Iso-uncertainty control in an experimental fluoroscopy system

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Purpose: X-ray fluoroscopy remains an important imaging modality in a number of image-guided procedures due to its real-time nature and excellent spatial detail. However, the radiation dose delivered raises concerns about its use particularly in lengthy treatment procedures (>0.5 h). The authors have previously presented an algorithm that employs feedback of geometric uncertainty to control dose while maintaining a desired targeting uncertainty during fluoroscopic tracking of fiducials. The method was tested using simulations of motion against controlled noise fields. In this paper, the authors embody the previously reported method in a physical prototype and present changes to the controller required to function in a practical setting.

Methods: The metric for feedback used in this study is based on the trace of the covariance of the state of the system, $\text{tr}(C)$. The state is defined here as the 2D location of a fiducial on a plane parallel to the detector. A relationship between this metric and the tube current is first developed empirically. This relationship is extended to create a manifold that incorporates a latent variable representing the estimated background attenuation. The manifold is then used within the controller to dynamically adjust the tube current and maintain a specified targeting uncertainty. To evaluate the performance of the proposed method, an acrylic sphere (1.6 mm in diameter) was tracked at tube currents ranging from 0.5 to 0.9 mA (0.033 s) at a fixed energy of 80 kVp. The images were acquired on a Varian Paxscan 4030A (2048 $\times$ 1536 pixels, ~ 100 cm source-to-axis distance, ~ 160 cm source-to-detector distance). The sphere was tracked using a particle filter under two background conditions: (1) uniform sheets of acrylic and (2) an acrylic wedge. The measured $\text{tr}(C)$ was used in conjunction with a learned manifold to modulate the tube current in order to maintain a specified uncertainty as the sphere traversed regions of varying thickness corresponding to the acrylic sheets in the background.

Results: With feedback engaged, the tracking error was found to correlate well with the specified targeting uncertainty. Tracking of the fiducial was found to be robust to changes in the attenuation presented by the varying background conditions. For a desired uncertainty of 5.0 mm, comparison of the feedback framework with a comparable system employing fixed exposure demonstrated dose savings of 29%.

Conclusions: This work presents a relation between a state descriptor, $\text{tr}(C)$, the x-ray tube current used, and an estimate of the background attenuation. This relation is leveraged to modulate the tube current in order to maintain a desired geometric uncertainty during fluoroscopy. The authors' work demonstrates the use of the method in a real x-ray fluoroscopy system with physical motion against varying backgrounds. The method offers potential savings in imaging dose to patients and staff while maintaining tracking uncertainty during fluoroscopy-guided treatment procedures. © 2014 American Association of Physicists in Medicine. [<http://dx.doi.org/10.1118/1.4900601>]

Key words: image-guided intervention, tracking, uncertainty, geometric performance, intrafraction motion, feedback

1. INTRODUCTION

X-ray fluoroscopy remains an important tool for image guided interventional procedures due to its high resolution, high frame rate, and high contrast of bone and fiducial markers. However, for a number of clinical procedures, there is concern regarding the imaging dose delivered to the patient over the duration of

the procedure or course of therapy.^{1–9} The selection of imaging parameters involves trade-offs. For example, when selecting the tube current (milliampere) in x-ray fluoroscopy, too low of an exposure could result in photon starvation which implies high noise, poor image quality, and poor localization. Conversely, too high of an exposure will deliver excessive dose to the patient. Developments in image-guided radiation therapy

have led to the tracking of markers over prolonged periods of treatment which raises concern regarding the magnitude of dose delivered to patients due to imaging.^{10,12}

In previous work, we showed that in x-ray fluoroscopy, a relationship can be established between the tube current used and the operating tracking uncertainty defined as the determinant of the covariance matrix of the state of the system.¹¹ In addition, we proposed a framework for feedback of uncertainty to modulate the tube current and limit radiation dose to the patient. We previously demonstrated this framework in simulation. In this paper, we (i) embody the previously reported framework for adaptive control of tube current in a physical prototype and (ii) present the changes required to the controller to function in a practical scenario.

2. METHODS

In this section, the framework (Fig. 1) for tube current modulation is described. This framework tracks an object of interest during an image-guided procedure via a state estimation module that also produces an estimate of the uncertainty in localizing the object of interest. This uncertainty estimate is then used as feedback by a control module to modulate the tube current in order to maintain an operator specified localization uncertainty.

