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Parallel Beam Approximation for Calculation of Detection Efficiency of Crystals in PET Detector Arrays

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Abstract

In this work we propose a parallel beam approximation for the computation of the detection efficiency of crystals in a PET detector array. In this approximation the detection efficiency of a crystal is estimated using the distance between source and the crystal and the pre-calculated detection cross section of the crystal in a crystal array which is calculated for a uniform parallel beam of gammas. The pre-calculated detection cross sections for a few representative incident angles and gamma energies can be used to create a look-up table to be used in simulation studies or practical implementation of scatter or random correction algorithms. Utilizing the symmetries of the square crystal array, the pre-calculated look-up tables can be relatively small. The detection cross sections can be measured experimentally, calculated analytically or simulated using a Monte Carlo (MC) approach. In this work we used a MC simulation that takes into account the energy windowing, Compton scattering and factors in the “block effect”. The parallel beam approximation was validated by a separate MC simulation using point sources located at different positions around a crystal array. Experimentally measured detection efficiencies were compared with Monte Carlo simulated detection efficiencies. Results suggest that the parallel beam approximation provides an efficient and accurate way to compute the crystal detection efficiency, which can be used for estimation of random and scatter coincidences for PET data corrections.

Keywords

Compton scattering; detection efficiency; positron emission tomography; random coincidence

I. INTRODUCTION

When computing random or scatter coincidences for data corrections (e. g., the Single Scatter Simulation correction technique [1]–[4]), one often needs to know the detection efficiency of crystals in a block detector of scintillator-based PET systems. In general, the crystal detection efficiency is a function of the incident angle, source location, position of the crystal in detector array, size and material of the crystals, energy of the incident gammas and a lower level discriminator (LLD) of energy window used. The complexity of this function requires simplified models in order to reduce the computation time to make any correction technique practical. Computation of crystal detection efficiencies can be based on

approaches used for system matrix computation such as: direct experimental measurement [5]; through the calculation of the lines integrals [6]; or Monte Carlo technique [7]. The correct modeling of the inter-crystal scattering, important for system matrix simulation since it leads to the image blurring [8], remains important for the detector efficiency simulation as well. The inter-crystal scattering in block detectors leads to the decreasing of the detection efficiency of the edge crystals in favor of the crystals in center of a detector block.

In our lab we are developing a high resolution insert device to clinical PET scanner [9]. Our insert device consists of 28 high resolution detectors (13×13 LSO crystals of $2 \times 2 \times 5$ mm³ each) arranged into two half rings ($R = 124$ mm). The acquired data are coincidences between scanner and scanner detectors, scanner and insert detectors, and insert and insert detectors. To minimize unnecessary attenuation of gamma rays by the insert device, there is no shielding at the side surfaces of the insert detector. In this design, some of the insert crystal arrays are subjected to the radiation at the large incident angles and can be located very close to the object being imaged. As a result, we needed to estimate the detection efficiencies of the crystals in our high-resolution PET insert detectors for gamma rays coming at a large variety of incident angles.

The detection efficiency of a crystal in an array $\varepsilon(E_\gamma(\mathbf{r}, \mathbf{a}), (N, m, n))$ is the probability for a gamma with energy E_γ emitted in a direction $\mathbf{a}$ from a point $\mathbf{r}$ to be detected by the crystal (m, n) in the N -th detector crystal array. For practical simulations of scatter or random coincidences, we will need to integrate this efficiency over all possible emission directions or, to be more specific, over a solid angle of the detector crystal (N, m, n) viewed from the emission point $\mathbf{r}$. For a point source with isotropic radiation and a distance d between the emission point and detection crystal that is significantly larger than the size of the crystal, the crystal detection efficiency can be expressed as an integral

$$\varepsilon_{eff} = \frac{1}{4\pi} \int \varepsilon d\Omega \approx \frac{1}{4\pi d^2} \int \sigma d\sigma \quad (1)$$

where, the integration of the solid angle is over the entire 4π , while the σ is a geometrical cross section of the detector crystal (projection of the crystal to the surface perpendicular to the line connecting the emission point and center of the crystal).

