A transmission-line model of back-cavity dynamics for in-plane pressure-differential microphones

Donghwan Kim,^{a)} Michael L. Kuntzman, and Neal A. Hall

Microelectronics Research Center and Department of Electrical and Computer Engineering,
 The University of Texas at Austin, 10100 Burnet Road, Building 160, Austin, Texas 78758

(Received 19 April 2014; revised 10 July 2014; accepted 23 September 2014)

Pressure-differential microphones inspired by the hearing mechanism of a special parasitoid fly have been described previously. The designs employ a beam structure that rotates about two pivots over an enclosed back volume. The back volume is only partially enclosed due to open slits around the perimeter of the beam. The open slits enable incoming sound waves to affect the pressure profile in the microphone's back volume. The goal of this work is to study the net moment applied to pressure-differential microphones by an incoming sound wave, which in-turn requires modeling the acoustic pressure distribution within the back volume. A lumped-element distributed transmission-line model of the back volume is introduced for this purpose. It is discovered that the net applied moment follows a low-pass filter behavior such that, at frequencies below a corner frequency depending on geometrical parameters of the design, the applied moment is unaffected by the open slits. This is in contrast to the high-pass filter behavior introduced by barometric pressure vents in conventional omnidirectional microphones. The model accurately predicts observed curvature in the frequency response of a prototype pressure-differential microphone $2\text{ mm} \times 1\text{ mm} \times 0.5\text{ mm}$ in size and employing piezoelectric readout. © 2014 Acoustical Society of America.

[<http://dx.doi.org/10.1121/1.4897401>]

PACS number(s): 43.38.Ar, 43.38.Fx, 43.38.Kb, 43.64.Yp [KG]

Pages: 2544–2553

I. INTRODUCTION

It has been discovered that the parasitoid fly, *Ormia ochracea*, is capable of locating sound sources in the audible frequency range. Pioneering work by Miles *et al.* discovered that the hearing mechanism is comprised of a structure resembling that of a teeter-totter with a first vibration mode that is excited by pressure gradients, and therefore has an inherently directional response to sound pressure.¹ The directional hearing structure has been successfully mimicked using silicon micromachining and optical-based motion detection. This early research inspired several additional teams to explore similar microphone structures using capacitive and piezoelectric readout.^{2–5} Figure 1 presents the essential mechanical features of the microphone, which include a beam anchored by rotational pivots and an air volume (i.e., back cavity) behind the pivoting beam. In several physical embodiments, the beam is surface-micromachined on silicon, and the back cavity is the open region created by a deep-silicon through-wafer etch process during fabrication of the device.^{3,6} The back cavity is enclosed when the silicon die is mounted onto a printed-circuit board. To realize a beam structure that is free to pivot, slits exist around the perimeter of the beam—also labeled in Fig. 1. Sound waves arriving in-plane and along the x -axis as depicted in Fig. 1 apply a moment or torque to the beam. This work is concerned with computation of the net moment applied to the beam by an incoming acoustic wave, taking dynamics of the back cavity into consideration.

An incoming acoustic plane wave loads the top surface of the beam and may be described as $P(x, t) = P_0 e^{-jkx} e^{j\omega t}$. A Taylor-series expansion about $x = 0$ results in

$$P(x, t) \approx P|_{x=0} + \left. \frac{\partial P}{\partial x} \right|_{x=0} x = P_0 e^{j\omega t} - jkx P_0 e^{j\omega t}. \quad (1)$$

Since the microphones of interest are typically small compared to the wavelength of sound, loading of pressure-differential microphones can be thought of as superposition of two separate loads appearing in Eq. (1): A uniform or omnidirectional component and a pressure-gradient component that varies linearly along x . Both loading components are relevant in the analysis of the structure in Fig. 1. The first omnidirectional component drives a vibration mode of the beam in which the two ends vibrate in-phase (i.e., flapping mode), while the gradient load creates a net torque on the beam that drives a rocking mode in which opposing ends move out-of-phase. Both modes are depicted schematically in Fig. 1. Depending on the designer's intent, it may be desirable to suppress as much as possible the contribution of the second vibration mode in the frequency response of the microphone so that a pure gradient microphone is realized. Alternatively, mimicking the hearing mechanism of the *Ormia ochracea* requires use of both vibration modes as discovered by Miles *et al.*¹ In either case, a need exists to model the net drive amplitude applied to the beam for each external-load component in Eq. (1).

