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Does Early Algebraic Reasoning Differ as a Function of Students' Difficulty with Calculations versus Word Problems?

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Abstract

According to national mathematics standards, algebra instruction should begin at kindergarten and continue through elementary school. Most often, teachers address algebra in the elementary grades with problems related to solving equations or understanding functions. With 789 2nd-grade students, we administered (a) measures of calculations and word problems in the fall and (b) an assessment of pre-algebraic reasoning, with items that assessed solving equations and functions, in the spring. Based on the calculation and word-problem measures, we placed 148 students into 1 of 4 difficulty status categories: typically performing, calculation difficulty, word-problem difficulty, or difficulty with calculations and word problems. Analyses of variance were conducted on the 148 students; path analytic mediation analyses were conducted on the larger sample of 789 students. Across analyses, results corroborated the finding that word-problem difficulty is more strongly associated with difficulty with pre-algebraic reasoning. As an indicator of later algebra difficulty, word-problem difficulty may be a more useful predictor than calculation difficulty, and students with word-problem difficulty may require a different level of algebraic reasoning intervention than students with calculation difficulty.

One of the five content strands for students across kindergarten through twelfth grade of the *Principles and Standards for School Mathematics* is algebra (National Council of Teachers of Mathematics, 2000). Algebraic reasoning is also a *Common Core* domain area in the elementary grades, an integral part of several domain areas in middle school, and one of six important areas of study for high school students (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). As the standards of these national organizations press for algebra instruction for all students across elementary, middle, and high school, as more states require all students to take and pass algebra courses in high school (Stein, Kaufman, Sherman, & Hillen, 2011), and given that many secondary students, including students with disabilities, are unprepared for algebra courses (Faulkner, Crossland, & Stiff, 2013; National Mathematics Advisory Panel, 2008), the elementary school curriculum needs to emphasize algebraic reasoning more than ever.

The widespread adoption of the *Common Core* indicates that districts are amending curricula to reflect the standards outlined in the *Common Core*. The *Common Core* emphasizes algebraic reasoning in the elementary grades with standards listed under the “Operations and Algebraic Thinking” domain. For example, at kindergarten, when given a number 1 through 9, students have to determine the other number (i.e., unknown) that adds to 10. In first grade, students solve addition and subtraction equations with a unknown (e.g., $8 + ? = 11$; $5 = + 3$). In grades 1, 2, 3, and 4, students use equations with a symbol for the unknown to represent and solve word problems. By grade 3, students solve multiplication and division equations with an unknown (e.g., $3 \times = 27$; $9 = 81 \div ?$). In grades 4 and 5, students generate patterns to describe the relationship between sets of numbers (e.g., $n + 3$). In the middle school grades, the *Common Core* standards explicitly related to algebraic reasoning fall under the “Expressions and Equations” domain (grades 6, 7, and 8) and the “Functions” domain (grades 8). Many algebraic reasoning skills, especially those related to problem solving, fall under other domains (e.g., Ratios and Proportional Relationships, Measurement) at grades 6, 7, and 8.

The *Common Core* standards influence statewide assessments, which in turn influence mathematics textbooks. Districts want to purchase curriculum materials aligned with the assessments to provide a cohesive mathematics program that enables students to succeed on the assessment. As more and more districts revise their mathematics curriculum to meet or exceed *Common Core* standards and as all students, including those with learning disabilities, are held accountable on statewide assessments (Yell, Katsiyannis, & Shiner 2006), it is important to understand how students, especially those with mathematics difficulties, interpret and solve algebraic problems.

In this introduction, we explain how most researchers approach algebraic reasoning in the elementary grades. We discuss solving algebraic problems using equations and functions. Then, we describe students with mathematics difficulties; the two main areas (i.e., calculations and word problems) in which they struggle during the elementary grades; and how algebraic reasoning may be connected to calculations and to word problems. Finally, we describe the rationale, purpose, and hypotheses for the present study.

Algebraic Reasoning

In its simplest form, algebraic reasoning is the manipulation of numerals and signs (e.g., $x + 5 = 12 - 4$) to solve for an unknown. Algebra is typically viewed as next step beyond arithmetic (i.e., calculations with addition, subtraction, multiplication, or division) and as the gateway to higher-level mathematics (Stein et al., 2011). Arithmetic and algebraic reasoning share the same numerals and signs, but the manipulation of those numerals and signs differs based on the question or expected outcome (Kieran, 1990). Algebraic reasoning typically requires a greater depth of mathematical thinking than arithmetic, and algebraic reasoning encourages problem solving (Richardson, Berenson, & Staley, 2009). Formal algebra instruction typically begins at eighth or ninth grade (Gamoran & Hannigan, 2000; Stein et al., 2011), but because national standards such as the *Common Core* emphasize algebraic reasoning as early as kindergarten, it is important to determine the vital components of

algebraic reasoning instruction in the elementary grades and how to assess such algebraic reasoning.

