Adaptation of the modified Bouc–Wen model to compensate for hysteresis in respiratory motion for the list-mode binning of cardiac SPECT and PET acquisitions: Testing using MRI

Paul K. R. Dasari and Mohammed Salman Shazeeb

Department of Radiology, Division of Nuclear Medicine, University of Massachusetts Medical School, Worcester, Massachusetts 01655 and Department of Biomedical Engineering, Worcester Polytechnic Institute, Worcester, Massachusetts 01609

Arda Könik, Clifford Lindsay, Joyeeta M. Mukherjee, Karen L. Johnson, and Michael A. King^{a)}

Department of Radiology, Division of Nuclear Medicine, University of Massachusetts Medical School, Worcester, Massachusetts 01655

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Purpose: Binning list-mode acquisitions as a function of a surrogate signal related to respiration has been employed to reduce the impact of respiratory motion on image quality in cardiac emission tomography (SPECT and PET). Inherent in amplitude binning is the assumption that there is a monotonic relationship between the amplitude of the surrogate signal and respiratory motion of the heart. This assumption is not valid in the presence of hysteresis when heart motion exhibits a different relationship with the surrogate during inspiration and expiration. The purpose of this study was to investigate the novel approach of using the Bouc–Wen (BW) model to provide a signal accounting for hysteresis when binning list-mode data with the goal of thereby improving motion correction. The study is based on the authors' previous observations that hysteresis between chest and abdomen markers was indicative of hysteresis between abdomen markers and the internal motion of the heart.

Methods: In 19 healthy volunteers, they determined the internal motion of the heart and diaphragm in the superior–inferior direction during free breathing using MRI navigators. A visual tracking system (vts) synchronized with MRI acquisition tracked the anterior–posterior motions of external markers placed on the chest and abdomen. These data were employed to develop and test the Bouc–Wen model by inputting the vts derived chest and abdomen motions into it and using the resulting output signals as surrogates for cardiac motion. The data of the volunteers were divided into training and testing sets. The training set was used to obtain initial values for the model parameters for all of the volunteers in the set, and for set members based on whether they were or were not classified as exhibiting hysteresis using a metric derived from the markers. These initial parameters were then employed with the testing set to estimate output signals. Pearson's linear correlation coefficient between the abdomen, chest, average of chest and abdomen markers, and Bouc–Wen derived signals versus the true internal motion of the heart from MRI was used to judge the signals match to the heart motion.

Results: The results show that the Bouc–Wen model generated signals demonstrated strong correlation with the heart motion. This correlation was slightly larger on average than that of the external surrogate signals derived from the abdomen marker, and average of the abdomen and chest markers, but was not statistically significantly different from them.

Conclusions: The results suggest that the proposed model has the potential to be a unified framework for modeling hysteresis in respiratory motion in cardiac perfusion studies and beyond. © 2014 American Association of Physicists in Medicine. [<http://dx.doi.org/10.1118/1.4895845>]

Key words: emission tomography, respiratory motion, modeling hysteresis, cardiac imaging, MRI

1. INTRODUCTION

Respiratory motion is inevitable in emission tomography due to the long acquisition times of emission scans. This leads to image artifacts and errors in quantification.^{1–3} With the emergence of multimodality imaging scanners (e.g., SPECT/CT and PET/CT), the problems from uncorrected motion have been amplified due to the artifacts present in the attenuation corrected images caused by an inconsistency in respiratory motions between the transmission/CT and the emission data.^{4–7} In

addition to inaccurate attenuation correction, respiratory motion leads to image blur resulting in loss of contrast and biased quantification.⁸ The artifacts caused by respiratory motion can differ between SPECT and PET. For example, in SPECT, since data are being acquired for only a couple of projection angles at any given time, drifts if respiration over time can cause inconsistencies between the location of the structures viewed at one time point versus another, with resulting artifacts. In PET, data are acquired from all angles all time. Thus drifting in

respiration results in increased blurring of structures, but not angular mismatch during reconstruction as in SPECT.