In our previously reported framework for modulating the tube current, we used the determinant of the covariance of the state matrix $\det(C)$ as the metric for feedback. To demonstrate the framework, we simulated the motion of the object of interest. The main objective of our current study is to implement a practical realization of the previously reported framework on an experimental system. In our attempt to build such a practical realization, we encountered two major challenges due to the limitations of our previously reported technique:

- (i) In the simulations used in our previous study, the mean background signal was normalized to simplify the algorithm. In a practical setting, the mean background signal changes and under such circumstances the previously reported algorithm fails to track the object of interest.
- (ii) For feedback, the previously reported technique used the determinant of the covariance matrix which is prone to collapsing and under estimating the uncertainty metric, $\det(C)$, when the uncertainty in any one dimension approaches zero.

FIG. 1. A system diagram showing an uncertainty feedback framework for image-guided therapy. A desired localization uncertainty is specified to a control unit that modulates the input parameters to an image acquisition system based on feedback of the estimated uncertainty with the goal of maintaining the specified uncertainty.

The previously reported technique requires further development for use in a practical setting. This paper builds on this framework by (i) integrating a new controller that incorporates a latent variable to account for large variations in the background signal and (ii) using a more robust uncertainty estimate based upon the trace of the covariance matrix of the state.

2.A. Experimental setup

We first define our metric for feedback (Sec. 2.A.1). We then describe an experimental setup used to develop an empirical relationship between uncertainty, tube current, and a latent variable to account for the unknown background thickness. We use acrylic sheets of uniform thickness to train our controller and then test the behavior of the controller in presence of a wedge.

2.A.1. Uncertainty dependence

A framework that modulates the tube current for maintaining a specified uncertainty would ideally use a measure of the disparity between the estimated location and ground truth (true location of the object of interest) as feedback. However, ground truth is not an accessible quantity in many clinical interventions. As an alternative, an estimate of the uncertainty in localization can be used. In this study, the trace of the state covariance matrix, $\text{tr}(C)$, is used to describe the uncertainty. This is an improvement over our previous use of the determinant of the state covariance matrix because it tolerates significant axial asymmetry in uncertainty (if the uncertainty in any one dimension is zero, the determinant collapses to zero). A mathematical definition of $\text{tr}(C)$ is presented below.

Consider the task of tracking a small sphere (a fiducial) using a flat panel x-ray fluoroscopy system. Assume that the motion of the sphere is limited to a plane parallel to the x-ray detector. Let u and v denote the axes of a 2D image obtained from this imaging system. Let $f[u,v]$ denote the probability density function (pdf) that defines the probability of observing the center of the sphere at location (u,v) in the image. This pdf can be used to construct a covariance matrix

$$C = \begin{bmatrix} \sigma_u^2 & \sigma_{uv}^2 \\ \sigma_{uv}^2 & \sigma_v^2 \end{bmatrix}.$$

The square root of the trace of this matrix, $\sqrt{\text{tr}(C)} = \sqrt{\sigma_u^2 + \sigma_v^2} \equiv \gamma$, is taken as the definition of uncertainty. Figure 2 illustrates a geometric interpretation of the trace.

Previous methods reported by our group were designed to be robust to the background signal variance but were not designed to be robust to changes in the background signal.¹¹ The uncertainty estimate defined above is unfortunately affected by the background signal. The expected behavior of the framework being developed here is to change the tube current in response to changes in the background. In a clinical context, overlapping structures (such as ribs when tracking a fiducial in the lung) present added attenuation and affect the ability to localize and track the object of interest. A relationship between the proposed metric for localization uncertainty, $\text{tr}(C)$, and the

FIG. 2. Geometric interpretation of uncertainty in 2D.

tube current, i , used in an x-ray fluoroscopy guided clinical procedure is desired to allow for the controller to modulate the tube current. This relationship needs to account for the effect of large scale changes in the background on $\text{tr}(C)$. An experiment is described below that develops a relationship between $\text{tr}(C)$, i , and changes in the background presented by uniform slabs of acrylic that manifest varying levels of x-ray quantum noise. A latent variable, α , for the unknown attenuation presented by the background is introduced and a model is developed that relates $\text{tr}(C)$, i , and α . Note that α is independent of the tube current used.

