Electrostatic focal spot correction for x-ray tubes operating in strong magnetic fields

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Purpose: A close proximity hybrid x-ray/magnetic resonance (XMR) imaging system offers several critical advantages over current XMR system installations that have large separation distances (~5 m) between the imaging fields of view. The two imaging systems can be placed in close proximity to each other if an x-ray tube can be designed to be immune to the magnetic fringe fields outside of the MR bore. One of the major obstacles to robust x-ray tube design is correcting for the effects of the MR fringe field on the x-ray tube focal spot. Any fringe field component orthogonal to the x-ray tube electric field leads to electron drift altering the path of the electron trajectories.

Methods: The method proposed in this study to correct for the electron drift utilizes an external electric field in the direction of the drift. The electric field is created using two electrodes that are positioned adjacent to the cathode. These electrodes are biased with positive and negative potential differences relative to the cathode. The design of the focusing cup assembly is constrained primarily by the strength of the MR fringe field and high voltage standoff distances between the anode, cathode, and the bias electrodes. From these constraints, a focusing cup design suitable for the close proximity XMR system geometry is derived, and a finite element model of this focusing cup geometry is simulated to demonstrate efficacy. A Monte Carlo simulation is performed to determine any effects of the modified focusing cup design on the output x-ray energy spectrum.

Results: An orthogonal fringe field magnitude of 65 mT can be compensated for using bias voltages of +15 and -20 kV. These bias voltages are not sufficient to completely correct for larger orthogonal field magnitudes. Using active shielding coils in combination with the bias electrodes provides complete correction at an orthogonal field magnitude of 88.1 mT. Introducing small fields (<10 mT) parallel to the x-ray tube electric field in addition to the orthogonal field does not affect the electrostatic correction technique. However, rotation of the x-ray tube by 30° toward the MR bore increases the parallel magnetic field magnitude (~72 mT). The presence of this larger parallel field along with the orthogonal field leads to incomplete correction. Monte Carlo simulations demonstrate that the mean energy of the x-ray spectrum is not noticeably affected by the electrostatic correction, but the output flux is reduced by 7.5%.

Conclusions: The maximum orthogonal magnetic field magnitude that can be compensated for using the proposed design is 65 mT. Larger orthogonal field magnitudes cannot be completely compensated for because a pure electrostatic approach is limited by the dielectric strength of the vacuum inside the x-ray tube insert. The electrostatic approach also suffers from limitations when there are strong magnetic fields in both the orthogonal and parallel directions because the electrons prefer to stay aligned with the parallel magnetic field. These challenging field conditions can be addressed by using a hybrid correction approach that utilizes both active shielding coils and biasing electrodes.

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Key words: XMR, finite element modeling

1. INTRODUCTION

The use of hybrid x-ray/magnetic resonance (XMR) suites to provide image guidance for interventional procedures has become more prevalent, with several vendors offering dual-

modality installations.¹⁻¹³ These systems take advantage of the complementary nature of the individual modalities by combining the device tracking capabilities and high spatial/temporal resolution of projection x-ray imaging with the 3D imaging capabilities and soft tissue contrast provided by

FIG. 1. Geometric parameters for the close proximity XMR system. (a) Cross section of the system geometry for neurologic procedures with no rotation of the C-arm. (b) Cross section of the system geometry for cardiac procedures with no rotation of the C-arm. The distance between the MR and x-ray imaging fields is longer than in the neuro case. (c) Cross sectional view with rotation of the C-arm illustrating the cardiac oblique (1) and cardiac oblique (2) system geometries.

MRI. In a typical XMR installation, the distance between the x-ray and MRI fields of view is on the order of 5 m or greater. The long separation distance eliminates hardware compatibility issues, but switching between the modalities requires shuttling the patient several meters, which can be cumbersome because all monitoring systems and intravenous lines for drug delivery and anesthesia that are attached to the patient must be relocated as well. The long separation distance serves as an impediment to repeated switching between the modalities, which limits the range of procedures that can be performed.

To circumvent moving the patient when switching between modalities, an XMR system geometry in which the fields of view of the individual modalities were fully colocalized was developed.¹⁴ In this system, a static anode x-ray tube and a digital flat-panel detector were placed in the gap of a split bore double-donut 0.5 T interventional MR system (Signa SP, GE Milwaukee, WI). To enhance the performance of the system, compatibility issues were closely examined and the necessary hardware modifications were implemented.^{15–17} This system was used successfully for clinical procedures such as brain biopsy, transjugular intrahepatic portosystemic shunts (TIPS), chemoembolization of hepatic tumors, hysterosalpingograms, and loopograms.^{18–21} However, the major limitations of this specific colocalized XMR geometry were the low field strength (0.5 T) of the double-donut MR system, and the limited x-ray flux output of the static anode tube. Static anode tubes have stricter limits on the instantaneous heat loading capability of the target area on the anode, which limits the maximum x-ray flux compared to their rotating anode counterparts. Typical maximum power ratings for static anode tubes are less than 1 kW, whereas rotating anode tubes can support power ratings well beyond 10 kW. For high flux angiographic procedures and abdominal interventions in larger patients, a rotating anode x-ray tube is required. Finally, due to discontinued manufacturer support

for the Signa SP magnet, the colocalized geometry is not available for installation at other institutions.

A theoretical close proximity hybrid system design, where a distance of approximately 1.2–1.8 m separated the x-ray and MR imaging fields of view, was proposed by Brzozowski *et al.*²² as a compromise between the typical XMR installations that have long separation distances and fully colocalized system geometry. The x-ray system in the close proximity design consisted of a rotating anode x-ray tube and a digital flat-panel detector mounted on opposite ends of a C-arm. To achieve the short separation distance between the imaging fields of view, the C-arm was positioned immediately next to the MRI shroud, thus creating the engineering challenge of operating a rotating anode x-ray tube in a relatively strong MR fringe field. A cross sectional diagram of the close proximity system is offered in Fig. 1 along with the proposed x-ray tube positions. The experiments performed by Brzozowski *et al.* demonstrate that a standard rotating anode x-ray tube purchased from any of the various vendors will fail in this environment because the fringe field will act as a magnetic brake on the induction motor that is used to rotate the anode at high speeds and will alter the direction of the electron trajectories as they traverse from cathode to anode.

To address the reduction in rotation speed of the anode, an alternate motor design was proposed that is analogous to a modified three pole brushed DC motor.^{23,24} Solutions to correct for the change in direction of the electron trajectories have been discussed previously.^{25,26} These methods are focused on designing a magnetostatic mechanism that corrects for the changes such that focal spot size, shape, and position are preserved. However, the magnetostatic approach is not infallible and suffers from limitations. The primary limitation is the maximum 15 min continuous fluoroscopic operation time that arises because of joule heating of the copper shielding coils if the coils do not have an external cooling mechanism (e.g., heat exchanger).²⁷ Furthermore, including

shielding coils in the x-ray tube design requires a modified x-ray tube housing since the coils need to be positioned inside the housing to be as close to the x-ray tube insert as possible. To correct for the proposed field strengths (~ 90 mT), the shielding coils have a combined weight in excess of 30 kg. The excess weight must be counterbalanced, and the C-arm must have enough structural strength to withstand the torque exerted on the x-ray tube housing and coil mounts when current is delivered to the coils. The torque is created from the magnetic moments of the shielding coils trying to align with the direction of the MR fringe field.

The combination of these factors led us to explore methods that consume less power, require less physical space, and are more compatible with an MR environment (i.e., avoid exerting torque on the x-ray tube housing). An electrostatic method in which a static external electric field is applied to correct for the changes in the electron trajectories could potentially fulfill all of the above requirements. Generating a static electric field over distances on the order of a centimeter consumes orders of magnitude less power than the active magnetic shielding approach. Furthermore, the components necessary to create the electric field could be placed inside the x-ray tube insert, which helps to minimize the size of the x-ray tube housing and eliminates the need to use shielding coils. The aims of this study are to describe the details of how to design and implement an electrostatic correction method, to demonstrate feasibility of the design with results derived from finite element models of the electron trajectories in the x-ray tube, and to determine the limitations for this approach when applied to the close proximity XMR system geometry.

