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A Theoretical and Experimental Comparison of 3-3 and 3-1 Mode Piezoelectric Microelectromechanical Systems (MEMS)

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Abstract

Two piezoelectric transducer modes applied in microelectromechanical systems are (i) the 3-1 mode with parallel electrodes perpendicular to a vertical polarization vector, and (ii) the 3-3 mode which uses interdigitated (IDT) electrodes to realize an in-plane polarization vector. This study compares the two configurations by deriving a Norton equivalent representation of each approach – including expressions for output charge and device capacitance. The model is verified using a microfabricated device comprised of multiple epitaxial silicon beams with sol-gel deposited lead zirconate titanate at the surface. The beams have identical dimensions and are attached to a common moving element at their tip. The only difference between beams is electrode configuration – enabling a direct comparison. Capacitance and charge measurements verify the presented theory with high accuracy. The Norton equivalent representation is general and enables comparison of any figure of merit, including electromechanical coupling coefficient and signal to noise ratio. With respect to coupling coefficient, the experimentally validated theory in this work suggests that 3-3 mode IDT-electrode configurations offer the potential for modest improvements compared against 3-1 mode devices (less than 2×), and the only geometrical parameter affecting this ratio is the fill factor of the IDT electrode.

Keywords

MEMS; piezoelectric; transduction

1. Introduction

Two common modes of piezoelectric transduction applied in microelectromechanical-system (MEMS) are the 3-1 and 3-3 modes, respectively. Fig. 1(a) provides an example schematic of each in which a piezoelectric material resides above a passive bending structure. In Fig. 1(b), horizontal strain in the film creates an electrical field in the vertical direction. In a sensor configuration, the electric potential resulting from the electric field

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may be read across the metallic electrodes which run parallel to the film as depicted in Fig. 1(b), or alternatively, charge shuffling between the electrodes may be measured in a short-circuit measurement configuration. Fig. 1(c) presents an example of a 3-3 mode device, in which case a set of interdigitated electrodes (IDT) may be used to directly measure the in-plane horizontal strain. As noted in Fig. 1(c), the direction of the polarization vector alternates between each IDT cell. This alternating polarity can only be accomplished with ferroelectric materials, which are poled after deposition. For non-ferroelectric materials such as AlN, crystal direction is aligned during deposition.

IDT electrodes patterned on top of piezoelectric layers have been used for surface acoustic-wave generation for many years [1]. 3-3 extension mode (i.e., non-bending mode) configurations have also been realized using IDT electrodes patterned on both sides of piezoelectric wafers, thereby realizing a wafer-level version of a piezoelectric “stack” [2, 3]. Use of IDTs in “bender” applications as summarized in Fig. 1(c) has also been demonstrated. Bending or “bender” configurations refer to those in which the S_{beam} strain labeled in Fig. 1 results from bending as opposed to net extension of the beam or plate. Kugel et al. modeled and compared 3-3 mode benders using IDTs against more traditional 3-1 mode benders [4]. An experimental comparison of several different bender configurations followed in [5], with emphasis placed on a force-generation figure of merit. With respect to bending applications, 3-1 mode devices are historically more common in MEMS and have been used in many applications including accelerometers [6, 7], microphones [8, 9], atomic force probes [10, 11], and energy harvesters [12, 13]. Bernstein et al. used 3-3 mode bending in an immersion ultrasonic transducer application [14]. An IDT electrode was patterned above a sol-gel deposited lead-zirconate-titanate (PZT) layer residing on a passive polyimide diaphragm. The use of IDT 3-3 mode in this application enabled the capacitance of the sensors to be tailored by the spacing between electrodes, rather than relying on thickness of the piezoelectric film as in the case of 3-1 mode benders. The small capacitance afforded by the 3-3 mode IDT configuration (~ 0.25 pF) was an advantage for impedance matching to the sensor to readout electronics. With respect to MEMS, a second advantage, as cited by Xu et al. [15], is reduction of fabrication steps since depositing and patterning of bottom electrodes are not required. In an energy harvesting MEMS application, Jeon et al. used cantilevers with 3-3 mode IDT electrodes and cited an advantage over 3-1 mode benders for the harvesting application [16]: the ability to achieve larger output voltages for overcoming threshold voltages in rectification electronics.