Many researchers believe that teaching elementary students to reason algebraically is feasible and sound (Richardson et al., 2009; Schifter, Russell, & Bastable, 2009). The two most common types of algebraic problems presented to elementary students include solving equations with an unknown and describing and working with functions (Carraher, Schliemann, Brizuela, & Earnest, 2006; Ketterlin-Geller, Jungjohann, Chard, & Baker, 2007; Rivera, 2006; Tani li, 2011). This makes sense because equations and functions are the first two topics taught in a typical algebra course (Kieran, 1990). Almost all of the *Common Core* standards related to algebraic reasoning in the elementary grades focus on solving equations in informal and formal settings. Although functions are not a focus of the *Common Core* standards until late elementary school, researchers believe functions should be prevalent in the early elementary grades because functions help students understand early algebraic reasoning concepts (Carraher et al., 2006).

At the elementary level, solving equations with an unknown can be presented to students in several ways. Students may be asked to solve for unknowns through situations using manipulatives (Cobb, 1987). For example, a teacher presents a student with four chips placed on a desk alongside a cup. The teacher asks how many chips are in the cup if there are nine chips total. For solving equations, students can also be presented with an equation (i.e., any number sentence with a relational symbol such as the equal sign [=]). The unknown in the equation may be marked with a line (e.g., $4 + _ = 9$), a question mark (e.g., $4 + ? = 9$), or a box (e.g., $4 + \square = 9$). The unknown, especially in the later elementary grades, may also be notated with a letter (e.g., x or n ; Malisani & Spagnolo, 2009). Any symbol that represents an unknown value, such as a question mark or letter, may also be referred to as a variable. Students solve for the unknown by employing an invented or taught strategy. Solving equations with an unknown helps bridge arithmetic and algebraic reasoning, which are often treated as separate entities (Cavanagh, 2009; Ketterlin-Geller et al., 2007). For example, students are thinking algebraically if they solve the subtraction problem " $11 - 6 =$ " by thinking, "What number can I add to 6 to reach 11?" Students can use invented drawings (Rivera, 2006), counting strategies (Fuchs, Powell, et al., 2009), or manipulatives (Beatty & Moss, 2007) to solve for an unknown.

Functions, by contrast, involve determining the relationship between two sets of numbers. At the elementary level, functions are often described in input/output tables (Schliemann & Carraher, 2002). A "function machine" where a number is put in (i.e., input) and a different number is produced (i.e., output) also help elementary students reason algebraically (Warren, Cooper, & Lamb, 2006). With functions, students determine the relationship between the input and output numbers and often solve for an unknown input or output based on the understanding of the relationship. Function tables with the numbers not presented in sequential order (e.g., input: 4, 8, 6, 2 and output: 6, 10, , 4) better test algebraic reasoning. If the numbers of the function table are in order, students pick up on the sequence or pattern without understanding the relationship of the input and output (Schliemann & Carraher, 2002).

Links Between Algebraic Reasoning and Calculations or Word Problems for Students with Mathematics Difficulties

Mathematics disabilities affect approximately 3 to 6% of the population (Shalev, Auerbach, Manor, & Gross-Tsur, 2000), and many additional students struggle with low mathematics performance without a formal diagnosis of a mathematics disability (often referred to as mathematics difficulty or MD; Martin et al., 2013). Regardless of the distinction between mathematics disability and mathematics difficulty, students with MD require supplementary instruction (Vukovic & Siegel, 2010). As many schools delay identification of a mathematics disability until late elementary school, using the MD category helps identify students early and provide important early intervention (Dowker, 2005). Before the Individual with Disabilities Education Act (IDEA) was amended in 2004, students had to qualify in at least one of two categories (i.e., calculations or problem solving) for identification of a learning disability in mathematics. Although students can now be identified with a mathematics learning disability without demonstrating a deficit specifically in one of these two areas, researchers continue to use the two difficulty categories frequently to study and understand strengths and weaknesses of students with MD (e.g., Andersson, 2008; Hanich & Jordan, 2004).

Several researchers have investigated the performance differences of students who struggle with calculations versus word-problem solving. In a longitudinal study of 314 sets of twins, Hart, Petrill, Thompson, and Plomin (2009) determined unique genetic factors for students who struggle with calculations versus those who struggle with word problems. Calculation skill was not related to reading skill or general cognitive ability, whereas word-problem solving was related to general cognitive ability as well as reading and mathematics skill. Fuchs et al. (2008) came to a similar conclusion in a study with 547 third-grade students. Students with calculation difficulty performed below students with word-problem difficulty on processing speed, whereas students with word-problem difficulty scored below peers with calculation difficulty on language comprehension. These profiles of cognitive and linguistic abilities led Fuchs et al. (2008) to conclude that calculation difficulty and word-problem difficulty are two distinct aspects of mathematical cognition. Moreover, in a longitudinal study of 279 students, Fuchs et al. (2012) examined how calculation and word-problem skill at the beginning of second grade predicted algebraic reasoning performance at the end of third grade. Fuchs et al. (2012) found that calculation skill and word-problem skill predicted algebraic reasoning performance; however, calculation and word-problem development mediated the effects of cognitive and linguistic variables on algebraic knowledge in different ways. Each of these studies demonstrates that calculations and word problems relate to algebraic reasoning but the nature of those relations differ.