2.A.2. Establishing the relationship between uncertainty and tube current

A simple linear motion model was developed and deployed on a motion controlled x-ray bench. The experimental setup for this study is shown in Fig. 3 and details of the geometry and equipment used are summarized in Table I. An acrylic (polymethyl methacrylate) sphere (16 mm in diameter; attenuation coefficient, $\mu = 0.02/\text{m}$ at 80 kVp) was chosen as the object of interest for tracking. This sphere was imaged under x-ray fluoroscopy at 80 kVp. The images were acquired on a Varian PaxScan 4030A (2048×1536 , $194 \mu\text{m}$) under the kilovolt imaging geometry typically found on linear accelerators

FIG. 3. Imaging apparatus and setup used for acquiring fluoroscopic images. (a) Top: A kV x-ray source is shown on the right and a flat panel x-ray detector on the left. An acrylic sphere used to simulate the object of interest is embedded in a foam block attached to a mechanical arm. SDD and SAD refer to source-to-detector distance and source-to-axis distance respectively. (b) Bottom: A schematic of the setup seen from above showing the relative locations of the kV x-ray source, x-ray detector, the acrylic sphere, and the background object [acrylic wedge or acrylic slab (s)].

equipped with a kilovolt-imaging modality (~ 100 cm source to axis distance, ~ 160 cm source to detector distance). The sphere was embedded in a low density uniform background Styrofoam™ block and attached to a mechanical arm that was moved with high precision ($\pm 40 \mu\text{m}$). To establish the relationship between $\text{tr}(C)$ and i , the object of interest (sphere) was tracked using the state estimation technique described later in Sec. 2.B.1 in near real-time and the estimated uncertainty was recorded. This was done at known values of the tube current ranging from 0.5 to 9 mA with acrylic slabs of thickness 54, 101.6, 128.6, and 155.6 mm corresponding to transmission, I/I_0 , of 0.3, 0.1, 0.08, and 0.04, respectively, and known background

TABLE I. Details on the geometry and equipment used for the experimental setup in this study.

Detector		Geometry		Source	
Make	Varian Medical Systems	SDD	~ 160 cm	Make	Varian Medical Systems
Model	PaxScan4030A	SAD	~ 100 cm	Model	Rad 94
Resolution	$194 \mu\text{m}$ pixel pitch	Range of motion, R	290 mm	Focal spot	0.4–0.8 mm
Scintillator	CsI	Object of interest		Target	W-Rh
Physical size	40×30 cm	Acrylic sphere with a 16 mm diameter		Angle	14°
Image size	1536×2048	Background object		Housing	Sapphire
BitDepth	14	Acrylic slabs of different thicknesses		Filter	0.1 mm Cu + 5×1.02 mm Al
		Acrylic wedge, max depth of 106 mm		Energy	80 kVp
				Tube current	0.5–9.0 mA
				Pulse width	0.033 s

attenuation, α , of 54, 101.6, 128.6, and 155.6 μ , respectively, where μ ($=0.02 \text{ mm}^{-1}$ at 80 kVp) is the attenuation coefficient of acrylic. These acrylic blocks of different thickness present uniform x-ray quantum noise in the background. Throughout this study, the x-ray pulse width was fixed at 1/30 s leaving the tube current–time product only a function of the tube current. The location of the background slabs relative to the object of interest is shown in Fig. 3(a). The sphere was moved on a line in a plane parallel to the detector according to a linear motion model with Gaussian perturbations to simulate physiological variation (e.g., heartbeat and breathing). The movement of the sphere was simple linear motion with drift in one dimension and additive white Gaussian noise in both dimensions. A simple linear motion model was chosen in order to let the background dictate changes to uncertainty and hence the tube current. The images of the sphere obtained while tracking were processed (dark and flood field corrections). From these images, the exposure manifold was empirically developed by computing a fit to the measured tracking uncertainty, the known background attenuation used, and the tube current employed.

2.A.3. Image guidance framework

For tracking performance, the programmed precision motion was recorded as ground truth and processed images were passed to the framework for tracking and feedback control.

