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Short-Lag Spatial Coherence Imaging on Matrix Arrays, Part I: Beamforming Methods and Simulation Studies

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Abstract

Short-lag spatial coherence (SLSC) imaging is a beamforming technique that has demonstrated improved imaging performance compared with conventional B-mode imaging in previous studies. Thus far, the use of 1-D arrays has limited coherence measurements and SLSC imaging to a single dimension. Here, the SLSC algorithm is extended for use on 2-D matrix array transducers and applied in a simulation study examining imaging performance as a function of subaperture configuration and of incoherent channel noise.

SLSC images generated with a 2-D array yielded superior contrast-to-noise ratio (CNR) and texture SNR measurements over SLSC images made on a corresponding 1-D array and over B-mode imaging. SLSC images generated with square subapertures were found to be superior to SLSC images generated with subapertures of equal surface area that spanned the whole array in one dimension. Subaperture beamforming was found to have little effect on SLSC imaging performance for subapertures up to 8×8 elements in size on a 64×64 element transducer. Additionally, the use of 8×8 , 4×4 , and 2×2 element subapertures provided 8, 4, and 2 times improvement in channel SNR along with 2640-, 328-, and 25-fold reduction in computation time, respectively. These results indicate that volumetric SLSC imaging is readily applicable to existing 2-D arrays that employ subaperture beamforming.

I. Introduction

Ultrasound images formed using conventional delay-and-sum (B-mode) beamforming are often degraded by clutter, a persistent haze that obscures potentially important anatomical structures. Clutter originates from off-axis echoes and near-field reverberations generated by the structural inhomogeneities of subcutaneous tissue [1]–[3]. Reducing clutter in ultrasound

images is becoming increasingly important as the number of inadequate ultrasound scans rises along with the incidence of obesity [4], [5].

We recently introduced a coherence-based beamforming technique, called short-lag spatial coherence (SLSC) imaging, that mitigates the impact of clutter [6]. SLSC imaging interrogates a fundamentally different property of the echo wavefront compared with B-mode imaging. B-mode images are a display of the magnitude of the echo wavefront, whereas an SLSC image is a direct image of the similarity of the echo wavefront over small spatial distances across the aperture, or how spatially coherent the echo is at short lags. SLSC imaging has demonstrated substantial improvements in contrast-to-noise ratio (CNR) and speckle SNR over B-mode in simulation experiments [6]–[8], as well as *in vivo* thyroid [6], liver [9], heart [10], [11], and carotid [8] studies. Similar improvements were obtained when SLSC images were formed with harmonic echoes in full-wave simulations and in experimental studies of *in vivo* livers, kidneys, and hearts [9], [10].

These previous studies were conducted using 1-D array transducers, which provide fine sampling of the echo wavefront in azimuth but cannot be used to measure spatial coherence in elevation. Because spatial coherence is fundamentally a 2-D function, a 2-D matrix array transducer with fine sampling in either dimension can yield more precise spatial coherence measurements of the entire field and potentially improve SLSC image quality. As with B-mode, SLSC imaging utilizes the focused channel data to generate images. The algorithm may be viewed as a black box that is analogous to the summation and demodulation steps of B-mode, and can be used with any of the same pulse sequences such as synthetic transmit aperture [12] and harmonic imaging [10], where channel signals are available.

There are several potential obstacles to the effective implementation of SLSC imaging on 2-D transducers. 2-D transducers are typically composed of thousands of elements, and access to each individual signal is impractical. In practice, manufacturers often reduce the channel count by employing micro or subaperture beamforming (SAB), a process in which small segments of the aperture are delay-and-summed within the probe handle [13]; the effects of SAB on SLSC imaging are not obvious. The computational time required to form an SLSC image is non-trivial and rises exponentially with the number of channels [14]. Additionally, the small surface area of each transducer element in a matrix array results in a worse electronic SNR than is typical of the larger 1-D transducer elements.

In Part I of this paper, we extend the SLSC algorithm to 2-D transducers and investigate imaging performance in a simulation study. A simulated 2-D phased array transducer is used to image hypoechoic lesion phantoms, and the acquired channel data are reconstructed into 3-D image volumes using various beamforming techniques: traditional B-mode, SLSC using the fully sampled array of signals, and SLSC after applying SAB using various subaperture configurations. For each method, imaging performance is quantified using lesion contrast, CNR, and texture SNR. Performance is also measured as a function of simulated clutter levels by modulating the channel SNR with spatially incoherent noise. In Part II, we implement SLSC imaging using a 2-D transducer on a clinical scanner and present imaging results from phantoms and *in vivo* liver [15].

II. Theory

The original formulation of SLSC imaging incorporated spatial coherence measurements as a function of only the lateral dimension [16]. Modifications are made here to incorporate the elevation dimension.