For each component, pressure loading at the top surface of the beam is completely characterized by Eq. (1). For the uniform or omnidirectional pressure load, computation of the net drive pressure may be computed using the simple lumped-element network depicted in Fig. 2. From the network,

^{a)}Author to whom correspondence should be addressed. Electronic mail: donghwan.kim@utexas.edu

FIG. 1. (Color online) 3-D rendered schematic and cutaway view with dimension variables of generic pressure-differential microphone with single slit around the perimeter.

$$\frac{P_0 - P_{cav}}{P_0} = \frac{P_{net}}{P_0} = j \frac{f/f_c}{1 + jf/f_c}, \quad f_c \equiv \frac{1}{2\pi R_{aslit} C_{acav}}, \quad (2)$$

where R_{aslit} and C_{acav} are defined in Table I along with all other variables appearing in this study, and f represents the frequency of the incident wave. The net omnidirectional drive pressure follows a high-pass filter (HPF) behavior. The analysis is identical to that commonly employed when studying the lower-limiting frequency of conventional omnidirectional microphones due to the barometric pressure vent.^{7,8} In the case of the pressure-differential microphones depicted in Fig. 1, and in contrast to omnidirectional microphones, the excessive slits along the perimeter are expected to result in a corner frequency above, rather than below, the audio frequency range.

The net torque or moment, M_{net} , applied to the beam by the gradient load is $M_s - M_{cav}$, where M_s is the moment applied by the source pressure at the top beam surface and M_{cav} is the moment applied by the distributed pressure inside

the microphone back cavity. Computation of M_s using the distributed pressure in Eq. (1) is straightforward. Computation of M_{cav} , however, requires first simulating the pressure distribution in the microphone back cavity that results from the gradient load along the top surface. Homencovschi *et al.* studied a similar problem using a wave model and taking into account reflection and diffraction of the incoming sound wave by the microphone.⁹ The approach assumed two open slits at opposing ends of the beam, but did not include the slits that run along the long axis of the beam (i.e., x -axis referring to Fig. 1). The model presented here takes into account all slits. Liu and Yu also studied the effect of the air cavity in a fly-inspired directional microphone, but for a structure using two coupled conventional parallel-plate piston transducers, which is substantially different than the structure in Fig. 1.¹⁰ In this work, we present a transmission-line (TL) model of the back cavity capable of computing M_{net} for the structure in Fig. 1, as well as for structures with a more complex distribution of etched slits (e.g., as in the case of piezoelectric embodiments presented by Kuntzman *et al.*³). An analytical formulation is first presented to establish motivation for the TL approach.

II. A 1-D WAVE EQUATION FOR THE BACK VOLUME

Figure 3 presents a control volume for a differential section of the back volume. The beam is held motionless in the analysis. $P_s(x, t)$ is a general source pressure that exists along the top surface of the beam. $P(x, t)$ is the distributed pressure within the cavity. All variables appearing in Fig. 3 are described in Table I. The small-signal (i.e., linearized) momentum equation for the differential element is the same as for plane waves in air,

$$-\frac{\partial P}{\partial x} = \rho_0 \frac{\partial v}{\partial t}. \quad (3)$$

The small-signal continuity equation for the control volume takes into account air entering the back cavity through the etched slit, which is driven by $P_s - P$ through an acoustic resistance R_a to create a source of volume flow into the differential element as noted in Fig. 3. The continuity equation is

$$\rho_0 A v_x + \frac{(P_s - P)}{R_a} \rho_0 - \rho_0 A v_{x+dx} = \frac{\partial \rho}{\partial t} A dx. \quad (4)$$

FIG. 2. A simple lumped-element network model for the omnidirectional pressure load.

TABLE I. Variable descriptions.

Variable	Description	Unit	Equation/value
ρ_0	Nominal density of air	kg/m ³	1.2
L	Length of back-cavity volume	m	2×10^{-3} ^a
W	Width of back-cavity volume	m	1×10^{-3} ^a
g	Width of slits	m	20×10^{-6} ^a , 32×10^{-6} ^b
t	Thickness of slits (i.e., thickness of beam)	m	20×10^{-6} ^a , 10×10^{-6} ^b
h_{cav}	Height of back-cavity volume	m	500×10^{-6} ^a
l	Length of TL segment	m	L/N
A	Cross-sectional area of back cavity	m ²	Wh_{cav}
R_{aslit}	Total acoustic resistance of slits around beam perimeter	N s/m ⁵	$\frac{1}{2} \left(\frac{12\mu t}{g^3 L} \right) \parallel \frac{1}{2} \left(\frac{12\mu t}{g^3 W} \right) = 9.30 \times 10^7$
C_{acav}	Acoustic compliance of total back-cavity volume	m ⁵ /N	$\frac{h_{\text{cav}} LW}{\rho_0 c^2} = 7.04 \times 10^{-15}$
v	Acoustic particle velocity	m/s	—
ρ	Small-signal acoustic density variation	kg/m ³	—
P_0	Amplitude of incident sound wave	Pa	—
$P_s(x, t)$	Small-signal source pressure on top Surface of beam	Pa	—
$P(x, t)$	Small-signal pressure inside back cavity	Pa	—
α	Piezoelectric proportionality constant	V/rad	23.7
R_a	Acoustic resistance of slit of length “dx”	N s/m ⁵	$\frac{12\mu t}{g^3 dx}$
R_{sp}	Normalized acoustic resistance	N s/m ²	$R_a A dx = \frac{12\mu A}{g^3}$
R_{aend}	Acoustic resistance of slits along y at opposing ends of beam	N s/m ⁵	$\frac{12\mu t}{g^3 W}$
M_{net}	Net moment applied to beam	N m	—
M_s	Moment applied to beam by top-surface source pressure	N m	$\int_{-L/2}^{L/2} P_s(x) x W dx$
Q_L	Volume velocity entering a TL element from left neighboring element	m ³ /s	—
Q_R	Volume velocity leaving a TL element at right	m ³ /s	—
Q_{in}	Volume velocity entering a TL element through slits in beam	m ³ /s	—
L_m	Rotational inertia of the beam	kg m ²	4.61×10^{-14}
R_m	Rotational damping coefficient of the beam	kg m ² /s	3.73×10^{-11}
C_m	Rotational compliance of the beam	N m/rad	6.56×10^3
m_{as}	Acoustic mass of two parallel slits in beam	kg/m ⁴	$\frac{1}{2} \left(\frac{\rho_0 t}{g l} \right)$
R_{as}	Acoustic resistance of two parallel slits in beam	N s/m ⁵	$\frac{1}{2} \left(\frac{12\mu t}{g^3 l} \right)$
m_{aL}	Acoustic mass of left portion of a TL element	kg/m ⁴	$\frac{1}{2} \left(\frac{\rho_0 t}{h_{\text{cav}} W} \right)$
m_{aR}	Acoustic mass of right portion of a TL element	kg/m ⁴	$\frac{1}{2} \left(\frac{\rho_0 t}{h_{\text{cav}} W} \right)$
C_a	Acoustic compliance of a TL element	m ⁵ /N	$\frac{h_{\text{cav}} l W}{\rho_0 c^2}$
R_{ac}	Acoustic resistance of slit at far end of beam	N s/m ⁵	$\frac{12\mu t}{g^3 W}$
m_{ac}	Acoustic mass of slit at far end of beam	kg/m ⁴	$\frac{\rho_0 t}{g W}$