2. METHODS

2.A. Motion of charged particles in combined electric and magnetic fields

2.A.1. Theoretical description

The Lorentz force equation [Eq. (1)] governs the motion of charged particles in combined electric and magnetic field environments.

$$\vec{F} = \frac{d\vec{p}}{dt} = q(\vec{E} + \vec{v} \times \vec{B}). \quad (1)$$

Here, $\vec{F}$ is the force on the particle, $\vec{p} = \gamma m \vec{v} = m \vec{v} / \sqrt{1 - |\vec{v}|^2/c^2}$ represents the relativistic momentum, $\vec{E}$ the electric field vector, $\vec{B}$ the magnetic field vector, q the charge of the particle, $\vec{v}$ the velocity, and c the speed of light in vacuum. When applying this equation to the particle dynamics inside an x-ray tube, the value of q is negative because the relevant particles are electrons. To gain an intuitive understanding of the electron dynamics, let us assume that $\vec{E}$ and $\vec{B}$ are time invariant and spatially uniform for simplicity. If $\vec{B} = 0$, then $\vec{E}$ exerts a force ($\vec{F}_E = q\vec{E}$) on the electrons that is constant in both magnitude and direction. The direction of $\vec{F}_E$ is antiparallel to the direction of $\vec{E}$, and the electrons accelerate along $\vec{F}_E$ until they reach the anode [Fig. 2(a)]. However, if $\vec{E} = 0$ then $\vec{B}$ exerts a force ($\vec{F}_B = q\vec{v} \times \vec{B}$) on the electrons that is constant in magnitude but not constant in direction. If we assume the electrons possess

FIG. 2. (a) Force diagram for the case when only $\vec{E}$ is present. The e^- accelerates along a direction parallel to $\vec{F}_E$. (b) Force diagram for the case when only $\vec{B}$ is present. The magnitude of $\vec{F}_B$ is constant but the direction changes as it is always orthogonal to the direction of the e^- . (c) Force diagram for the case with $\vec{E}$ and $\vec{B}$ orthogonal to each other. In this case, the trajectory shape is a trochoid that can be thought of as having a drift velocity in the $\vec{E} \times \vec{B}$ direction. (d) Force diagram for when an electric field ($\vec{E}_{EXT}$) is applied along the x-direction. The drift direction is changed making it possible for the trajectory to intersect the anode at the same position when only $\vec{E}$ is present.

a nonzero initial velocity that is orthogonal to $\vec{B}$, then they travel in a circular path where $\vec{F}_B$ is always orthogonal to their trajectory [Fig. 2(b)].

If both $\vec{E}$ and $\vec{B}$ are nonzero, then the electron trajectory depends on the orientation of the fields relative to each other. The trajectory when $\vec{E}$ is parallel to or slightly misaligned with $\vec{B}$ is helical with variable pitch. The mathematical details of this case are rigorously examined by Wen *et al.*¹⁶ For the system geometries proposed in Fig. 1 and with the x-ray tube operating at diagnostic voltages (20–150 kVp), the relevant field conditions for this study are

- (i) $\vec{E}$ orthogonal to $\vec{B}$.
- (ii) $|\vec{B}| > |\vec{E}|/c$.

A specific example is illustrated in Fig. 2(c) where $\vec{E}$ is parallel to the -z-axis and $\vec{B}$ is parallel to the +y-axis. In this case, $\vec{F} = \vec{F}_E + \vec{F}_B$ changes in both magnitude and direction along the trajectory. At the beginning of the trajectory, $|\vec{F}_E| > |\vec{F}_B|$ and the electrons accelerate primarily along $\vec{F}_E$. As the electrons accelerate, $|\vec{v}|$ grows larger causing $|\vec{F}_B|$ to increase thus bending the electrons toward the +x direction. Eventually, the direction of the electrons will be entirely in the +x direction with $|\vec{v}| = v_{max}$ and $|\vec{F}_B| > |\vec{F}_E|$. From here, the electrons are bent further toward -z until eventually $|\vec{v}|$ decreases and $|\vec{F}_E| > |\vec{F}_B|$, at which point the behavior repeats. This type of trajectory is referred to as a trochoid, and there is a “drift” velocity associated with the motion of the center of the trochoid in the +x direction. In general, when $\vec{E}$ and $\vec{B}$ are orthogonal, the drift velocity is in the $\vec{E} \times \vec{B}$ direction.²⁸ The specific shape of the trajectory depends on the initial velocities ($\vec{v}(t_0)$) of the electrons, $|\vec{B}|$ and $|\vec{E}|$. In the limiting case where $|\vec{B}| \gg |\vec{E}|/c$, the radius of the trochoid is very small

FIG. 3. Diagram of the electric and magnetic field directions. (a) Illustration of the situation when there is an external magnetic field orthogonal to the electric field created by the potential difference between the anode and cathode of the x-ray tube. This electric and magnetic field geometry results in a drift velocity in the $\vec{E} \times \vec{B}$ direction. (b) Illustration of the possible correction technique in which another electric field ($\vec{E}_{EXT}$) is introduced in the direction parallel to the drift velocity. The presence of $\vec{E}_{EXT}$ corrects for the electron drift.

and the electrons do not reach the anode before being bent back toward the cathode. Also, in highly relativistic cases, the electrons do not reach the anode because their path is stretched in the $\vec{E} \times \vec{B}$ direction causing the trajectory to no longer resemble a trochoid. However, for nonlimiting cases, the electrons do reach the anode, but there is a significant deflection in the $+x$ -direction relative to the case when $\vec{B} = 0$ [Fig. 3(a)].

The previously mentioned magnetostatic approaches attempt to correct for the electron deflection by using shielding coils that create a magnetic field equal in magnitude, but in a direction opposite to the MR fringe field (i.e., trying to replicate the case where $\vec{B} = 0$). The approach proposed in this study is to introduce another component to the electric field, which is denoted as $\vec{E}_{EXT}$, in the direction of $\vec{E} \times \vec{B}$. The new net electric field ($\vec{E}'$) can be defined as $\vec{E}' = \vec{E} + \vec{E}_{EXT}$. The purpose of $\vec{E}_{EXT}$ is to exert a force on the electrons in the direction opposite to $\vec{E} \times \vec{B}$ as they traverse from the cathode to the anode. As the electrons accelerate, $|\vec{v}|$ grows larger and $\vec{F}_B$ pulls the electrons back in the direction of $\vec{E}' \times \vec{B}$, and the trajectory behavior is very similar to that shown in Fig. 2(c), with the important difference that $\vec{E}' \times \vec{B}$ now has components in both the $\hat{x}$ and $\hat{z}$ directions instead of just being parallel to $\hat{x}$ as before [Fig. 2(d)]. If $\vec{E}_{EXT}$ is chosen precisely, the forces on the electrons can be balanced in such a way that the final position of the electrons when they intersect the anode is nearly identical to the case where $\vec{B} = 0$ [Fig. 3(b)]. Hence, this method can be thought of as “precorrecting” the position of the electron trajectories with $\vec{E}_{EXT}$ such that they eventually end up in the correct position due to the $\vec{E}' \times \vec{B}$ drift.

2.A.2. Magnetic field parameters

The design constraints for the electrostatic approach will depend on the values of the MR fringe field at the position of the electron trajectories inside the x-ray tube. As shown in Fig. 1, there are four x-ray tube geometries that are of interest for the close proximity XMR system geometry. The magnetic field values associated with each position are enumerated in Table I.