Early algebraic reasoning may be challenging, especially in the early grades, for students with MD. Unfortunately, there is a scarcity of research related to students with MD and pre-algebra. Most available studies focus on pre-algebraic reasoning and word problems. Xin et al (2011) provided 9- and 10-year-old students with MD with word-problem instruction. Half received word-problem instruction with a focus on word-problem schema (i.e., problem type), while learning to use equations to represent word problems. Students in the word-

problem schema condition learned to use any letter (e.g., c , x , m) to represent the unknown in the equation. The other half of students learned general problem-solving steps without schema or equations. Students who learned to use equations performed significantly better on word-problem outcomes as well as outcomes related to solving equations and modeling equations from word problems. In a similar way, Powell and Fuchs (2010) provided third-grade students with MD word-problem instruction focused on identifying word problems by schema and using equations to represent the schema. All students learned to use the letter x to represent the unknown in an equation. Half of students received an embedded component focused on understanding the equal sign relationally (i.e., a balance between two sides of an equation). Students who received word-problem instruction demonstrated significantly greater gains on word-problem outcomes than students who did not receive instruction, and students with the embedded equal-sign component demonstrated higher performance on measures of equation solving and equal-sign understanding. Neither of these studies, however, was designed to compare the role of word problems versus calculation in algebra. Together, the small collection of studies indicates that students with MD can reason algebraically in the elementary grades. What is needed, however, is a study investigating whether performance differences on different algebraic tasks vary by calculation or word-problem MD in the early grades.

Rationale, Purpose, and Hypotheses

Even though the NCTM and *Common Core* standards emphasize algebraic reasoning in the elementary grades, available research provides an inadequate basis for understanding the algebraic reasoning of students with MD. That is, is calculation or word-problem MD a better predictor of algebraic reasoning and does calculations or word problems provide a better target for early intervention to better prepare students with MD to face the challenges of algebra? In this study, we investigated whether performance differences on algebraic tasks differ by MD difficulty with calculations versus word problems. The goal was to provide insight into which type of difficulty is more strongly associated with early algebraic reasoning to help schools identify students for early intervention and to help practitioners identify instructional practices that may better support the development of early algebraic reasoning in students with MD.

We hypothesized that students with a history of word-problem MD would find the equations and functions tasks more difficult than students with prior calculation MD. We also hypothesized that students with comorbid MD (i.e., calculation difficulty with concurrent word-problem difficulty) would perform more similarly to students with specific word-problem MD than to students with specific calculation MD. This hypothesis was based on the following reasoning and prior work. First, word problems may involve more complex reasoning about unknown quantities than reasoning for solving calculation problems, which are already set up for solution (e.g., Kieran, 1992; Sfard & Linchevski, 1994). Second, the manipulation of numbers in solving equations and function tables may rely on spatial ability, which is relevant because students with word-problem MD exhibit weaker spatial ability than those with calculation MD (Fuchs et al., 2012). This finding was echoed by Tolar, Lederberg, and Fletcher (2009) who found spatial skill was associated with algebraic competence in a representative sample of college students without disabilities.

In testing our hypothesis, we assessed calculation and word-problem performance in fall of second grade among a large sample of 789 students and then assessed pre-algebraic reasoning in spring of second grade in these same children. We used the fall data to establish, among the 789 students, MD status groups, and we contrasted spring pre-algebraic reasoning as a function of calculation MD ($n = 14$) versus word-problem MD ($n = 23$) versus comorbid MD (i.e., difficulty with calculations and word problems; $n = 21$) versus typical development (i.e., without difficulty in calculations and word problems; $n = 90$). This is necessary for assessing the specific connection among word problems, calculation, and pre-algebraic reasoning in the MD population. However, given potential problems in looking at extreme groups (Preacher, Rucker, MacCallum, & Nicewander, 2005), such as MD categories, we then conducted path analytic mediation analyses to examine whether the connection between pre-algebra reasoning and word problems was also stronger than with calculations in the full, representative sample of 789 children.

Method

Participants

Within an experimental study assessing the efficacy of multi-tier instruction focused on calculation or word problems (Fuchs et al., in press; Powell et al., in press), we assessed 789 second-grade students from 61 classrooms in 12 schools on calculation and word-problem screening measures in the fall. We administered a test of pre-algebraic reasoning in the spring to all 789 students. These 789 students did not participate in multi-tier instruction provided in the larger study (i.e., they had all been randomly assigned to the control group; we excluded children who received intervention because intervention was designed to disrupt the naturally occurring relations among calculations or word problems with pre-algebraic reasoning).

