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ROTATIONAL GRADIENT FIELD FOR INTERPOLATION OF FIBER ORIENTATION DISTRIBUTION IN CONNECTIVITY ANALYSIS

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Abstract

Advances in diffusion weighted MR imaging have made it possible to non-invasively study the structural connectivity of human brains at high resolution. To model crossing fibers in white matter, a popular choice is the reconstruction of fiber orientation distributions (FODs) from diffusion data. For this sophisticated image representation of brain connectivity, classical image operations such as differentiation and interpolation must be redefined. In this paper, we introduce rotational gradient fields (RGF) as the spatial differential of FODs' orientations. By taking into account the rotational effect of traveling in the RGF, an FOD at one location can be transported and aligned with the FOD at the target location. We propose a method for inducing RGFs with FOD metrics. We also show how RGFs can be used for interpolation, yielding intuitively more reasonable results than the Frechet mean on the unit sphere in a Hilbert space [1].

1. INTRODUCTION

With the cutting-edge diffusion weighted MR imaging techniques from the Human Connectome Project [2], brain anatomical connectivity now can be scanned in vivo at unprecedented spatial and angular resolution. This exciting advancement urges the development of novel methods to accurately extract information on neural fibers.

Reconstructed from diffusion weighted images, fiber orientation distribution (FOD) images are widely used to represent the spatial distribution and orientation of fibers. At each voxel location p , the likelihood of neural fibers along a direction u is represented by a real-valued and nonnegative spherical function $f(u)$. (Here we do not require $f(u)$ to be a probability density function, because in some areas, for example, the cerebrospinal fluid, there should not be any fibers.)

As the mathematical property of FODs is more complicated than that of intensity values or diffusion tensors, their fundamental operations such as differentiation and smoothing cannot be conducted in the same way as those for intensity or tensor images. For example, linear interpolation does not yield satisfactory results, as shown in Section 4.3. In [1, 3, 4], $\sqrt{f}$ is modeled as a point on the unit sphere in a Hilbert space, a manifold whose distance, velocity

map and exponential map are well studied. Later in [5, 6], FODs are separated as two parts (o, s) where o is its orientation and s is its shape. FODs' comparison, interpolation and smoothing are conducted by matching the two parts with rotation.

In this paper, we introduce rotational gradient fields (RGF) for FOD images. The gradient field of an intensity image is the spatial differential of intensity values. Similarly, the RGF of an FOD image is the spatial differential of FODs' orientations. By taking the rotational effect accumulated of traveling in the RGF, an FOD at one location can be transported and aligned with that at another location in its neighborhood. Contrast to the work in [5, 6], we focus on the infinitesimal difference of FODs in a continuous space, instead of the weighted average of FODs. We propose a method for inducing the RGF with FOD metrics. We also show how RGF can be used for interpolation. In our experiment, the method achieved more reasonable interpolation than that using the spherical weighted mean of $\sqrt{f}$ [1].

2. ROTATIONAL ALIGNMENT OF FODS

2.1. Rotation and Angular Velocity

Any rotation in $\mathcal{R}^3$ can be defined with a pivot axis $\vec{a}$ ($\|\vec{a}\|=1$) and a rotation angle $\theta \in [0, \pi]$ according to the right-hand rule. If the pivot axis $\vec{a}$ and the angle θ of a rotation are combined into a rotation vector $\vec{r} = [r_x, r_y, r_z] = \vec{a}\theta$, then the rotation matrix R and the rotation vector satisfy Eq. (1)

$$R = e^{[\vec{r}]_{\times}}, \text{ where } [\vec{r}]_{\times} \equiv \begin{bmatrix} 0 & -r_z & r_y \\ r_z & 0 & -r_x \\ -r_y & r_x & 0 \end{bmatrix}, \quad (1)$$

where $[\cdot]_{\times}$ is the cross-product matrix of a vector. Eq. (1) defines the exponential and logarithm map between the rotation group $SO(3)$ and its Lie algebra $so(3)$.

An angular velocity vector $\vec{\omega}$ describes the pivot axis and spinning speed of a rotating object with its direction and amplitude respectively. Angular velocity and its accumulated rotation effect R (parameterized as a rotation matrix) at time t follows the relationship defined in Eq. (2)

$$\frac{dR}{dt} = [\vec{\omega}]_{\times} R. \quad (2)$$

2.2. Rotational Alignment of FODs

As proposed in [5, 6], given a difference metric J between two spherical functions f_1 and f_2 , we can rotationally align them by minimizing $J(f_1, f_2 \circ R)$ regarding a rotation R , where $\circ$ is the composition operator and $f_2 \circ R(u) = f_2(Ru)$ is f_2 inversely rotated by R .