A. 2-D Spatial Coherence of Diffuse Scatterers

Mallart and Fink demonstrated [17] that the spatial coherence of the echoes returning from a diffuse scattering medium can be characterized by the van Cittert–Zernike (VCZ) theorem. This theorem, when applied to acoustic signals, shows that the normalized spatial covariance of a backscattered wave from an incoherent source can be expressed as the Fourier transform of the square of the transmit beam amplitude $H(u, v)$:

$$C(\Delta x, \Delta y) = \frac{1}{\mathcal{C}(0, 0)} \iint_{-\infty}^{+\infty} |H(u, v)|^2 \times \exp\left(-j2\pi \frac{\Delta x u + \Delta y v}{zc}\right) du dv, \quad (1)$$

where (u, v) and (x, y) denote coordinates in the source and the aperture planes, respectively, z is the distance between the two planes, c is the speed of propagation, and $\mathcal{C}(0, 0)$ is the spatial covariance at $x = y = 0$. Because the echo signals are zero-mean, the spatial covariance is related to the spatial correlation, $R(x, y)$:

$$R(\Delta x, \Delta y) \propto \iint_{-\infty}^{+\infty} |H(u, v)|^2 \times \exp\left(-j2\pi \frac{\Delta x u + \Delta y v}{zc}\right) du dv. \quad (2)$$

When transmitting a narrowband pulse with an unapodized rectangular aperture, the transmit beam amplitude is approximated by $H(u, v) = \text{sinc}(u) \text{sinc}(v)$. The spatial correlation for the diffuse uniform scattering medium $R_d(x, y)$ then reduces to the product of the triangle functions $\Lambda(x)$ and $\Lambda(y)$:

$$\langle R_d(\Delta x, \Delta y) \rangle = \left(1 - \left|\frac{\Delta x}{D_x}\right|\right) \left(1 - \left|\frac{\Delta y}{D_y}\right|\right), \quad (3)$$

where D_x and D_y denote the transmit aperture size in x and y , and $\langle \cdot \rangle$ denotes the expected value.

B. Spatial Coherence Measurements

The spatial coherence between two signals received at elements i and j , at a given time t , can be quantified using normalized cross-correlation:

$$\hat{R}_{ij}(t) = \frac{\hat{C}_{ij}(t)}{\sqrt{\hat{C}_{ii}(t)\hat{C}_{jj}(t)}}, \quad (4)$$

where $\hat{C}_{ij}(t)$ is the estimated spatial covariance between the two signals, defined as

$$\hat{C}_{ij}(t) = \sum_{\tau=-T/2}^{T/2} s_i(t+\tau) s_j^*(t+\tau), \quad (5)$$

where $s_i(t)$ is the signal at position (x_i, y_i) with zero mean over the time interval T , and s^* denotes the complex conjugate of s .

A pixel in the SLSC image is formed by summing the measured spatial correlations between all signals from element pairs at short lags:

$$V_{\text{slsc}}(t) = \sum_{i=1}^N \sum_{j \in \xi_i} \hat{R}_{ij}(t), \quad (6)$$

where N is the total number of signals and ξ_i is the set of all elements that are within the short-lag region relative to the i th signal.

C. Definition of the Short-Lag Region

On a 1-D transducer, a short-lag region ξ_i can easily be defined for each element i using a simple threshold value M for the largest permissible lag:

$$\xi_{i,1D} = \{j: |j - i| \leq M\}. \quad (7)$$

Defining ξ_i on a 2-D transducer is more complex because signals at short lag in one dimension are not necessarily at short lag in the other dimension; for example, signals in neighboring rows but in columns at opposite ends of the array. Although distances such as the l_1 or l_2 norms can be used to define the short-lag region, they are not necessarily representative of the inherent structure of the coherence of the backscatter.

Instead, given that many imaging targets in medical ultrasound are composed primarily of diffuse sub-resolution scatterers, it is reasonable to use the spatial coherence of diffuse scatterers predicted by theory as a single surrogate value for the lags in two dimensions. Short-lag regions can be defined as the set of signals that would in theory have an inter-element correlation of P or greater when imaging diffuse scatterers.

By using the response to a diffuse scattering medium as the baseline (3), the image contrast is maximized for structures of interest, such as hypoechoic cysts and blood vessels. If the scattering medium is homogeneous and filled with sub-resolution scatterers, the measured correlations will be close to the theoretical value. If the scattering medium contains a strong point reflector, the measured correlation will be greater than the theoretical value, and if the echo signals from the scattering medium contain incoherent noise, the measured correlation will be less than the theoretical correlation.

The short-lag region for a given element i , centered at (x_i, y_i) , is therefore defined as

$$\xi_i = \{j: \langle R_d(x_i - x_j, y_i - y_j) \rangle \geq P\}, \quad (8)$$

where P is the correlation threshold for the short-lag region. Fig. 1 illustrates the short-lag region for the element in row 50, column 20 on a 64×64 2-D transducer when using $P = 0.75$.

D. Subaperture Beamforming (SAB)

SAB is the technique in which the echoes received over a portion of the aperture (i.e., a subaperture) are beamformed into a single signal. Although SAB typically refers to the summing of signals from a subset of transducer elements, the signal transduced by each element is itself an integration of the pressure received on the element surface, and can therefore be considered a result of SAB.

In SLSC imaging, SAB tends to remove short-lag correlations and introduce long-lag correlations. To demonstrate this, consider a simple 1-D array of 4 point receivers with uniform spacing d . To generate an SLSC value using a maximum lag of 2 elements, corresponding to $P = 1/2$, we use (6):

$$V_{1 \times 4} = \hat{R}_{12} + \hat{R}_{23} + \hat{R}_{34} + \hat{R}_{13} + \hat{R}_{24}, \quad (9)$$

where $\hat{R}_{ij}$ is the correlation coefficient between elements i and j . Note that $V_{1 \times 4}$ is the sum of three lag 1 correlations ($\hat{R}_{12}$, $\hat{R}_{23}$ and $\hat{R}_{34}$) and of two lag 2 correlations ($\hat{R}_{13}$ and $\hat{R}_{24}$).

