

The covariance matrix in Table 5 produces analogous formulas to determine the expected values of $\hat{c}_T$, $\sigma_{c'T}^2$, and $\hat{\sigma}_{e1T}^2$, from Equation 1 as shown in Equations A.10, A.11, and A.12, respectively.

$$\hat{c}_T = \frac{\text{Cov}[X, Y]}{\text{Var}[X]} \quad (\text{A.10})$$

$$\sigma_{\hat{c}T}^2 = \frac{\sigma_{e1}^2}{(N-2) \text{Var}[X]} \quad (\text{A.11})$$

$$\hat{\sigma}_{e1T}^2 = \hat{\sigma}_Y^2 - c^2 \hat{\sigma}_X^2 \quad (\text{A.12})$$

Statistical Power to detect the Mediated Effect

Statistical power is determined using these expected values as shown in Equations A.13, A.14, and A.15 where ϕ_t is the probability from the cumulative t distribution. The statistical power of the joint significance test for mediation consists of the power to detect a nonzero a path and the power to detect a nonzero b path. If both paths are statistically significant, the hypothesis of no mediated effect is rejected (MacKinnon et al., 2002).

$$P(\hat{a}_T) = 1 - \phi_t(\hat{a}_T / \sigma_{\hat{a}T} - t_{1-\alpha/2, N-2}) \quad (\text{A.13})$$

$$P(\hat{b}_T) = 1 - \phi_t(\hat{b}_T / \sigma_{\hat{b}T} - t_{1-\alpha/2, N-3}) \quad (\text{A.14})$$

$$P(\hat{c}_T) = 1 - \phi_t(\hat{c}_T / \sigma_{\hat{c}T} - t_{1-\alpha/2, N-2}) \quad (\text{A.15})$$

So a case where power to detect the mediated effect is greater than the power to detect the total effect is given by the following inequality

$$P(\hat{a}_T) P(\hat{b}_T) > P(\hat{c}_T) \quad (\text{A.16})$$

All power values were computed analytically with a SAS program available from the authors. Analytical power values checked empirically in a statistical simulation study for the sample sizes and parameter values described in this article. Note that when both a and b are zero, for the joint significance test of a and b , the Type I error rate is .0025.

Appendix B

Effect Size Formulas for the Parallel Two Mediator Model

$$\rho_{M_1 M_2} = \frac{\sigma_{M_1 M_2}}{\sqrt{\sigma_{M_1}^2} \sqrt{\sigma_{M_2}^2}}$$

$$\rho_{M_1 Y} = \frac{\sigma_{M_1 Y}}{\sqrt{\sigma_{M_1}^2} \sqrt{\sigma_Y^2}}$$

$$\rho_{M_2Y} = \frac{\sigma_{M_2Y}}{\sqrt{\sigma_{M_2}^2} \sqrt{\sigma_Y^2}}$$

$$\rho_{XY} = \frac{\sigma_{XY}}{\sqrt{\sigma_X^2} \sqrt{\sigma_Y^2}}$$

For a_1 :

$$\rho_{XM_1} = \frac{\sigma_{XM_1}}{\sqrt{\sigma_X^2} \sqrt{\sigma_{M_1}^2}}$$

For a_2 :

$$\rho_{XM_2} = \frac{\sigma_{XM_2}}{\sqrt{\sigma_X^2} \sqrt{\sigma_{M_2}^2}}$$

For b_1 :

$$\rho_{M_1Y.XM_2} = \frac{\rho_{M_1Y.X} - \rho_{M_1M_2.X} \rho_{M_2Y.X}}{\sqrt{1 - \rho_{M_1M_2.X}^2} \sqrt{1 - \rho_{M_2Y.X}^2}}, \text{ where}$$

$$\rho_{M_1Y.X} = \frac{\rho_{M_1Y} - \rho_{XM_1} \rho_{XY}}{\sqrt{1 - \rho_{XM_1}^2} \sqrt{1 - \rho_{XY}^2}},$$

$$\rho_{M_1M_2.X} = \frac{\rho_{M_1M_2} - \rho_{XM_1} \rho_{XM_2}}{\sqrt{1 - \rho_{XM_1}^2} \sqrt{1 - \rho_{XM_2}^2}},$$

$$\rho_{M_2Y.X} = \frac{\rho_{M_2Y} - \rho_{XM_2} \rho_{XY}}{\sqrt{1 - \rho_{XM_2}^2} \sqrt{1 - \rho_{XY}^2}}$$

