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Magnetic Pressure as a Scalar Representation of Field Effects in Magnetic Suspensions

Maciej Zborowski^{a,*}, Lee R. Moore^a, P. Stephen Williams^a, and Jeffrey J. Chalmers^b

^aDepartment of Biomedical Engineering, Lerner Research Institute, Cleveland Clinic, 9500 Euclid Ave., Cleveland, Ohio 44195, U.S.A

^bDepartment of Chemical and Biomolecular Engineering, The Ohio State University, Columbus, Ohio 43210, U.S.A

Abstract

Magnetic microsphere suspensions undergo complex motion when exposed to finite sources of the magnetic field, such as small permanent magnets. The computational complexity is compounded by a difficulty in choosing a suitable choice of visualization tools because this often requires using the magnetic force vector field in three dimensions. Here we present a potentially simpler approach by using the magnetic pressure. It is a scalar quantity, $p_m = B^2/2\mu_0$, and its usefulness has been already demonstrated in applications to magnetohydrodynamics and ferrohydrodynamics (where B is the applied field and $\mu_0 = 4\pi \times 10^{-7}$ T.m/A). The equilibrium distribution of the magnetic bead plug in aqueous suspension is calculated as an isosurface of the magnitude of the magnetic pressure $p_m = \text{const}$, in the field of two permanent magnet blocks calculated from closed formulas. The geometry was adapted from a publication on the magnetic bead suspensions in microsystems and the predicted bead plug distribution is shown to agree remarkably well with the experiment.

Keywords

magnetic separation; superparamagnetic bead; magnetic body force

INTRODUCTION

The purpose of this paper is to find a visual representation of magnetostatic field effects on a cloud of superparamagnetic particles suspended in a viscous medium. The secondary purpose is to find a representation that could be easily implemented and provide a rapid illustration of key features in an easily understood, intuitive manner. There are many excellent treatments on the subject in the physics and electrical engineering literature [1]. However, due to the complexity of the problem and the associated mathematics, those sources are not readily accessible to a reader who is not fluent in the science of electrodynamics of continuous media but who, nevertheless, is interested in their application to magnetic separation. Yet, there is a growing need of having access to a computational

*zborowm@ccf.org.

apparatus that would provide easily understood simulations for real-life permanent magnet assemblies, in applications to magnetic separation in microchannels. Here we propose to use the concept of the magnetic pressure, as first introduced by Faraday and Maxwell [2] and now routinely used in the electrical engineering applications to calculate force and torque acting on extended, magnetically permeable magnetic objects, including magnetic fluids (ferrofluids) [3]. Such treatment also applies to microscale, where the size of the microfluidic channels is not far removed from the magnetically labeled cells (on the order of 100 μm as compared to 10 μm , correspondingly).

The paper is organized into sections that first, provide expression for force acting on a labeled cell treated as an extended object, followed by taking a limit of the small cell size, for which the magnetic pressure at a point is defined, leading to interpretation of the magnetic pressure isosurfaces (isobars in 2D) as envelopes of the superparamagnetic particle cloud at equilibrium. The results are illustrated in Figures that are amenable to a direct comparison with experimental results and provide *ex post* validation of the proposed approach. The application of the magnetic pressure to predict spatial distribution of superparamagnetic particle suspensions is hoped to be “simple but not too simple”, and the simplifying assumptions are identified in the text. Ultimately, the usefulness of the proposed approach will be tested by future applications to more complex field and flow geometries, and its amenability to superposition with other forces, such as viscous stresses and capillary pressure.

THEORY

Field Equations

The closed-form formulas for the $\mathbf{H}$ field of rectangular block permanent magnets were adapted from the available literature sources [4].

$$H_x(x, y, z) = \frac{M}{4\pi} \sum_{k=1}^2 \sum_{m=1}^2 (-1)^{k+m} \ln \frac{y-y_1 + \sqrt{(x-x_m)^2 + (y-y_1)^2 + (z-z_k)^2}}{y-y_2 + \sqrt{(x-x_m)^2 + (y-y_2)^2 + (z-z_k)^2}} \quad (1)$$

$$H_y(x, y, z) = \frac{M}{4\pi} \sum_{k=1}^2 \sum_{m=1}^2 (-1)^{k+m} \ln \frac{x-x_1 + \sqrt{(x-x_1)^2 + (y-y_m)^2 + (z-z_k)^2}}{x-x_2 + \sqrt{(x-x_2)^2 + (y-y_m)^2 + (z-z_k)^2}} \quad (2)$$

$$H_z(x, y, z) = \frac{M}{4\pi} \sum_{k=1}^2 \sum_{n=1}^2 \sum_{m=1}^2 (-1)^{k+n+m} \arctan \frac{(x-x_n)(y-y_m)}{(z-z_k) \sqrt{(x-x_n)^2 + (y-y_m)^2 + (z-z_k)^2}} \quad (3)$$

where the position vectors $[x_1, y_1, z_1]$ and $[x_2, y_2, z_2]$ define the diagonally opposed vertices of rectangular block of the permanent magnet. The magnet block dimensions and their spatial disposition were selected so that they corresponded to the experimental values reported by Gassner *et al.* [5].