If SAB was applied using 1×2 element subapertures, the SLSC value of the resulting 1×2 array at $P = 1/2$ would be

$$V_{1 \times 2} = \hat{R}_{12,34}, \quad (10)$$

where $\hat{R}_{12,34}$ is the correlation coefficient of the two subapertures. Using (4), this can be rewritten as

$$\hat{R}_{12,34} = \frac{\sum (s_1 + s_2)(s_3 + s_4)^*}{\sqrt{\sum (s_1 + s_2)^2 \sum (s_3 + s_4)^2}}, \quad (11)$$

which can be expanded to

$$\hat{R}_{12,34} = \frac{\hat{C}_{13} + \hat{C}_{24} + \hat{C}_{23} + \hat{C}_{14}}{\sqrt{(\hat{C}_{11} + \hat{C}_{22} + 2\hat{C}_{12})(\hat{C}_{33} + \hat{C}_{44} + 2\hat{C}_{34})}}. \quad (12)$$

Therefore, the correlation coefficient of two beamformed subapertures is the weighted sum of the covariances between each element in one subaperture with each element in the other subaperture.

The Cauchy–Schwarz inequality places an upper bound on the cross-terms in the denominator:

$$(\hat{C}_{ij})^2 \leq \hat{C}_{ii} \hat{C}_{jj}. \quad (13)$$

Assuming all signals have the same power $\hat{C}_{ii}$, (12) can be combined with (13) to be rewritten as

$$\hat{R}_{12,34} \geq \frac{\hat{C}_{13} + \hat{C}_{24} + \hat{C}_{23} + \hat{C}_{14}}{\sqrt{(4\hat{C}_{ii})(4\hat{C}_{ii})}}. \quad (14)$$

Replacing the covariances with correlations, the SLSC pixel value with SAB is

$$V_{1 \times 2} \geq \frac{1}{4} [\hat{R}_{13} + \hat{R}_{24} + \hat{R}_{23} + \hat{R}_{14}], \quad (15)$$

with equality achieved only when $\hat{C}_{12} = \hat{C}_{34} = \hat{C}_{ii}$, or equivalently, $R_{12} = R_{34} = 1$. In theory, this would only occur when imaging a coherent target, such as a point target.

Unlike $V_{1 \times 4}$, $V_{1 \times 2}$ is the sum of one lag 1 (R_{23}), two lag 2 (R_{13}), and one lag 3 (R_{14}) correlations. In this simple example, the net effect of SAB was to replace two lag 1 correlations with a lag 3 correlation. The trend of removing short-lag correlations for long-lag correlations is observed when using larger subaperture sizes as well.

III. Methods

A. Field II Simulations

A 64×64 element 2-D matrix array transducer with a center frequency of 5 MHz and a bandwidth of 60% was simulated with Field II [18]. The element pitch was $\lambda/2 = 0.154$ mm in both the azimuthal and elevation dimensions, resulting in a total aperture size of 10×10 mm. A fixed focus of 5 cm was used on transmit, and dynamic focusing was applied on receive. 3-D volumes were acquired by transmitting and receiving a 30×30 grid of beams with a beam spacing of 0.42° in both dimensions, corresponding to a 0.368 mm beam spacing at the focal depth.

A 3-D tissue phantom was simulated by randomly distributing scatterers with random scattering strength throughout a $12 \times 12 \times 12$ mm volume with a density of 20 scatterers per resolution cell. The echogenicities were reduced proportionally for all scatterers within a 5 mm diameter sphere at the center of the phantom to simulate an anechoic, a -12 -dB, and a -6 -dB lesion. Four such scatterer distributions were realized for a total of 12 lesion phantoms. Each phantom was centered directly beneath the transducer at the focal depth.

B. Beamforming

Several beamforming approaches were implemented using the same dynamically-focused received signals. Conventional B-mode image volumes were generated by summing the signals over the entire aperture, demodulating, and computing the magnitude. SLSC image volumes were formed by taking the sum of the correlation values between all signals at short-lag (6). Additionally, SLSC image volumes were formed from the signals from beamformed subapertures; this was accomplished by summing 2×2 , 4×4 , 8×8 , and 16×16 element subapertures, resulting in 32×32 , 16×16 , 8×8 , and 4×4 sets of subaperture signals, respectively. SLSC image volumes were also generated using 1-D analogs of the 2-

D transducer, which were created by summing 1×64 or 64×1 element subapertures. Fig. 2 illustrates the layout of the 64×64 , 8×8 , and 1×64 set of signals that result from subaperture beamforming.

Each image volume was scan converted to account for the beam steering and normalized by the maximum value within the volume. The B-mode image volumes were logarithmically compressed for display on a decibel scale, whereas the SLSC images were not compressed and were displayed on a linear scale. The computation time was also recorded for each beamforming operation.

C. Performance Metrics

For each image volume generated, image quality metrics were obtained using 3-D regions of interest (ROIs). The ROI for the lesion was selected as a sphere inside of and at the center of the spherical lesion; the ROI for the speckle background was chosen as a toroidal volume around the lesion, limited in axial range to match the lesion ROI. The contrast, texture SNR, and CNR were calculated as

$$\text{Contrast} = 20 \log_{10} \left(\frac{\mu_L}{\mu_B} \right), \quad (16)$$

$$\text{TextureSNR} = \frac{\mu_B}{\sigma_B}, \text{ and} \quad (17)$$

$$\text{CNR} = \frac{\mu_B - \mu_L}{\sqrt{\sigma_B^2 + \sigma_L^2}}, \quad (18)$$

where μ_B and μ_L denote the sample means of the background ROI and the lesion ROI, respectively, and σ_B and σ_L denote the sample standard deviations of these regions. The B-mode image metrics were measured from the uncompressed image data. Note that the CNR is a combination of the other two metrics, and can be represented as a scaled product of the texture SNR and the contrast on a linear scale, as shown in Bottenus *et al.* [19, Eq. (9)]. Smith and Wagner have shown that the CNR is essentially equivalent to their measure of lesion detectability, and is therefore used as an imaging performance metric in this study [20].