From this representative sample, we identified a subset of 148 students from 15 second-grade classrooms in 11 schools who met our criterion for difficulty with calculations (CD), word problems (WPD), calculations and word problems (C&WPD), or typical (TYP) performance on calculations and word problems. As with the representative sample, these 148 students did not participate in multi-tier instruction of the larger study. Our screening measures were *Addition Facts* (Fuchs, Hamlett, & Powell, 2003) and *Story Problems* (Jordan & Hanich, 2000). Students with CD ($n = 14$) scored 5 or below on calculations and 7 or above on word problems; students with WPD ($n = 23$) scored 7 or above on calculations and 4 or below on word problems; students with C&WPD ($n = 21$) scored 5 or below on calculations and 4 or below on word problems; and TYP students ($n = 90$) scored 7 or above on calculations and 6 or above on word problems. Students scoring a 6 on calculations or 5 on word problems were not included in the final sample because these students scored in a buffer zone, so the final sample included 148 students. For calculations, a raw score of 5 was approximately the 29th percentile, and a raw score of 7 was approximately the 45th percentile. For word problems, a raw score of 4 was approximately the 28th percentile, whereas a raw score of 6 was approximately the 49th percentile. These percentile ranges fall within the ranges (i.e., 25th, 31st, 35th, or 45th percentiles) commonly used for identifying students struggling with MD in the literature (Martin et al., 2013; Mazzocco, 2005).

The 148 students were identified in two cohorts, recruited across two consecutive school years. The pattern of results did not differ based on cohort; therefore, the first ($n = 94$) and second ($n = 54$) cohorts were combined for data analyses. Participants did not differ on demographics (sex, ethnicity, subsidized lunch status, special education status, and English learner status) as a function of difficulty status. See Table 1 for demographic information by difficulty status. Retained status did differ based on difficulty status, $\chi^2(3, N = 148) = 10.76$, $p = .013$, but including retained status in the final model did not alter the pattern of results.

Screening Measures

Addition Facts (Fuchs et al., 2003) comprises 25 addition fact problems with sums 0 to 12. Students have 1 min to write answers. The examiner instructs students to work easy problems first and then go back and try the harder problems. The student's score is the number of correct answers. Coefficient alpha on this sample was .90. The 25 addition fact problems break down so one addend is always less than 6: one +0/0+ problem, two +1/1+ problems, seven +2/2+ problems, seven +3/3+ problems, five +4/4+ problems, two +5/5+ problems, and one double (6 + 6). *Addition Facts* has been utilized in a number of research projects at first, second, and third grade as a screening and outcome measure (Fuchs et al., 2006; 2008; 2010; Powell, Driver, & Julian, in press; Powell et al., in press).

Story Problems (Jordan & Hanich, 2000) comprises 14 additive word problems with single-digit sums or minuends (e.g., Dennis has 7 pennies. Molly has 5 pennies. How many pennies does Dennis have more than Molly?). Each story problem is read aloud by the examiner, and students write answers before the examiner moves to the next problem. The score is the number of correct answers. Coefficient alpha on this sample was .83. The 14 word problems reflect the most common additive word-problem schema in the elementary grades (Jitendra et al., 1998; Riley & Greeno, 1988), and the breakdown by schema is: 2 combine problems, 4 compare problems, 6 change problems, and 2 equalize problems. Similar to *Addition Facts*, *Story Problems* has been used as a screening and outcome measure at first, second, and third grade (Fuchs, Powell, et al., 2009; Hanich, Jordan, Kaplan, & Dick, 2001).

Algebraic Reasoning Measures

The *Test of Pre-Algebraic Reasoning* (TPAR; Fuchs, Seethaler, & Powell, 2009) comprises two subtests: *Equations* and *Functions*. We understand that pre-algebraic reasoning encompasses more than manipulating symbols to solve for an unknown, but we selected these two subdomains of early algebraic reasoning because they align with the *Common Core* across grades K through 5. Also, equations and functions, of the types included on this measure, are the two most common algebraic reasoning tasks presented to elementary students (Ketterlin-Geller et al., 2007).

On *Equations*, students solve for a variable (i.e., x or y) on 20 equations. See Figure 1 for test items. Six equations are standard with the equal sign in a standard position (e.g., $y + 4 = 9$); 14 equations are nonstandard (e.g., $5 - 2 = y - 1$; $6 = 2 + x$). Students solve for x on half of the equations and for y on the other half. Nine problems use a plus sign, and nine use a minus sign. Two problems involve plus and minus signs. After working through a sample

problem with the examiner, students have 8 min to solve as many equations as possible. The score is the number of correct answers. Coefficient alpha for this sample was .87.

With *Functions*, students fill in the missing amount in a function table. The functions within each table are not presented in numerical order. See Figure 2 for test items. Students work through a sample problem with the examiner and then work independently on six problems. The score is the number of correct answers. Coefficient alpha for this sample was .71.

We understand second graders are not frequently asked to solve equations and functions with the unknown marked with x or y , but we felt comfortable using x and y for three reasons. First, students at second grade are typically unfamiliar with the multiplication sign ($\times$), which could be confused with x , because multiplication instruction typically begins in third grade as outlined by the *Common Core* and the curriculum of this sample's school district (Greenes et al., 2005). Second, as part of test administration, the examiner worked an example of solving for x with an equation and filling in the missing box within function table. Third, in other studies, elementary students learned to use a variable (e.g., x) to represent an unknown and to solve for the unknown with little difficulty (e.g., Fuchs et al., 2010; Hewitt, 2012; Jitendra et al., 2007; Powell & Fuchs, 2010; Powell et al., in press; Xin et al., 2011).