The $\mathbf{B}$ field was calculated from definition of the magnetization vector $\mathbf{M}$

$$\begin{aligned} \mathbf{B} &= \mu_0(\mathbf{H} + \mathbf{M}) && \text{inside the magnet} \\ \mathbf{B} &= \mu_0\mathbf{H} && \text{outside the magnet} \end{aligned} \quad (4)$$

The magnetization of the block permanent magnets was along the z -axis, $\mathbf{M} = [0, 0, M_s]$.

Superparamagnetic Microparticle Suspension

The superparamagnetic microparticles in viscous fluid suspension are approximated by a collection of noninteracting, freely suspended, inducible magnetic dipoles whose magnetization is directly proportional to the applied magnetic field:

$$M = \chi H = \chi \frac{B}{\mu_0} \quad (5)$$

where χ is the magnetic susceptibility that is independent of the applied field at low fields but becomes a function of the applied field at high fields, considered here. Letting χ depend on the applied field, H , results in an added computational complexity but contributes little to the local features of the p_m isosurface limited to the small volume of the capillary (as observed in the course of preliminary simulations, not shown here). Therefore, for the sake of clarity and without the loss of key local features of the p_m isosurfaces, we let the parameter χ stay constant for the rest of the simulations. Ultimately, the qualitative agreement between the predicted and the experimental results, shown below, justified our approach. It is a matter of further study to test if such a simplifying assumption is tenable for visualization of more complex magnetic systems and larger suspension volumes than those considered here. It is expected that the χ dependence on the applied field will lead to appreciable distortion from the linear model considered here, in the case of large suspension volumes (relative to the characteristic size of the magnet system). Subsequently, the second equation follows from the assumption that the superparamagnetic microparticles do not interact, so that $B = \mu_0 H$.

Magnetic Pressure

The magnetic field exerts a stress, $\mathbf{T} \cdot \mathbf{n}$, on a surface of a magnetically susceptible, extended body:

$$\mathbf{T} \cdot \mathbf{n} = \mathbf{H}(\mathbf{B} \cdot \mathbf{n}) - \frac{1}{2}(\mathbf{H} \cdot \mathbf{B})\mathbf{n} \quad (6)$$

where $\mathbf{T}$ is the Maxwell stress tensor and $\mathbf{n}$ is the unit surface normal vector at a point of stress [2]. The dot indicates the vector dot product. The magnetic force acting on such a body is equal to the surface integral of the magnetic stress over the surface of the body, A . In the limit of a small cell volume, the magnetic body force, $\mathbf{f}_m$, becomes a gradient of a scalar quantity that acts as pressure in the equations of motion of continuous media:

$$\mathbf{f}_m = \frac{\oint_A \mathbf{T} \cdot \mathbf{n} dS}{V} \xrightarrow{V \rightarrow 0} \nabla \left(\frac{1}{2} MB \right) = \nabla p_m \quad (7)$$

The magnetic pressure, p_m , acting on a freely suspended, superparamagnetic microparticle is obtained by substituting expression for M from Equation (5) into Equation (7)

$$p_m = \frac{1}{2} MB = \chi \frac{B^2}{2\mu_0} \quad (8)$$

The isosurface of the magnetic pressure is the surface on which $B^2 = \text{const}$. It determines an envelope of a set of non-interacting, superparamagnetic microparticle suspension in equilibrium with the colloidal pressure and other types of pressure that may exist in the viscous fluid suspension.

Field Geometry

Three different magnet configurations were considered: two parallel magnets, two anti-parallel magnets, and a single magnet. The spatial disposition of the magnets and their magnetization vectors is illustrated in Figure 1. Also shown is the location of a glass capillary used to hold the magnetic microparticle suspension (magnetic bead plug).

RESULTS AND DISCUSSION

Two Magnets Magnetized in Parallel Direction

The magnetic field of two permanent magnets magnetized in a parallel direction (Figure 1(a)) is shown in Figure 2 as represented by $\mathbf{B}$ vector field (panel (a)), magnetic pressure (panel (b)), magnetic pressure gradient (panel (c)), and B_z field component (panel (d)).

The magnetic pressure isosurface, $p_m = \text{const}$, was calculated from the magnetic pressure distribution shown in panel (b) and is shown in panel (e). Additional constraints included the capillary side walls. The value of the constant p_m used to calculate the pressure isosurface was selected so that the volume enclosed between the pressure isosurfaces and the capillary side walls corresponded to the volume of the magnetic microparticle suspension reported by Gassner *et al.*

The resulting, predicted location of the magnetic bead plug agrees remarkably well with that reported experimentally, shown in panel (f). Interestingly, the shape of the magnetic pressure isosurface (concave, in panel (e)) is different from that observed experimentally (convex, in panel (f)), which is suggestive of contribution from other type of forces than magnetic (such as capillary forces).