The values are derived from a manufacturer field map for a 1.5 T short bore magnet (Conquest, GE Milwaukee, WI). All of the distances and field values in Table I are in centimeters and milliteslas, respectively. The variable R represents the radial distance from the MR imaging FOV, while Z represents the longitudinal distance. The neuro geometry refers to the illustration in Fig. 1(a), the cardiac geometry refers to Fig. 1(b), and the cardiac oblique geometries refer to Fig. 1(c). When there is no rotation of the x-ray tube, the longitudinal component of the fringe field (B_Z) is parallel to the x-ray tube axis (i.e., the axis between the cathode and anode), while the radial component (B_R) is orthogonal to the tube axis. When the tube is rotated, the magnetic field geometry conditions described above are no longer satisfied. Hence, we define rotated coordinate systems for each of the rotated tube positions, where Z_{ROT} and R_{ROT} represent the directions parallel and orthogonal to the rotated tube axis, respectively. These coordinate transformations enable us to define $B_{Z\ ROT}$ and $B_{R\ ROT}$ as the magnetic field components parallel and orthogonal to the rotated tube axis, respectively.

2.B. Constraints on the electrostatic correction technique

A potential solution to the electron drift problem is to introduce another electric field, $\vec{E}_{EXT}$, into the x-ray tube, but

TABLE I. Close proximity XMR system geometry and field parameters. The bold values are used to demonstrate the two system geometries with the largest fringe magnetic field magnitudes.

Geometry	Z (cm)	R (cm)	B_Z (mT)	B_R (mT)	$B_{Z\ ROT}$ (mT)	$B_{R\ ROT}$ (mT)	$\ B\ $ (mT)
Neuro	126	66	-7.9	88.1	N/A	N/A	88.5
Cardiac	150	66	8.2	40.1	N/A	N/A	40.9
Cardiac oblique (1)	194	57	12.3	15.7	2.8	19.7	19.9
Cardiac oblique (2)	128	57	24.7	102.2	72.5	76.2	105.1

FIG. 4. A simplified diagram of the x-ray tube cathode demonstrating a design utilizing biasing electrodes to generate an electric field orthogonal to the electric field between the anode and cathode.

the details of how to do so without compromising x-ray tube operation have not been previously described. It is possible to create $\vec{E}_{EXT}$ using bias electrodes that have potential differences (ΔV_1 and ΔV_2) relative to the cathode voltage [Fig. 4]. Adding bias electrodes inside the x-ray tube insert leads to new constraints relating to the geometry of the focusing cup and high voltage standoffs. These constraints are summarized below as follows:

- (i) The potential difference between the bias electrodes must create an electric field that is strong enough to counter balance the effects from the MR fringe field.
- (ii) There must be adequate high voltage standoff distances in the cathode assembly between the bias electrodes and the filament.
- (iii) There must be adequate high voltage standoff distances between the cathode assembly and the anode.
- (iv) The size of the cathode/bias electrode assembly should be comparable to the size of a conventional focusing cup so that a similar mounting technique can be used inside the x-ray tube.
- (v) Avoid introducing sharp structures that can concentrate local electric field lines. Concentration of local field lines can cause undesired arcing inside the x-ray tube insert.

To quantify the standoff distances, a maximum dielectric strength must be established for the conditions inside the x-ray tube. The x-ray tube insert is pumped down to a hard vacuum with pressures on the order of 10^{-6} Torr or lower before the insert is sealed. The dielectric strength of the vacuum inside the insert is affected by various factors such as temperature, electrode geometry, and electrode conditioning. Hence, it is not possible to derive the dielectric strength value theoretically from first principles. However, there are several empirical studies in the literature that have determined the dielectric strength for different electrode geometries in a vacuum. Results for cylindrical, electropolished stainless steel electrodes with radii between 5 and 8 mm demonstrate a dielectric strength between 30 and 40 kV/mm for a separation distance of 10 mm at a test pressure of 10^{-6} Torr.²⁹ Hrach et al.,³⁰ tested an experimental geometry with different gauges of tungsten wire (0.1–0.4 mm diameter) separated from a copper plane parallel to the wire at pressures ranging from 10^{-6} to 10^{-9} Torr. In this study design, the copper

plane was held at a positive potential relative to the tungsten wire. The results show breakdown occurring at macroscopic dielectric strengths ranging from 15 kV/mm for the 0.1 mm diameter tungsten to 32 kV/mm for the 0.4 mm diameter tungsten for a separation distance of 1 mm between the electrodes (for reference the dielectric strength of air is ~ 3 kV/mm). Another important conclusion from this study was that the breakdown voltage increased linearly up to a distance of 1 mm, but began to increase less than linearly (approximately logarithmically) for distances larger than 1 mm. This implies that the dielectric strength decreases with increasing separation distance, and agrees with other literature that states that the “macroscopic gap field required to maintain a prebreakdown current decreases with increasing separation distance.”³¹ These findings lead to the general guideline that at separation distances greater than 2 mm, the dielectric strength of the vacuum becomes weaker, so including separation distances that are this large between electrodes in the electrostatic design should be avoided.

2.C. Split focusing cup design

From the constraints given in Sec. 2.B, a feasible design for the split focusing cup can be developed. The design choices for each individual component in the focusing cup are explained in detail in Sec. 2.C.1 and 2.C.2. All conducting structures in the assembly should be made out of an austenitic stainless steel (e.g., SS316L) because it is nonmagnetic [μ_r ranging from 1.003 to 1.005 (Ref. 32)]. A schematic drawing of the split focusing cup design is given in Fig. 5(a) and a list of the electrical and geometrical parameters associated with the design is provided in Table II. Figure 5 uses a Cartesian coordinate system that is different from the rotated cylindrical coordinate system that is referred to in Table I. The relationship between these coordinate systems is defined as follows: the y -axis is aligned with the radial direction (R or R_{ROT} for the rotated tube geometries), while the z -axis is aligned with the longitudinal direction (Z or Z_{ROT}).

FIG. 5. A cross sectional schematic of a split cathode x-ray tube design with biasing electrodes placed above and below the x-ray tube filament. The parameters in the design that determine the exact geometry are shown, and their specific values are listed in Table II. The figure is drawn to scale.

TABLE II. List of the electrical and geometrical parameters of the split focusing cup design.

Voltages (kV)	
Cathode voltage (V_C)	-120
Anode voltage (V_A)	0
Max $ \Delta V_1 $	20
Max $ \Delta V_2 $	20
Distances (mm)	
Cathode anode separation (δ_1)	10
Electrode cathode separation (δ_2)	1.5
Filament mount thickness (t_F)	2
Electrode thickness (t_E)	7.5
Curvatures (mm^{-1})	
R_1	0.91
R_2	0.27
R_3	0.18
R_4	0.47

2.C.1. The filament and filament mount

The tungsten filament is positioned at the center of the focusing cup assembly. The filament is positioned inside the filament mount, which serves several purposes. The filament mount is a conducting structure that is held at the same voltage as the common side of the filament, which is typically called the cathode voltage, V_C . The first purpose of the filament mount is to shape the local electric field lines around the filament such that when the filament begins to emit electrons via thermionic emission, they are primarily accelerated toward the anode. The filament mount also allows for exact positioning of the filament so that it is held at the correct distance from the anode, as well as providing voltage feed-through connections for both ends of the filament. There is no concern about dielectric breakdown between the filament and the filament mount because they are both held virtually at the cathode voltage. To be precise, one end of the filament (i.e., the common side) is held at the cathode voltage. The other end, which will be referred to as the high side, is held at the cathode voltage plus a small DC bias of 5–10 V. This potential difference induces a DC current that heats the filament to a sufficient temperature for the thermionic emission process. For the purposes of the focusing cup design, both the common and high side of the filament are considered to be at the cathode voltage, V_C . Hence, the filament and filament mount can be positioned very close to each other.