Several acquisition methods based on respiratory gating have been proposed for mitigating the effects of respiratory motion.^{9–11} Such gated acquisitions yield a number of images corresponding to distinct parts of the respiratory cycle. Each gated set of images contains only a small part of the total respiratory motion resulting in images with minimal motion artifacts. Respiratory gating of the emission data can be performed employing (a) phase-based gating and/or (b) amplitude-based gating. Both are based on a respiratory signal obtained by tracking some aspect of respiratory motion in the subject. In phase-based gating, the emission data are sorted based on the phase or relative time point within the subject's respiratory cycle. In this approach, however, the extent of respiratory motion is not considered thereby resulting in inconsistent distribution of motion among the set of gated images. On the other hand, in amplitude-based gating, the sorting of emission data is based on the measured amplitude of the respiratory signal, irrespective of the phases in the subject's respiratory cycle. This method captures the extent of respiratory motion more accurately. This is especially important when patient respiration is variable. Experimental analysis from various studies^{12–16} indicates that the amplitude-based gating is the preferred method, with the exception of when respiration is very regular. The amplitude-based gating method assumes that the organs move in the same trajectory for both inspiration and expiration. However, studies have shown that certain individuals can exhibit hysteresis in which the organs follow different trajectories during the inspiratory and expiratory phases of the respiratory cycle.^{17–19} The effects of hysteresis have been investigated and reported for lung tumor motion,^{20–22} and to a lesser extent for heart motion.^{17,18}

In the presence of hysteresis, for a given displacement of an external respiratory signal (e.g., from markers or bellows on patient's surface), the actual internal motion of the heart can show a variation in its displacement. This was previously illustrated by Dasari *et al.*¹⁹ using the MRI navigator data from a volunteer and is shown in Fig. 1, wherein the position of the heart is very different with respect to the position of the abdomen during expiratory and inspiratory phases of the respiratory cycle. Therefore, performing binning using a hysteretic respiratory signal based on amplitude alone results in suboptimal correction. This complex internal behavior indicates that estimating the heart motion from a single parameter (such as the respiratory signal related to abdominal motion) may not fully describe the motion trajectories of the heart thereby introducing errors and reducing the effectiveness of amplitude-based binning method.

Dasari *et al.*,¹⁹ investigated the relationship between the anterior–posterior (AP) motion of external markers on the chest and the abdomen of free-breathing human volunteers using a visual tracking system (vts)²³ (described in Sec. 2.B). In addition, the relationship between the superior–inferior (SI) motion of the heart and the diaphragm was investigated by internal tracking using MR-navigators in synchrony with external tracking vts. This study inferred that if hysteresis is present internally between the heart and diaphragm, a

FIG. 1. 2D scatter plot illustrating the phenomenon of hysteresis in the respiratory motion of the heart relative to that of an external abdominal marker. Each dot represents the position of the heart (superior/inferior direction) with respect to the position of the external abdomen marker position (anterior/posterior) for each of 300 time points measured. The MRI navigator methodology was employed to measure the superior/inferior motion of the heart in a free-breathing human volunteer. The inspiration and expiration phases depict distinct trajectories of the heart motion. The shaded area shows the extent of residual motion present in the middle-bin if the abdomen signal were to be selected in amplitude binning. Note that the sign choice for the abdomen signal is positive for the posterior direction and negative for anterior direction. This makes the direction of the hysteresis counterclockwise.

similar behavior might manifest externally in the chest and the abdomen signals. The most direct approach to address the problem of hysteresis is to treat the inspiratory and expiratory phases separately using the amplitude-based gating method.¹² However, the resulting gated images are deteriorated by lower signal-to-noise ratio (SNR), since each gated image possesses a smaller portion of the total counts available at that signal amplitude. This also nearly doubles the number of reconstructions necessary to estimate respiratory motion for later correction in a combined iterative reconstruction with motion correction of all acquired events.¹⁶ Therefore, a method that accounts for the effects of hysteresis without sacrificing the SNR of the reconstructed images or significantly increasing processing time would be desirable. One such method can be based on using the respiratory information from both the chest and the abdomen regions²⁴ in combination with a better motion model to account for the effects of hysteretic respiratory motion.

Simplest form of hysteresis occurs in a linear system as a result of the existence of phase lag between input and output. It is commonly referred as “rate-dependent” hysteresis, which can be accounted by introducing a simple lag. However, systems with complex internal structure which can occupy one of the many possible states for a given input value can remain in that state until the external conditions are modified. The behavior of such systems with persistent memory effects is referred to as following rate-independent hysteresis. The behavior of rate-independent hysteresis is encountered in a diverse range of processes such as magnetism, ferroelectricity, mechanics, structures, biology, and other areas. The differences in heart positions between the inspiratory and expiratory phases during respiration are examples of this rate-independent hysteretic phenomenon. To describe the behavior of rate-independent hysteretic processes, several

mathematical models²⁵ have been put forth which includes the Duhem model,²⁶ the Preisach model,²⁷ and the Bouc–Wen (BW) model.^{28–30} The Bouc–Wen model is popular in mechanical and structural engineering due to its simple numerical implementation and its ability to represent a wide range of hysteretic loop shapes. However, in the plot of cardiac motion with respect to the abdomen marker, the direction of the hysteresis loops appears to be in the counterclockwise direction¹⁹ as illustrated in Fig. 1, while the loops generated by the Bouc–Wen model in its original form are clockwise hysteretic loops. Therefore to account for counterclockwise hysteresis in this study, we utilized a “modified Bouc–Wen model”^{31,32} to estimate the heart position relative to the external signals influenced by hysteretic respiratory motion. We tested the proposed modified Bouc–Wen model in accounting for rate-independent hysteresis using the MRI data of the volunteers, as detailed in Sec. 2.