For b_2 :

$$\rho_{M_2Y.XM_1} = \frac{\rho_{M_2Y.X} - \rho_{M_1M_2.X} \rho_{M_1Y.X}}{\sqrt{1 - \rho_{M_1M_2.X}^2} \sqrt{1 - \rho_{M_1Y.X}^2}}, \text{ where}$$

$$\rho_{M_2Y.X} = \frac{\rho_{M_2Y} - \rho_{XM_2}\rho_{XY}}{\sqrt{1 - \rho_{XM_2}^2} \sqrt{1 - \rho_{XY}^2}},$$

$$\rho_{M_1M_2.X} = \frac{\rho_{M_1M_2} - \rho_{XM_1}\rho_{XM_2}}{\sqrt{1 - \rho_{XM_1}^2} \sqrt{1 - \rho_{XM_2}^2}},$$

$$\rho_{M_1Y.X} = \frac{\rho_{M_1Y} - \rho_{XM_1}\rho_{XY}}{\sqrt{1 - \rho_{XM_1}^2} \sqrt{1 - \rho_{XY}^2}}$$

For c' :

$$\rho_{XY.M_1M_2} = \frac{\rho_{XY.M_1} - \rho_{XM_2.M_1}\rho_{YM_2.M_1}}{\sqrt{1 - \rho_{XM_2.M_1}^2} \sqrt{1 - \rho_{YM_2.M_1}^2}}, \text{ where}$$

$$\rho_{XY.M_1} = \frac{\rho_{XY} - \rho_{XM_1}\rho_{M_1Y}}{\sqrt{1 - \rho_{XM_1}^2} \sqrt{1 - \rho_{M_1Y}^2}},$$

$$\rho_{XM_2.M_1} = \frac{\rho_{XM_2} - \rho_{XM_1}\rho_{M_1M_2}}{\sqrt{1 - \rho_{XM_1}^2} \sqrt{1 - \rho_{M_1M_2}^2}},$$

$$\rho_{YM_2.M_1} = \frac{\rho_{M_2Y} - \rho_{M_1Y}\rho_{M_1M_2}}{\sqrt{1 - \rho_{M_1Y}^2} \sqrt{1 - \rho_{M_1M_2}^2}}$$

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Figure 1.
Path diagrams for the regression and one mediator models. Adapted from MacKinnon, 2008.

Figure 2. Path diagram for the parallel two mediator model. Adapted from MacKinnon, 2008.

Figure 3.
Path diagram for the sequential two mediator model. Adapted from MacKinnon, 2008.

Table 1

Analytical Power of the Mediated and Total Effects for the Single Mediator Model at $c' = 0$ and $c' = .14$