The sum of squared errors $J_{SSE} = \int_{S^2} [f_1(u) - f_2(u)]^2 du$ is a classical choice of the metric, where S^2 denotes the unit sphere in $\mathcal{R}^3$. For spherical probabilistic functions

$\{f: \int_{S^2} f(u) du = 1; f(u) \geq 0\}$, the Fisher-Rao metric

$$J_{FR} = \arccos \int_{S^2} \sqrt{f_1(u) f_2(u)} du \quad (3)$$

proposed in [4, 1] is a popular metric. It regards $\sqrt{f_1}$ and $\sqrt{f_2}$ as points on the unit sphere in a Hilbert space, and measures their difference with the length of great arc connecting them.

3. ROTATIONAL GRADIENT FIELD OF FOD IMAGE

A 3-dimension FOD image is a function mapping from spatial domain $\mathcal{R}^3$ to the set of nonnegative spherical functions. It is reasonable to assume that the FODs change smoothly across the image and those in a neighborhood can be aligned by rotation. We can assume that each FOD is associated with an element in $SO(3)$ defining its orientation. In this way, an FOD image is associated with an orientation image, mapping from $\mathcal{R}^3$ to $SO(3)$. Here we propose a method to estimate the differential of this underlying orientation image for an FOD image.

3.1. Induced Rotational Gradient Field

Given a difference metric J between FODs, a rotational gradient field g can be induced as follows.

1. For any point $p \in \mathcal{R}^3$, a local helper function h_p can be defined as

$$h_p(q) \equiv \arg \min_{\vec{r}} J \left(f_p, f_q \circ e^{[\vec{r}]_{\times}} \right) \quad (4)$$

where q is a point near p , and f_p and f_q are the FODs at p and q respectively.

Remarks on h_p :

- It is the rotation parameterized in $so(3)$ to align f_p with f_q .
 - It equals $[0, 0, 0]^T$ at $p = q$.
 - It maps from a neighborhood around p in $\mathcal{R}^3$ to a neighborhood around $[0, 0, 0]^T$ in $so(3)$.
 - It is the logarithm map of o_q around o_p in $SO(3)$, where o_p and o_q are the underlying orientations associated with f_p and f_q respectively.
2. The rotational gradient at point p can be defined as the gradient of $h_p(q)$ around $q = p$:

$$g_p \equiv \nabla h_p(q) \big|_{q=p}.$$

Remarks on g_p :

- Because $h_p(q)$ logarithmically maps o_q to an element in the tangent space at o_p in $SO(3)$, g_p as its derivative around p linearly maps a velocity vector at p in $\mathcal{R}^3$ to a velocity vector at o_p in $SO(3)$.
- Its value is a 3×3 matrix.

- The angular velocity which g_p exerts on an object traveling through p at spatial velocity $\vec{v}$ is $\vec{\omega} = g_p \vec{v}$.

3.2. Estimation of RGF

Section (3.1) introduces the RGF for spatially continuous images. Given a discretized FOD image with voxel spacing $[x, y, z]^T$, its rotational gradient at voxel location p can be estimated with the finite-difference method, as shown in Eq. (5).

$$g_p = \begin{bmatrix} \frac{h_p(p+e_x\Delta_x)}{\Delta_x} & \frac{h_p(p+e_y\Delta_y)}{\Delta_y} & \frac{h_p(p+e_z\Delta_z)}{\Delta_z} \end{bmatrix} \quad (5)$$

where h_p is defined in Eq. (4) and e_x, e_y and e_z are unit vectors along the x, y , and z axes respectively.

3.3. Rotational Transportation of FODs

As the RGF exerts rotational force on objects traveling in it, it can be used to transport an FOD at one location to another location with appropriate rotation, for the purpose of comparison, interpolation, and denoising, etc.

Given a path $p(t)$ in $\mathcal{R}^3$ where $t \in [0, 1]$, the rotational effect R accumulated by traveling through the RGF g follows Eq. (6)

$$\begin{cases} \frac{dR}{dt} = \left[g_p \frac{dp}{dt} \right]_{\times} R \\ R|_{t=0} = I \end{cases}, \quad (6)$$

where $\left[g_p \frac{dp}{dt} \right]_{\times}$ is the angular velocity exerts on objects traveling through point p at spatial velocity dp/dt . To transport the FOD f_{p0} at point p_0 to another point p_1 , we can accumulate the rotational effect $R_{p0 \rightarrow p1}$ of traveling along path $p(t) = p_0(1-t) + p_1t$, and apply the rotation to f_{p0} as $f_{p0 \rightarrow p1} = f_{p0} \circ R_{p0 \rightarrow p1}^{-1}$.