2. MATERIALS AND METHODS

2.A. Subjects

Nineteen volunteers without counter-indications to MRI were recruited for this study. The participation of volunteers for motion-tracking using both MRI and vts was in compliance with IRB approval and informed consent. The volunteers were prepared for the vts stereo imaging and MRI acquisition by wrapping bands with markers (2-cm diameter retroreflective markers filled with copper sulfate solution) about their chest and abdomen as illustrated in Fig. 2. The volunteers were instructed to breathe slowly and regularly for approximately a 5 min period during which the motion associated with the markers and the internal organs was tracked by vts and MRI, respectively.

2.B. External respiratory motion-tracking system: vts

The vts employed in this study allows near-infrared tracking of multiple markers within the 3D imaging volume of the MRI as shown in Fig. 3. The components of the vts include the near-infrared cameras, the controlling hardware module, and the software to analyze and present the data. Three

FIG. 2. Layout of markers used on volunteers for tracking external chest and abdomen motions. The 2 cm diameter retroreflective markers were used for vts tracking.

FIG. 3. Marker layout view as seen in vts software. The top window shows a 3D rendering of the markers placed on the subject's body within the imaging volume as determined by the analysis of the three stereo cameras. 2D renditions of the marker locations from each of the cameras are shown in bottom windows.

cameras of the vts, mounted on the wall of the MRI room, were arranged in a manner such that the center of the volume to be tracked is near the isocenter of the MRI scanner. Before each experimental session, the vts was calibrated for stereo motion tracking and tested for minimal radio-frequency (RF) interference with the MRI as discussed in Ref. 33. The vts performs stereo imaging of the external retroreflective markers placed on the thoracic/abdominal regions at 30 frames/s throughout the MR acquisition. The MRI was configured as an external remote-control device which was used to trigger data capture automatically on the vts. Once established in this way, we have previously determined¹⁹ that synchrony between the clocks of the MRI and vts is maintained during the duration of acquisition by determining the Spearman rank-order correlation coefficient between the chest wall navigator signal and that from the chest marker nearest the navigator position. Example plots of the AP motion of the markers for one subject are shown in Fig. 4(a). Note the strong similarity of motion between the five chest markers. This trend was observed for all 19 volunteers of our previous study.¹⁹ Therein a strong pairwise correlation was observed over all-pairs of comparison between the motions of the five chest markers which implies that any of the chest markers could be used for data analysis. Herein, the signal obtained from the chest marker closest to the heart was considered for the purpose of modeling and analysis.

2.C. Internal respiratory motion-tracking system: MR navigator technique

MR imaging of the participants was performed using a Philips Healthcare ACHIEVA 3.0-T whole body MRI scanner. The MRI software provided three independent pencil-beam navigator radio-frequency pulses.³⁴ Each pulse can be freely placed in space at any desired angle as guided by patient anatomy in pilot scans. As illustrated in Fig. 5, the three navigator windows were localized on the superior left-ventricular wall/lung boundaries, the right diaphragm,

FIG. 4. (a) Plot of the AP displacements of the external chest and abdomen markers as a function of time as measured by the vts for a subject. (b) Plot of the displacements of the internal heart, diaphragm and chest wall respiratory motions obtained by the respective MRI navigators as a function of time.

and the anterior wall of the chest to track the motion of the corresponding structures. An example of the observed displacements is shown in Fig. 4(b). The multinavigator MR dynamic imaging through the left ventricle of the heart was performed for 300 dynamic scans during continuous breathing using the ECG-triggered single shot, 2D fast-field-echo sequence, with TR/TE = 5.5/2.1 ms, 128×128 image matrix, 3 mm voxel size, 250 ms acquisition window at mid-diastole to reduce cardiac motion during acquisition, and navigator TR = 5 ms. This multinavigator sequence was used to record the respiratory motion of the anatomic regions mentioned

above for each cardiac cycle with 5 ms time lag between the three navigator acquisitions.