Procedure

The two screening measures were administered during the first two weeks of September. Testing occurred in one 45-min whole-class testing session. Additional measures were administered during the testing session, but the order of administration was the same across all classrooms in that *Addition Facts* was administered first and *Story Problems* was administered second. Two examiners conducted each session. One examiner read verbatim from a test script while the other examiner walked around the room to answer questions, ensure students were working on the correct page, and make sure students had sharpened pencils. Examiners were research assistants from a local university. All examiners were working or had finished graduate degrees in education-related fields. Examiners were trained to administer the tests following the same testing procedures and read from the same testing scripts.

The *TPAR* was administered during the first two weeks of March. Testing occurred in one 45-min whole-class testing session. Additional measures were administered during the testing session, but the order of administration was the same across all classrooms. Similar to the screening measures, two examiners conducted each testing session. One examiner read verbatim from a test script while the other walked around the room to answer questions, ensure students were working on the correct page, and make sure students had sharpened pencils. Examiners were research assistants from a local university, all of whom were working on or had finished graduate degrees in education-related fields. Examiners were trained to administer the tests following the same testing procedures and read from the same testing scripts.

Data Analysis

A MANOVA was applied to the two *TPAR* subtests, *Equations* and *Functions*, with difficulty status (TYP, CD, WPD, or C&WPD) as the factors. The raw scores for each subtest were used as the dependent variable. Post-hoc tests of least significant differences were run to determine which difficulty status group, if any, demonstrated significantly greater scores on the *TPAR* subtests. Effect sizes (ESs) were calculated by subtracting the means and dividing by the pooled *SD* (g ; Hedges & Olkin, 1985) as suggested by the What Works Clearinghouse (2011).

To examine whether the pattern of effects based on the MD status groups would be corroborated in the full sample (i.e., to address concerns about analyses on extreme groups, as discussed in Preacher et al., 2005), we conducted two path analytic mediation analyses, as follows. In the analysis, the independent variable was the fall calculation score in the first analysis and the fall word-problem score in the second analysis; the mediator was the fall word-problem score in the first analysis and the fall calculation score in the second analysis; and the dependent variable in both mediation analyses was the spring *TPAR* score. For a variable to be a mediator, four conditions must be met. First, the independent variable must be associated with the dependent variable (path c). The c path establishes there is an effect to mediate. Second, the independent variable must be associated with the mediator (path a); this establishes the action theory involved in mediation. Third, the mediator must also affect the dependent variable when the independent variable is controlled in the model (path b). (Path c' is the effect of the dependent variable on the independent variable when the mediator is included in the model.) Fourth, the mediator (or indirect) effect, which is the product of paths a and b , must be significant. This is equivalent to testing whether adding the mediator changes the relation between the independent and dependent variable. In our first mediation analysis, we assessed whether word-problem ability mediated the relation between calculation skill and pre-algebraic reasoning; in the second mediation analysis, whether calculation skill mediated the relation between word-problem ability and pre-algebraic reasoning. Based on the potential connections between calculations and word problems with pre-algebra, but with the possibility that word problems has a stronger connection than calculations, we expected both mediation effects to be significant, but expected word-problem ability to be a stronger mediator of the relation between calculation skill and pre-algebraic reasoning than would be the case for calculation skill as a mediator of the relation between word-problem ability and pre-algebraic reasoning. In these analyses, we relied on a total *TPAR* score (results were analogous when run separately on the *Equations* versus *Functions* subtests).

Results

See Table 2 for screening and *TPAR* scores as a function of difficulty status. As expected, because of the nature of using the screening measures for determining student difficulty status, there were group differences on *Addition Facts*, $F(3, 147) = 54.23, p < .001$, and *Story Problems*, $F(3, 147) = 93.40, p < .001$. Follow-up tests indicated that TYP and WPD students performed significantly above CD and C&WPD students on *Addition Facts*. On

Story Problems, TYP and CD students performed significantly higher than WPD and C&WPD students.

On *TPAR*, there were differences among difficulty groups, Wilks' $\lambda = .811$, $F(6, 286) = p < .001$. On *Equations*, the effect of difficulty status was significant, $F(3, 147) = 7.78$, $p < .001$. Follow-up tests indicated that TYP students outperformed WPD and C&WPD students with ESs of 0.71 and 0.96, respectively. Similarly, CD students demonstrated superior performance over WPD students (ES = 0.66) and C&WPD students (ES = 0.91). Interestingly, TYP and CD students performed comparably (ES = 0.16) as did WPD and C&WPD students (ES = 0.36).

An identical pattern emerged on the *Functions* subtest, $F(3, 147) = 5.49$, $p < .001$. Follow-up tests showed TYP students performed significantly higher than WPD students (ES = 0.55) and C&WPD students (ES = 0.77). CD students also outperformed WPD and C&WPD students with ESs of 0.84 and 1.20, respectively. There were no significant differences between TYP and CD students (ES = 0.11) or WPD and C&WPD students (ES = 0.34).