^aFor case-study simulations of device in Fig. 1.^bFor prototype device appearing in Figs. 8 and 9.

FIG. 3. A control volume for a differential section of the back volume.

The equation may be simplified to

$$\frac{\partial v}{\partial x} + \frac{1}{\rho_0} \frac{\partial \rho}{\partial t} = \frac{1}{R_{\text{sp}}} (P_s - P), \quad (5)$$

where the following definition/substitution has been used:

$$R_{\text{sp}} = R_a A dx. \quad (6)$$

Using the small-signal equation of state $P = c_0^2 \rho$, a final form of continuity may be expressed as

$$\frac{\partial v}{\partial x} + \frac{1}{\rho_0 c_0^2} \frac{\partial P}{\partial t} = \frac{1}{R_{\text{sp}}} (P_s - P). \quad (7)$$

When Eq. (3) is differentiated with respect to “x” and Eq. (7) is differentiated with respect to “t,” the resulting pair enables substitution of mixed partials for v , resulting in the forced wave equation for the back cavity,

FIG. 4. (Color online) (a) A schematic of a small section of the back cavity and (b) the corresponding TL element.

$$\frac{\partial^2 P}{\partial x^2} - \frac{1}{c_0^2} \frac{\partial^2 P}{\partial t^2} - \frac{\rho_0}{R_{sp}} \frac{\partial P}{\partial t} = -\frac{\rho_0}{R_{sp}} \frac{\partial P_s}{\partial t}. \quad (8)$$

The solution of Eq. (8) for $P(x, t)$ is challenging due to the inclusion of boundary conditions on each opposing end of the structure, which are characterized by

$$\left. \frac{P_s - P}{R_{aend}} \right|_{x=-L/2} = Q_{LE}(t) \text{ and } \left. \frac{P_s - P}{R_{aend}} \right|_{x=+L/2} = Q_{RE}(t), \quad (9)$$

where Q_{LE} and Q_{RE} are additional sources of volume flow into the back cavity from slits that run along the y -axis at the far left and right ends of the beam, respectively. Further, many microphone structures of interest are more complex than that depicted in Fig. 1 and contain additional slits and perforations at various locations along the length of the beam. A versatile model of the back cavity based on a lumped-element TL approach is now presented. It should be noted that the intent of the forgoing analysis, and also of the TL analysis to follow, is to create a relatively simple model capable of addressing the special case of a cavity or channel actuated by a distributed source pressure, specifically the case in which the distributed pressure acts through a slit. The general formulation and solution of sound propagation through tubes and channels has been addressed with more careful consideration to non-uniform acoustic velocity profiles within the tube and viscous and thermal boundary layer effects at the tube walls.^{11–13}

III. TRANSMISSION-LINE MODEL OF BACK VOLUME

A schematic of a small section of the back cavity with length “ l ” and its corresponding network model are presented in Fig. 4. Table I summarizes the description of each network element and also includes equations used in this work to compute numerical values for the elements. A complete model of the back cavity may be assembled using many elements connected end-to-end, owing to continuity of pressure and acoustic volume velocity at boundaries of contiguous elements. A complete model for the system in Fig. 1 has been assembled using ten elements ($N = 10$) configured in the circuit simulation software LTSPICE.¹⁴ The dimensions used in the case-study simulation are summarized in Table I. Figure 5 presents half of the model in LTSPICE, with elements

computed using expressions provided in Table I. Each segment, l , is $L/10 = 200 \mu\text{m}$ since ten elements are used in this particular case. With the TL approach in Fig. 5, boundary conditions are easily incorporated as shown, with additional sources of volume flow into the cavity through the slit impedances at opposing ends, represented by R_{ae} and m_{ae} .