For a clinical PET scanner with large ring diameter, the incident angle of gamma rays relative to the norm direction of the detector crystal array can be relatively large ($> 45^\circ$). The angular dependency of crystal detection efficiency is even more prominent for PET systems such as animal PET scanners or PET insert systems [9] in which the object may be very close to the edge of the detector rings or to the side surfaces of PET insert detectors. The relatively extensive computation of (1) makes it unfeasible to calculate it “on the fly” during simulation without some approximation. On the other hand, a look-up table for the crystal detection efficiencies, pre-calculated (or measured) for all image voxels, will be extremely large and difficult to apply because of the large number of detectors and large scanner field-of-view (FOV). The radiation coming from outside the scanner FOV can also contribute to scatter and random coincidences and therefore the crystal detection efficiencies should be calculated for the region much larger than the actual scanner FOV.

In this work we propose to establish a detection cross section look-up table instead of a detection efficiency look-up table to simplify computation of (1), allowing it to be done “on the fly” during simulation. This approach reduces the spatial dependency (source and crystal array position) of the look-up table and only the gamma energy, incident angle and position of the crystal in the crystal array are the parameters used in a look-up table. The crystal detection cross sections for this new look-up table can be estimated using a parallel beam approximation.

In the parallel beam approximation, the integral on the right side of (1) can be considered as a detection cross section of the selected crystal (N, m, n)

$$\sigma_{m,n}^N = \int \varepsilon d\sigma. \quad (2)$$

With a parallel beam of gammas illuminating the crystal array, the crystal detection cross section will be independent of the distance between the crystal and emission point, which is a reasonable approximation for the condition used in (1) ($d \gg \sigma_{m,n}^N$). The crystal detection cross section will depend only on gamma energy, the gamma incident angles (θ, ψ) (see Fig. 1) and crystal position (m, n) in the crystal array. Thus using the crystal detection cross section $\sigma(E_\gamma, \theta, \psi, m, n)$ instead of crystal detection efficiency ε , we can significantly reduce the size of the look-up table. The incident angles (θ, ψ) and distance d can be calculated “on the fly” and the crystal detection cross section can be easily transformed into crystal detection efficiency using (1).

The crystal detection cross section is a relatively smooth function of the incident angles and energy. We can create a look-up table using a set of discrete values E_i , θ_j , and ψ_k

$$\sigma_{i,j,k,m,n} = \sigma(E_i, \theta_j, \psi_k, m, n) \quad (3)$$

The crystal detection cross sections for any intermediate angles and energies can be obtained from this look-up table through interpolation.

Further reduction of the size of the look-up table can be accomplished using the symmetries of a square crystal array. To cover all possible incident angles, we need only the crystal detection cross sections for $\theta_j \in [0, \pi/2]$ and $\psi_k \in [0, \pi/4]$.

There are at least three possible ways to compute the proposed look-up table: 1) experimental measurement of the crystal detection efficiencies using a point source located at different positions around the crystal array, followed by conversion of the measured crystal detection efficiencies into the detection cross section using the known distance between source and crystals; 2) direct calculation of the line integrals (geometrical approach); and 3) MC simulation of the parallel beam of gammas illuminating the crystal array.

Although direct experimental measurement is the best way to create a look-up table, it can be difficult in practice due to the relatively large number of calibration points needed and low statistics as the point source is moved away from the detector. If the symmetries of the

square crystal array are to be used to minimize the look-up table size, the experimentally measured crystal detection efficiency should also be averaged over a few modules to minimize variation in crystal detection efficiencies among different modules. Our alternative approach is to use the simulated look-up table cross-calibrated with experimental data measured by using only one point source position inside a scanner. This can significantly simplify the calibration procedure and lead to a more practical solution.

In a simple geometrical approach (see Fig. 1) the crystal detection cross section would be given by the double integral

$$\sigma_{Geom} = \cos \theta \iint \left(1 - e^{-\mu(E)L_+}\right) e^{-\mu(E)L_-} dx dy, \quad (4)$$