Comparisons between 3-3 mode and 3-1 mode benders have been made, but these have typically focused on a single figure of merit. Several macro-scale (non MEMS) commercial elements were experimentally compared for battery-charging applications [17, 18]. More recently in a MEMS energy harvesting application, Kim et al. fabricated 3-3 mode (IDT electroded) and 3-1 mode (parallel electroded) cantilevers of similar size and volume, and compared output power extracted from each beam [19]. Yu et al. and Wang et al. present results from both 3-3 mode and 3-1 mode PZT MEMS accelerometers in two separately published works [7, 20]. A similar fabrication process was used in each case, and it was noted that 3-3 mode IDT configurations have potential to achieve small signal-to-noise ratio (SNR) improvements over 3-1 mode devices of similar size. Mo et al. provide a theoretical

comparison between IDT and 3-1 mode devices [21]. The analysis is general and not limited to thin films, and no experimental results are presented.

In this work, the approach to modeling, while still being rooted in the fundamental piezoelectric equations, is different than previously presented. Norton equivalent network models of single 3-3 and 3-1 mode cells are presented, and the results are then extended to full-length electrodes using a parallel summation of single-cell equations. The single-cell equations use effective material parameters, which are common in MEMS as typically only effective properties can be measured and extracted from microfabricated structures. Rigorous definition of effective properties in terms of actual material properties is reviewed. Together, the Norton equivalent cell approach combined with the use of effective material parameters keeps the analysis simple and streamlined. Further, an analysis based on a Norton (or Thévenin) equivalent is the most general analysis that can be made, and from there a comparative analysis for any application can be made, including comparison of coupling coefficients, signal-to-noise ratio (SNR), and actuator strength. In this study, analytical analysis of the IDT cell assumes electric fields exist only in regions between electrodes, are horizontal, and are of constant magnitude through the thickness of the piezoelectric film. The accuracy of these assumptions for various film thicknesses is quantified with finite element analysis, and show that the assumptions are accurate for cases where films are thin relative to the electrode pitch, as is the case for many MEMS structures.

A case-study device is presented for model verification. The device is a micromachined structure with beams of identical dimensions, identical PZT volume and layer thicknesses, and anchored to the same moving mass. The only difference between the beams is electrode configuration. Charge output and capacitance measurements are made and results agree exceptionally well with theory (within 3% difference). In what follows, theory is first presented, followed by a description of the case-study device and related measurements. With the theory validated, a discussion comparing coupling coefficients and SNR is presented.

2. Theory

Analysis of the 3-3 mode IDT structure may be performed by analyzing a unit cell, and then recognizing that the full electrode set is comprised of many cells electrically connected in parallel. Analysis of the 3-1 mode cell may also be performed on a unit-cell basis in an identical fashion to facilitate a direct comparison against the 3-3 mode case. In the following analysis, subscript IDT denotes the 3-3 mode configuration and subscript PP (for “parallel-plate” electrodes) denotes 3-1 mode device operation. In comparing Fig. 1(b) and Fig. 1(c), it is noted that the numbered coordinate systems are oriented differently. This must be the case to maintain the convention that the three axes be aligned with the polarization axis of the material. To facilitate the following comparative analysis, Fig. 1(a) includes a non-numbered coordinate system that is the same for both cases, in which the “*x*” and “*z*” axes are parallel and perpendicular to, respectively, the beams. For simplicity of presentation, the following analysis proceeds under the assumption that the horizontal strain in the piezoelectric film (i.e., *x*-axis strain) is approximately uniform along the vertical axis (i.e., *z*-axis), as is the case for thin piezoelectric films residing on top of much thicker passive

beams, i.e., referring to Fig. 1(b), the S_1 strain within the film has no z -axis dependence. Similarly, referring to Fig. 1(c), the S_3 strain has no z -axis dependence within the film. In the 3-1 mode case, $S_1 = S_{beam}$ and in the 3-3 mode case, $S_3 = S_{beam}$, where S_{beam} is the horizontal strain at the top surface of the beam. It is shown in Section 5 that conclusions from the comparative analysis apply for general film thicknesses.