A. Uniform-loading case

Any source pressure $P_s(x)$ may be used in the simulation. To test the network simulation for accuracy against a known analytical solution, an initial simulation was performed for the uniform-loading case, in which all P_s values were set equal to 0.5 Pa in the network simulation. The amplitude of the transfer function relating the net drive force on the beam to the applied source force on the top side, F_{net}/F_s , is computed and plotted in Fig. 6(a). The simple analytical solution following Eq. (2) is also included for comparison, with expressions and numerical values for R_{aslit} and C_{acav} included in Table I. The solutions match closely and demonstrate the anticipated HPF behavior, suggesting correctness of the TL model in LTSPICE. It is interesting to note that the TL model captures additional accuracy at high frequencies, showing an amplified net force in the vicinity of 150 kHz. A spatial plot of the pressure distribution at 100 kHz, presented in Fig. 6(b), shows that this is due to a cavity resonance in which the pressure inside the cavity becomes greater than the source pressure, P_s .

B. Net moment from gradient loading

The gradient-loading case was then studied using the TL model. The load consists of a linearly-ramped external source pressure, as depicted by the P_s values presented in Fig. 5. After running the simulation, the node pressures inside the microphone back cavity are extracted. Figure 7(a) plots the spatial distribution of applied source pressure and internal pressure at the particular frequency of 15 kHz. The simulation was run for frequencies in the range of 100 Hz–1 MHz. At each simulated frequency, a source moment, M_s , is computed from P_s values, and an internal moment, M_{int} , is computed from the simulated back-volume pressure values. The net applied moment to the beam is $M_{net} = M_s - M_{int}$. Figure 7(b) presents the transfer function (TF) for M_{net}/M_s . The TF is markedly different from the

Symm.

FIG. 5. (Color online) Schematic of half of the model in LTSPICE (right-hand side with respect to the rotational pivot).

uniform pressure case, showing a low-pass filter (LPF) behavior with a corner frequency at approximately 10 kHz for the particular device parameters summarized in Table I. While the large slits along the perimeter of the beam tend to eliminate the net drive force applied by the uniform-pressure component of incoming sound waves in the audio frequency range, the same slits do not reduce the moment applied by the gradient-load component of incoming sound waves throughout the majority of the audio frequency range.

IV. MODEL VERIFICATION

The model is shown to accurately predict the on-axis frequency response of a prototype microphone. A micrograph of the prototype taken from the top-side view is presented in Fig. 8. The beam structure is anchored by two torsional pivots at the center as labeled. Four end springs anchor near the pivot and wrap around the perimeter of the beam to connect at the beam's ends. Each spring is comprised of a thin piezoelectric film (1- μm thick) on top of a much thicker silicon layer (10- μm thick). Angular rotation of the beam about its pivot creates bending stress in the top layer of a spring, in-turn creating an electric potential across the thickness of the film which operates in the 3-1 mode of

piezoelectric transduction. Small wirebond connections for reading the signals are also visible in Fig. 8. Details of the microfabrication process as well as the acoustic characterization of prototypes including directivity measurements have been the subject of previous investigations.^{3,15} The beam and springs are etched into a 10- μm -thick epitaxial silicon device layer for the particular prototype presented in Fig. 8, and the etched slits are approximately 32- μm wide. The relatively complex distribution of etched slits, as compared to the simple case in Fig. 1, necessitates division of the TL model into different regions along the x -axis. Figure 9(a) provides a sketch of the beam's top surface with cavity-pressure nodes numbered. The TL model in LTSPICE for the right half of the back cavity is presented in Fig. 9(b), with pressure nodes also numbered to communicate mapping of the network model to regions of the physical structure. Figure 10(a) plots the spatial variation of the back-volume pressure amplitude at the particular frequency of 1 kHz, and Fig. 10(b) plots the magnitude and phase of M_{net}/M_s . The LPF effect is observed for the structure, but in this case the corner frequency is approximately 2 kHz owing to the additional slits and wider slit gap as compared to the simple case-study device depicted in Fig. 1.

(a)

(b)

FIG. 6. (Color online) Plots of (a) TF of the net drive force on the beam to the applied source force on the top side (F_{net}/F_s) and (b) the spatial distribution at 100 kHz.

FIG. 7. (Color online) Plots of (a) the spatial distribution of applied source pressure and internal pressure at 15 kHz and (b) TF of net moment, M_{net} , to source moment, M_s .