2.D. Modeling respiratory motion using the BW model

In this study, we utilized a variant of the BW model^{31,32} to estimate the internal motion of the heart by utilizing the respiratory motion from the chest and the abdomen regions. This model was experimentally modified to describe counterclockwise hysteric loops as observed in respiratory dynamics (Fig. 1). This modified form is given as

$$\dot{w} = \rho(\dot{x} + \delta x|\dot{w}| - \sigma|\dot{x}|w - \gamma w|\dot{w}|), \quad (1)$$

FIG. 5. (a) Positioning of navigator beams (boxes) on the dome of the right hemidiaphragm, the left ventricle, and the chest wall. The lines in the transaxial and the coronal slices indicate the position of the sagittal slice acquired along with the navigator beams. (b) The output from the three navigator beams are shown as plots of 1D gray-scale information perpendicular to each body interface versus time. These portray the respiratory motions of the diaphragm, heart, and chest wall.

where $x(t)$ and $w(t)$ represent the AP displacements of the abdomen marker and chest marker as a function of time, respectively. Also in this equation

$$|\dot{w}| = \dot{w} \cdot \text{sgn}(\dot{w}), \quad |\dot{x}| = \dot{x} \cdot \text{sgn}(\dot{x}), \quad \dot{w} = dw/dt, \\ \dot{x} = dx/dt, \quad \text{sgn}(\dot{x}) = \begin{cases} -1 & \text{if } \dot{x} < 0 \\ 0 & \text{if } \dot{x} = 0 \\ 1 & \text{if } \dot{x} > 0, \end{cases}$$

and ρ , δ , σ , and γ are the parameters that determine the shape and extent of hysteresis in the modified BW model.

Instead of the respiratory rates in Eq. (1), we are interested in the variation of respiratory motion between the chest and the abdomen regions, given by dw/dx . This can be derived by rearranging Eq. (1) and dividing it by the respiratory rate dx/dt

$$w_{\text{Est}} = dw/dx \\ = \rho(1 - \sigma w \text{sgn}(\dot{x})) / (1 - \rho \delta x \text{sgn}(\dot{w}) + \rho \gamma w \text{sgn}(\dot{w})). \quad (2)$$

The differential term (w_{Est}) represents the estimated data points on the hysteretic loop, which describes the estimated internal heart motion information in the superior-inferior direction. w_{Est} will be the new respiratory signal, which replaces the abdomen signal in performing the amplitude binning of the list-mode data to estimate the heart motion with minimal hysteresis.

2.E. Parameter identification scheme

A critical first step when employing an iterative fitting process is the selection or identification of the initial values of the parameters. Although the BW model is widely used for modeling hysteresis in other disciplines, to the best of our knowledge, its application to fit respiratory hysteresis has not been reported. Thus, the following parameter identification scheme was employed to determine the optimal initial set of parameter values for a given respiratory pattern (either monotonic or hysteretic).

Herein, the sum-squared error (E) between the model response (w_{Est}) and the actual experimental value (w) was used as the objective function and expressed as

$$E = \sum_{m=1}^M (w_{\text{Est}}(m) - w(m))^2, \quad (3)$$

where M is the total number of samples, $w(m)$ denotes the observed experimental value of the m th sample of the chest marker, and $w_{\text{Est}}(m)$ denotes the corresponding m th estimated value from the model expressed by Eq. (2). The optimization problem can be stated as the minimization of this objective function. To solve the optimization problem, the interior-point algorithm was adopted in Eq. (3) iteratively.³⁵ The MATLAB optimization algorithm (*fmincon*) was used to implement the iterative process. Since hysteresis is a nonlinear phenomenon, fitting to experimental data with a multiparameter nonlinear model is done using an iterative procedure which require initial values. The procedure must start with the given initial

values for each parameter, inappropriate starting values can prevent the procedure from finding a best fit by converging to a local minimum rather than the global minimum. Therefore, it is imperative for the optimization algorithm to begin the iterations with a good initial estimate of the four parameters to avoid being trapped in local minimums. To circumvent the use of arbitrary values as initial parameters, a systematic search was performed to determine a parameter set that results in the best fit using an exhaustive search algorithm through initial parameter values coupled with the optimization algorithm as illustrated in the flowchart in Fig. 6. A permutation matrix of parameter values consisting of non-negative real numbers with a row sum of unity was used to sweep through all possible values in steps of 0.05 for each of the four parameters. The estimated values from the model were then correlated with the navigator heart motion (“truth”). A Pearson product-moment linear correlation coefficient (r) was computed to assess the strength of the linear association between the BW model-estimated heart motion and the true heart motion from the navigators, which in-turn depicts the effectiveness of the parameter estimation. The best estimate of the true heart position was obtained using the set of initial parameters from the permutation matrix that resulted in the maximum correlation coefficient (r_{Max}) in the optimization algorithm. Apart from this parameter set, the search resulted in multiple parameter sets that resulted in similar correlational values. The iterations for interior-point algorithm proceed until they meet the stopping criterion, when the size of the current search direction is less than twice the value of the step size tolerance (1×10^{-6}) and constraints are satisfied to within the value of the constraint tolerance (1×10^{-6}); this is an inner iteration, not the number of iterations of the exhaustive search algorithm through initial parameter space.