See table 3 for path coefficients for the mediation analysis with word-problem skill as the mediator. In path c , calculation skill was significantly related to *TPAR* performance, and path a shows calculation skill was significantly related to word-problem skill. With path b , word-problem skill was significantly related to *TPAR* performance (with calculation skill controlled), and the mediation (indirect) effect was also significant. Therefore, the word-problem mediator reduced the direct effect of calculation skill on *TPAR* performance from .35 (c , without the mediator in the model) to .10 (c' , with the mediator in the model), a reduction of 71.4% $([.35 - .10]/.35)$.

See Table 4 for path coefficients for the mediation analysis with word-problem ability as the mediator. Word-problem skill was significantly related to *TPAR* performance (path c), and word-problem skill was significantly related to calculation skill (path a). In path b , when word-problem skill was controlled, calculation skill was significantly related to *TPAR* performance, and the mediation effect was significant. Calculation as mediator reduced the direct effect of word-problem skill on *TPAR* performance from .58 (c) to .53 (c'), a reduction of 8.6% $([.58 - .53]/.58)$.

Discussion

The purpose of this study was to examine whether the algebraic reasoning of students with MD differs as function of whether their history of difficulty is with calculations versus word problems. Toward that end, we recruited 148 students at the start of second grade and administered screening measures to classify students in terms of calculation difficulty without word-problem difficulty (CD), word-problem difficulty without calculation difficulty (WPD), calculation and word-problem difficulty (C&WPD), or no difficulty (i.e., typical performance; TYP). We later assessed students, in spring of that academic year, on a measure of algebraic reasoning. The assessment included two subtests related to algebraic reasoning required of elementary students in the *Common Core*. First, students solved

equations with x or y as the unknown. Second, students completed unknowns in function tables.

We found that CD and TYP students significantly outperformed WPD and C&WPD students on both types of algebraic reasoning (i.e., *Equations* and *Functions*). CD and TYP students performed comparably, and both groups of students with word-problem difficulty, WPD as well as C&WPD, performed comparably. ESs favoring CD students over WP and C&WPD students ranged from 0.66 to 1.20. ESs favoring TYP students over WP and C&WPD students ranged from 0.55 to 0.96. Adding support to these findings, the path analytic mediation analyses indicated that word-problem skill as a mediator reduced the direct effect of calculation skill by 71.4%, whereas calculation skill as a mediator reduced the direct effect of word-problem skill by only 8.6%. The large reduction favoring word-problem skill as a mediator indicates that word-problem skill influences *TPAR* performance dramatically more than calculation skill influences *TPAR* performance, thereby corroborating the MD status analyses. Although no experimental work exists on the algebraic reasoning of elementary students with MD, the present study's findings support our hypothesis that students with word-problem difficulty, whether alone or concurrent with calculation difficulty, experience greater challenges with equations and function tables.

Findings add to the literature on mathematics development and MD in several important ways. First, elementary students can solve algebraic problems. Without prior instruction and with only brief examples on the assessment, students, including those with MD, successfully solved at least some equations and functions items. This is encouraging, especially as algebraic standards become more prevalent in the elementary grades. It indicates that elementary age students can think algebraically and reinforces the notion that arithmetic and algebraic reasoning are connected, as per Fuchs et al. (2012). It also suggests that with effective instruction, these students may solve even more of these items correctly, thereby paving the way to stronger algebra performance later in school.

Second, performance differences clearly emerged as a function of students' type of difficulty. Students with word-problem difficulty (whether alone or combined with calculation difficulty) answered fewer algebraic reasoning problems correctly than students without word-problem difficulty. By contrast, algebraic reasoning was not related to calculation difficulty. This trend held for both solving items related to equations and functions. Results therefore indicate an association between word problems and algebraic reasoning. Findings also suggest that screening students for word-problem difficulty may provide a means to identify students for early algebra intervention, as early as second grade. Results also indicate that word-problem instruction, which incorporates algebra equations and stresses the algebraic nature of word problems, may be used productively to simultaneously remediate or prevent difficulty with word problems and algebraic reasoning and that this form of early intervention may be more productively allocated to students with word-problem difficulty than to students with specific calculation difficulty.

This finding also underscores the notion that not all students with MD struggle with solving algebraic problems in the same way. In fact, students with calculation difficulty performed comparably to students without MD. This result indicates that an undifferentiated approach

to teaching students with MD is not the most viable instructional model for students with MD. Instead, before instruction begins, a thorough assessment of the mathematical strengths and weaknesses of students should be conducted to determine which areas require instruction and which types of instruction may benefit students the most. We caution, however, that whereas such assessment may generate instructional hypotheses for teachers to pursue, progress monitoring to determine the effects of resulting programs must be collected to determine student response to instruction (Stecker, Fuchs, & Fuchs, 2008).