The sphere was tracked in the presence of a variety of backgrounds including acrylic slabs that presented a uniform background (54 and 137.6 μ) and an acrylic wedge of linearly increasing thickness. The sphere was tracked both with and without feedback and the results were compared. The feedback case was evaluated for specified uncertainties (γ_{spec}) of 2, 3, 5, and 17 mm. The conventional case without feedback was evaluated at tube current levels of 1.8, 2, 2.8, 3.2, 4, and 5 mA. Note that for the feedback case, direct control of the tube current was not an option on the generator. As a surrogate, the tube current was incremented or decremented by a human operator in response to the controller's output (~ 2 s intervals). For both the feedback case and the conventional case, each tracking run was repeated three times with a different random seed each time to evaluate the robustness of the stochastic state estimator used. The state estimation module and the controller are described next.

2.B. Framework

2.B.1. State estimation module

A particle filter algorithm similar to that used in our previous work¹¹ was used for state estimation. The state of the system, $X[k] = [x_k, y_k]^T$, $X[k] \in \mathbb{R}^2$, was described by the 2D location of the center of the acrylic sphere being tracked. The steps involved in state estimation are summarized in Fig. 4(a). The particle filter was initialized by sampling uniformly from a rectangular region containing the sphere in the first frame. A previously reported observation process was employed (see Appendix A). Note that the uncertainty that

FIG. 4. (a) Flow chart showing details of the state estimation module. $X[k] = [x_k, y_k]^T$ represents the state of the system at the k th time step and $Y[k]$ represents the image observed at the k th time step. (b) Flow chart showing details of the tube current controller.

can be captured by this observer is limited by size of the window over which the observer operates.

The following dynamic model was used to describe the motion of the sphere:

$$x_{k+1} = x_k + D_k + \varphi_x, \quad (1)$$

$$y_{k+1} = y_k + \varphi_y \quad (2)$$

for $k=0,1,2,\dots$, where φ_x and φ_y are zero mean Gaussian random variables with variance σ_x^2 and σ_y^2 , respectively. The x and y axes lie on a 2D plane parallel to the detector at the object stage where the sphere being tracked is located.

$$D_k = \begin{cases} E[X_k] - E[X_{k-2}]/2 & k > 3 \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

2.B.2. Control module

The control module produces the tube current for the next time step, i_{k+1} , using the measured uncertainty, $\text{tr}(C_k)$, and the

specified operating uncertainty, γ_{spec} , based on the relationship developed from the experiment described in Sec. 2.A.2. A two-step process [4(b)] was used in which the background attenuation, α_k , is first estimated using

$$\alpha_k = \frac{1}{g} \log \left[\frac{1}{f i_k} \log \left(\frac{c}{\gamma_{\text{spec}} - h} - 1 \right) - d \right], \quad (4)$$

where g , f , c , h , and d are fitted constants and i_k is the tube current at the current time step. This estimate of the latent variable α_k , is then used to estimate i_{k+1}

$$i_{k+1} = \frac{1}{f e^{g \alpha_k}} \log \left[\log \left(\frac{c}{\gamma_{\text{spec}} - h} - 1 \right) - d \right]. \quad (5)$$

The equation above was derived from the results of the experiment described in Sec. 2.A.2. Section 3.A presents the results and the relation obtained.

A sample image sequence in which feedback of uncertainty is employed by the control module is shown in Fig. 5. As the sphere being tracked moves from left to right with an acrylic wedge in the background, the tube current changes in response to the estimated uncertainty.

FIG. 5. (a) A schematic of the acrylic wedge as seen from above. (b)–(d) Sample x-ray fluoroscopic image sequence showing an acrylic sphere being tracked with an acrylic wedge in the background. Feedback of uncertainty is employed. The ellipse overlaid on the sphere represents the observed uncertainty with major and minor axis drawn at 1σ . Note that the underlying distribution is not necessary Gaussian, however, an ellipse is used as a simple visual representation.

2.B.3. Validation metrics

The root mean square error (RMSE) in world coordinates and the entrance surface exposure (ESE), both at the object stage were used as metrics for assessing the performance of the feedback platform. These are defined in Appendix B.