2.F. Average-area algorithm for respiratory motion classification

In our preliminary tests of the BW model, we determined that there existed multiple initial parameter sets producing near identical response for a given respiratory pattern and identified a global set of initial parameter values that allowed the model to adequately adapt between monotonic and hysteretic respiratory patterns. However, the performance of the model was further improved when two distinct sets of initial parameter values were employed for each of the respective patterns. The selection process of the initial parameter values unique to monotonic and hysteretic patterns of respiratory motion is described in Sec. 2.G. To automate the determination of which set of initial parameters was to be employed with each dataset, we developed a measure of the extent of hysteresis H . The motivation for our definition of H is illustrated in Fig. 7. H is calculated as

$$H(\%) = \left[\sum_{i=1}^N (\text{Area}_{\text{Loop}} / \text{Area}_{\text{Rect}})_i / N \right] \times 100, \quad (4)$$

where N is the number of respiratory cycles, $\text{Area}_{\text{Loop}}$ is the area of the individual respiratory cycle, and $\text{Area}_{\text{Rect}}$ is the

FIG. 6. Flowchart to determine the optimum set of Bouc–Wen model initial parameters for a given external chest and abdomen respiratory signal.

area of the rectangle with sides “ p ” and “ q ” corresponding to the depth of breathing shown by the chest and abdomen markers, respectively.

2.G. Training and testing of the BW model for respiratory cardiac motion

Our objective was to determine and validate initial parameter sets for use in estimating respiratory cardiac motion such that the error in estimating the heart motion could be minimized. To achieve this, the subject data were divided into training and testing sets to determine the model’s initial parameters and test the estimating ability of the model when employing these initial values, respectively.

2.G.1. Training set

A training set of 7 from the 19 available subjects was chosen retrospectively based on the H value by subsetting the

original data in a manner that it included examples of four monotonic patterns and three hysteretic patterns. This selection was *ad hoc* with the greater number of studies being placed in the testing set to facilitate statistical testing with it. The algorithm described in Sec. 2.E was applied to these studies to select the initial sets of parameters (one for monotonic and one for hysteretic) to be used with the testing studies to judge the utility of our proposed methodology. The method for selecting the initial parameter sets was performed as follows:

- (i) A pool of parameter value sets was selected from the permutation matrix such that the Pearson product-moment linear correlation coefficient (r) was within 5% of the maximum correlation coefficient (r_{Max}) for each subject as well as across the seven subjects in the training set.
- (ii) For each of the parameter value sets in the pool, the percentage difference was calculated for each subject

FIG. 7. Schematic plot of the motion of chest versus abdomen markers during a single respiratory cycle from end expiration (EE) to end inspiration (EI) and back to EE when hysteresis is present. The shaded region shows the area of the respiratory loop ($\text{Area}_{\text{Loop}}$). The ratio of $\text{Area}_{\text{Loop}}/\text{Area}_{\text{Rect}}$ averaged over all respiratory cycles is proposed as the parameter to describe the extent of hysteresis in the subject's breathing pattern.

by calculating the difference between (r_{Max}) and (r) for the corresponding parameter value set.

- (iii) The mean of the percentage differences corresponding to each parameter value set from the pool was calculated independently for the monotonic subjects, the hysteretic subjects, and the combined seven subjects in the training set.
- (iv) The parameter sets resulting in the lowest mean percentage difference unique to the monotonic subjects, the hysteretic subjects, and the combined subjects were chosen accordingly.

2.G.2. Testing set

The remaining 12 subjects unused in the training set were considered as the testing set. The proposed average-area algorithm discussed in Sec. 2.F was applied on the testing set to determine the algorithm's ability to select the initial set of parameters determined using the training dataset, which when employed gave similar correlations to the internal heart motion as that of the optimal set obtained through the exhaustive search.