2.1 Analysis of IDT Unit Cell (i.e., 3-3 mode)

A 3-3 mode IDT electrode may be analyzed by considering a unit cell depicted in Fig. 2(a), with critical geometrical parameters defined in the figure. The corresponding Norton equivalent circuit for a single cell is depicted in Fig. 2(b) and is characterized by a short-circuit charge generated by the cell, $q_{i,IDT}$, and the capacitance looking into the cell, $C_{i,IDT}$, under blocked conditions, or $S_3 = S_{beam} = 0$. Defining a fill factor for the cell assists in subsequent analysis. The fill factor, Γ , is defined as

$$\Gamma = \frac{a}{a+g} \quad (1)$$

The fill factor is the ratio of active cell volume, defined by the region in between the electrodes and spanning the distance “ a ” in Fig. 2(a), to the total cell volume which spans a distance “ $a + g$ ”. If the electric field, E_3 , is assumed to be uniform between the electrodes and contained within the region defined by “ a ” in Fig. 2(a), the expression for the capacitance of the cell is

$$C_{i,IDT} = \frac{\epsilon_{33f,IDT}(wb)}{a} \quad (2)$$

$\epsilon_{33f,IDT}$ is the effective 3-3 mode dielectric constant. Effective material properties are discussed in a section to follow. The electric field that results upon a strain, S_3 , is

$$E_3 = h_{33f} S_3 \quad (3)$$

where h_{33f} is an effective property of the piezoelectric material, and S_3 is, in general, a function of x . The open-circuit voltage output of the cell, V_i , is obtained by integrating the electric field as,

$$V_i = \int_0^a E_3 dx = \int_0^a (h_{33f} S_3) dx = h_{33f} \Delta l_{active} = h_{33f} \Gamma \Delta l_i \quad (4)$$

l_{active} is the change in length of the active region of the cell and is equal to the total change in cell length, l_i , scaled by the cell fill factor. The charge generated by the cell under short-circuit conditions is therefore,

$$q_{i,IDT} = V_i C_i = e_{33f} \frac{wb}{a} \Gamma \Delta l_i \quad (5)$$

The substitution $e_{33f} = h_{33f} \epsilon_{33f, IDT}$ has also been made in Eq. (5). A complete IDT electrode may be comprised of “ N ” unit cells, which are electrically connected in parallel. As such, and following the Norton circuit in Fig. 2(b), the total capacitance and total charge output may be expressed as,

$$C_{IDT} = \frac{\epsilon_{33f, IDT}(wb)}{a} N = \frac{\epsilon_{33f, IDT}(wb)}{a^2} \Gamma L_e \quad (6)$$

$$q_{IDT} = e_{33f} \frac{wb}{a} \Gamma \sum_{i=1}^N \Delta l_i = e_{33f} \frac{wb}{a} \Gamma \Delta l \quad (7)$$

where, L_e is the electrode length as noted in Fig. 1 l and is the total elongation of the piezoelectric film. The second form of Eq. (6) makes use of the relation $N = L_e/(a + g)$. A Norton equivalent representation of the full-length electrode is presented in Fig. 2(c).

2.2 Analysis of Parallel Plate Unit Cell (i.e., 3-1 mode)

Using a similar analysis, a 3-1 mode device may be analyzed on a unit-cell basis. The unit cell is defined to occupy the same volume and span the same total cell length as the IDT cell to facilitate a direct comparison. Fig. 3(a) depicts the unit cell spanning the length $a + g$. The fill factor of the cell is inherently unity, since the entire volume of piezoelectric material between the electrodes is active and generates an electric field. Assuming a uniform field, E_3 , the capacitance of the cell under blocked conditions (i.e., $S_1 = S_{beam} = 0$) is,

$$C_{i, PP} = \frac{\epsilon_{33f, PP} w(a+g)}{b} \quad (8)$$

where $\epsilon_{33f, PP}$ is the effective ϵ_{33} material property in the parallel-plate electrode configuration. The Norton equivalent circuit for the cell is depicted in Fig. 3(b). The short-circuit charge density across the electrodes, D_3 , resulting from the S_1 strain is

$$D_3 = e_{31f} S_1 \quad (9)$$

where, e_{31f} is the effective e_{31} property of the piezoelectric material, and, in general, S_1 is a function of x . Integration of the surface charge density across the electrode area yields the total short-circuit charge output by the cell,