where $\mu(E)$ is the mean linear attenuation coefficient of the crystal material, $L_-(x,y,\theta,\psi)$ and $L_+(x,y,\theta,\psi)$ are the attenuation and detection lengths, respectively. This integral is simple and the calculation of the look-up table can be done very efficiently without any statistical error. On the other hand, this geometry-based calculation does not model all the detection physics correctly and is subject to some inherent limitations. For example, it is assumed in (4) that any interaction in a given crystal is a valid detection and any interaction before this crystal contributes to attenuation. In reality, some gammas scattered in a crystal can deposit small amounts of energy and do not qualify as valid events. This leads to an overestimation of the crystal detection cross sections, especially for detector arrays made of small crystals. Another problem is multiple scattering inside the crystal array. If the sum energy deposit is higher than LLD, this event will be considered as a valid event with energy weighted position averaged over all interactions. Generally, in this situation, the crystal associated with the detected location is different from the crystal of the first interaction. Gammas that are scattered by crystals near the edge of an array and travel outwards will fail the energy windowing and become rejected, while gammas that are scattered and travel inwards will have a higher probability to become a valid event, although the final detected position will be shifted toward the center of the crystal array. As a result, the detection efficiencies of the crystals at the edges of a crystal array will be smaller than the detection efficiencies of the central crystals. This “block effect” will be especially prominent for the corner crystals.

To overcome these problems, we used the MC simulation approach that includes the modeling of LLD, Compton scattering and “block effects”. To find the limitation for the parallel beam approximation, we have performed additional MC simulations of the point sources at different distances from the crystal array and compare the numbers of counts obtained in simulations and estimated using the MC simulated cross section look-up table. To validate the MC simulation, we also compared some of simulated results with experimental measurements.

II. MC SIMULATION OF THE DETECTION CROSS SECTION IN PARALLEL BEAM APPROXIMATION

We developed a MC code for gamma tracking to simulate the crystal detection cross section in the parallel beam approximation. The crystal arrays in these simulations were illuminated by a uniform, wide and continuous beam of parallel gammas with fixed energy E_γ at the

incident angles θ and ψ (the same angle notations as in Fig. 1). Only the photoelectric interaction and Compton scattering were taken into account in simulations. To emulate a real detector array, the LLD was set to 450 keV as it was in our scanner set-up [5].

Multiple interactions inside a crystal array were transformed into a single interaction with a sum energy deposit at an energy-weighted detection position (similar to the algorithm used by typical block detectors)

$$\vec{r} = \frac{\sum_n E_n \vec{r}_n}{\sum_n E_n}. \quad (5)$$

Only events with a total energy deposit over the given LLD are considered to be valid events. A large flux of gammas $\sim 10^5/\text{mm}^2$ was used in simulation to minimize statistical error.

The ratio of the number of detections in the crystal (m,n) of the array relative to the gamma flux used in the simulation gives the detection cross section σ_{MC} for this crystal.

The calculations were done for two types of detector crystal arrays: 1) a clinical PET “scanner” detector— 13×13 LSO crystals $4 \times 4 \times 20 \text{ mm}^3$ each; and 2) a high resolution PET “insert” detector— 13×13 LSO crystals $2 \times 2 \times 5 \text{ mm}^3$ each. (Both types of crystal arrays were used in a PET system with a prototype high resolution insert device described in [5].) The results were stored as two different look-up tables. Each look-up table consists of the crystal detection cross sections simulated for $\theta_j \in [0, \pi/2]$ and $\psi_k \in [0, \pi/4]$ with $\pi/12$ steps in both angles. So for a 13×13 crystal array, we have $169 \times 7 \times 4$ elements in each look-up table. To have the energy dependent crystal detection cross section, the look-up tables were simulated for two gamma energies (450 and 511 keV). Using interpolation and the symmetries of the square crystal array, the simulated look-up tables provide crystal detection cross sections for all incident angles and gamma energies between 450 and 511 keV.

III. POINT SOURCE MC SIMULATION

To validate and find the limitation for using the parallel beam approximation, a few additional MC simulations were performed for both crystal arrays (“scanner” and “insert”) illuminated by a point source located in a few positions (OS, θ, ψ) around the crystal array (see Fig. 2) and for two gamma energies (450 and 511 keV). The same set of angles (θ_j, ψ_k) as in look-up tables was used in these simulations; however, in this case these angles are the elevation and azimuth of the source S in the coordinate system related to the center of the crystal array. The simulated distances $|OS|$ were 4, 5, 6 and 7 cm. For every combination of energy and source position, 10^8 gammas were simulated. Similar to the previous simulations, only Compton and photoelectric interactions were used for gamma tracking. Multiple interactions of one gamma inside the crystal array were transformed into single interaction with the energy weighted position (5). The LLD was set to 450 keV.