2.H. Data analysis

For both training and testing data sets, pairwise comparisons along with correlational analysis were performed between the navigator heart motion and (1) the external abdomen marker, (2) the external chest marker, (3) average of the external abdomen and chest markers, and (4) the Bouc–Wen model derived signals. The measure used for assessing the performance of the model in the testing sets was Pearson product-moment linear correlation coefficient (r). A one-way ANOVA (Ref. 36) was used to test the presence of a statistically significant difference in the correlation with the heart among the different external surrogate respiratory signals. If

ANOVA indicated there was a significance difference, then Tukey's honestly significant difference (HSD)³⁷ test was used to determine which of the surrogates were different. A p -value of less than 0.05 indicates that the results differ from each other significantly.

3. RESULTS

3.A. Determination of initial model parameter values

Figure 8(a) shows the plot of the number of subjects whose H (measure of extent of hysteresis) fell within binned values versus H . Based on these results, we selected a threshold value of 20% to categorize their respiratory chest versus abdomen motion as either monotonic or hysteretic. Figure 8(b) shows examples of the extent of hysteresis for one respiratory cycle along with their corresponding H values for three subjects with varying H .

Table I lists the initial parameter values for each of the seven studies in the training set which resulted in the highest (r) (between the BW model predicted and the navigator signals) obtained from the exhaustive search of the studies in the training set. Also listed are the corresponding values of (r) for four signals based on external markers versus navigator signal, the extent of observed hysteresis (H) between the abdomen and chest markers, and the pattern determined by using 20% as a threshold value on H .

Note that the set of initial parameters varies from one study to another. As stated in Sec. 2.F, no common starting set of initial parameters was observed which resulted in the highest (r) for all studies in the training set. However, the exhaustive search on the training studies resulted in multiple parameter value sets with nearly maximal (r). In clinical applications, besides the computational burden in the use of exhaustive search, the true cardiac motion is unavailable to determine the optimum initial parameters. Hence, using the training set, three different parameter sets based on the criteria discussed in Sec. 2.G were selected. These initial parameter value sets along with their corresponding mean percentage differences for the monotonic, the hysteretic, and the combined subjects are reported in Table II. Notice that using the "monotonic" parameter value set resulted in the lowest percentage difference for the monotonic subjects. Similarly the "hysteretic" and the "global" parameter value sets resulted in the lowest percentage difference for the hysteretic subjects and for the combination of all subjects, respectively, in the training set.

The performance of the BW model, in estimating the heart motion using the chest and abdomen markers is assessed by correlating with the true cardiac motion determined by the navigator, is shown for five selected subjects from the training set in Fig. 9. The first two rows in Fig. 9 show plots for studies with monotonic patterns, and the last three rows show plots for the hysteretic patterns. The first column shows plots of the chest marker motion as a function of the abdomen marker motion. The corresponding BW model fits obtained using the set of initial parameter values through the average-area algorithm are overlaid with the experimental data to illustrate the impact of BW model fitting. The correlation

FIG. 8. (a) Histogram of subjects with respect to the H value. Respiratory data with $H < 20\%$ was considered monotonic and the ones with $H > 20\%$ was considered hysteretic. (b) Plots illustrating the extent of hysteresis (H) in one respiratory cycle for monotonic (left) and hysteretic (center and right) respiratory patterns.

of each of the plotted signals in the first column with the navigator-measured SI motion of the heart internally is shown in the rest of the scatter plots within the figure in columns 2–4. In each of these plots, the correlation coefficients for the two plotted signals are displayed in the upper left corner of each plot. Notice that both visually and by the (r) values, the BW model derived signal shows consistently stronger correlation with the internal heart motion than either the chest or abdomen marker signal alone. It should be noted that the exact agreement in the magnitude of these external signals with the internal motion of the heart is not being sought; rather, the goal is to determine a signal which provides the best correlation with the true heart motion so that it can be used as an external surrogate for binning list-mode acquisitions.

3.B. Testing methodology to select the initial parameters of the BW model

Table III summarizes the correlation coefficient results for the 12 subjects in the testing set with respect to the