$$q_{i, PP} = w \int_0^{a+g} D_3 dx = w e_{31f} \Delta l_i \quad (10)$$

Similar to the case of the IDT analysis, the full span of a 3-1 mode device may be analyzed by summing “ N ” cells in parallel, in which case the total capacitance and total charge output are

$$C_{PP} = N \frac{\varepsilon_{33f,PP} w(a+g)}{b} = \frac{\varepsilon_{33f,PP} w L_e}{b} \quad (11)$$

$$q_{PP} = w e_{31f} \sum_{i=1}^N \Delta l_i = w e_{31f} \Delta l \quad (12)$$

The Norton equivalent circuit for a full-length parallel-plate electrode is depicted in Fig. 3(c).

2.3 Effective Material Properties

Effective piezoelectric material properties are functions of the complete set of actual material properties and the particular set of stress/strain assumptions applicable to the film in the transducer configuration. Piezoelectric equation sets may be presented in many forms. The following pair is most useful this particular analysis:

$$T = c^E S - e^{tr} E \quad (13)$$

$$D = e S + \varepsilon^S E \quad (14)$$

Four effective material properties appear in Eqs. (6), (7), (11), and (12) and are relevant to this study. The stress/strain assumptions for the IDT configuration in Fig. 1(c) are

$$\begin{array}{llll} S_1=0 & S_2 \neq 0 & S_3=S_{beam} \neq 0 & S_4=S_5=S_6=0 \\ T_1 \neq 0 & T_2=0 \text{ (free surface)} & T_3 \neq 0 & T_4=T_5=T_6=0 \end{array}$$

Evaluation of $C_{i,IDT}$ is performed under a zero source or “clamped” condition, so that the additional constraint $S_3 = S_{beam} = 0$ applies in evaluation of the capacitance. By definition,

$$\varepsilon_{33f,IDT} \equiv \left. \frac{D_3}{E_3} \right|_{S_1=0, S_2 \neq 0, S_3=0, T_1 \neq 0, T_2=0, T_3 \neq 0} \quad (15)$$

Under these conditions, (13) and (14) yield two equations that may be solved simultaneously to yield

$$\varepsilon_{33f,IDT} = \frac{e_{32}^2}{c_{22}^E} + \varepsilon_{33}^S \quad (16)$$

Similarly, definition of e_{33f} for the IDT configuration and its evaluation following Eqs. (13) and (14) are:

$$e_{33f,IDT} \equiv \frac{D_3}{S_3} \Big|_{S_1=0, S_2 \neq 0, T_1 \neq 0, T_2=0, T_3 \neq 0} \quad (17)$$

$$e_{33f,IDT} = e_{33} - e_{32} \frac{c_{23}^E}{c_{22}^E} \quad (18)$$

The relevant set of stress/strain assumptions applicable to the film in the parallel-plate (PP) configuration summarized in Fig. 1(b) are:

$$\begin{array}{llll} S_1 = S_{beam} \neq 0 & S_2 = 0 & S_3 \neq 0 & S_4 = S_5 = S_6 = 0 \\ T_1 \neq 0 & T_2 \neq 0 & T_3 = 0 \text{ (free surface)} & T_4 = T_5 = T_6 = 0 \end{array}$$

Since evaluation of $C_{i,PP}$ is under zero-excitation conditions, $S_1 = S_{beam} = 0$ is used in evaluation of $\varepsilon_{33f,PP}$. The definition of $\varepsilon_{33f,PP}$ is

$$\varepsilon_{33f,PP} \equiv \frac{D_3}{E_3} \Big|_{S_1=0, S_2=0, S_3 \neq 0, T_1 \neq 0, T_2 \neq 0, T_3=0}, \quad (19)$$

and its evaluation following (13) and (14) is:

$$\varepsilon_{33f,PP} = \frac{e_{33}^2}{c_{33}^E} + \varepsilon_{33}^S \quad (20)$$

e_{31f} for the parallel-plate electrode configuration is, by definition:

$$e_{31f,PP} \equiv \frac{D_3}{S_1} \Big|_{S_2=0, S_3 \neq 0, T_1 \neq 0, T_2 \neq 0, T_3=0}, \quad (21)$$