3. RESULTS

3.A. Establishing the relation between uncertainty and tube current

Estimates of the uncertainty, $\sqrt{\text{tr}(C)}$, measured as described in Sec. 2.A.1 for different values of tube current, i , and different background attenuation levels, α , are shown in Fig. 6 along with a 2D fit given by

$$\sqrt{\text{tr}(C)} = \frac{c}{1 + e^{b_\alpha i + d}} + h, \quad (6)$$

where b_α , c , d , and h are constants. A 3D fit to the measured data assuming a continuous latent variable, α , (representing the background attenuation that is independent of the tube current) was captured and given by substituting $b_\alpha = f e^{g\alpha}$ in the above equation, with f , and g being constants. A manifold representing this relation is shown in Fig. 7. Note that at lower tube currents and higher values of the background attenuation, α , the observer saturates in accordance with our discussion earlier (Sec. 2.B.1). Conversely, at lower values of α and higher values of the tube current, the uncertainty plateaus. During tracking, only a subregion of the manifold was used that was limited by a maximum and a minimum cutoff uncertainty. Uncertainty estimates that were beyond these cutoff values were clipped and the tube current values at the corresponding extrema were used.

3.B. Image guidance framework

In Fig. 8, a sample tracking run using feedback is shown for the case with a wedge background and is compared with two cases without feedback. In all cases, the sphere moves from left to right as it traverses an increasing background

FIG. 6. Relation between uncertainty, $\text{tr}(C)$, and tube current, i , at 80 kVp shown for background attenuation, α , corresponding to 54, 101.6, 128.6, and 155.6 μ , from left to right, respectively.

FIG. 7. Measured data (dotted) and computed curve for the relation between uncertainty, $\text{tr}(C)$, tube current, i , and the latent background attenuation, α , at 80 kVp shown in 3D.

attenuation followed by a decreasing background attenuation presented by the acrylic wedge. A desired tracking uncertainty of 5 mm at the object stage was chosen (black dashed line). This is specified as an input to the feedback algorithm. On the first frame of the feedback case, the tube current starts off at 2.8 mA and then falls because of the lower attenuation at the ends of the wedge. As the background attenuation gradually increases, so does the tube current. After crossing the highest attenuation point, the background attenuation lowers as the sphere moves to the right and correspondingly the tube current also drops. The RMSE for this case is 5.08 mm (which is tolerable for the specified uncertainty of 5 mm) and the ESE for

this run is 0.15 mR/frame. This feedback case is compared to a conventional tracking case without feedback ($i = 2$ mA) with a comparable ESE of 0.12 mR/frame. However, the RMSE for the case without feedback is much higher (26.1 mm) and the sphere is considered lost (as it exceeds the highest measurable instantaneous uncertainty of 24.1 mm). The feedback case is also compared with a case without feedback ($i = 3.2$ mA) that is able to successfully track the sphere without getting lost. In this case, the RMSE of 2.84 mm is within the specified uncertainty of 5 mm. However, the ESE for this case is 0.21 mR/frame. Comparing the feedback case with the case without feedback ($i = 3.2$ mA) that is able to successfully track the sphere without getting lost as it traverses the varying background attenuation presented by the acrylic wedge, we note that the overall dose saving for the feedback case is 29%. This saving corresponds to the transition over the entire wedge.

The RMSE metric used in this study captures the average error over a run. Higher order statistics relating to the operating error need to be investigated. The instantaneous uncertainty at any given frame can be used as a surrogate to alert the operator when the operating precision during a procedure is within specification.

In Figs. 9(a) and 9(b), the RMSE and ESE metrics introduced in Sec. 2.B.2 are plotted as a function of the specified uncertainty, γ_{spec} , for background attenuation levels, α , of 54 and 137.6 μ presented by uniform acrylic slabs while tracking with feedback of uncertainty. A linear fit between γ_{spec} and RMSE is observed and given by

$$\text{RMSE} = p\gamma_{\text{spec}} + q, \quad (7)$$

FIG. 8. Comparison of a run from the feedback case with runs from two cases without feedback for background attenuation presented by an acrylic wedge. Note that the plots in this figure show the *instantaneous* uncertainty, tube current, and RMSE for each frame in the run. The instantaneous RMSE is defined as $\|\hat{X}_r[k] - \bar{X}[k]\|_2$. Refer to Appendix B for validation metrics.

FIG. 9. Results of tracking a sphere while employing feedback of uncertainty for a background attenuation of 54 and 137.6 μ presented by uniform acrylic slabs. (a) RMSE and (b) ESE as a function of the specified uncertainty.

where p and q are constants. An exponential relation between ESE and γ was observed given by

$$ESE = re^{-s\gamma_{\text{spec}}} + t, \quad (8)$$

where r , s , and t are constants.