The cross sectional dimension of the filament mount (t_F) in the x -direction determines the separation distance between the bias electrodes and the filament. The separation distance is critical because, as stated earlier, the dielectric strength of vacuum begins to decrease as the separation distance between the electrodes increases. Hence, for the split focusing cup design to be effective, it is necessary that the cross sectional width of the filament mount is kept as small as possible in the x -direction. These requirements become troublesome when trying to decide on the thickness of the two protrusions on

the filament mount, which are necessary for shaping the local electric field. The protrusions should be as thin as possible to maximize the dielectric strength of the gap. There are limits on how thin the protrusions can be made because as they are made thinner there will be local concentration of electric field lines which can lead to potential arcing. Furthermore, there are manufacturing limitations on how thin these protrusions can be. A reasonable filament diameter for a fluoroscopic procedure is 1 mm, so a 2 mm cross sectional width is chosen for the filament mount. Out of this 2 mm, the gap between the protrusions will be kept at 1.6 mm to give adequate space to mount the filament without potentially causing an inadvertent short to the filament mount. These dimensions imply that the protrusions will have cross sectional widths of 0.2 mm each, which is easily achievable using electrical discharge machining (EDM). In standard x-ray tube design, the filament mount and focusing cup are one continuous structure that are not electrically isolated since the entire structure can be held at the cathode voltage V_C . Hence, in the standard design, the only high voltage standoff that must be considered is the separation between the focusing cup and the anode.

2.C.2. The biasing electrodes and electrode separation distances

The biasing electrodes are located above and below the filament mount (Fig. 5). The faces of the electrodes that face the anode are rounded to avoid the presence of sharp edges. The electrodes are the closest structures to the face of the anode, with a maximum potential difference of 150 kV relative to the anode. With such a large potential difference, any sharp edge could be a potential source of arcing. Furthermore, the biasing electrodes are not identical; their front faces have different radii of curvature (i.e., the focusing cup design is not symmetric about the z -axis). From Fig. 5, it is evident that the biasing electrode below the filament has a smaller radius of curvature on one side versus the other. The reasoning behind this design choice is to create more physical space for the trajectory deflection that will occur due to the Lorentz force that arises from the presence of $\vec{E}_{\text{EXT}}$.

The separation distance between the bias electrodes and anode (δ_1) is a key design parameter. On the one hand, this distance should be short because it allows $\vec{E}_{\text{EXT}}$ to affect a larger proportion of the electron trajectory. However, as δ_1 decreases the electric field between the anode and biasing electrodes increases. To keep the electric field below the previously reported breakdown voltages, the value of δ_1 is set to 10 mm. With a maximum potential difference of 150 kV, this sets the maximum macroscopic field strength to 15 kV/mm, which adds a safety margin, as it is less than half of the values quoted earlier for rounded stainless electrodes at this separation distance. The other critical separation distance is the distance between the biasing electrodes and the filament mount (δ_2). It is ideal to keep this distance less than the 2 mm threshold. However, it is also not practical to make this distance too small because of concerns about any local electric field concentration around the protrusions from the filament mount. In this

design, the value of δ_2 is set to 1.5 mm, which then effectively sets the distance between the bias electrodes ($2\delta_2 + t_F$) to 5 mm at their closest point.

The curvature of the top bias electrode (i.e., R_2) is chosen such that δ_1 is not smaller than 10 mm and the cross sectional thickness of the electrode (t_E) is equal to 7.5 mm. The 7.5 mm value for t_E is derived from the fact that the thickness of the entire assembly ($2t_E + 2\delta_2 + t_F$) should be equal to 20 mm, a value that is comparable to a standard single filament x-ray tube focusing cup. To give the electron trajectories more clearance from the surface of the bottom bias electrode, while also satisfying the constraints $\delta_1 > 10$ mm and $t_E = 7.5$ mm, it is necessary to have two different curvature values (R_3 and R_4) for the bottom bias electrode. However, having two different curvatures can lead to a discontinuity in the slope of surface contour at the transition point between R_3 and R_4 . A discontinuity could cause local concentration of electric field lines leading to a dielectric breakdown between the anode and the bias electrode. The front of the bias electrode can be represented by a function $z(x)$ with $x = x_t$ representing the location of the transition point between R_3 and R_4 . To minimize any local concentration of electric field lines, it is specified that $(dz/dx)|_{x_t} = 0$ and that the value of $(dz/dx)|_{x_t}$ is continuous. The location of x_t can be determined from the value of the secant angle that is formed between the starting point of the arc to the transition point x_t . The starting point of the arc for each electrode can be represented by an (x, z) coordinate pair. The x coordinate is offset from the center of the filament by $+(\delta_2 + t_F/2)$ and $-(\delta_2 + t_F/2)$ for the top and bottom bias electrodes, respectively. The z coordinate for both electrodes is offset from the end of the protrusions by 1 mm. This value is chosen because it is comparable to the filament inset distance in traditional focusing cups. In prior iterations of the design, different values were experimented with, for this angle, and 55° was deemed sufficient to create enough space for the electron trajectories. With the location of the starting point of the arc, the location of x_t and the constraint $(dz/dx)|_{x_t} = 0$ the value of R_3 can be uniquely determined. Similarly, the location of x_t , the constraint $(dz/dx)|_{x_t} = 0$, and $t_E = 7.5$ mm provide the necessary information to derive the value of R_4 .

The 1.5 mm separation distance (δ_2) between the bias electrodes and filament mount can be used to derive the maximum bias voltages. Using the 15 kV/mm maximum electric field strength restriction mentioned previously, the maximum potential difference between the bias electrode and filament mount is limited to 22.5 kV. However, as the filament mount has protrusions with dimensions of 0.2 mm, it is possible that this potential difference could still lead to dielectric breakdown. Hence, in practice, the potential difference will be limited to 20 kV to add an additional safety factor. All of the safety calculations above are based upon macroscopic electric field value comparisons to the literature, but do not account for the local concentration of electric field lines around protrusions of the filament mount. The additional safety margins are added because the local field strengths in this electrode design will not be identical to those in the experimental designs studied in the literature.

2.D. Finite element model

2.D.1. Mathematical formulation

To validate the split focusing cup design proposed in Sec. 2.C, a finite element model of the focusing cup and anode was created to perform a 3D space charge simulation (SCALA, OPERA 3D v 11.0, Cobham, Dorset, UK). The simulation computes the electric field ($\vec{E}$) along with the position and velocity of the electron trajectories inside the x-ray tube. When numerically computing $\vec{E}$, it is customary to first compute the scalar electrostatic potential function V , which is related to $\vec{E}$ by the gradient operator as shown below [Eq. (2)].

$$\vec{E} = -\nabla V. \quad (2)$$

By combining Eq. (2) with the differential form of Gauss's law [Eq. (3)], the electrostatic version of Poisson's equation is derived [Eq. (4)], where ϵ is the dielectric permittivity and ρ is the charge density including the space charge distribution.

$$\nabla \cdot \epsilon \vec{E} = \rho, \quad (3)$$

$$\nabla \cdot \epsilon \nabla V = -\rho. \quad (4)$$

The simulation numerically approximates the solution for V that satisfies Eq. (4) subject to a set of boundary conditions, which are discussed in more detail in Sec. 2.D.1. The method of weighted residuals is used to^{33,34} formulate a set of linear equations for V over all nodes of the finite elements within the 3D model geometry. The algorithm used to arrive at the solution of this system of equations is comprised of two nested iterative loops. The inner loop inverts the system of equations to arrive at a solution for V using an iterative conjugate gradient technique. The outer loop computes $\vec{E}$ from V via the gradient operator and then proceeds with the electron trajectory calculation, including full relativistic effects, using the computation for $\vec{E}$ along with any external magnetic field ($\vec{B}$) that the user has specified. Once trajectory calculation is completed, the space charge from each trajectory is calculated at each node in the model. The space charge distribution is used to update the system of equations for V , at which point the system is solved again using the conjugate gradient inner loop for an updated solution for V . The updated V is then used to update the trajectory calculation by the steps described above. This updating process repeats indefinitely (i.e., updated $V \leftrightarrow$ updated trajectories) until the magnitude of the residual vector for the linear system being solved for V (i.e., $|Ax - b|$) is less than 1×10^{-8} . At first glance, the convergence value may seem arbitrary, but it is empirically determined by the software manufacturer to be sufficient for all known applications.³⁵

2.D.2. Boundary conditions

The boundary conditions applied to the solution for V in the finite element model are listed below [Eqs. (5) and (6)].

$$\begin{aligned} V|_{\text{anode}} &= V_A, & V|_{\text{cathode}} &= V_C, \\ V|_{E_1} &= V_C + \Delta V_1, & V|_{E_2} &= V_C + \Delta V_2. \end{aligned} \quad (5)$$

In Eq. (5), V_A , V_C , ΔV_1 , and ΔV_2 represent scalar constants, while E_1 and E_2 are being used to define the surfaces of the two biasing electrodes. The purpose of these conditions is to specify the different voltages on the conducting structures within the model (i.e., the anode, cathode, and biasing electrodes).

$$\hat{n} \cdot \nabla V = 0. \quad (6)$$

In Eq. (6), $\hat{n}$ represents the unit vector normal to the exterior boundaries of the finite element mesh. This condition, referred to as the tangential electric boundary condition, specifies that the electric field component normal to the exterior boundaries be set to zero. However, this condition is only applied to those exterior boundaries that do not have voltages already specified on them from Eq. (5) [e.g., the anode does not have Eq. (6) applied to it].