and its evaluation following (13) and (14) is:

$$e_{31f,PP} = e_{31} - e_{33} \frac{c_{31}^E}{c_{33}^E} \quad (22)$$

2.4 Comparison

A direct comparison of capacitance and charge output can be made on a unit-cell basis or by comparing complete electrodes of equal length and width. Each approach yields identical results. Referring to Figs. 2(b) and 3(b), the ratio of device capacitance is

$$\frac{C_{IDT}}{C_{PP}} = \frac{\varepsilon_{33f,IDT} \frac{w_0}{a}}{\varepsilon_{33f,PP} \frac{w(a+g)}{b}} = \frac{\varepsilon_{33f,IDT}}{\varepsilon_{33f,PP}} \frac{b^2}{a(a+g)} = \frac{\varepsilon_{33f,IDT}}{\varepsilon_{33f,PP}} \frac{b^2}{a^2} \Gamma, \quad (23)$$

and the ratio of short-circuit charge output is

$$\frac{q_{IDT}}{q_{PP}} = \frac{e_{33f}}{e_{31f}} \left(\frac{b}{a} \right) \Gamma. \quad (24)$$

While values of individual effective material properties can vary significantly for different compositions of PZT, the ratios of effective material properties appearing in Eqs. (23) and (24) do not vary significantly across many well-known PZT compositions. Table I illustrates this point by presenting ratios for three different types of PZT. The (e_{33f}/e_{31f}) and $(\epsilon_{33f, IDT}/\epsilon_{33f, PP})$ ratios vary by less than 5% across the different materials.

For the case study device presented in this work, the dimensions of the IDT electrode design are $a = g = 5 \mu\text{m}$ such that the fill factor $\Gamma = 0.5$. The PZT-film thickness, b , is equal to $1.2 \mu\text{m}$. When directly compared against a 3-1 mode device, the anticipated comparisons are: $q_{PP}/q_{IDT} = 7.08$, and $C_{PP}/C_{IDT} = 46.08$.

2.5 Quantifying Accuracy of electric field assumptions via FEA

Analysis of the IDT cell has assumed parallel electric field lines in regions between electrodes (i.e., active regions), with uniform magnitude independent with respect to z . In reality field lines are not purely horizontal and parallel within the active region since electrodes exist only at the surface of the active film. Further, fringe fields exist within the underlying $4\text{-}\mu\text{m}$ -thick oxide layer and in the air region above the piezoelectric film. These complexities can (i) result in a non-uniform field during poling, and therefore spatially dependent polarization and material properties, and (ii) affect cell capacitance. An FEA model was constructed using the Multiphysics module in ANSYS to investigate these effects and to quantitatively bound the accuracy of the simple analytical formulation. Fig. 4(a) summarizes the model geometry which comprises two unit cells. To study the electric field during poling, the center electrode is set to 1 V and outer electrodes are set to 0 V. Fig. 4(b) presents the electric field and potential field for the prototype device geometry. Although vertical field components reside in the oxide layer, the electric field in the piezoelectric layer is approximately parallel and of uniform magnitude in regions between electrodes, and approximately null in regions directly beneath the electrodes. The maximum deviation in electric field amplitude in regions between the electrodes is 2% as noted in Fig. 4(b). Fig. 4(c) presents the identical simulation for a thicker PZT layer equal to $4 \mu\text{m}$. In this case, the field strength diminishes to 21% of its maximum value at the bottom of the film, as highlighted in Fig. 4(c). The model has been used to show that the field strength diminishes to 10% of its maximum value for a film thickness equal to $3 \mu\text{m}$.

Assuming spatially independent polarization and therefore spatially independent material properties within a cell, the capacitance of an IDT cell vs. film thickness may be explored using the model presented in Fig. 4. The result is summarized in Fig. 5. Between $1 - 2\text{-}\mu\text{m}$ piezoelectric film thickness, FEA and analytical models are within 11% deviation. The slightly increased values revealed by the FEA are due to the additional capacitance of the underlying SiO_2 layer. Above $2\text{-}\mu\text{m}$ film thickness, FEA reveals greatly reduced capacitance compared to the analytical formulation, which is due to the decaying electric-field strength

with depth into the film. In summary, when considering poling uniformity and cell capacitance, FEA and the analytical formulation are within 11% agreement for b/a ratios between 0.2 and 0.4.