c'	N = 50			N = 100			N = 200			N = 500			N = 1000			N = 5000		
	π_c	π_{ab}	$\pi_{c'}$	π_c	π_{ab}	$\pi_{c'}$	π_c	π_{ab}	$\pi_{c'}$	π_c	π_{ab}	$\pi_{c'}$	π_c	π_{ab}	$\pi_{c'}$	π_c	π_{ab}	$\pi_{c'}$
$a = .14, b = .14$	0	.034	.025	.038	.078	.046	.249	.063	.766	.089	.986	.278	.986	.986	.986	.986	.986	.986
0.14	.189	.025	.341	.078	.598	.249	.939	.766	.999	.986	.986	.986	.986	.986	.986	.986	.986	.986
$a = .14, b = .39$	0	.054	.117	.072	.269	.106	.499	.204	.875	.361	.993	.949	.993	.993	.993	.993	.993	.993
0.14	.235	.117	.427	.269	.716	.499	.980	.875	.993	.993	.993	.993	.993	.993	.993	.993	.993	.993
$a = .14, b = .59$	0	.070	.153	.104	.279	.168	.499	.354	.875	.612	.993	.999	.993	.993	.993	.993	.993	.993
0.14	.257	.153	.467	.279	.763	.499	.989	.875	.993	.993	.993	.993	.993	.993	.993	.993	.993	.993
$a = .39, b = .14$	0	.056	.117	.077	.269	.115	.499	.225	.875	.400	.993	.969	.993	.993	.993	.993	.993	.993
0.14	.259	.117	.471	.269	.767	.499	.990	.875	.993	.993	.993	.993	.993	.993	.993	.993	.993	.993
$a = .39, b = .39$	0	.161	.551	.285	.928	.509	.999	.883	.994	.994	.994	.994	.994	.994	.994	.994	.994	.994
0.14	.454	.551	.754	.928	.965	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999
$a = .39, b = .59$	0	.271	.720	.492	.963	.789	.999	.992	.992	.992	.992	.992	.992	.992	.992	.992	.992	.992
0.14	.575	.720	.870	.963	.993	.993	.993	.993	.993	.993	.993	.993	.993	.993	.993	.993	.993	.993
$a = .59, q = .14$	0	.081	.153	.124	.279	.208	.499	.445	.875	.732	.993	.993	.993	.993	.993	.993	.993	.993
0.14	.323	.153	.577	.279	.866	.499	.998	.875	.993	.993	.993	.993	.993	.993	.993	.993	.993	.993
$a = .59, q = .39$	0	.308	.720	.554	.963	.848	.999	.997	.997	.997	.997	.997	.997	.997	.997	.997	.997	.997
0.14	.640	.720	.915	.963	.997	.997	.997	.997	.997	.997	.997	.997	.997	.997	.997	.997	.997	.997
$a = .59, b = .59$	0	.526	.941	.829	.986	.986	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999
0.14	.800	.941	.981	.986	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999	.999

Note. Analytical power of the total effect is represented here as ' π_c ', and analytical power calculated using regression coefficients and their standard errors is represented here as ' π_{ab} '.

Table 2

Comparison of Power and Standard Errors of ab and c for the Single Mediator Model Where $c' = 0$ and $N = 100$

a	b	ab	c	s_c	s_{ab}	π_c	π_{ab}
.14	.14	.020	.020	.102	.020	.038	.078
.14	.39	.055	.055	.108	.042	.072	.269
.14	.59	.083	.083	.117	.061	.104	.279
.39	.14	.055	.055	.102	.042	.077	.269
.39	.39	.152	.152	.108	.056	.285	.928
.39	.59	.230	.230	.117	.072	.492	.963
.59	.14	.083	.083	.102	.062	.124	.279
.59	.39	.230	.230	.108	.072	.554	.963
.59	.59	.348	.348	.117	.085	.829	1

Table 3

Covariance Matrix for the Parallel Two Mediator Model

	M_2	M_1	X
M_2	$a_2^2\sigma_X^2 + \sigma_{\epsilon_2}^2$	$a_1a_2\sigma_X^2$	$a_2\sigma_X^2 + \sigma_{\epsilon_2}^2$
M_1	$a_1a_2\sigma_X^2$	$a_1^2\sigma_X^2 + \sigma_{\epsilon_1}^2$	$a_1\sigma_X^2 + \sigma_{\epsilon_1}^2$
X	$a_2\sigma_X^2 + \sigma_{\epsilon_2}^2$	$a_1\sigma_X^2 + \sigma_{\epsilon_1}^2$	$\sigma_X^2 + \sigma_{\epsilon}^2$