The purpose of these simulations was to allow comparison of the number of counts for each crystal determined by MC simulations with the number of detections estimated using the crystal detection cross sections from the look-up tables simulated using the parallel beam approximation. The predicted number of counts that should be detected by the selected crystals was calculated using the crystal detection cross section from the look-up table for the incident angles (θ_C, ψ_C) where θ_C and ψ_C are the elevation and azimuth of the source S in the coordinate system related to the center of the selected crystal C (see Fig. 2).

IV. EXPERIMENT

To validate the result of MC simulations we performed a series of experiments with “insert” crystal array and a ^{22}Na source ($\sim 18\mu\text{Ci}$) at the distance $d = 7$ cm from the center of crystal array and the incident angles $\psi = 0^\circ$ and $\theta = 0^\circ, 15^\circ, 30^\circ, 45^\circ, 60^\circ, 75^\circ, 90^\circ$ (using the same geometry as shown in Fig. 2). For every incident angle we collect the data for 4 min. The flood image for crystal identification and energy calibration was obtained from the $\psi = 0^\circ$ and $\theta = 0^\circ$ experiment. These results were used to define the lookup table for crystal identification and photopeak locations for energy discriminator for all experiments. The LLD was set to 450 keV (as in MC calculations) and the upper level discriminator (ULD) was set to 700 keV. The only significant difference between the MC simulation and the actual experiments was the non-uniform detector response of different crystals. That is, the central crystals have higher signal output levels than edge crystals (see Fig. 3). This effect is mainly caused by the design of the light guide that couples the LSO array to PMT. The light guide used in this experiment has multiple cuts (filled with white reflective powder) along its edges to control light distribution in order to improve the identification of the edge crystals in flood image. On the other hand these cuts lead to the loss in photon collection efficiency of the edge crystals. For events that involve multiple interactions in a crystal array, energy weighted position from real experiment will be different from that calculated in MC simulation. This effect also causes problem for energy discrimination because the energy calibration of the erroneously identified crystals will be applied to the detected sum signal. As a result, Compton scattered events may be accepted and tend to be placed toward the center of the array. Nevertheless, for the crystal array used in simulations, the majority of events with multiple interactions happen in neighboring crystals where the difference in signal response for the energy deposited is not significant and the misposition of the detection will be within the crystal determined by the algorithm used in MC simulation.

V. RESULTS AND DISCUSSION

The comparison of the experiment and MC simulation results is presented in Fig. 4. In this figure, N_{EXP} is the number of detections obtained in experiment by each crystal in a 13×13 “insert” crystal array; N_S is the number of counts obtained in point source MC simulations; N_σ and is a number of counts predicted using the detection cross sections from the MC simulated look-up table. The MC simulated result for every incident angle was scaled to match the total numbers of detection (sum over all 169 crystals) in experiment and simulations. The scaling coefficient k for every incident angle θ is shown on the top of the column (incident angle θ is shown on the bottom of the column). Mean scaling coefficient was 2.56 with standard deviation 0.066. With this scaling, the corresponding source activity

used in MC simulation would be $16 \mu\text{Ci}$. This agrees well with the corresponding ^{22}Na source activity used in the experiment ($\sim 18 \mu\text{Ci}$) if we had taken into account the loss of detector materials due to reflectors between crystals in a real crystal array which were not modeled in our MC simulations.

The relative error between the experiment and MC simulation data using a point source is shown in the 4th row of Fig. 4. The results suggest that the experiment and simulation are in good agreement (within 10% for most cases). Only a few crystals have their detection efficiency estimation deviated from the measured results by $\sim 20\%$. This error is mostly caused by the contamination of the experimental data from the prompt 1275 keV gamma and by the statistical errors of both the experimentally measured data and the MC simulated data, compounded by fluctuations in properties of individual crystals and the PMT. This is evidenced by the fact that the largest errors happen at the crystals with low detection efficiency—corner crystals and crystals shadowed from radiation when the incident angle is large ($\theta = 75^\circ$ and 90°).

With the reasonable agreement between the experimental results and MC simulated point source results, we also compared the crystal detection efficiency estimated by the parallel beam approximation and that estimated from MC simulation using a point source in the last row of Fig. 4. For the majority of crystals in the array, the relative error is well under 10%. Only a few crystals have an error larger than 20% and their absolute error is below the statistical error of the point source simulation, since these crystals are shadowed from radiation by neighboring crystals and have only a few detected events.