CNR was examined as a function of the ROI diameter. Additionally, SLSC imaging performance was measured as a function of short-lag region size, as specified by the cutoff value P . Otherwise, a constant ROI diameter of 3.75 mm and a value of $P = 0.75$ was used throughout the remainder of the study.

D. Simulation of Clutter

To simulate the acoustical and electrical noise that might be encountered in a clinical setting, additive white noise was band-pass filtered at the cutoff frequencies of the probe and added to the raw received channel signals. The rms amplitude of the noise was modulated to investigate the imaging performance as a function of element and subaperture SNR for each

beamforming configuration, measured as the ratio of the rms of the RF signal received from a speckle region at the focal depth to the noise rms amplitude. This model of acoustical noise generates a delta-function-like spatial coherence function and is compatible with acoustic reverberation or multiple scattering [2], random electrical or acoustical noise, or bright off-axis reflectors.

IV. Results

Fig. 3 shows the average measured spatial correlations as a function of the correlation values predicted by the VCZ theorem for diffuse scatterers, as found in (3). Plots are shown for the 64×64 , 8×8 , and 1×64 subaperture arrays inside and outside an anechoic lesion. For each beamforming method, the spatial correlations measured in the image background (squares) lie on the diagonal, indicating good agreement between observation and theory. The correlation values inside of the hypoechoic lesion (circles) fall below the diagonal, signifying that the signals are less correlated than the background. The 1×64 subaperture array demonstrates significantly more variance than the 64×64 and 8×8 arrays both in the background and inside the lesion and more rapid decorrelation of echoes inside the lesion, corresponding to a greater contrast.

The CNRs of the image volumes of the -12 dB lesion from several beamforming configurations are plotted in Fig. 4 as a function of the diameter of the lesion ROI. A sample mean and standard deviation were computed over 4 realizations of the lesion. In general, the SLSC images attained higher CNR values than B-mode. The measured CNR of the B-mode images did not change as a larger volume was considered, except when the diffraction-limited resolution of the imaging system degraded CNR at diameters of roughly 4 mm or greater. The CNR of the SLSC images, however showed a consistent decrease as a larger volume was incorporated, suggesting a gradual decrease in coherence from the lesion borders to the center. To avoid differences resulting from the resolution characteristics of B-mode and SLSC imaging, a 3.75 mm ROI diameter (dashed line) was selected and used throughout the rest of the study. The 64×64 , 32×32 , and 16×16 arrays had nearly identical imaging performance, yielding the highest CNR values, followed closely by the 8×8 array. The CNRs of the 1×64 and 4×4 arrays were much lower than the 8×8 array, but were slightly better than B-mode.

The CNR of the anechoic lesion is shown as a function of the threshold P in Fig. 5, with each curve showing the CNR for SLSC images generated using a different subaperture size. For reference, the CNR value of the B-mode image is included (dashed line). At low P , when all lags are incorporated into the SLSC image, the CNR was roughly 2. The CNR value increased as the short-lag region was reduced in size, with a maximum CNR achieved around $P = 0.65$, before falling again at high P . The curves are similar for increasing amounts of SAB, with only the 4×4 subaperture array showing significant deviations in CNR values. In all cases, the SLSC images display CNR superior to the B-mode image.

Fig. 6 shows the coronal, sagittal, and transverse planes through 3-D scan converted image volumes of data acquired from the anechoic, -12 -dB, and -6 -dB lesions. Each row corresponds to a different beamforming configuration; from top to bottom, images were

formed using standard B-mode, SLSC using a 1×64 set of subaperture signals, SLSC using an 8×8 set of subaperture signals, and SLSC using the full 64×64 set of signals. The B-mode images are compressed and show 40 dB of dynamic range, whereas the SLSC images are not compressed and show normalized values ranging from 0 to 1.

The anechoic and -12 -dB lesions are easily detected in all imaging cases, whereas the -6 -dB lesion is more difficult to see. The background has a smoother texture in the SLSC images compared with the dark speckle patterns in the B-mode image. The 8×8 and 64×64 SLSC images are nearly indistinguishable. A spurious bright spot in the center of the -12 -dB and -6 -dB lesions is visible in the coronal and transverse planes of the SLSC images and is most apparent in the 1×64 SLSC image. The spot is also visible in the B-mode as a brighter speckle, but is not as pronounced.

Using a P of 0.75 and a lesion ROI diameter of 3.75 mm, the contrast, CNR, and SNR were calculated for each beamforming method across four realizations of each lesion type, and are presented in Table I. The first row corresponds to the SLSC images formed using the full 64×64 set of received signals, and the last row to the B-mode images. All remaining rows refer to the SLSC images formed using arrays of beamformed subaperture signals.

For the anechoic lesion, the measured contrasts improved with increasing subaperture size (i.e., moving down the table), with the maximum contrast being achieved in the B-mode images. Notably, the measured contrasts in the B-mode images matched the true echogenicities of the lesion for the -12 dB and -6 dB contrast lesions. The texture SNR in the B-mode images aligned with the theoretical prediction of 1.9, whereas the speckle SNR of the SLSC images was roughly 7-fold higher when using either the full 64×64 data set or the 32×32 subaperture array. The lesion CNR also decreased with increased SAB, although it should be noted that the 32×32 and 16×16 subaperture arrays attained the same CNR values as the full 64×64 array of data, and that the 8×8 subaperture array was within one standard deviation. Additionally, the 8×8 subaperture array outperformed the 64×1 and 1×64 arrays, despite having subapertures of equal surface area.