Table 4

Empirical Power Values for the Test of the Total Mediated Effect and the Test of the Total Effect for the Parallel Two Mediator Model

		a_1b_1					
		SS		MM		LL	
a_2b_2		$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c
$N = 50$							
SS	$c'=0$	.006	.064	<u>.146</u>	<u>.102</u>	<u>.830</u>	<u>.546</u>
	$c'=131$	.012	.174	.140	.320	<u>.852</u>	<u>.768</u>
MM	$c'=0$	<u>.168</u>	<u>.096</u>	<u>.510</u>	<u>.240</u>	<u>.950</u>	<u>.686</u>
	$c'=131$	.162	.368	.484	.504	<u>.942</u>	<u>.882</u>
LL	$c'=0$	<u>.860</u>	<u>.542</u>	<u>.950</u>	<u>.692</u>	<u>.992</u>	<u>.942</u>
	$c'=131$	<u>.868</u>	<u>.800</u>	<u>.950</u>	<u>.890</u>	<u>.994</u>	<u>.980</u>
$N = 100$							
SS	$c'=0$	.026	.066	<u>.576</u>	<u>.148</u>	<u>.998</u>	<u>.832</u>
	$c'=131$	.030	.354	.614	.630	<u>1</u>	<u>.982</u>
MM	$c'=0$	<u>.560</u>	<u>.170</u>	<u>.936</u>	<u>.438</u>	<u>1</u>	<u>.914</u>
	$c'=131$	.540	.586	<u>.948</u>	<u>.846</u>	<u>1</u>	<u>.996</u>
LL	$c'=0$	<u>1</u>	<u>.828</u>	<u>1</u>	<u>.938</u>	1	1
	$c'=131$	<u>1</u>	<u>.974</u>	<u>1</u>	<u>.996</u>	1	1
$N = 200$							
SS	$c'=0$	<u>.088</u>	<u>.058</u>	<u>.962</u>	<u>.310</u>	<u>1</u>	<u>.982</u>
	$c'=131$	.106	.536	<u>.962</u>	<u>.896</u>	1	1
MM	$c'=0$	<u>.974</u>	<u>.302</u>	<u>1</u>	<u>.716</u>	<u>1</u>	<u>.998</u>
	$c'=131$	<u>.974</u>	<u>.886</u>	<u>1</u>	<u>.980</u>	1	1
LL	$c'=0$	<u>1</u>	<u>.982</u>	1	1	1	1
	$c'=131$	1	1	1	1	1	1
$N = 500$							
SS	$c'=0$	<u>.546</u>	<u>.070</u>	<u>1</u>	<u>.602</u>	1	1
	$c'=131$	.546	.918	<u>1</u>	<u>.996</u>	1	1
MM	$c'=0$	<u>1</u>	<u>.618</u>	<u>1</u>	<u>.982</u>	1	1
	$c'=131$	1	1	1	1	1	1
LL	$c'=0$	1	1	1	1	1	1
	$c'=131$	1	1	1	1	1	1
$N = 1000$							
SS	$c'=0$	<u>.962</u>	<u>.110</u>	<u>1</u>	<u>.884</u>	1	1
	$c'=131$	.972	.998	1	1	1	1
MM	$c'=131$	<u>1</u>	<u>.898</u>	<u>1</u>	<u>.998</u>	1	1
	$c'=131$	1	1	1	1	1	1

		a_1b_1					
		SS		MM		LL	
a_2b_2		$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c	$a_1b_1+a_2b_2$	c
		$N = 50$					
LL	$c'=0$	1	1	1	1	1	1
	$c'=131$	1	1	1	1	1	1

Note. For effect sizes, S = 0.101, M = 0.314, and L = 0.577, as per the path parameters used in the simulation. Different values of c' are listed down the lefthand column and the different tests are listed across the top. Combinations where power of the test of mediation exceeds power of the test of the total effect are underlined.