2.D.3. Model construction and implementation

To construct the finite element model [Fig. 6], a 2D profile of the anode, cathode, and biasing electrode was constructed in the xz -plane using the geometry parameters specified in Table II. This profile was then extruded in the y -direction to create a 3D volume. In this approach, a small segment of the anode is simulated instead of the full circular shape to make the simulation space a manageable size. This simplification has been used in a prior work by Wen *et al.*¹⁶ and experimentally validated.³⁶ The final volume was a hexahedron with x , y , and z dimensions of 90, 18, and 25 mm, respectively. The dimensions were chosen to be sufficiently large enough such that boundary conditions applied to the exterior planes of the model did not affect the results. The x -dimension was specifically found to be the most sensitive and hence, it was

chosen to be larger than the others. The anode in the model had a tilt angle of 12° , while a 6.9 mm long cylinder oriented along the y -axis with a radius of 0.55 mm was used to model the filament. The front surface of this cylinder was used as an electron emitter using the 1D Langmuir/Fry thermal emission model (emitter temperature = 2350 K, work function = 4.5 eV, and total current = 6.8 mA).

Meshing was performed using hexahedral elements with linear basis functions used to model V in each element. At the filament where the electron trajectories originate, the resolution of the finite element mesh was very fine (~ 0.1 mm isotropic). To reduce the problem complexity (i.e., the total number of finite elements in the model), the x - and z -resolutions of the mesh was gradually increased toward the boundaries of the simulation space to final values of 5.0 and 1.0 mm, respectively. The final mesh had a total of $\sim 6.1 \times 10^5$ finite elements and a total of $\sim 6.3 \times 10^5$ nodes. The tolerance of the mesh generator was set to 1×10^{-5} mm.

For all simulations, the filament and the filament mount were kept at an electric potential of $V_C = -120$ kV and the anode at a potential of $V_A = 0$ kV. A grounded anode configuration was chosen because it was better suited for the motor design mentioned previously.²⁴ The external magnetic fields in these simulations represent the MR fringe field; hence radial fields are parallel with the y -axis, and longitudinal fields are parallel with the z -axis. Electron trajectories were calculated for the following cases, and the potential differences for the biasing electrodes relative to the cathode were varied depending on the strength of the magnetic field in the simulation as described below:

- (i) No external magnetic fields and no correction mechanism.

FIG. 6. (a) 3D Geometry of the split focusing cup design with filament/filament mount at the center and adjacent bias electrodes. (b) View of the relevant simulation space from y -axis showing the xz -plane. The electron beam originates at the filament and terminates at the anode. (c) View of the relevant simulation space from the x -axis showing the yz -plane. The electron beam has a finite extent in the y -direction.

- a. $B_y = 0.0$ mT, $B_z = 0.0$ mT, $\Delta V_1 = -0.5$ kV, and $\Delta V_2 = +0.5$ kV.
- (ii) External magnetic fields in the radial direction with no correction mechanism.
 - a. $B_y = 19.7$ mT, $B_z = 2.8$ mT, $\Delta V_1 = -0.5$ kV, and $\Delta V_2 = +0.5$ kV.
- (iii) External magnetic fields in the radial direction with electrostatic correction.
 - a. $B_y = 65.0$ mT, $B_z = 0.0$ mT, $\Delta V_1 = +15$ kV, and $\Delta V_2 = -20$ kV.
 - b. $B_y = 76.2$ mT, $B_z = 0.0$ mT, $\Delta V_1 = +15$ kV, and $\Delta V_2 = -20$ kV.
- (iv) External magnetic fields in the radial direction with both magnetostatic (i.e., shielding coils present) and electrostatic correction.
 - a. $B_y = 88.1$ mT, $B_z = -7.9$ mT, $B_{y \text{ ext.coil}} = -57.5$ mT, $\Delta V_1 = +15$ kV, and $\Delta V_2 = -20$ kV.
- (v) External magnetic fields in both the radial and longitudinal directions with electrostatic correction.
 - a. $B_y = 40.1$ mT, $B_z = 8.2$ mT, $\Delta V_1 = +15$ kV, and $\Delta V_2 = -20$ kV.
 - b. $B_y = 19.7$ mT, $B_z = 2.8$ mT, $\Delta V_1 = +15$ kV, and $\Delta V_2 = -20$ kV.
 - c. $B_y = 76.2$ mT, $B_z = 72.5$ mT, $\Delta V_1 = +15$ kV, and $\Delta V_2 = -20$ kV.

All external magnetic fields used in the simulations were treated as spatially uniform. The finite elements in the simulation space where the accuracy of the external magnetic field is critical (i.e., the volume where $|\vec{B}|$ affects the electron trajectories) have x , y , and z dimensions of $6 \times 8 \times 16$ mm. Based on the data from manufacturer field maps, the average percent variation in $|\vec{B}|$ over this volume of interest for the “neuro” and “cardiac oblique (2)” system geometries are 1.9% (4.1% max $\Delta|\vec{B}|$) and 1.9% (4.0% max $\Delta|\vec{B}|$), respectively. These values drop off further for the “cardiac” and “cardiac oblique (1)” system geometries. Hence, over this critical volume of interest the spatial variance of $|\vec{B}|$ is small and it is safe to assume that effects that would arise from $\nabla \vec{B}$ are negligible. The validity of this assumption was experimentally verified for $|\vec{B}| \sim 20$ mT.³⁶

2.E. Monte Carlo model

As mentioned previously, the electrostatic technique will create curved electron trajectories, as there will be a region between the cathode and anode where the Lorentz force contribution from the electric field dominates, followed by a region where the contribution from magnetic field dominates as the velocity of the electrons increases. Hence, when the electrons reach the anode their velocity will not be normal to the xy -plane (i.e., their velocity will not be solely in the z -direction). Instead, the electrons will have a significant component of their velocity in the x -direction as well. The angle of incidence can have an effect on the output spectrum from the x-ray tube.³⁷ In order for the electrostatic correction technique to be practical, it cannot drastically alter the output

spectrum of the x-ray tube. Some changes to the spectrum would be acceptable if the change was well understood and did not dramatically reduce the x-ray output. For example, if the mean energy of the spectrum was lowered substantially (i.e., the spectrum was “softer”) this would *not* be acceptable because additional filtration would have to be added, which would reduce the useful x-ray output. To test the effect of the changed velocity distribution on the x-ray spectra, a Monte Carlo model was implemented using a validated software package (GEANT4 v8.p01).^{38,39} The goal of this model was to create two x-ray spectra that could be compared against each other. The first spectrum was generated from the solution with no external magnetic fields solution where high voltage was only applied between the anode and cathode (i.e., $|\Delta V_1|$ and $|\Delta V_2| = 0.5$ kV) so the electron velocities were primarily in the z -direction. The second x-ray spectrum was from the electrostatically corrected case ($B_y = 65.0$ mT, $B_z = 0.0$ mT, $\Delta V_1 = +15$ kV, and $\Delta V_2 = -20$ kV) that had a modified velocity distribution. The details of the implementation are given below.