3.4. Application to FOD Interpolation

Let $c_1, \dots, c_{2^d}$ denote the corners of a d -dimensional cube, and $f_1, \dots, f_{2^d}$ the FOD values respectively associated with them. The FOD value at a point p in the cube can be interpolated with Eq. (7)

$$f_p = \sum_{i=1}^{2^d} w_i \left(f_i \circ R_{c_i \rightarrow p}^{-1} \right) \quad (7)$$

where w_i is the weight by which corner c_i contributes to p , and the rotational effect $R_{c_i \rightarrow p}$ of traveling from c_i to p is defined in Eq. (6).

In [1], the interpolation of FODs is defined as the Frechet mean regarding the Fisher-Rao difference metric J_{FR} . It is formulated as

$$f_p = \arg \min_f \sum_{i=1}^{2^d} w_i J_{FR}(f, f_i). \quad (8)$$

where J_{FR} is defined in Eq. (3).

Eq. (7) can also be extended with the Frechet mean as

$$f_p = \arg \min_f \sum_{i=1}^n w_i J_{FR}(f, f_i \circ R_{c_i \rightarrow p}^{-1}). \quad (9)$$

4. EXPERIMENTS

4.1. Data

We applied our method to the diffusion data of a subject from the Human Connectome Project, acquired with a multi-shell sampling scheme. The FOD image, represented with the 8-th order spherical harmonics, was reconstructed with the method in [7]. Figure 1 shows the image. We sampled 2-by-2 axial patches from this image to test RGF-based FOD transportation and interpolation.

4.2. FOD Transportation with Rotation

Figure 2 shows the result of transporting the four corners of a 2-by-2 patch to points evenly distributed in the patch. Each sub-figure shows the transportation of an FOD at a corner, with the original FOD highlighted with a red circle. The transportation generates a smooth rotational transition for every corner FOD. The FODs transported from the four corners align at every point.

4.3. Interpolation

In this experiment, we compare three different interpolation methods: the linear interpolation, the Frechet mean with the Fisher-Rao metric, defined in Eq. (8) and proposed in [1], and the RGF-based interpolation, defined in Eq. (7). The generalized fractional anisotropy (gFA) defined in Eq. (10) is used to evaluate the sharpness of the interpolated FODs. As the gFA is an index of fiber integrity, we expect the gFA values of the interpolated FODs are similar to those of the corner FODs.

$$gFA(f) = \frac{J_{FR}(f, f_0)}{\pi/2} \text{ where } f_0(u) = \frac{1}{4\pi} \int_{S^2} f(u) dv. \quad (10)$$

Figures (3) and (4) show examples of 1D interpolation and 2D interpolation respectively. The results based on the RGF are visually more reasonable, as the interpolated FODs show that fibers gradually turn at the sub-voxel level. Consequently, its gFA values are higher than those of the linear interpolation and the Frechet mean with the Fisher-Rao metric.

5. CONCLUSIONS AND DISCUSSIONS

The rotational gradient field (RGF) is a useful concept for processing FOD images. Given a difference metric for FODs, an RGF can be induced with the method proposed in Section (3.1).

The application of RGF to FOD interpolation yields better results, showing that fibers gradually turn at the sub-voxel level. As a classical operation widely involved in image processing, better interpolation may benefit many tasks, for instance, registration, segmentation, and tractography.

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Fig. 1.

The FOD image of a subject from the Human Connectome Project, reconstructed with the method in [7]. Fiber orientations are color-coded.

Fig. 2.

Transportation of FODs with rotational gradient field. Each subfigure is the transportation of an FOD from a corner, with the original FOD highlighted with a red circle.

Fig. 3.

Example of 1D Interpolation. The top three rows are the results of the linear interpolation, the Frechet mean with the Fisher-Rao metric, defined in Eq. (8), and the RGF-based interpolation defined in Eq. (9). The left and right ends of each row are the corner FODs. The bottom sub-figure shows the gFA values of the three interpolation methods.

Fig. 4.

Two examples of 2D Interpolation. The upper and lower panels are two interpolation cases. Columns show the results of the linear interpolation, the Frechet mean with the Fisher-Rao metric, defined in Eq. (8), and the RGF-based interpolation, defined in Eq. (9).