Two Magnets Magnetized in Anti-Parallel Direction

The type of analysis described above was repeated for two magnets magnetized in the anti-parallel direction, Figure 3. Here also the magnetic pressure isosurfaces (after taking into

account the spatial constraints of the capillary walls) led to a correct prediction of the location of the magnetic bead plug when compared to the experiment (as reported by Gassner *et al.*). The magnetic bead plug separated into two parts, panels (e) and (f) corresponding to the local maxima of the magnetic pressure, panel (b).

Interestingly, here the shape of the magnetic pressure isosurfaces (concave, panel (e)) agree with the shape of the magnetic bead plug surfaces (also concave, panel (f)). This suggests that the magnetic forces predominate. The agreement is unlike that reported in Figure 2. This may suggest differences in the capillary glass surface treatment and the sample preparation between the experiments shown in Figures 2(f) and 3(f).

Single Permanent Magnet

The type of analysis described above was repeated for a single permanent magnet, Figure 4. Here again the magnetic pressure isosurface predicted correctly the position and shape of the magnetic bead plug reported by Gassner *et al.*

The shape of the plug in panel (f) is slightly skewed to the left indicating possible experimental artifacts, such as related to the viscous shear of the suspending fluid (assuming that the magnetic microparticle suspension was introduced into the glass capillary from left to right).

CONCLUSIONS

The magnetic pressure approach to calculate spatial distribution envelope of discrete, superparamagnetic objects, leads to a remarkably similar picture as that obtained experimentally. For simple field source geometries such as those considered in this work, consisting of rectangular blocks of permanent magnets, closed form expressions for the external B field exist, allowing for a rapid development of computationally inexpensive algorithms [4]. The advantage of using magnetic pressure to visualize volume occupied by the set of the paramagnetic microparticles at equilibrium is that it treats the entire microparticle set globally, without the computational expense of simulating the motion of individual microparticles. This is different from current approaches to predicting the magnetic microparticle set evolution over time and its final spatial distribution by a type of Monte Carlo simulations that predict the behavior of the entire set by selecting a statistically representative microparticle subset and calculating the motion of each element in the subset [6]. Although the computer time increases only linearly with the number of elements in the subset, such approach may result in a considerable expense for even simplest 3D geometries if simulated on a desktop PC. The added complication is the reconstruction of the final set distribution from the individual cell trajectories. Therefore, the proposed treatment of the entire microparticle set as a whole by means of the magnetic pressure isosurfaces is appealing. It could be particularly useful for a quick survey of the effects of various magnet geometries on the expected microbeads equilibrium concentration distribution for which the inter-particle interactions could be neglected.

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FIGURE 1.

Three different magnet configurations used for simulation of the magnetic particle suspension distribution inside a glass capillary: (a) two magnets magnetized in a parallel direction, (b) two magnets magnetized in an anti-parallel direction, and (c) a single magnet. Arrows show direction of the magnetization vector, $\mathbf{M}$.

FIGURE 2.

Magnetic field representations of two permanent magnets magnetized in parallel. Thin blue line indicated location of a glass capillary used for holding magnetic microparticle suspension. For spatial orientation refer to Figure 1. (a) $\mathbf{B}$ field; (b) false color map of the magnetic pressure, p_m (red – low, violet – high); (c) magnetic pressure gradient, ∇p_m ; (d) B_z field component along the capillary axis; (e) magnetic bead plug volume inside the capillary predicted from the magnetic pressure isosurfaces; (f) experimentally observed magnetic bead plug (from Gassner A-L, Abonnenc M, Chen H-X, Morandini J, Josserand J, Rossier JS, Busnel J-M, Girault HH. Magnetic forces produced by a rectangular permanent magnets in static microsystems. *Lab Chip* 2009 **9** 2356–63, Fig. 7(b) DOI: 10.1039/b901865d – Reproduced by permission of The Royal Society of Chemistry).

FIGURE 3.

Magnetic field representations of two permanent magnets magnetized in anti-parallel direction. Refer to Figure 2 legends for description of Figure panels (a) through (f). The experimental results shown in panel (f) was copied from Gassner A-L *et al. Lab Chip* 2009 **9** 2356–63, Fig. 7(c) DOI: 10.1039/b901865d – Reproduced by permission of The Royal Society of Chemistry). Note agreement between the predicted and experimentally observed distribution and shape of the magnetic bead plugs, panels (d) and (e), respectively.

FIGURE 4.

Magnetic field representations of a single permanent magnet. Refer to Figure 2 legends for description of Figure panels (a) through (e). The experimental results shown in panel (f) was copied from Gassner A-L *et al. Lab Chip* 2009 **9** 2356–63, Fig. 7(d) DOI: 10.1039/b901865d – *Reproduced by permission of The Royal Society of Chemistry*). Note agreement between the predicted and experimentally observed location and shape of the magnetic bead plug, panels (d) and (e), respectively.