Limitations

Before concluding, we note our study's limitations. First, we only administered one screening calculation measure, only one word-problem screening measure, and only one pre-algebraic reasoning measure. Although a one-time assessment to screen for mathematics difficulty is common in similar research projects (e.g., Bryant, Bryant, Gersten, Scammacca, & Chavez, 2008; Powell & Fuchs, 2010), administering multiple forms of the screening measures would improve the reliability of the CD, WPD, or C&WPD designations. Similarly, administering multiple forms of the *TPAR* would provide the basis for a more reliable, composite score on pre-algebraic reasoning. Even so, readers should note that the one-form *TPAR* provided high reliability coefficients and thus appears to represent a reliable index of the pre-algebraic reasoning of second-grade students.

A second limitation is that we screened students for mathematics difficulty in September of the school year and administered the *TPAR* the following March. While MD remains fairly constant, a few students may have significantly increased or decreased their mathematics performance in ways that could have changed their assigned condition. The goal in the present study, however, was to examine predictive relations, which requires a gap in time between diagnosis and outcome. Also, to minimize reliability problems in diagnoses, we set stringent cut-points for determination of CD or WPD and employed a buffer zone between students with CD and/or WPD and typical-performing students. A third limitation is that we did not counterbalance the screening measures, and future related work should incorporate this design feature. Finally, on the *Equations* and *Functions* subtests of the *TPAR*, we used the letters x and y to represent unknown quantities. In future work, we would like to vary the use of letter variables with other markers for variables (e.g., $?$, $?$, $[]$) to determine the effect of variables on the equation and function solving of second-grade students.

Future Directions

Future research needs to investigate why students with word-problem difficulty experience greater challenges with algebraic items with missing values. Perhaps there are elements to solving algebraic problems, like visual and spatial reasoning, that contribute to difficulty with conceptualizing and understanding algebraic problems in the form of equations or functions (Fuchs et al., 2012; Swanson & Jerman, 2006). Geary, Hoard, Byrd-Craven, and DeSoto (2004) hypothesized that students with word-problem difficulty may struggle because this type of mathematics problem requires organizing and completing multiple steps. Similarly, solving equations and function tables are multi-step processes. Of course, both domains also make high demands on working memory and possible spatial reasoning

(Lee, Ng, Ng, & Lim, 2004; Passolunghi & Cornoldi, 2000; Swanson & Beebe-Frankenberger, 2004). Only future research can disentangle the sources of difficulty common to both domains.

Other possible lines of research include determining how to teach algebraic principles and understanding to students with MD as elementary teachers often feel unprepared to teach algebraic principles in elementary school (Stephens, 2008). Several researchers have suggested approaches for teaching algebra to middle and high school students with MD (Impecoven-Lind & Foegen, 2010; Strickland & Maccini, 2010; Witzel, Mercer, & Miller, 2003), and some studies provide suggestions for teaching algebra at the elementary level (Hewitt, 2012; Lee, 2010; Ormond, 2012). Yet, we know little about the efficacy of algebra instruction at the elementary level. The present study shows that elementary students with MD can solve simple algebra problems. The next step is developing and determining which practices may enhance the performance of students with MD, especially those with word-problem difficulty, on solving equations and functions.

Implications for Students with MD

As early MD is a strong predictor for MD in the later grades (Jordan, Glutting, Ramineni, & Watkins, 2010; Mazzocco & Thompson, 2005), it is necessary to identify students with MD and provide appropriate, targeted instruction for this subset of students (Dowker, 2005). As algebraic reasoning has become an integral part of elementary mathematics curricula, it is necessary to understand how calculation MD may differ from word-problem MD and how these differences may affect algebraic reasoning. Therefore, when teachers provide instruction to students with MD, it is important to understand whether the MD is rooted in calculation difficulty, word-problem difficulty, or both. Based on the results from this study, students with calculation MD (without word-problem MD) may not require as intensive instruction on algebraic tasks such as solving equations and function tables as students with word-problem MD (combined with calculation MD or alone). The most efficient and effective routes for providing elementary algebra instruction, however, still need to be determined.

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Solving Equations

Example:			
$2 + 3 = X$		$X = \underline{5}$	
$1 + 5 = X$	$X = \underline{\hspace{1cm}}$	$Y + 4 = 9$	$Y = \underline{\hspace{1cm}}$
$8 - 3 = Y$	$Y = \underline{\hspace{1cm}}$	$X = 8 - 2$	$X = \underline{\hspace{1cm}}$
$6 = 2 + X$	$X = \underline{\hspace{1cm}}$	$Y = 1 + 7$	$Y = \underline{\hspace{1cm}}$
$3 = Y - 4$	$Y = \underline{\hspace{1cm}}$	$4 + 3 = 9 - X$	$X = \underline{\hspace{1cm}}$
$1 + 5 = 4 + X$	$X = \underline{\hspace{1cm}}$	$8 - 4 = 6 - Y$	$Y = \underline{\hspace{1cm}}$
$5 - X = 3$	$X = \underline{\hspace{1cm}}$	$5 = Y + 4$	$Y = \underline{\hspace{1cm}}$
$2 + Y = 7$	$Y = \underline{\hspace{1cm}}$	$8 - X = 3 + 3$	$X = \underline{\hspace{1cm}}$
$1 = 8 - X$	$X = \underline{\hspace{1cm}}$	$Y + 4 = 5 + 2$	$Y = \underline{\hspace{1cm}}$
$5 - 2 = Y - 1$	$Y = \underline{\hspace{1cm}}$	$X - 1 = 2$	$X = \underline{\hspace{1cm}}$
$3 + 4 = X + 5$	$X = \underline{\hspace{1cm}}$	$3 - Y = 4 - 2$	$Y = \underline{\hspace{1cm}}$