The frequency response of the prototype in Fig. 8, defined as the angular displacement of the beam about its pivots per 1-Pa on-axis incident sound pressure, may be modeled using the network presented in Fig. 11. The network uses an impedance analog with moment, M , the effort variable, and angular velocity of the rotating beam, $\dot{\theta}$, the flow variable. Impedances in the network are therefore defined as $Z = M/\dot{\theta}$. The mechanical beam itself is characterized by the L_m , R_m ,

FIG. 8. (Color online) Micrograph of the prototype used for verification of the TL model of the back-cavity dynamics.

FIG. 9. (Color online) (a) A sketch of the prototype's top surface and (b) the corresponding TL model of the right half of the structure in LTSPICE.

and C_m series combination: rotational inertia, resistance, and compliance respectively, with subscript “m” emphasizing that these are mechanical impedances associated with the beam. The physical system and the network may be organized into two subsystems: (i) the mechanical beam and (ii) the remaining acoustical system comprising the air in the back cavity and the source pressure distribution acting on the top surface

FIG. 10. (Color online) Plots of (a) the spatial variation of the back-volume pressure at 1 kHz and (b) the amplitude and phase of the net moment applied to the beam vs frequency.

FIG. 11. (Color online) Network model of the prototype.

of the beam. The acoustical system may be represented by its Thévenin equivalent consisting of the net applied moment, M_{th} , computed with the beam constrained from rotating, and the source impedance, Z_{th} . The source impedance is the rotational impedance seen by the rotating beam in the absence of an acoustic wave passing across the device (i.e., with the source pressures turned off). Replacement of a subsystem by its Thévenin equivalent is common in the study of transducers.^{16–19} Each subsystem is labeled in Fig. 11. M_{th} , the net moment applied to the beam with the beam motion constrained, is provided as

$$M_{th} = M_s \left(\frac{M_{net}}{M_s} \right). \quad (10)$$

The expression for M_s is included in Table I. Its derivation is straightforward considering the gradient portion of the pressure load summarized in Eq. (1).³ Computation of the M_{net}/M_s transfer function is the focus of this study and is summarized in Fig. 10(b). It is shown in a subsequent section that the Thévenin source impedance, Z_{th} , is small compared to the beam impedance, and it is therefore neglected in the analysis. The resonance frequency, ω_n , and Q of the beam structure have been measured in previous work as 9.15 kHz and 70, respectively.¹⁵ C_m and R_m are extracted using these measured quantities in combination with the computed rotational inertia of the beam, L_m [i.e., C_m and R_m may be considered fitted to previously measured data according to $C_m = 1/(L_m \omega_n^2)$, and $R_m = \omega_n L_m / Q$]. Numerical values for all three system parameters are summarized in Table I. To complete the description of the model, it is noted that a proportionality constant exists, α , between measured open-circuit voltage from a piezoelectric sensing port, V_{oc} , and the angle of beam rotation, θ . This constant has been studied in prior investigations.^{20,21} Its numerical value is fitted in this particular case and is included in Table I for completeness but is inconsequential for the present purpose, which is to observe the influence of the M_{net}/M_s TF on shaping the measured response data. A goal, in particular, is to observe evidence of the LPF behavior predicted in Fig. 10(b).

From the model in Fig. 11,

$$\dot{\theta} = \frac{M_s \left(\frac{M_{net}}{M_s} \right)}{Z_{beam}}, \quad Z_{beam} = j\omega L_m + \frac{1}{j\omega C_m} + R_m, \quad (11)$$

$$\theta = \frac{\dot{\theta}}{j\omega}, \quad V_{oc} = \alpha \theta. \quad (12)$$

Figure 12 presents three on-axis acoustic frequency response traces, summarized as follows.

- (i) As measured in a 10 ft × 10 ft × 10 ft walk-in anechoic chamber using swept continuous-wave signals. At each frequency, the open-circuit voltage amplitude of the prototype device is captured and normalized to the known source pressure which is measured separately using a broadband free-field microphone (G.R.A.S. Type 40AC). This sweep measurement sequence is automated using a Prism dScope Series III fast-Fourier-transform audio analyzer.
- (ii) As simulated following Eq. (12), but with M_{net}/M_s set to unity and thereby ignoring the effect of the back-cavity dynamics on the applied net moment. This trace is labeled as “Sim. w.o. Back-cavity effect” in Fig. 12.
- (iii) As simulated following Eqs. (11) and (12), and using the M_{net}/M_s TF simulated in this work. This trace is labeled as “Sim. w. Back-cavity effect” in Fig. 12.

Trace (iii) correctly predicts the bend in the measured frequency response beginning near 1 kHz and also shows close matching with measurements at and beyond the first resonance peak. For a simple pressure-differential sensor, one expects a 20-dB/decade slope up to the first resonance of the device as captured by the simpler model in Fig. 12 [i.e., trace (ii)], leaving the observed bend in the measured response beginning at 1 kHz unexplained. The trace incorporating the M_{net}/M_s TF studied in this work [i.e., trace (iii)] correctly predicts the curvature in the response. The departure from the 20-dB/decade behavior beyond 1 kHz is attributed to a reduction in driving moment introduced by the back-cavity dynamics. From Fig. 12, at 7 kHz and beyond, trace (ii) and trace (iii) simulations are more than 10-dB

FIG. 12. (Color online) Measured and simulated acoustic frequency response functions.

apart—quantifying the importance of the M_{net}/M_s model for this particular prototype device.