The Monte Carlo model consisted of a target, a detector, and many monoenergetic electron guns positioned inside a physical volume referred to as the world (Fig. 7). Each of these components is described further in more detail. The world was a cube (6 cm length on each side) with material properties equivalent to vacuum, while the target was a block of tungsten ($2 \times 2 \times 1$ cm) positioned at the center of the world and rotated by 12° . The detector was a rectangular volume ($6 \times 6 \times 1$ cm) with material properties of vacuum positioned at the top of the world volume. It was a virtual detector as it simply recorded the energies of x-ray photons that intersected it (i.e., there are no physics models of x-ray detection).

FIG. 7. A depiction of the geometry of the Monte Carlo simulation showing the world volume, tungsten target, and virtual detector. The electron guns are positioned in front of the target. Each gun has an associated position, energy, and direction vector that are computed from the position and velocity of the electron trajectory associated with that gun.

Once the energy of a photon trajectory was recorded, it was terminated within the model so that it was no longer tracked.

The monoenergetic electron guns were a crucial part of the model as they provided the link between the finite element and Monte Carlo models. In the finite element model, each trajectory represented a fraction of the total current that was passed between the cathode and anode of the x-ray tube. Upon intersecting the anode, each trajectory had a final position (x, y, z) , a final velocity (v_x, v_y, v_z) , and a fractional current (i_k) . The position and velocity information was tabulated for each trajectory in the finite element solution, and was referred to as the phase space for that solution. The phase space for the solution where there are no external magnetic fields was referred to as the control phase space, while the phase space for the electrostatically corrected case was referred to as the modified phase space. The phase space information from the finite element model was input into the Monte Carlo model through the electron guns. For each phase space, a series of electron guns were created in a macro file, one for every trajectory in the finite element model solution. For a given trajectory, it was possible to calculate the energy for the corresponding electron gun using Eq. (7) shown below.

$$K = mc^2 \left[\left(1 / \sqrt{1 - |\vec{v}|^2 / c^2} \right) - 1 \right]. \quad (7)$$

Here, K represents the relativistic kinetic energy of the particle, m is the mass of the particle, $\vec{v}$ is the velocity vector, and c is the speed of light. The energy range of the electrons used in the model ranged from 119.4 to 120.6 keV. Furthermore, from the velocity information, a normalized direction vector ($\hat{n}$) for each electron gun can be computed using Eq. (8).

$$\hat{n} = \frac{1}{\|\vec{v}\|} (\vec{v}_x, \vec{v}_y, \vec{v}_z). \quad (8)$$

The number of electrons associated with each electron gun was not constant. Instead, given a total number of electrons to be used in the simulation (N), it was possible to calculate the number of electrons that should be associated with each electron gun (N_k) from the fractional currents of the trajectories as shown in Eq. (9).

$$N_k = N \frac{i_k}{I}. \quad (9)$$

Here, i_k was the current associated with the k th trajectory, and I was the total current summed over all trajectories from the finite element solution (~ 5.5 mA). The value used for N in the simulation was 1.2×10^8 , while each phase space consisted of $\sim 1.7 \times 10^3$ trajectories. The electron guns were positioned on a plane directly in front of the tungsten target. The collision of these electrons with the tungsten led to the creation of x-ray photons via Bremsstrahlung and characteristic radiation. The energies of x-ray photons that intersected the virtual detector were recorded, and the energy distribution for these photons was then used to create histograms for the two desired phase spaces for comparison.

3. RESULTS

3.A. Electric field results

As mentioned earlier, the solution from the finite element simulation yields V that satisfies Eq. (4) along with the boundary conditions specified by Eqs. (5) and (6). The value of $\vec{E}$ is then calculated by applying the gradient operator to the solution for V . 2D contour maps of V are shown below for $\Delta V_1 = -0.5$ kV, $\Delta V_2 = +0.5$ kV [Fig. 8(a)] and for $\Delta V_1 = +15$ kV, $\Delta V_2 = -20$ kV [Fig. 8(b)]. In both cases, $V_c = -120$ kV and $V_A = 0$ kV. The maps are evaluated on the xz -plane that passes through the center of the filament. The 2D vector field maps associated with the contour maps are also shown [Figs. 8(c) and 8(d)]. As expected, the $\vec{E}$ vectors in Figs. 8(c) and 8(d) are orthogonal to the contour lines in Figs. 8(a) and 8(b), respectively. Furthermore, in Fig. 8(c), $\vec{E}$ is primarily oriented in the z -direction while in Fig. 8(d), $\vec{E}$ has components in both the x - and z -directions because the magnitudes of the bias voltages are much larger in the latter case. The x component of the electric field diminishes closer to the anode as expected because the biasing electrodes have a limited extent in z due to the high voltage standoff constraints mentioned previously. Finally, the maximum $|\vec{E}|$ at the protrusions of the filament assembly is 2.0 kV/mm ($\pm 21\%$) for $\Delta V_1 = -0.5$ kV, $\Delta V_2 = +0.5$ kV versus 21.4 kV/mm ($\pm 18\%$) for $\Delta V_1 = +15$ kV, $\Delta V_2 = -20$ kV. The percentage values in the parentheses above are the local errors reported by the simulation. These errors represent the difference between the gradient of V and nodal averaged field that the simulation reports as the solution. The nodal averaged field is the field calculated from averaging field values between adjacent nodes. It can be thought of as a smoothing operation to remove sharp discontinuities of the field value at element boundaries. The global solution error for $\vec{E}$ over all simulations ranges from 12% to 35%, which according to software documentation³⁵ indicates that the simulation space is well posed. Furthermore, varying the mesh resolution to create different models with $\sim 3 \times 10^5$ to $\sim 1.2 \times 10^6$ finite elements introduced changes in the maximum $|\vec{E}|$ of ± 1.0 kV/mm for $\Delta V_1 = +15$ kV, $\Delta V_2 = -20$ kV demonstrating relative mesh independence for the mesh resolution used in these simulations.

3.B. Electron trajectory results

The solution for the electron trajectories for the simulation in the absence of an external magnetic field is shown in Fig. 9(a). All trajectories are superimposed on a single xz -plane for ease of visualization and the boundary conditions used for the electrode voltages are shown in the figure. Again the anode is held at ground for all simulations as mentioned previously. Due to the asymmetric geometry of the two bias electrodes, small potential differences (± 0.5 kV) are applied such that the electron trajectories intersect the anode at approximately $x = 0$ mm. The locations where the trajectories intersect the anode can be used to calculate a current density map [Fig. 10(a)], which can be thought of as a simulated focal spot. The brighter regions correspond to

FIG. 8. (a) Contour map for the solution to the scalar potential (V) for the split focusing cup geometry with $\Delta V_1 = -0.5$ kV and $\Delta V_2 = +0.5$ kV. The potential difference between anode and cathode was 120 kV, which is in the upper range of voltages used for fluoroscopic x-ray imaging. (b) Contour map for the solution to the scalar potential (V) for the split focusing cup geometry with $\Delta V_1 = +15$ kV and $\Delta V_2 = -20$ kV. (c) Field map for $\vec{E}$ corresponding to the scalar potential solution shown in (a). (d) Field map for $\vec{E}$ corresponding to the scalar potential solution shown in (b). The presence of the bias voltages introduces an x component to $\vec{E}$ near the filament.

larger values of current density (i.e., more trajectories intersect the anode at this location). A solution with magnetic fields representative of the cardiac oblique (1) geometry, but with no electric field correction is shown in Fig. 9(b) with the associated simulated focal spot shown in Fig. 10(b). The solution with no external magnetic fields is treated as a negative control, while the solution with magnetic fields and no

electric field correction is treated as a positive control. These two cases can be used as references for the remainder of the simulation cases to determine the effectiveness of the correction technique.