Figure 1.
TPAR Equations Subtest

Functions

Example:

X	X + 1
5	6
2	3
8	9
6	7

X	X + 3
8	11
1	4
7	
3	6

Y	3Y
7	21
4	12
10	30
5	

Y	Y - 6
11	5
7	1
12	
14	8

X	2X + 1
3	
9	19
7	15
2	5

X	X + 4 - 2
8	10
5	
12	14
2	4

Y	1/2Y
16	8
18	
12	6
8	4

Figure 2.
TPAR Functions Subtest

Table 1

Student Demographics

Variable	Typical (<i>n</i> = 90)		CD (<i>n</i> = 14)		WPD (<i>n</i> = 23)		C&WPD (<i>n</i> = 21)	
	<i>n</i>	(%)	<i>n</i>	(%)	<i>n</i>	(%)	<i>n</i>	(%)
Ethnicity								
African American	29	(32.2)	3	(21.4)	8	(34.8)	7	(33.3)
Caucasian	28	(31.1)	5	(35.7)	7	(30.4)	7	(33.)
Hispanic	20	(22.2)	5	(35.7)	6	(26.1)	4	(19.0)
Other	13	(14.4)	1	(7.1)	2	(8.7)	3	(14.3)
Males	43	(47.8)	5	(35.7)	7	(30.4)	11	(52.4)
Subsidized lunch	71	(78.9)	12	(85.7)	17	(73.9)	18	(85.7)
School-identified disability	4	(4.4)	1	(7.1)	2	(8.7)	2	(9.5)
English learners	11	(12.2)	4	(28.6)	4	(17.4)	5	(23.8)
Retained	3	(3.3)	0	(0.0)	4	(17.4)	4	(19.0)

Table 2

Screening and Pre-Algebraic Reasoning Data

Variable	<u>Typical (<i>n</i> = 90)</u>		<u>CD (<i>n</i> = 14)</u>		<u>WPD (<i>n</i> = 23)</u>		<u>C&WPD (<i>n</i> = 21)</u>	
	<i>M</i>	(<i>SD</i>)	<i>M</i>	(<i>SD</i>)	<i>M</i>	(<i>SD</i>)	<i>M</i>	(<i>SD</i>)
Screening Measures								
Addition Facts	12.67	(4.30)	3.93	(1.33)	10.39	(2.52)	3.33	(1.39)
Story Problems	9.3	(2.52)	7.79	(1.63)	3.13	(1.01)	2.38	(1.24)
Pre-Algebraic Reasoning Measure								
Equations	8.86	(4.42)	8.14	(4.13)	5.87	(2.83)	4.76	(3.27)
Functions	1.28	(1.45)	1.43	(1.28)	0.52	(0.90)	0.24	(0.70)

Table 3

Path Analytic Mediation Analysis with Word-Problem Skill as Mediator

Path	Coefficient	SE	<i>t</i>	<i>p</i>
<i>c</i> : Calculation skill on <i>TPAR</i>	.35	.03	10.58	<.001
<i>a</i> : Calculation skill on word-problem skill	.47	.03	14.96	<.001
<i>b</i> : Word-problem skill on <i>TPAR</i> with calculation skill controlled	.53	.03	16.08	<.001
<i>c'</i> : Calculation skill on <i>TPAR</i> with mediator in the model	.10	.03	3.19	.002

Indirect effect (<i>a*b</i>)				
Effect	SE	<i>t^a</i>	<i>p^a</i>	Bootstrapped 95% CI ^b
Indirect effect via word-problem skill	.25	.02	16.08	<.001
				.2111 to .2860

^a *t* value and *p* are for Total Effect.^b Bootstrapped 95% CI is for indirect effects.

Table 4

Path Analytic Mediation Analysis with Calculation Skill as Mediator

Path	Coefficient	SE	t	p
c: Word-problem skill on <i>TPAR</i>	.58	.03	19.81	<.001
a: Word-problem skill on calculation skill	.47	.03	14.96	<.001
b: Calculation skill on <i>TPAR</i> with word-problem skill controlled	.10	.03	3.19	.002
c': Word-problem skill on <i>TPAR</i> with mediator in the model	.53	.03	16.09	<.001

Indirect effect (<i>a*b</i>)					
Effect	SE	t ^a	p ^a	Bootstrapped 95% CI ^b	
Indirect effect via calculation skill	.05	.02	3.19	.002	.0235 to .0754

^a t value and p are for Total Effect.

^b Bootstrapped 95% CI is for indirect effects.