V. DISCUSSION

A. Insights revealed by model

In light of the well-known HPF nature of the omnidirectional-load component for vented microphones, the LPF behavior of M_{net}/M_s is potentially counterintuitive. Figure 13 duplicates $|M_{\text{net}}/M_s|$ from Fig. 10(b) and attempts to explain various regions of the TF. At low frequencies, the P_s gradient drives air from the top surface of the beam into the back volume through the slits, and this air then moves between right and left halves of the back volume to equalize pressure. An approximately uniform and approximately zero net cavity-pressure profile results in the back volume, and M_{int} is approximately zero in this frequency range so that the net moment is M_s applied at the top side of the beam. Beyond the cut-off of 2 kHz, air enters the back cavity through the slits but ceases to move between left and right regions of the cavity, thereby creating a pressure profile within the cavity that closely matches P_s , in-turn resulting in a substantially reduced net moment applied to the beam. To support this explanation, the normalized acoustic volume velocity at the center of the back cavity (i.e., at $x = 0$) is also plotted in Fig. 13, denoted Q_0 . The magnitude of Q_0 indicates the amount of air moving between left and right halves of the cavity. For clarity, Q_0 is also labeled on the network in Fig. 9(b). As expected, Q_0 closely follows M_{net}/M_s in the first two frequency regions highlighted in Fig. 13, indicating that the cessation of movement of air within the cavity is responsible for the reduction of M_{net} . The computations presented cannot be expected to yield accurate results at frequencies beyond 40 kHz, and for this reason the plot in Fig. 13 is shaded at frequencies above 40 kHz. In all simulations presented, the distribution of pressure along the top surface of the beam is based on the linear Taylor-series expansion in Eq. (1). The wavelength of sound in air at 40 kHz is 8.6 mm, and the device presented in this

work thus approaches $\lambda/4$ in size at 40 kHz, rendering the simple Taylor approximation inaccurate. Nevertheless, inclusion of high-frequency computations in Fig. 13 highlights an interesting trend that would be expected to be present in corrected, higher-frequency versions of the model. The return of $|M_{\text{net}}/M_s|$ to unity and beyond in the third frequency region labeled in Fig. 13 is due to local Helmholtz resonances between slit mass and cavity compliance. Referring to Fig. 9(b), each segment of the TL experiences its own local Helmholtz resonance. It can be verified, using values of slit mass and cavity compliance presented in Fig. 9(b), that these local resonances occur approximately at 150 kHz, which is the peak-frequency value in Fig. 13. The inset in Fig. 13 in this region shows that the cavity pressure is amplified well beyond the source pressure due to the local resonances. As a final supporting note, the phase of M_{net}/M_s , presented in Fig. 10(b), tends to -180° in this frequency region, as is expected for a structure driven at resonance.

B. Additional modeling considerations

1. Applied moment source impedance, Z_{th}

The source impedance, Z_{th} in Fig. 11, has been simulated using the TL depicted in Fig. 9(b) with two key modifications. The source pressures are set equal to zero, and the motion of the beam segments are modeled using sources of volume flow. A representative segment of the model is depicted in Fig. 14. Each flow source is assigned a volume velocity equal to the area of the beam section comprising the TL segment multiplied by the velocity of the beam segment. The velocity of the beam is the product of $\dot{\theta}$ (set equal to 1 rad/s) and the distance of the segment from the rotational axis. Upon solution of the network, the internal-cavity node pressures are used to compute the moment on the beam resulting from the applied $\dot{\theta} = 1$ rad/s, which is by definition Z_{th} . Figure 15 presents the amplitude and phase of Z_{th} for the prototype device in Fig. 8. The impedance of the beam, defined in Eq. (11), is also plotted in Fig. 15. The beam

FIG. 13. (Color online) Details of the $|M_{\text{net}}/M_s|$ behavior over various frequency ranges: A magnified view of the amplitude plot from Fig. 10(b) plotted along with the normalized volume velocity, Q_0 .

FIG. 14. (Color online) Network model used for computation of Z_{th} .

impedance is significantly larger than Z_{th} everywhere except in the immediate vicinity of the beam's mechanical resonance frequency, where Z_{th} influences the resonance Q . Z_{th} is mass-like in the frequency range 10 Hz–1 kHz, as can be noted by the 180° phase of Z_{th} in Fig. 15. This is intuitive and consistent with the insights elaborated on above—i.e., in this frequency range, motion of the beam shifts the air back and forth between left and right halves of the cavity.