A series of simulations were performed with varying values for the external magnetic field, and with the biasing electrodes set to -105 kV ($\Delta V_1 = +15$ kV) and -140 kV

FIG. 9. (a) Electron trajectories plotted for the control case where there is no external magnetic field, and small potential differences (± 0.5 kV) are used for the bias electrodes to fine-tune the position of the trajectories on the anode. (b) Trajectories simulated with external magnetic fields taken from the cardiac oblique (1) system geometry (19.7 mT y , 2.8 mT z) with no electrostatic correction present. The trajectories demonstrate a deflection in the $+x$ -direction. (c) Trajectories simulated with a 65 mT external field along y and $\Delta V_1 = +15$ kV and $\Delta V_2 = -20$ kV. The trajectories intersect the plane of the anode at the same position as the trajectories in (a), but the velocity distribution of the electrons is modified. (d) Trajectories simulated with a 76 mT external field along y , and $\Delta V_1 = +15$ kV and $\Delta V_2 = -20$ kV demonstrating incomplete correction.

FIG. 10. (a) Simulated focal spot for the trajectories in Fig. 9(a). The focal spot has a rectangular shape with dimensions of 1.0×0.7 mm. (b) Simulated focal spot for the trajectories in Fig. 9(b). The focal spot shape has a slight parallelogram shape and there is a deflection in position of about 2.7 mm in x . (c) Simulated focal spot for the trajectories in Fig. 9(b). The focal spot shape has a parallelogram shape and there is a deflection in position of about 0.5 mm in x . (d) Simulated focal spot for the trajectories in Fig. 9(c). The deflection has increased to ~ 2.5 mm in x .

($\Delta V_2 = -20$ kV). The external magnetic field was always applied along the y -axis (equivalent to the radial direction in the MR scanner reference frame) of the finite element model. The purpose of these simulations was to determine the maximum magnitude of the magnetic field the biasing electrodes could correct. The solution for the electron trajectories with an external magnetic field strength of 65 mT is shown in Fig. 9(c), with the corresponding simulated focal spot shown in Fig. 10(c). The focal spot dimensions are similar to the control, and there is a small (between 0.5 and 1 mm) deflection in the horizontal direction. The most significant changes are that the focal spot has a slanted orientation so it has a parallelogram like shape instead of a rectangular shape, along with a 32% decrease in emission efficiency from the filament.

At magnetic field strengths larger than 65 mT, the magnitudes of ΔV_1 and ΔV_2 must be increased to achieve complete correction. Increasing the potential differences has negative effects as explained in Sec. 4. The results from a simulation with a magnetic field strength of 76.2 mT with $\Delta V_1 = +15$ kV and $\Delta V_2 = -20$ kV are shown in Fig. 9(d) to demonstrate incomplete correction. The trajectory solution displays an offset of between 2 and 2.5 mm in the $+x$ -direction with further slanting of the focal spot shape and a 30% decrease in emission efficiency [Fig. 10(d)]. The simulation is performed at 76.2 mT, because this is equivalent to the magnitude of the radial magnetic field for the cardiac oblique (2) geometry in Table I.

Solutions for the simulated electron trajectories for each of the four proposed system geometries [neuro, cardiac, cardiac oblique (1) and (2)] are presented in Figs. 11(a)–11(d), and the associated simulated focal spots in Figs. 12(a)–12(d).

The neuro geometry has the largest external magnetic field magnitude along the y -axis (88.1 mT), which is ~ 23 mT larger than the maximum correctable magnetic field demonstrated earlier. Hence, it is proposed for this geometry to combine the magnetostatic and electrostatic correction techniques. External correction coils are introduced into the simulation and programmed to compensate for 57.5 mT of the external field; the correction coils can provide this field strength for between 45 and 60 min continuously without overheating.²⁷ The remaining 30.6 mT of the external magnetic field is compensated using the electrostatic technique with $\Delta V_1 = +7.1$ kV and $\Delta V_2 = -9.4$ kV. The horizontal dimension of the focal spot dimension is slightly enlarged, but the shape is still rectangular. There is also a small change (< 0.5 mm) in focal spot position in the $-x$ -direction and a 29% decrease in emission efficiency [Fig. 12(a)].

The cardiac and cardiac oblique (1) system geometries present less challenging magnetic field conditions than the neuro geometry. Hence, the external correction coils are not required, and a purely electrostatic correction is performed. The bias voltages used for the cardiac geometry are $\Delta V_1 = +9.3$ kV and $\Delta V_2 = -12.3$ kV, whereas the bias voltages used for the cardiac oblique (1) geometry are $\Delta V_1 = +4.5$ kV and $\Delta V_2 = -6.1$ kV. The simulated trajectories and simulated focal spot for the cardiac geometry are shown in Figs. 11(b) and 12(b). The results for the cardiac oblique (1) geometry are similarly displayed in Figs. 11(c) and 12(c). For both geometries, the horizontal focal spot dimension is slightly enlarged but the shape is still rectangular. There is no significant deflection in the x - or y -directions of the focal spot position, and both cases have a 20% decrease in emission efficiency.

FIG. 11. (a) Electron trajectories plotted for neuro system geometry (88.1 mT y , -7.9 mT z) with $\Delta V_1 = +7.1$ kV and $\Delta V_2 = -9.4$ kV. External correction coils are present shielding 57.5 mT along y . (b) Trajectories for the cardiac system geometry (40.1 mT y , 8.2 mT z) with $\Delta V_1 = +9.3$ kV and $\Delta V_2 = -12.3$ kV. (c) Trajectories for the cardiac oblique (1) system geometry (19.7 mT y , 2.8 mT z) with $\Delta V_1 = +9.3$ kV and $\Delta V_2 = -12.3$ kV. (d) Trajectories for the cardiac oblique (2) system geometry (76.2 mT y , 72.5 mT z) with $\Delta V_1 = +15$ kV and $\Delta V_2 = -20$ kV.

All geometries above have only included weak longitudinal magnetic fields (i.e., the magnitude of the magnetic field in the z -direction has been < 10 mT). The cardiac oblique (2) system geometry has a much larger longitudinal magnetic field (72.5 mT along z), along with a strong radial magnetic field (76.2 mT along y). An electrostatic correction is attempted for these magnetic field conditions with the values of ΔV_1 and ΔV_2 held to +15 and -20 kV, respectively, so an offset in the x -direction is expected [Fig. 11(d)]. The results demonstrate a focal spot size

change with the vertical dimension elongated from 1.0 to 1.3 mm, and the horizontal dimension contracted from 0.7 to 0.5 mm. The change in shape is also more pronounced than any of the previous cases. There is a deflection of 3.5–5 mm in the $+x$ -direction, and a small deflection (< 0.5 mm) in the $-y$ -direction [Fig. 12(d)]. Furthermore, there is a decrease in the emission efficiency by approximately 28%. A summary of the magnetic field conditions, bias voltages, focal spot changes, and emission efficiencies are presented in Table III.

FIG. 12. (a) Simulated focal spot for the trajectories in Fig. 11(a). The focal spot has a rectangular shape with dimensions of 1.0×0.9 mm and no significant deflection. (b) Simulated focal spot for the trajectories in Fig. 11(b). The focal spot shape is rectangular with dimensions of 1.0×0.9 mm with no significant deflection. (c) Simulated focal spot for the trajectories in Fig. 11(c). The focal spot shape is rectangular with dimensions of 1.0×0.8 mm with no significant deflection. (d) Simulated focal spot for the trajectories in Fig. 11(d). The shape and size of the focal spot have noticeably changed, and there is a deflection in x that is greater than 4 mm along with a minor deflection in y .

TABLE III. Summary of trajectory and focal spot results across all simulations.