Images were formed in the presence of clutter-mimicking noise, and are shown in Fig. 7 in the same layout as in Fig. 6. The rms amplitude of the added noise was modified for each beamforming configuration so that the electronic SNR of the beamformed subaperture signals was 12 dB for the anechoic lesion and 6 dB for the -12 -dB and -6 -dB lesions. Depending on the amount of SAB, this corresponded to different element SNRs which are listed in Table II. The B-mode images are indistinguishable from those in the noise-free case; this is examined in greater detail in Fig. 8.

Table II provides imaging performance metrics for each beamforming configuration when imaging in noisy conditions. The table demonstrates the existence of a specific subaperture SNR for each array at which the SLSC lesion CNR is maximized above the value in the noiseless case. At this level of noise, the increase in lesion contrast outweighs the loss in texture SNR to yield a higher CNR. The 1×64 and 64×1 arrays are found to have comparable performance to the 4×4 array rather than the 8×8 array. Note that the level of

noise added to achieve the maximum CNR is dependent on the level of subaperture beamforming. The B-mode CNR does not improve at any level of added noise.

Fig. 8 highlights the differences in the characteristic responses to noise between B-mode and 64×64 SLSC imaging. From left to right, images of a 5-mm-diameter anechoic lesion were formed in the presence of increasing noise. Generally, the SLSC images become darker as noise degrades echo coherence whereas the B-mode images grow brighter. In the SLSC images, the lesion becomes dark at low amplitudes of noise. Beginning around 0 dB of noise, the background also darkens, eventually approaching the darkness of the lesion. The SLSC images are lightly compressed at higher noise levels to emphasize qualitative characteristics. In the B-mode images, the noise appears as a bright haze throughout the image, filling in the lesion. However, note that the B-mode image obtains a $\sqrt{N}$ improvement in electronic SNR compared with the SLSC image due to delay-and-sum beamforming. The background is dark at 6 dB of noise in the SLSC image, whereas 6 dB noise does not impact the B-mode image significantly.

The lesion contrasts of the B-mode and 64×64 , 32×32 , 16×16 , 8×8 , and 4×4 array SLSC images are plotted in Fig. 9 as a function of element SNR for the -12 -dB lesion. As with the images in Fig. 8, the plots show the contrast with increasing noise level from left to right. The contrast of the B-mode images is reduced, gradually decreasing from the true echogenicity of the lesion to 0 dB. The contrast of the SLSC images, while low initially, increased above the true contrast with the addition of noise. The contrast computation becomes unstable at high levels of noise, as the correlation both inside and outside the lesion approaches zero. Note that the onset of contrast improvement occurs at a different point for each subaperture array; these offsets correspond to the differences in subaperture electronic SNR.

This effect is further illustrated in Fig. 10. In the left column, the CNR is plotted as a function of individual element SNR for the beamforming configurations used in Figs. 6 and 7 for each lesion echogenicity. As observed qualitatively in Fig. 8, the CNR in SLSC images improved with small amounts of noise, whereas the CNR in B-mode images was fairly constant up to 6 dB of noise. The CNR tailed off to zero with higher levels of noise in all imaging cases. The right column shows similar plots for the SLSC images from the 64×64 , 32×32 , 16×16 , 8×8 , and 4×4 subaperture arrays and for the B-mode images as a function of subaperture SNR. The plots align closely for each subaperture array when plotted this way, with the 64×64 , 32×32 , 16×16 , and 8×8 arrays nearly identical at subaperture SNR values less than 6 dB.

For the anechoic (not pictured) and -12 -dB lesion, the CNR of the SLSC images is higher than that of B-mode in all cases. When imaging the low contrast -6 -dB lesion, however, both the 1-D analog (1×64) and the 4×4 arrays demonstrated similar CNR to B-mode in low-noise environments.

The average runtimes to compute a single SLSC beam are shown for each of subaperture size in Table III. Although these computation times were all measured using a single-threaded C++ implementation on a Dell T7500 workstation (Dell Inc., Round Rock, TX)

with Intel Xeon X5690 CPUs (Intel Corp., Santa Clara, CA) running at 3.47 GHz, the absolute runtimes were highly machine-dependent. The 64×64 array took the longest time to beamform a single array. As the number of channels was reduced by subaperture beamforming, the computation time was also reduced exponentially. The 32×32 array demonstrated a 25-fold gain in computation speed. Likewise, the 16×16 array showed an improvement of 328-fold, and the 8×8 showed an improvement of 2640-fold in runtime over the fully sampled array. The time required to generate a single B-mode line was on the order of 10 ms.

V. Discussion

SLSC imaging on 1-D arrays has demonstrated improved lesion CNR over B-mode imaging in previous studies. The gains in imaging performance were attained primarily because of an improved texture SNR, often in spite of a loss in lesion contrast. SLSC imaging on matrix arrays extends these effects further; by incorporating sampling in elevation, texture SNR and lesion CNR were further improved over both B-mode and 1-D SLSC imaging, despite a loss in lesion contrast (Table I). As is characteristic of SLSC imaging, the losses in lesion contrast were minimal when spatially incoherent clutter was present throughout (Table II). Furthermore, for a set number of channels, an even sampling of the aperture in both dimensions (8×8 array) generated SLSC images with superior CNR to those formed with fine sampling along one dimension and coarse sampling along the other (1×64 and 64×1 arrays). These results show that, because the spatial coherence function is sampled in both dimensions, SLSC imaging performance on matrix arrays is superior to that on 1-D arrays.