2. Inclusion of cavity resistance

TL segments may be modified to include the acoustic resistance of air segments vibrating laterally within the cavity. Such resistances appear in series with m_{aL} and m_{aR} , respectively, in Figs. 4(b) and Fig. 14. Numerical values for the resistances may be computed assuming no-slip conditions as $1/2(12\mu l/(h_{cav}^3 W))$.¹² Figure 15 includes Z_{th} simulations with TL resistances included. For the particular device dimensions under study, a significant difference is only observed in the frequency range of 10–100 Hz, and no significant difference is observed on the simulated frequency response of the device. The M_{net}/M_s TF was also computed with acoustic resistances included and, at any frequency, the effect was less than 3% difference from the results presented in Fig. 10(b). In future embodiments employing cavities with smaller cross sections

and/or thinner depth, as might be realized with surface-micromachined design variants, resistance is expected to play a more dominant role. For such designs, TL segments can be crafted to take advantage of prior studies where careful attention has been given to physics at the boundary layer.^{11,13,22}

3. Radiation impedance

Computation of Z_{th} has neglected acoustic radiation impedance on the top surface of the beam. This is justified since radiation impedance is small in comparison to the beam impedance presented in Fig. 15. While analytical solutions have not been derived for the radiation impedance on a rotating structure, an over-estimated value may be computed assuming piston radiation from a single side of the rocker. Such an approach ignores the cancellation of pressure that would actually exist due to the opposing side of the rocker vibrating out-of-phase. Beranek summarizes the case of an un baffled piston radiating into free space.¹² At 20 kHz, for example, half of the beam would have a “ ka ” value equal to 0.23. In this regime the reactive component of the specific acoustic impedance dominates over the real component, and has a magnitude equal to $0.15\rho c_0$.¹² This figure is converted to a rotational impedance via scaling by the area of the right half of the beam, the moment arm, $L/4$, and by $L/4$ once more to refer the piston velocity to angular velocity of the beam. The value is 9.3×10^{-12} Nms/rad. Referring to Fig. 15, this is significantly less than the beam impedance at 20 kHz. This estimation approach, while anticipated to grossly over-estimate the actual radiation impedance, is effective for establishing an upper limit and justifying the exclusion of radiation impedance for the computations presented. Radiation impedance is expected to become more important for thinner, more compliant beams and/or for ultrasonic frequencies.

4. Miscellaneous considerations

The Taylor series approximation discussed above is one of several approximations limiting the valid frequency range of the model. The model is inherently 1-D, neglecting pressure profiles that might exist within the cavity along the y -axis or z -axis. In particular, the first cavity resonance along y occurs when $\lambda/2$ coincides with the cavity width, or 96 kHz for the device studied. Further, each TL segment

FIG. 15. (Color online) Plot of amplitude and phase responses of Z_{beam} and Z_{th} with/without including TL resistances.

employs lumped parameters for mass and resistance, so a TL segment itself should be kept short compared to wavelength for lumped approximations to accurately hold (e.g., smaller than $\lambda/10$). For the results presented, each segment is approximately $300\text{ }\mu\text{m}$, so the number of elements would need to be increased at frequencies above 100 kHz. Finally, nonlinearities are not taken into account that might arise from large beam rotations, e.g., the dependence of acoustic slit resistance on the instantaneous rotation of the beam.

VI. CONCLUSION

A TL model for back-cavity dynamics in pressure-differential microphones has been presented. The model is adaptable to structures with multiple slits and perforations as in the case of the piezoelectric prototype used for model verification. For omnidirectional (i.e., uniform) pressure loading, the model predicts a net drive pressure that behaves like a HPF as expected. It has been discovered that the net moment applied from gradient acoustic loads, however, follows a LPF behavior. In some design scenarios, it may be desirable to realize a pressure-differential microphone that is selective to only gradients and is as irresponsive as possible to omnidirectional sound pressure. For these cases, the presented research offers an important design insight: Wider-etched slits reduce the omnidirectional driving force while leaving the net gradient excitation force unaffected below a certain cut-off frequency, which may be computed using the presented model. In other design scenarios, a balance of uniform and gradient response is desirable. Early work by Miles and coworkers discovered that the hearing mechanism employed by the *Ormia ochracea* detects both components simultaneously. Here again, the relative contribution of the omnidirectional response may be tuned in the design stage by the etched slit width without affecting the first mode up to a particular corner frequency.

ACKNOWLEDGMENTS

This work was supported by Award No. 1R43DC013746-01 from the National Institute on Deafness and Other Communication Disorders. The content is solely the responsibility of the authors and does not necessarily represent the official views of the National Institute on Deafness and Other Communication Disorders or the National Institutes of Health.

¹R. N. Miles, D. Robert, and R. R. Hoy, "Mechanically coupled ears for directional hearing in the parasitoid fly *Ormia ochracea*," *J. Acoust. Soc. Am.* **98**, 3059–3070 (1995).

²M. Touse, J. Sinibaldi, K. Simsek, J. Catterlin, S. Harrison, and G. Karunasiri, "Fabrication of a microelectromechanical directional sound sensor with electronic readout using comb fingers," *Appl. Phys. Lett.* **96**, 173701 (2010).