Fig. 5 shows an example of the crystal detection cross sections simulated by MC using the parallel beam approximation for the “insert” detector with 511 keV gammas coming at the normal incident angle ($\theta = 0^\circ$). In this very simple case, the calculations of the crystal detection cross sections using the simple geometrical approach (4) gives 1.38 mm^2 for all 169 crystals in array, that is much greater than the corresponding cross sections obtained from the MC simulation, the highest of which is roughly 0.85 mm^2 . Fig. 5 also clearly demonstrates the “block effect”—the crystals on the edge of the array (and especially those in the corners) have significantly smaller detection cross sections than the crystals in the middle of the array.

Examples of the crystal detection cross section of the corner (12,12) and the central (6,6) crystals for the incident angle $\theta(\psi = 15^\circ)$ are presented in Fig. 6. The MC simulated detection cross sections of the “scanner” and “insert” crystals are shown for the two gamma energies 450 and 511 keV.

The relative error between the numbers of the detections simulated in point source MC simulation and predicted by the parallel beam approximation are jointly presented in relative error charts in Fig. 7. All simulated angle combinations are presented in a single chart. Every row in these charts corresponds to the relative error of all crystals in a crystal array for a particular angular source position (θ, ψ). The index of the x-axis corresponds to the crystal $ID = 13n+m$ for the crystal (n, m) in a 13×13 array.

In Fig. 7(a), the relative error for most of the “insert” crystals is only a few percent and the parallel beam approximation gives reasonable values for all simulated incident angles even for the smallest distance ($OS = 4$ cm). Similar to the results shown in Fig. 4 with , only a few crystals have an error larger than 20% and their absolute error is below the statistical error of the point source simulation, since these crystals are shadowed from radiation by neighboring crystals and have only a few detected events.

On the other hand, the parallel beam approximation fails to estimate the “scanner” crystal detection cross section for some crystals at the large incident angles (at $\theta = 75^\circ$). Even for the largest simulated distance between source and “scanner” crystal array (7 cm), the error for some crystals can be more than $\sim 50\%$. These large errors happen only for the “deep” crystals—crystals for which radiation is strongly attenuated by neighboring crystals [see Figs. 8(b) and 8(c)]. However, although the relative error is very high for some crystals in an array, the absolute error remains relatively small and the predicted and simulated total numbers of the detections for whole crystal array are similar. For example, for the worst case presented in Fig. 8(b) (“scanner” at $\theta = 75^\circ$), the relative error between predicted and simulated total numbers of the detections for entire array is $\sim 10\%$ and the “front” crystals (that detect the most of interactions) have a relative error of only a few percent ($\sim$ statistical error).

The large relative error of the “deep” crystals for the large incident angles could not be attributed completely to the statistical error of simulation. An explanation of this error is presented in Fig. 9. In this figure, we can see that for the crystal C, all gammas emitted from the point source S should be attenuated by the whole crystal array, while with the parallel beam approximation, there is a small fraction of gammas that would enter from the top of the crystal array without significant attenuation. This small fraction of gammas can dominate the strongly attenuated gammas. For this reason the parallel beam approximation overestimates the cross section for the “deep” crystals in a large crystal array when the source is located close to the plane passing throughout the top surface of the crystal array. Thus, this error is the largest for some critical angle θ determined by the array dimension (e. g., for this particular “scanner” crystal array, the critical angle is $\sim 75^\circ$). A potential solution to this problem is to employ θ -dependent “effective center of the crystal” instead of the crystal center as used for the current simulations. On the other hand, this correction is not critical for practical application due to the extremely low detection efficiencies of these crystals.

In our model, we computed detector's response at 2 energies: 450 keV and 511 keV, and use interpolation for all energy in between these two levels. The finite energy resolution of scintillation detectors was not modeled in the Monte Carlo study because we did not want the model to be tied to a particular type of scintillator. Nevertheless, if one would like to apply this model to a particular PET scanner, one would have to factor in the finite energy resolution of the type of scintillator used in the system.

The simulated cross section can be used for image based correction procedures. We successfully used this method for 3D SSS scatter correction in our system [9]. These cross sections also help to directly compute the singles rates that may be used for estimation of

randoms. In general, the standard randoms corrections methods (such as delayed window technique or randoms estimated from measured single rates) work well in typical PET scanners and there is no need for additional image based random correction. Nevertheless, the presented method can be used for simulation of developing new systems. It can also be used for non-conventional PET isotope imaging applications where the positron decay is accompanied by a prompt gamma, which may cause triple coincidence or coincidence events that involve prompt gamma rays.