Similarly, in Figs. 10(a) and 10(b), the RMSE and ESE are plotted as a function of γ_{spec} with an acrylic wedge in the background using feedback of uncertainty and similar relations are observed. In contrast, plots of RMSE and ESE as a function of the tube current when tracking the acrylic sphere using a conventional approach not employing feedback are shown in Figs. 10(c) and 10(d) for an acrylic wedge background.

4. DISCUSSION

One of the main goals of image guidance in clinical treatment procedures is to localize objects of interest within a desired level of uncertainty. A trade-off exists between the desired tracking uncertainty and the tube current used for imaging. Figures 6 and 7 provide valuable insight into this trade-off and show that a relationship exists between these variables. This relationship is also a function of a latent variable for the unknown background attenuation presented by structures that overlap with the object of interest, such as anatomical or patient support structures. In this study, we assume that the background presents uniform quantum noise. From Fig. 6, it is observed that as the thickness of the acrylic slab increases from 54 to 155.6 mm, corresponding to higher attenuation presented by the background, the slope of the uncertainty-tube

FIG. 10. Results of tracking a sphere *with* feedback of uncertainty for a background attenuation presented by an acrylic wedge (a) RMSE and (b) ESE as a function of the specified uncertainty. Results of tracking a sphere *without* feedback of uncertainty for a background attenuation presented by an acrylic wedge (c) RMSE and (d) ESE as a function of the tube current used.

current curve gets shallower, offering a broader range for optimizing the trade-off between the imaging dose and localization uncertainty. The relationship developed from Fig. 7 is significant as it allows one to modulate the tube current based on the desired operating tracking uncertainty and the automatically estimated background attenuation. Note in Fig. 6

that the measured uncertainty for the curve corresponding to $\alpha = 54 \mu$ starts to increase for tube current values larger than 6 mA. At these higher tube current levels, the detector reaches saturation and correspondingly the localization uncertainty is artificially elevated. The other curves corresponding to higher values of α are expected to show similar behavior for currents greater than 9.0 mA. Depending on the attenuation presented by the background in the neighborhood of the object of interest, an appropriate tube current needs to be engaged in order to maintain the specified uncertainty. Use of a tube current that is too low will result in elevated quantum noise in the image and consequently will make localization difficult. On the other hand, use of a tube current that is too high will result in saturation of the detector and also produce poor localization results. The relation developed between i , α , and γ can help control dose by modulating the tube current while tracking to achieve the desired tracking uncertainty. The incorporation of the latent variable α allows us to automatically adapt the tube current to background changes without prior knowledge of the background.

Figure 9(a) shows the important result that we are able to achieve a correlation between a specified uncertainty estimate and the observed error. A linear relationship appears to exist between the uncertainty that is specified, γ_{spec} , and the RMSE that is observed. Figure 9(a) shows that it is possible to specify a desired uncertainty estimate and have the system modulate the tube current so as to control the localization error despite changes in the background attenuation.

From Fig. 9(b), we can infer that the higher the background attenuation, α , the more slowly varying the curve, offering more potential dose savings. In a clinical context, when tracking an object of interest that moves in and out of a region of high signal-to-noise ratio or SNR (presented by higher background attenuation) and a region of low SNR (corresponding to lower background attenuation), the greater the disparity between the region of high signal-to-noise ratio and low signal-to-noise ratio, the greater the potential dose savings that can be achieved. The framework proposed here is expected to be more valuable in clinical contexts with a greater disparity between regions of high and low SNR, for example, when the object of interest moves across bones. This framework is of particular benefit for procedures that require the use of x-ray fluoroscopy over longer treatment times.

From Fig. 10(a), it is understood that despite the use of a nonuniform background (wedge that presents a higher attenuation in the middle), it is possible for the method to achieve an RMSE similar to that for the case of uniform acrylic background slabs for different values of the specified uncertainty. Comparing Figs. 10(a) and 10(b) with Figs. 10(c) and 10(d), we see that because there is a correlation between the uncertainty metric used and the RMSE, and the fact that the purpose of image guidance is to localize objects of interest, the proposed uncertainty metric is a natural choice as an input parameter for the framework. With this correlation, we do not need the error explicitly; we can use the uncertainty metric as a surrogate.