Table 5
Comparison of Bootstrapped and Empirical Power Values of the Total and Total Mediated Effects for the Parallel Two Mediator Model

N	a_1	q_2	a_2	b_2	c'	Power of $a_1b_1 + a_2b_2$			Power of c		
						Empirical	Bootstrap	Difference	Empirical	Bootstrap	Difference
50	.101	.314	.577	.577	0	.890	.876	.038	.576	.535	.041
50	.577	.577	.314	.101	0	.864	.864	.000	.554	.599	-.045
100	.101	.101	.314	.577	0	.820	.852	-.032	.342	.371	-.029
100	.101	.314	.577	.314	0	.842	.859	-.017	.514	.518	-.004
200	0	0	.314	.314	0	.934	.962	-.028	.272	.260	.012
200	.314	.314	.101	0	0	.934	.925	.009	.254	.281	-.027
500	.577	.101	.577	.101	0	.856	.881	-.025	.724	.713	.011
500	.101	.314	.314	.101	0	.892	.863	.029	.270	.237	.033
1000	.101	.101	.101	.101	0	.962	.968	-.006	.110	.095	.015
1000	.101	.577	.314	.101	0	.994	.998	-.004	.680	.698	-.018

Table 6Type I Error Rates for the Parallel Two Mediator Model, $a_1b_1 = a_2b_2 = 0$

<i>N</i>	<i>a</i> ₁	<i>b</i> ₁	<i>a</i> ₂	<i>b</i> ₂	<i>c</i> '	Product of Coefficients
50	0	0	0	0	0	0
	0	0	0	0	.14	0
	0	0	0	0	.39	0
	0	0	0	0	.59	0
100	0	0	0	0	0	0
	0	0	0	0	.14	0
	0	0	0	0	.39	0
	0	0	0	0	.59	0
200	0	0	0	0	0	0
	0	0	0	0	.14	.002
	0	0	0	0	.39	0
	0	0	0	0	.59	.002
500	0	0	0	0	0	0
	0	0	0	0	.14	0
	0	0	0	0	.39	0
	0	0	0	0	.59	0
1000	0	0	0	0	0	0
	0	0	0	0	.14	0
	0	0	0	0	.39	0
	0	0	0	0	.59	0

Table 7

Analytical Power of the Joint Significance Test of Mediation and Test of the Total Effect for the Sequential Two Mediator Model

	50		100		200		500		1000		5000	
	c	JS	c	JS	c	JS	c	JS	c	JS	c	JS
SSS	.026	.004	0.27	.021	0.27	.124	.029	.673	.031	.979	.039	1
SSM	.028	.018	.030	.074	.032	.249	.037	.768	.043	.986	.078	1
SSL	.030	.024	.032	.077	.036	.249	.044	.768	.055	.986	.127	1
SMS	.028	.018	.030	.074	.032	.249	.037	.768	.043	.986	.078	1
SMM	.035	.087	.040	.261	.048	.500	.069	.877	.099	.993	.325	1
SML	.041	.114	.050	.269	.066	.500	.107	.877	.173	.993	.624	1
SLS	.030	.024	.032	.077	.036	.249	.044	.768	.055	.986	.127	1
SLM	.041	.114	.050	.269	.066	.500	.107	.877	.173	.993	.624	1
SLI	.052	.149	.069	.278	.101	.500	.191	.877	.337	.993	.931	1
MSS	.028	.018	.030	.074	.032	.248	.037	.767	.043	.986	.078	1
MSM	.035	.086	.040	.259	.048	.498	.069	.876	.099	.993	.325	1
MSL	.041	.113	.050	.267	.066	.498	.107	.876	.173	.993	.624	1
MMS	.035	.086	.040	.259	.048	.498	.069	.876	.099	.993	.325	1
MMM	.060	.419	.084	.906	.129	.999	.261	1	.465	1	.987	1
MML	.088	.549	.140	.937	.241	1	.515	1	.808	1	1	1
MLS	.041	.113	.050	.267	.066	.498	.107	.876	.173	.993	.624	1
MLM	.088	.549	.140	.937	.241	1	.515	1	.808	1	1	1
MLL	.150	.720	.265	.969	.477	1	.856	1	.990	1	1	1
LSS	.030	.023	.032	.076	.036	.248	.044	.767	.055	.986	.127	1
LSM	.041	.112	.050	.267	.066	.498	.107	.876	.173	.993	.624	1
LSL	.052	.147	.069	.276	.101	.498	.191	.876	.337	.993	.931	1
LMS	.041	.112	.050	.267	.066	.498	.107	.876	.173	.993	.624	1
LMM	.088	.544	.140	.936	.241	1	.515	1	.808	1	1	1
LML	.150	.713	.265	.967	.477	1	.856	1	.990	1	1	1
LLS	.052	.147	.069	.276	.101	0.5	.191	0.88	.337	.993	.931	1
LLM	.150	.713	.265	.967	.477	1	.856	1	.990	1	1	1
LLL	.286	.936	.521	1	.820	1	.996	1	1	1	1	1