navigator-determined heart motion for the abdomen signal, the chest signal, average of the abdomen and chest signals, the average-area algorithm ($BW_{\text{Average-Area}}$), and the exhaustive search ($BW_{\text{Exhaustive-Search}}$). The extent of hysteresis (H) in percentage and pattern categorization are also shown. Note that in all cases shown in this table, the signal from the $BW_{\text{Exhaustive-Search}}$ provided the largest or tied for largest correlation coefficient. Tukey post-hoc comparisons of the r -values from the $BW_{\text{Exhaustive-Search}}$ when compared to those of the abdomen signal, average of abdomen and chest external markers, and $BW_{\text{Average-Area}}$ were statistically not significant ($p = 0.85$, $p = 0.77$, and $p = 0.98$, respectively). However, the r -values of the heart with the external chest signal gave significantly lower values than the signals from the abdomen ($p < 0.05$), $BW_{\text{Average-Area}}$ ($p < 0.05$), and the $BW_{\text{Exhaustive-Search}}$ ($p \ll 0.05$) signals. This indicates that usage of solely the chest signal to bin cardiac list-mode data is not a good choice. The average correlation of the $BW_{\text{Average-Area}}$ signal was not statistically different when compared to that of the abdomen signal and average of external markers ($p = 0.98$ and $p = 0.96$).

TABLE I. Shown are the initial four-parameter values along with the corresponding highest correlation coefficient (r) values for seven volunteers in the training set. The extent of hysteresis, H (%) and the pattern categorization are also tabulated.

Subject #	Initial parameter value				Correlation coefficient (r)				H (%)	Pattern
	ρ	δ	σ	γ	Abdomen	Chest	Average	$BW_{\text{Exhaustive-Search}}$		
1	0	0.1	0.15	0.75	0.94	0.92	0.94	0.95	12	Monotonic
2	0	0.05	0.85	0.20	0.98	0.95	0.98	0.97	12	Monotonic
3	0.1	0.4	0.05	0.45	0.96	0.95	0.96	0.97	7	Monotonic
4	0.05	0.15	0.1	0.7	0.91	0.90	0.91	0.95	8	Monotonic
5	0.05	0.6	0.1	0.25	0.76	0.86	0.79	0.85	22	Hysteresis
6	0.05	0.00	0.60	0.35	0.78	0.96	0.87	0.93	54	Hysteresis
7	0	0	0.05	0.95	0.90	0.71	0.90	0.90	56	Hysteresis

TABLE II. Shown are the initial parameter value sets with corresponding mean percentage differences for the monotonic (4 subjects), the hysteretic (3 subjects), and the combined (7 subjects) sets.

Parameter set type	Parameter set				Mean percentage difference (%)		
	ρ	δ	σ	γ	Monotonic set	Hysteretic set	Combined set
Monotonic	0.05	0.15	0.05	0.75	0.70	1.74	1.14
Hysteretic	0.4	0.1	0.4	0.1	1.73	0.56	1.23
Global	0.25	0.15	0.25	0.35	0.94	1.12	1.02

Note: The numbers in bold indicate the lowest percentage difference resulted when the subjects were classified into monotonic and hysteretic patterns and treated with corresponding parameter sets.

FIG. 9. Pairwise scatter plots of external, BW model derived, and internal navigator-measured signals for five of the seven subjects from the training set. First column: external chest marker and BW derived signals AP signals versus abdomen marker AP signal. Second column: Navigator-determined internal heart SI motion versus abdomen marker AP signal. Third column: internal heart motion versus chest marker signal. Fourth column: internal heart motion versus BW derived signal. In each case the correlation coefficient (r) is indicated for the columns 2–4.

TABLE III. Correlation coefficient (r) between the navigator determined true heart motion and the signal from the abdomen, chest, average of abdomen and chest, BW model using the set of initial parameters selected through average-area algorithm (BW_{Average–Area}), and exhaustive search algorithm (BW_{Exhaustive–Search}). The extent of hysteresis (H) and the pattern categorization are also shown.

Subject #	Correlation coefficient (r)					H (%)	Pattern
	Abdomen	Chest	Average	BW _{Average–Area}	BW _{Exhaustive–Search}		
1	0.96	0.90	0.96	0.97	0.97	7	Monotonic
2	0.91	0.90	0.91	0.95	0.95	8	Monotonic
3	0.97	0.93	0.97	0.98	0.99	14	Monotonic
4	0.93	0.71	0.91	0.93	0.94	38	Hysteretic
5	0.98	0.68	0.95	0.97	0.99	43	Hysteretic
6	0.81	0.80	0.82	0.86	0.87	7	Monotonic
7	0.96	0.93	0.96	0.97	0.97	30	Hysteretic
8	0.96	0.96	0.96	0.98	0.98	17	Monotonic
9	0.96	0.95	0.97	0.96	0.98	14	Monotonic
10	0.92	0.93	0.92	0.93	0.97	2	Monotonic
11	0.95	0.88	0.94	0.95	0.97	18	Monotonic
12	0.97	0.94	0.97	0.97	0.97	17	Monotonic
Mean $\pm$ SD	0.94 $\pm$ 0.05	0.88 $\pm$ 0.09	0.94 $\pm$ 0.04	0.95 $\pm$ 0.03	0.96 $\pm$ 0.03		