VI. CONCLUSION

In this work, we proposed to establish a compact look-up table for crystal detection cross sections that can simplify calculation of crystal detection efficiencies used in scatter and random data correction techniques. The proposed parallel beam approximation for computing the look-up table gives a reasonable result for the crystal detection efficiencies. For relatively small crystal arrays (such as the “insert” crystal array simulated in this work), this approximation makes a correct estimation of the crystal detection efficiency at distances larger than ~ 8 times the crystal length for all incident angles. There is a some deviation between the “real” (point source MC simulation) and parallel-beam approximated crystal detection efficiencies for the large incident angle θ for “deep” crystals (crystals shadowed from radiation by neighboring crystals) in a large crystal array (such as the “scanner” crystal array used in this simulation). Nevertheless, this deviation is not critical for practical purposes because the crystals with large relative errors have extremely low detection efficiencies, and thus contribute minimally to detection of gamma rays when compared with the “front” crystals of the same array that share almost the same line-of-response.

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Fig. 1.

Simple geometrical calculation of the detection cross section of the crystal in a detector array based on the mean linear attenuation coefficient (see (4)).

Fig. 2.

MC simulation for the point source. The distance $|OS|$ between crystal array center and source was 4, 5, 6, and 7 cm. The simulations were done for θ from 0° to 90° and for ψ from 0° to 45° with 15° steps in both angles.

Fig. 3.

Difference in raw energy spectra (before the background subtraction) detected by the central (6,6) and corner (0,0) crystals from “insert” crystal array obtained in an experiment with a ^{22}Na source at distance $d = 7$ cm from the center of the crystal array and the incident angles $\psi = 0^\circ$ and $\theta = 0^\circ$. The top histogram is the detected photopeak positions (in relative units before energy calibration) for all crystals in “insert” array.

Fig. 4.

Comparison of the point source experiment and MC simulation for the “insert” crystal array at $d = 7$ cm and incident angles $\psi = 0^\circ$ and $\theta = 0^\circ, 15^\circ, 30^\circ, 45^\circ, 60^\circ, 75^\circ, 90^\circ$. N_{EXP} is a number of counts obtained in experiment. N_s is a number of counts obtained in point source MC simulation. N_σ is a number of counts predicted using the cross sections from the MC simulated look-up table. The scaling coefficient k is used to fit to each other the total numbers of detection in experiment and simulation.

Fig. 5. Detection cross sections of the crystal in 13×13 crystal array (“insert”) for the 511 keV gammas with normal incident angle ($\theta = 0^\circ$) MC simulated in parallel beam approximation.

Fig. 6. Detection cross sections of crystals (12,12) (circles) and (6,6) (squares) in “insert” and “scanner” crystal arrays for 511 (solid line) and 450 keV (dotted line) for azimuth angle $\psi = 15^\circ$.

Fig. 7.

The joint error charts for a 511 keV gamma source: (a) “insert” crystal array at $|OS| = 4$ cm; (b) “scanner” crystal array at $|OS| = 4$ cm and (c) “scanner” crystal array at $|OS| = 7$ cm. The gray-scale color table shows the relative error between the number of counts predicted from the look-up table N_σ and the number of counts obtained in point source MC simulation N_s . Crystal ID = $13n + m$ is the index of the crystal (n,m) in a 13×13 array. Each row of a chart represents the relative error between “real” (point source simulation) and “predicted” numbers of counts for the angular position (θ, ψ) of the source relative to the crystal array (see Fig. 2.). (Note that the gray-scale color tables for “scanners” (b) and (c) are saturated-errors $> 50\%$ are shown black.).

Fig. 8.

Comparison of the numbers of detections predicted using the crystal detection cross sections simulated in the parallel beam approximation N_σ (top) and actually simulated N_S (middle): (a) is for “insert” at 4 cm from the source and incident angles $(0^\circ, 0^\circ)$; (b) “scanner” at 4 cm; and (c) “scanner” at 7 cm and incident angles $(75^\circ, 30^\circ)$. Bottom plots are the relative errors between predicted and simulated numbers of the detections $Err = (1 - N_\sigma/N_S)$.

Fig. 9.
The difference between parallel beam and point source fluxes for large incident angles.