In the studies conducted here, the tube current was incremented or decremented by a human operator based on

the controller's output. If this process is made automatic and faster, the tube current adaptation could be improved and more closely follow changes in the wedge's thickness. This is the subject of future work. Ultimately, we would like to explore this in real-time with $\Delta t \cong 30$ ms. The main barriers to real-time performance are the observer computations, particle filter parameters (number of particles), and generator control. Fortunately, the observation process is parallelizable [$O(n)$ in the number of particles used by the particle filter] and real-time performance can potentially be achieved.

5. CONCLUSIONS

In this paper, a relationship has been developed between the tube current used during an x-ray fluoroscopy guided procedure, the localization uncertainty associated with the object of interest, and a latent variable representing the unknown background attenuation. This relationship could be used to modulate the tube current while imaging during treatment in order to lower the imaging dose to patients while maintaining the localization uncertainty required for treatment. The framework proposed here is particularly useful when an object of interest goes through regions of high and low SNR presented by varying background regions of different attenuation. The framework offers the potential for reducing the imaging dose delivered particularly in procedures that involve long treatment times.

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APPENDIX A: STATE ESTIMATION MODULE

This section describes the observation process that forms part of the state estimation module used in this study. This observation process is described in more detail in our previously published paper.¹¹ Using the acquired x-ray images and prior information in the form of a geometric model (disc of a specified radius to represent the sphere being tracked), the observation process updates the weights of the particles for the next tracking cycle. A tracking cycle consists of a *prediction* step (based on a dynamic model) and an *update* step (based on the observation process).

A Canny edge detector is first applied to the image obtained from the current tracking cycle to produce an edge map that highlights regions where edges are observed, and an orientation map that shows the orientation of the observed edge elements. For each sample point of the particle filter, four radially symmetric profiles across the expected locations of edges of a sphere centered at the sample point are examined based on the object's geometric model. Next, the likelihood of the sphere's center being at the sample point given

the model is evaluated

$$\begin{aligned} \text{Likelihood} &= P(\text{Object is present at location } (u, v) \\ &\quad \text{in image} \mid \text{Object model}) \\ &= P(g_1 = m_{g1}, g_2 = m_{g2}, g_3 = m_{g3}, g_4 = m_{g4} \mid \\ &\quad \text{Object model}), \end{aligned}$$

where $g_j = m_{g_j}$ represents the event that an edge of the right orientation, within a tolerance, is found at location m_{g_j} on profile g_j . The following independence assumption was made:

$$\begin{aligned} P(g_i \mid g_j, \text{Object model}) &= P(g_i \mid \text{Object model}) \\ &\quad \times P(g_j \mid \text{Object model}) \end{aligned}$$

for $i, j = 1, \dots, 4$, $i \neq j$. This independence assumption simplifies the likelihood above to

$$\text{likelihood} = \prod_{j=1}^4 P(g_j = m_{g_j} \mid \text{Object model}).$$

These likelihood values were then normalized to produce the weights for the sample points. The set of samples along with the normalized weights constitute a sampled representation of the pdf of the location of the center of the sphere in the image. This representation of the pdf was used to compute the covariance matrix, C , and the location of the center of the sphere using Monte Carlo integration.

APPENDIX B: VALIDATION METRICS

1. RMSE

The RMSE is defined here as the Euclidean distance between the estimated location, $\hat{X}_r[k]$, of the sphere for run r at frame k , and the corresponding ground truth value averaged over the number of frames, K , and the total number of runs, R

$$\text{RMSE} = \frac{1}{KR} \sum_{k=1}^K \sum_{r=1}^R \|\hat{X}_r[k] - \bar{X}[k]\|_2.$$

2. ESE

Using an exposure duration of 0.033 s/frame and a measured exposure rate of 2.0 mR/mA s at the detector, the ESE at the

object was computed as

$$\text{ESE} = \frac{1}{KR} \sum_{k=1}^K \sum_{r=1}^R 0.033 i_r[k] \left(\frac{\text{SDD}}{\text{SAD}} \right)^2 2.0,$$

where $i_r[k]$ is the tube current used for the r th run and k th frame, SAD is the source to axis distance and SDD is the source to detector distance.

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