A. Differences Between SLSC and B-Mode Imaging

The contrast in SLSC images arises from differences in the level of coherence of the backscatter over short lags. Fig. 3 demonstrates the difference in spatial coherence of echoes from inside and outside the lesion. Although the spatial coherence of the texture background outside the lesion adheres closely to that described by (3), the coherence within the lesion shows significant decorrelation. In the noiseless simulation environment, the only source of decorrelation was off-axis scattering. *In vivo*, the backscattered wave is typically further degraded by a spatially incoherent noise arising from random electrical and acoustical noise and multi-path scattering. The extent of degradation dictates the darkness of the pixel value in the image.

Because SLSC imaging utilizes the normalized cross-correlation, the magnitude of the backscatter does not play a direct role in generating contrast as it does in B-mode images. However, the hypoechoic regions of B-mode images generally align with the hypocorrelated regions of SLSC images, as is observed in Fig. 6. This is because the low-amplitude backscattered waves from hypoechoic regions of tissue are more vulnerable to off-axis scatterers and noise sources and typically exhibit weaker correlations than expected of diffuse scatterers (3). Contrast in SLSC images is generated by this indirect mechanism, and is dependent on the strength of the decorrelating source. The only source of decorrelation in these particular images was off-axis scattering, and resulted in lower lesion contrasts in the SLSC images than in B-mode (Table I).

This phenomenon is observable not only in the lesions but also in the texture of the SLSC images, where the peaks and troughs appear at the same spatial positions as the speckle in the B-mode images. However, this speckle-like texture in the SLSC images does not obey the same first-order random walk statistics that place a fundamental limit of approximately 1.9 on the B-mode texture SNR [21]. As with the lesions, the contrast within the texture is much lower in SLSC than in B-mode images, which is beneficial in this case and yields a much improved texture SNR. In our simulations, the 64×64 array generated SLSC images with 7-fold higher texture SNR than B-mode. The enhanced texture SNR in SLSC images was found to offset the loss in lesion contrast to provide a significantly improved lesion CNR, a metric for lesion detectability [8]. SLSC imaging attained greater lesion CNR than B-mode across all lesions with the 64×64 , 32×32 , 16×16 , and 8×8 arrays.

The high SNR of the texture in the SLSC images causes the speckle spots to appear enlarged or blurred. However, although speckle size is a good measure of resolution in B-mode images, it is not representative of the true resolution of SLSC imaging. As with the minimum variance beamformer [22], it is difficult to establish a single expression or value as the true resolution of the technique because it is target and noise dependent and a function of many additional parameters.

Two particularly significant sources of noise have been quantified in the past: thermal noise and acoustical clutter. Thermal noise, which increases with depth, was isolated in an experiment by Dahl *et al.* [23]. In this study, the levels of purely thermal noise in each channel were measured to be -12.5 dB, -3.7 dB, and -2.8 dB at the focus when imaging *in vivo* breast, liver, and thyroid tissue, respectively. In a study by Lediju *et al.*, reverberation clutter in beamformed data was found to range from -30 dB to 0 dB in magnitude relative the mean tissue signal in *in vivo* bladders and fetal phantoms [1]. Note that the measurable reverberation was capped at 0 dB in this study because the clutter and the tissue signal could not be separated in the tissue regions surrounding the bladder; the true level of the reverberation clutter relative to the tissue could be greater. Most medical ultrasound images are subject to a combination of both noise sources, though the magnitude and impact vary widely with application and patient. To illustrate the response across this entire range, B-mode and SLSC imaging performance is presented along the wide range of noise levels in Figs. 9 and 10.

In this study, acoustical clutter was modeled using white noise band-pass filtered at the transducer bandwidth cutoff frequencies. This model is based on previous work by Pinton *et al.* [2], who demonstrated that the spatial coherence of backscatter containing significant reverberation clutter approaches a delta function. By introducing this additional source of decorrelation, the contrast of the SLSC images was improved, as seen in the lesion images in Figs. 7 and 8(a). The SLSC lesion contrasts, plotted in Fig. 9, increased with added noise level. The noise also increased contrast within the texture, reducing the texture SNR. These competing effects combined to yield an *increase* in lesion CNR at low noise levels, and a gradual decline in CNR at higher noise levels, as seen in Fig. 10.

B-mode, however, showed a loss in contrast with increasing noise level. This is because B-mode has an intrinsically different response to incoherent noise. When summing together N

channels containing uncorrelated noise to form a B-mode image, the improvement in channel SNR is $\sqrt{N}$. However, the overall signal magnitude is increased by a factor of $(1 + A_n/A_s)$, where A_n is the amplitude of the noise after summation and A_s is the amplitude of the original signal. Because B-mode images display echo magnitude, this means that hypoechoic regions are more sensitive to the increase in magnitude than the background, appearing brighter and losing contrast on a logarithmic scale, as shown in Fig. 8(b). As in the case without additional clutter, the CNR of the B-mode images was worse than that of the SLSC images generated with the 64×64 , 32×32 , 16×16 , and 8×8 arrays over various noise levels, as shown in Fig. 10.

B. Effects of SAB

The correlation of beamformed subapertures is a weighted sum of the correlations between each element from one subaperture with each from the other, some of which are at a longer lag than desired. SAB also removes the correlations between elements within a subaperture, which are at short lags. In doing so, SAB introduces decorrelation and effectively increases the contrast throughout the SLSC image. This is corroborated by the increase in lesion contrast and the loss in texture SNR with increasing subaperture size in Table I.