Scenario	B_z (mT)	B_y (mT)	ΔV_1 (kV)	ΔV_2 (kV)	Ext. coil (mT)	F.S. shift (mm)	F.S. shape	Efficiency (%)
Control without B [Figs. 9(a) and 10(a)]	0.0	0.0	-0.5	+0.5	N/A	0.0	Rectangular	100
Control with B [Figs. 9(b) and 10(b)]	2.8	19.7	-0.5	+0.5	N/A	2.7	Parallelogram	90
Maximum B [Figs. 9(c) and 10(c)]	0.0	65.0	+15	-20	N/A	0.6	Parallelogram	68
Incomp. correction [Figs. 9(b) and 10(d)]	0.0	76.2	+15	-20	N/A	2.4	Parallelogram	70
Neuro [Figs. 11(a) and 12(a)]	-7.9	88.1	+7.1	-9.4	-57.5	0.2	Rectangular	71
Cardiac [Figs. 11(b) and 12(b)]	8.2	40.1	+9.3	-12.3	N/A	0.1	Rectangular	80
Cardiac oblique 1 [Figs. 11(c) and 12(c)]	2.8	19.7	+4.5	-6.1	N/A	0.2	Rectangular	80
Cardiac oblique 2 [Figs. 11(d) and 12(d)]	72.5	76.2	+15	-20	N/A	4.1	Parallelogram	72

3.C. Monte Carlo results

The simulation for each phase space took ~ 48 h using a single PC (single core Pentium 4, 3.2 GHz, 2 GB RAM) with N chosen to be 1.2×10^8 for each phase space. For this value of N , there are $\sim 3 \times 10^5$ x-ray photons detected for each phase space. The comparison of the x-ray photon energy spectra created from the control and modified electron phase space is shown in Fig. 13. The control phase space is generated from the trajectories shown in Fig. 9(a), while the modified phase space is generated from the trajectories shown in Fig. 9(b). The shapes of the two energy spectra are nearly identical. The mean energies of the spectra are also similar ($33.84 \text{ keV} \pm 0.6\%$ for the control phase space versus $33.59 \text{ keV} \pm 0.7\%$ for the modified phase space). The photon output of the modified phase space spectrum is reduced by approximately 7.5% relative to the control phase space spectrum.

4. DISCUSSION

The results from the electric field map demonstrate that the proposed electrostatic design introduces an electric field component in the x -direction as proposed. The macroscopic electric field strengths in the design are below the dielectric breakdown thresholds. There is concern about localized electric field strengths, such as the $21.4 (\pm 18\%) \text{ kV/mm}$ seen at

the protrusions of the filament mounting assembly. Whether or not these localized electric field strengths will cause a dielectric breakdown must be tested in an experimental setting.

The trajectory results shown in Figs. 9(a)–9(d) demonstrate that to completely correct for larger magnetic field strengths ($B_y > 65 \text{ mT}$) the potential differences for the bias electrodes must be increased. Increasing the bias voltages would lead to problems with dielectric breakdown. Furthermore, as the potential differences are increased, the trajectories gain even more curvature drawing the electrons closer to the lower biasing electrode, and eventually causing the electrons to collide with the electrode surface. Having any fraction of the x-ray tube current not intersect the anode is undesirable because this leads to the loss of potential x-ray photons that can be used for imaging as well as potentially damaging the electrode at the point where the trajectories intersect. There is a concern since the melting point of 316-grade stainless steel (1400°C) is substantially lower than that of tungsten (3400°C), which is the anode target material. However, any electron that collide with the bias electrode will only have a fraction of the kinetic energy relative to the electrons that collide with the anode because they have not accelerated through the entire electric field between the cathode and anode.

As shown in Figs. 9(b) and 9(d), trajectories that are not corrected or have incomplete correction will have deflected focal spots with changes in shape and size. Bracken *et al.*⁴⁰ addressed the question of how much focal spot deflection is significant. With a standard fluoroscopy x-ray tube collimator, it was demonstrated that deflections of 3 mm in the focal spot produce a 6 mm field of view shift on the detector, and deflections of 5 mm or greater were necessary for field of view truncation. Field of view shifts can be accommodated for using a precalibration procedure such that the true field of view can be known *a priori* based on the x-ray tube position. The change in shape (i.e., from rectangular to parallelogram) occurs because the anode has a 12° angle. In general, conventional x-ray tubes will all have anode angles in this range to take advantage of the line focusing principle to reduce the effective focal spot size. In this simulation, the anode angle causes the electrons at the top of the focal spot travel $\sim 1 \text{ mm}$ further in z relative to the electrons at the bottom of the focal spot. The extra travel distance leads to longer time

FIG. 13. A comparison of the x-ray photon spectra simulated using a Monte Carlo model with the control phase space generated from the trajectories in Fig. 9(a), and the modified phase space generated from the trajectories in Fig. 9(b).

of flights during which the external magnetic field causes additional deflection in the x -direction (i.e., longer time for $\vec{F}_B$ to act on the electrons). Significant changes in focal spot size could have an effect on the resolving power of the x-ray system, while changes in focal spot shape could introduce local changes in the modulation transfer function (MTF) that would need to be measured experimentally.

Using these findings as a frame of reference, the trajectory results for the neuro, cardiac, and cardiac oblique (1) geometries have negligible deflections and would not require correction for field of view shift. However, the cardiac oblique (2) geometry does have a significant deflection (4.1 mm) that would require correction. Furthermore, the changes in shape and size are only a concern for the cardiac oblique (2) case, demonstrating that a limitation of the electrostatic technique is the presence of strong longitudinal magnetic fields. In the cardiac oblique (2) geometry, there are strong magnetic field components in the y - and z -directions along with electric field components in the x - and z -directions. The presence of the z component of the magnetic field reduces the effectiveness of the correction from the x component of the electric field, because the electrons prefer to stay aligned with the z -axis. The electron trajectories do not deflect past -2 mm on the x -axis as they are emitted from the filament and have 3.5–5 mm of deflection in $+x$ when they reach the anode. The preference of the electrons to stay aligned with z -axis also causes the trajectories to be bunched together reducing the x -dimension of the focal spot. These changes in focal spot shape and size could change the resolving power and alter the MTF of the x-ray system.

One approach to the problem of correcting for strong magnetic fields in both the radial and longitudinal directions is to use magnetostatic correction in the radial direction. This solution can work reasonably well but does not offer perfect correction, as there is an offset in the focal spot position and an alteration in the focal spot shape.²⁷ Another possible approach would be to use the hybrid magnetostatic/electrostatic technique as done for the neuro system geometry. The advantages to this approach are that it requires lower current densities in the correction coils because they have to shield only a fraction of the external fringe field, hence alleviating the heating concerns. For the cardiac oblique (2) geometry, the magnitude of the bias voltages for the electrodes would need to be empirically determined. The voltages likely need to be larger than if no longitudinal field was present, because the longitudinal magnetic field reduces the effectiveness of the electric field created by the bias electrodes. However, the voltages would not need to be as large as a purely electrostatic approach. The major disadvantage of a hybrid approach is that the extra weight and torque from the shielding coils still need to be accounted for in the design of the C-arm, and the x-ray tube housing would need to be redesigned.

For all proposed system geometries, there is a 20%–29% decrease in efficiency of electron emission from the filament.

This could decrease the image quality of the x-ray imaging system as the signal to noise ratio depends on the number of x-ray photons generated by the x-ray tube, which is directly related to the number of electrons striking the anode. The decrease in efficiency occurs because the presence of the electric field created by the bias electrodes perturbs the thermionic emission process of the electrons at the filament. This problem can be corrected in practice by simply delivering a higher current value to the filament based on the expected decrease in emission efficiency. This would ensure the same electron current density would strike the anode relative to the control case. Finally, as seen in Fig. 13, the electrostatic correction technique does have an effect on the x-ray energy spectrum, with the corrected trajectories being more inefficient in the production of x-ray photons relative to the control case. It is hypothesized that larger incidence angles increase the path length traveled by the x-ray photons in the tungsten target thus increasing the self-absorption and reducing the output efficiency. This limitation is not as easily correctable as the decrease in electron emission from the filament because it is tied to the x-ray production process, but the decrease in efficiency is not sufficient to make the technique impractical. Experimental validation of the change in efficiency is a necessary step for future work.

5. CONCLUSIONS

An electrostatic correction technique is described for x-ray tubes operating in the fringe field of an MR system. The results from the finite element and Monte Carlo simulations validate that the proposed design is feasible for the neuro, cardiac, and cardiac oblique (1) hybrid system geometries. The technique suffers from limitations when there are strong magnetic fields in both the radial and longitudinal direction as seen in the cardiac oblique (2) geometry, but using a hybrid magnetostatic/electrostatic scheme can potentially overcome these limitations. Further work is required to experimentally validate the design, and to empirically determine when dielectric breakdown occurs for the split focusing cup geometry.

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