Note. For effect sizes, $S = .14$, $M = .39$, and $L = .59$, as per the path parameters used in the simulation. The path parameters are ordered as b_1, b_2, b_3 . The tests are listed across the top. Combinations where power of the test of the three-path mediated effect exceeds power of the test of the total effect are underlined.

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Table 8

Comparison of Power and Standard Errors of $b_1b_2b_3$ and c for the Sequential Two Mediator Model Where $N = 100$

b_1	b_2	b_3	$b_1b_2b_3$	c	S_c	$S_{b_1b_2b_3}$	π_c	$\pi_{b_1b_2b_3}$
0.14	0.14	0.14	.003	.003	.101	.003	.027	.022
0.14	0.14	0.39	.008	.008	.101	.008	.029	.075
0.14	0.14	0.59	.012	.012	.101	.012	.031	.078
0.14	0.39	0.14	.008	.008	.101	.008	.030	.075
0.14	0.39	0.39	.021	.021	.101	.017	.039	.259
0.14	0.39	0.59	.032	.032	.101	.025	.045	.269
0.14	0.59	0.14	.012	.012	.101	.012	.032	.078
0.14	0.59	0.39	.032	.032	.101	.025	.047	.269
0.14	0.59	0.59	.049	.049	.101	.037	.059	.279
0.39	0.14	0.14	.008	.008	.101	.008	.030	.075
0.39	0.14	0.39	.021	.021	.101	.017	.039	.259
0.39	0.14	0.59	.032	.032	.101	.025	.046	.269
0.39	0.39	0.14	.021	.021	.101	.017	.040	.259
0.39	0.39	0.39	.059	.059	.101	.027	.078	.894
0.39	0.39	0.59	.090	.090	.101	.036	.112	.928
0.39	0.59	0.14	.032	.032	.101	.025	.050	.269
0.39	0.59	0.39	.090	.090	.101	.036	.124	.928
0.39	0.59	0.59	.136	.136	.101	.048	.195	.963
0.59	0.14	0.14	.012	.012	.101	.012	.032	.078
0.59	0.14	0.39	.032	.032	.101	.025	.048	.269
0.59	0.14	0.59	.049	.049	.101	.037	.061	.279
0.59	0.39	0.14	.032	.032	.101	.025	.050	.269
0.59	0.39	0.39	.090	.090	.101	.036	.126	.928
0.59	0.39	0.59	.136	.136	.101	.048	.202	.963
0.59	0.59	0.14	.049	.049	.101	.037	.069	.279
0.59	0.59	0.39	.136	.136	.101	.048	.228	.963
0.59	0.59	0.59	.205	.205	.101	.061	.382	.999

Table 9

Comparison of Power for the Single and Sequential Two Mediator Models and the Total Effect When All Effects are Equal

Effect Value	π_c	π_{ab}	π_{b1b2b3}
0.05	0.071	0.163	0.697
0.09	0.141	0.697	0.936
0.15	0.312	0.835	0.997