- ³M. L. Kuntzman, J. G. Lee, N. N. Hewa-Kasakarage, D. Kim, and N. A. Hall, "Micromachined piezoelectric microphones with in-plane directivity," *Appl. Phys. Lett.* **102**, 054109 (2013).
- ⁴H. J. Liu, M. Yu, and X. M. Zhang, "Biomimetic optical directional microphone with structurally coupled diaphragms," *Appl. Phys. Lett.* **93**, 243902 (2008).
- ⁵S. Ando, T. Kurihara, K. Watanabe, Y. Yamanishi, and T. Ooasa, "Novel theoretical design and fabrication test of biomimicry directional microphone," in *The 15th International Conference on Solid-State Sensors, Actuators and Microsystems, TRANSDUCERS 2009*, Denver, CO (2009), pp. 1932–1935.
- ⁶R. N. Miles, Q. Su, W. Cui, M. Shetye, F. L. Degertekin, B. Bicen, C. Garcia, S. Jones, and N. Hall, "A low-noise differential microphone inspired by the ears of the parasitoid fly *Ormia ochracea*," *J. Acoust. Soc. Am.* **125**, 2013–2026 (2009).
- ⁷M. W. Alfons Dehé, Marc Földner, and Ulrich Krumbein, "The Infineon Silicon MEMS Microphone," in *AMA Conferences 2013* (AMA Service GmbH, Nuremberg, Germany), pp. 95–99 (2013).
- ⁸S. C. Thompson, J. L. LoPresti, E. M. Ring, H. G. Nepomuceno, J. J. Beard, W. J. Ballad, and E. V. Carlson, "Noise in miniature microphones," *J. Acoust. Soc. Am.* **111**, 861–866 (2002).
- ⁹D. Homencovschi, M. J. Aubrey, and R. N. Miles, "A two-dimensional model of a directional microphone: Calculation of the normal force and moment on the diaphragm," *J. Acoust. Soc. Am.* **119**, 756–768 (2006).
- ¹⁰H. Liu and M. Yu, "Effects of air cavity on fly-ear inspired directional microphones: a numerical study," in *Sensors and Smart Structures Technologies for Civil, Mechanical, and Aerospace Systems 2011*, San Diego, CA (2011), pp. 79811V1–79811V9.
- ¹¹M. R. Stinson, "The propagation of plane sound waves in narrow and wide circular tubes, and generalization to uniform tubes of arbitrary cross-sectional shape," *J. Acoust. Soc. Am.* **89**, 550–558 (1991).
- ¹²L. L. Beranek and T. Mellow, *Acoustics: Sound Fields and Transducers* (Academic Press, Saint Louis, MO, 2012).
- ¹³S. C. Thompson, T. B. Gabrielson, and D. M. Warren, "Analog model for thermoviscous propagation in a cylindrical tube," *J. Acoust. Soc. Am.* **135**, 585–590 (2014).
- ¹⁴LTspice IV, Linear Technology Corporation, www.linear.com (Last viewed 4/21/2014).
- ¹⁵M. L. Kuntzman, D. Kim, N. N. Hewa-Kasakarage, K. D. Kirk, and N. A. Hall, "Network modeling of multiple-port, multiple-vibration-mode transducers and resonators," *Sensor. Actuat. A: Phys.* **201**, 93–100 (2013).
- ¹⁶D. H. Keefe, R. Ling, and J. C. Bulen, "Method to measure acoustic impedance and reflection coefficient," *J. Acoust. Soc. Am.* **91**, 470–485 (1992).
- ¹⁷S. E. Voss, J. J. Rosowski, C. A. Shera, and W. T. Peake, "Acoustic mechanisms that determine the ear-canal sound pressures generated by ear-phones," *J. Acoust. Soc. Am.* **107**, 1548–1565 (2000).
- ¹⁸J. D. Lewis, R. W. McCreery, S. T. Neely, and P. G. Stelmachowicz, "Comparison of *in-situ* calibration methods for quantifying input to the middle ear," *J. Acoust. Soc. Am.* **126**, 3114–3124 (2009).
- ¹⁹K. Fukudome and M. Yamada, "Influence of the shape and size of a dummyhead upon Thevenin acoustic impedance and Thevenin pressure," *J. Acoust. Soc. Jpn. (E)* **10**, 11–22 (1989).
- ²⁰N. N. Hewa-Kasakarage, D. Kim, M. L. Kuntzman, and N. A. Hall, "Micromachined piezoelectric accelerometers via epitaxial silicon cantilevers and bulk silicon proof masses," *J. Microelectromech. Syst.* **22**, 1438–1446 (2013).
- ²¹D. Kim, N. N. Hewa-Kasakarage, S. H. Yoon, K. D. Kirk, M. Kuntzman, and N. A. Hall, "Electrical Admittance Spectroscopy for Piezoelectric MEMS," *J. Microelectromech. Syst.* **22**, 295–302 (2013).
- ²²S. C. Thompson and J. L. LoPresti, "Thermal boundary layer effects on the acoustical impedance of enclosures and consequences for acoustical sensing devices," *J. Acoust. Soc. Am.* **123**, 1364–1370 (2008).