The VCZ theorem predicts that the shape of the subapertures can significantly affect the correlations between the resulting subaperture signals. Consider the subaperture signals generated by beamforming 64×1 element and 8×8 element subapertures; though the two subapertures have the same surface area, (3) shows that, when imaging diffuse scatterers, the 64×1 element subaperture contains signals at opposing ends of the aperture that are essentially uncorrelated ($\langle R_d(D_x, D_y/64) \rangle = 0$), whereas the signals within an 8×8 element subaperture will have a worst-case correlation of $\langle R_d(D_x/8, D_y/8) \rangle = 0.77$. The plots in Fig. 3 show that compared with the 8×8 array, the 1×64 array correlation estimates have a greater variance in the texture, corresponding to lower texture SNR, as well as an improved contrast between the estimates inside and outside of the lesion. However, as Table I shows, the two effects combine to result in lower CNR values for the 1×64 array. These results suggest that SLSC imaging is more effective when using an even spatial sampling of the aperture in two dimensions rather than fine sampling in one dimension and coarse sampling in the other.

SAB effectively combines the improved channel SNR and low computational cost of B-mode with the high texture SNR of SLSC beamforming. Fig. 10 shows that a subaperture consisting of N_{SA} elements can be beamformed to obtain an improvement of $\sqrt{N_{SA}}$ in SNR while maintaining comparable SLSC imaging performance to that of the full 64×64 array, especially when using the 32×32 , 16×16 , and 8×8 arrays. These showed negligible losses in CNR while improving channel SNR by a factor of 2, 4, and 8 times, respectively. Moreover, these arrays drastically decreased the number of correlations to be computed, and in the case of the 8×8 array, resulted in a 2640-fold improvement in the SLSC computation time.

These results are encouraging for the feasibility of SLSC imaging on existing 2-D arrays which employ SAB because it maintains imaging performance, improves channel SNR, and

reduces the overall computation time. These benefits imply that SAB may be desirable even when access to the full channel data set is available. The performance of the 8×8 array also suggests that 1.75-D arrays may provide sufficient elevational sampling for effective SLSC imaging in two dimensions.

VI. Conclusion

We have presented a method to apply SLSC imaging to a 2-D matrix-array transducer and implemented it in simulation in conjunction with SAB. The results showed that SLSC images using a full 64×64 set of channel signals had higher lesion CNR than B-mode images in all cases. SLSC imaging with 2-D subapertures (e.g., 8×8 element subapertures) was found to be superior to imaging with 1-D subapertures (e.g., 1×64 , 64×1 element subapertures). Additionally, SLSC imaging with beamformed 8×8 , 4×4 , and 2×2 element subapertures (yielding 8×8 , 16×16 , and 32×32 sets of signals, respectively) showed nearly identical performance to that using the full signal set, while benefiting from 8, 4, and 2 times increase in channel SNR and 2640-, 328-, and 25-fold improvement in computation time. These results demonstrate that moderate amounts of subaperture beamforming applied in the handles of 2-D transducers do not degrade SLSC images but instead confer additional benefits, and suggest that volumetric SLSC imaging is readily applicable to existing technology.

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Fig. 1.

An image of the short-lag region (shaded boxes) with respect to the element in row 50, column 20. The short-lag region with respect to the element is delineated according to $\langle R_d \rangle$, the predicted correlation coefficients for echoes returning from a diffuse scattering medium. Isocohere curves are shown for theoretical correlation values of $\langle R_d \rangle = 0.8, 0.6, 0.4$, and 0.2 . A short-lag region is defined for each element using a threshold theoretical correlation value ($\langle R_d \rangle = P = 0.75$ in this example). To form an SLSC image, the measured spatial correlations between each element and all elements within this short-lag region are summed.

Fig. 2.

Several sets of subaperture signals were generated from the original 64×64 array. Pictured are some of the element groupings used to form subaperture arrays. From left to right: (a) the full 64×64 array, (b) an 8×8 set of pre-beamformed subapertures, and (c) a 1-D analog of the original array composed of a 1×64 set of subaperture signals.

Fig. 3.

The spatial correlation $\hat{R}$ was measured inside (circles) and outside (squares) of an anechoic lesion, and is plotted as a function of the predicted coherence function of diffuse scatterers $\langle R_d \rangle$, as defined in (3), for 3 beamforming cases. High values of $\langle R_d \rangle$ correspond to short lags and low values to long lags. The signals from within the lesion are less correlated than those from the image background. The (a) 64×64 and (b) 8×8 arrays show nearly identical correlations, whereas (c) the 1×64 array shows much greater variance.

Fig. 4.

The CNR of a -12 -dB lesion is plotted as a function of ROI diameter for images generated using several beamforming configurations, with error bars showing one standard deviation above and below over 4 scatterer realizations. The CNR decreases as the ROI diameter approaches the true lesion diameter of 5 mm. The ROI diameter of 3.75 mm used throughout this study is marked by the dashed line.

Fig. 5.

CNR is plotted as a function of P for the SLSC images of an anechoic lesion formed with various amounts of subaperture beamforming. Higher P corresponds to a smaller short-lag region. In all instances, CNR appears to peak at $P \approx 0.65$. Note the nearly identical response of the 64×64 , 32×32 , 16×16 , and 8×8 subaperture arrays. For reference, the CNR of B-mode is also provided. A P of 0.75 is shown for reference by the dashed line.

Fig. 6.