4. DISCUSSION

We used experimental respiratory signals acquired from subjects to test the proposed BW model in estimating the heart position over different respiratory hysteretic patterns. The possible reasons for hysteresis are due to the complex and intricate interplay between the diaphragm and intercostal muscles responsible for inspiration and expiration. Furthermore, it is believed that the viscoelastic nature of the lungs¹⁷ and internal imbalances in lung pressure³⁸ also play a part in retarding the motion of the heart mimicking the delayed periodic motion as observed in a forced damped oscillation.³⁹

This study used the true heart motion from the MRI navigator data to select the initial parameter set and test the BW model estimations of the heart motion. In general, for both monotonic and hysteretic cases, the response of the BW model based on the correlation values (r) accurately captured the trends of respiration in all subjects. The respiratory motion from the external markers and the BW signals showed a good correlation with the heart motion. However, from the examples seen in Fig. 9, one cannot rely on the chest or the abdomen marker alone for amplitude binning due to the effects of hysteresis. For example, in our limited number of studies, we observed that the chest marker performed better in some cases while the abdomen marker performed better in other cases in estimating the heart motion. This is an undesirable situation to estimate the heart motion with just a single surrogate signal. For instance, if amplitude binning was to be performed on the respiratory signal obtained from the abdomen marker (hysteretic) corresponding to Fig. 9, the middle bins would inherit motion in the range of 15–20 mm resulting in residual blurring of the images corresponding to those bins. Such undesirable effects can be overcome to some extent by applying a suitable model with the information from both the thoracic and abdomen regions as we demonstrated in this study using the BW model. Although we did not find a statistically significant difference in r -values between its usage and that of either the chest signal or average of the

abdomen and chest signals in the 12 subjects of our test set, we did observe that the BW_{Average–Area} r -value was smaller only in one subject in comparison to usage of the chest signal, and was smaller in a different subject in comparison to the average of the abdomen and chest signals. Conversely the r -values for those signals were smaller than those for BW_{Average–Area} in seven subjects for the chest signal and ten for the average of the abdomen and chest signals. Thus our BW model may offer a more robust alternative to the other signals investigated.

The cost of the commercial vts used in these exploratory studies is likely prohibitive for wide-spread clinical usage. There are many number of much lower cost alternatives such as placement of pneumatic bellows or similar device about the chest and abdomen,⁴⁰ usage of low-cost vts systems to track markers on both the abdomen and chest,⁴¹ or employing of one of the newer inexpensive cameras that could potentially view the chest and abdomen simultaneously thereby tracking both without usage on markers.⁴²

One alternative to our approach is to use the change in the location of the counts within a structure with time to provide a signal related to the actual internal motion of that structure with respiration. In planar scintigraphy, this approach was implemented in analog circuitry⁴³ and digitally on computer.⁴⁴ Its application was shown in a receiver operating characteristics (ROC) study of 102 patients to improve the detection accuracy of hepatic masses. In SPECT, the application of this approach to correcting respiratory motion cardiac perfusion imaging was shown to work very well in phantoms where the simulated motion is the same for all activities.⁴⁵ However, with application to clinical images where the heart and nearby extra-cardiac concentrations of activity can move differently during respiration, its application was not as successful. This was further complicated by the low-count rate from structures like the heart and angular variation in counts with changing attenuation as the camera-heads rotate. Given that the detectors do not rotate about the patient in PET and the count-rates are much higher, one would expect the data-driven approach to be more successful, as recent studies have shown.^{46–48} However,

with small structures or low uptake the addition of an external radioactive marker has been found to be useful with data-driven respiratory gating in PET.⁴⁹ Thus even in PET usage of the BW model to account for hysteresis internal motion from external surrogate signals may be useful.

5. CONCLUSION

The modified Bouc–Wen model has been used to describe and predict the responses of nonlinear systems to transient inputs due to its ability to incorporate and reproduce a wide range of real cases of hysteretic behavior. To the best of our knowledge, this study introduces a novel approach in estimating the heart motion using the Bouc–Wen model applied to respiratory signals from the external body markers. The results of our study suggest that the proposed model may serve as a robust unified framework for modeling hysteresis seen in respiratory motion of the heart, and perhaps other organ systems.

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^{a)}Author to whom correspondence should be addressed. Electronic mail: Michael.King@umassmed.edu; Telephone: 508-856-4255; Fax: 508-856-6363.

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