The cross-sectional planes are shown through the image volumes for each of 3 lesion contrasts: (a) axial-by-azimuth, (b) axial-by-elevation, and (c) azimuth-by-elevation. For each set of images, anechoic, -12 -dB, and -6 -dB lesions are shown in columns 1, 2, and 3, respectively. The images in the top row are log-compressed B-mode images showing 40 dB of dynamic range. The remaining rows are normalized SLSC images, formed using a 1×64 set of subapertures (second row), an 8×8 set of subapertures (third row), and the full 64×64 set of signals (bottom row). The SLSC images show values from 0 to 1.

Fig. 7.

Cross-sectional planes are shown for image volumes formed in the presence of noise: (a) azimuth plane, (b) elevation plane, (c) transverse plane. Different amplitudes of noise are added to each beamforming configuration to illustrate the characteristic responses to noise. The images are presented in the same order as in Fig. 6, with B-mode in the top row and SLSC images formed using (second row) a 1×64 subaperture array, (third row) a 8×8 subaperture array, and (bottom row) the full 64×64 data set. The B-mode images show 40 dB of dynamic range. The SLSC images show values from 0 to 1.

Fig. 8.

An image of an anechoic lesion using the (a) 64×64 SLSC beamforming configuration and (b) B-mode are shown with increasing channel noise from left to right. The electronic SNR at each element is shown below each image. The SLSC images are compressed for higher noise levels and show a range from 0 to 1, while the B-mode images show 40 dB of dynamic range.

Fig. 9.

The lesion contrast is plotted as a function of element SNR for SLSC images made with the 64×64 , 32×32 , 16×16 , 8×8 , and 4×4 arrays and the B-mode image for the -12 -dB lesion, with the level of noise increasing from left to right (as in Fig. 8). The contrast of the SLSC images increases with noise whereas the contrast of B-mode images decreases.

Fig. 10.

The CNR values of each imaging configuration are plotted as a function of (a) the element and (b) the subaperture SNR for each lesion contrast. Note that element SNR and subaperture SNR are related by a factor of $\sqrt{N}$, where N is the number of elements per subaperture.

TABLE I

Imaging Performance in Noise-Free Environment.

Signal array	Anechoic lesion		-12-dB lesion		-6-dB lesion		Texture SNR
	Contrast (dB)	CNR	Contrast (dB)	CNR	Contrast (dB)	CNR	
64×64	-6	3.8 ± 0.2	-4	2.9 ± 0.2	-2	1.4 ± 0.2	13.9 ± 0.4
32×32	-6	3.8 ± 0.2	-4	2.9 ± 0.2	-2	1.4 ± 0.2	14.1 ± 0.4
16×16	-7	3.8 ± 0.2	-4	2.9 ± 0.2	-2	1.4 ± 0.2	13.0 ± 0.3
8×8	-8	3.6 ± 0.2	-5	2.7 ± 0.2	-2	1.2 ± 0.2	11.2 ± 0.2
64×1	-10	3.1 ± 0.2	-4	1.9 ± 0.1	-1	0.8 ± 0.1	10.6 ± 0.6
1×64	-11	3.2 ± 0.1	-4	1.8 ± 0.1	-1	0.8 ± 0.1	10.7 ± 0.2
4×4	-11	3.1 ± 0.1	-4	1.8 ± 0.1	-1	0.7 ± 0.1	7.7 ± 0.1
B-mode	-20	1.7 ± 0.0	-12	1.4 ± 0.1	-6	0.9 ± 0.1	1.9 ± 0.0

TABLE II

Imaging Performance in Noisy Environment.

Signal array	Anechoic lesion (Subap. SNR 12 dB)			-12-dB Lesion (Subap. SNR 6 dB)			-6-dB Lesion (Subap. SNR 6 dB)		
	Contrast (dB)	CNR	Texture SNR	Contrast (dB)	CNR	Texture SNR	Contrast (dB)	CNR	Texture SNR
64×64	-9	4.2 ± 0.2	10.1 ± 0.3	-8	3.3 ± 0.1	6.5 ± 0.2	-4	1.7 ± 0.1	6.5 ± 0.2
32×32	-9	4.2 ± 0.2	10.2 ± 0.3	-8	3.3 ± 0.1	6.5 ± 0.2	-4	1.7 ± 0.1	6.5 ± 0.2
16×16	-10	4.2 ± 0.2	9.5 ± 0.3	-9	3.2 ± 0.1	6.2 ± 0.2	-4	1.7 ± 0.1	6.2 ± 0.2
8×8	-11	4.1 ± 0.2	8.5 ± 0.2	-9	3.1 ± 0.1	5.6 ± 0.1	-4	1.6 ± 0.1	5.7 ± 0.1
64×1	-16	4.0 ± 0.2	7.0 ± 0.3	-12	2.7 ± 0.2	4.3 ± 0.1	-5	1.4 ± 0.1	4.3 ± 0.1
1×64	-16	4.2 ± 0.2	7.0 ± 0.2	-12	2.7 ± 0.1	4.2 ± 0.1	-5	1.3 ± 0.1	4.3 ± 0.1
4×4	-15	3.5 ± 0.1	7.0 ± 0.1	-10	2.4 ± 0.1	4.5 ± 0.1	-4	1.2 ± 0.1	4.4 ± 0.1
B-mode	-20	1.7 ± 0.0	1.9 ± 0.0	-12	1.4 ± 0.1	1.9 ± 0.0	-6	0.9 ± 0.1	1.9 ± 0.0

TABLE III

Average SLSC Computation Time Per Beam.

Array	Runtime (s)	Speedup
64×64	950	1
32×32	38	25
16×16	2.9	328
8×8	0.36	2640
64×1	0.36	2640
1×64	0.36	2640
4×4	0.04	23 800
B-mode	0.009	110000