3. Measurements and Model Verification

A device with cross section schematic presented in Fig. 1 was fabricated as a case-study device for model verification. A 3-D computer aided design (CAD) image of the device is presented in Fig. 6(a). The device is comprised of a 500- μm -thick bulk-silicon cylinder at the center attached to six epitaxial silicon bending beams, or “springs”, at the surface. The electrode configuration (i.e., IDT vs. PP) alternates with each spring, such that three of the springs contain IDT electrodes and the other three springs contain PP electrodes. Figs. 6(b) and 6(c) present zoomed-in CAD images of springs with PP and IDE electrodes, respectively, while Fig. 6(d) is a further zoomed-in section of the IDT electrode with a 10- μm pitch. Insets in Figs. 6(b) and 6(d) show cross-section schematics of the device with the targeted layer thicknesses labeled, and cross-sectional scanning-electron-microscope (SEM) images in Fig. 7 present the measured layer thicknesses upon fabrication completion. Step-by-step details of the fabrication process flow have been presented in prior work [22]. Top-side micrographs of two directly opposing springs are presented in Fig. 8, with Figs. 8(a) and 8(b) illustrating an IDT device at varying zoom levels, and Fig. 8(c) illustrating a 3-1 mode PP device. Each device is poled at 160°C for 30 minutes using an electric field equal to 5V/ μm , which equates to a potential of 6 V and 25 V for the 3-1 and 3-3 springs, respectively. Capacitance and short-circuit charge measurements are made on two such springs.

3.1 Capacitance Measurements

Capacitance measurements are made using electrical admittance spectroscopy [23]. The transimpedance amplifier (TIA) depicted in Fig. 9 is used to measure the current admitted by each device in response to an input voltage, V_{in} , swept with frequency. V_{in} is constant in amplitude at each frequency, and the capacitance may be computed at any frequency, ω ($= 2\pi f$), within the measurement range as,

$$C_{eb} = \frac{q}{V} = \frac{i}{j\omega V_{in}} = \frac{1}{j\omega R_f} \frac{V_{out}}{V_{in}}, \quad (25)$$

Three admittance measurements are performed – the first on a ceramic-disk “dummy” capacitor with a known capacitance of 6 pF with 10% tolerance, followed by measurement of the IDT- and PP-electrode springs of Fig. 8. Measurement of the dummy capacitance is performed to ensure a proper functioning setup. The input voltage and measured TIA output voltage for each measurement are presented in Fig. 10. A PrismSound dScope Series III fast-Fourier-transform (FFT) analyzer was used to generate and measure the voltage signals. The input voltage amplitude is the same for the dummy-capacitor and IDT measurements, but smaller for the parallel-plate electrode. The high capacitance of the PP device admits significantly larger current than the other two devices, which requires scaling down the input to avoid saturation of the TIA. Table II summarizes the measured capacitance following

V_{out}/V_{in} data obtained from Fig. 10 at 10 kHz. The dummy capacitance is close to the anticipated value and within the tolerance of the part. The measured C_{PP}/C_{IDT} ratio is 46.08 which is, to within 2-decimal places, equal to the value predicted by theory.

3.2 Charge Measurements

Output-charge measurements are made using the setup presented in Fig. 11. The device is mounted onto a shaker and the shaker frequency is swept over a narrow band of frequencies in proximity of the first resonance frequency of the device. The first resonance mode corresponds to simple up-and-down vertical translation of the center mass. A laser Doppler vibrometer (LDV) measures the motion of the center cylinder region while the short-circuit charge output from the springs is measured under zero-impedance conditions using a TIA, as shown in Fig. 11. Since the goal is to make a fair comparison of charge output for the two electrode configurations, the LDV ensures that the two opposing springs are vibrating with the same amplitude – as one would expect from the first-mode vibration. Further, in subsequent discussion, it is helpful to normalize the charge output to spring tip displacement amplitude to compute and compare a ϕ parameter, defined as:

$$\phi = \frac{q_{sc}}{x_{tip}} = \frac{i_{sc}}{v_{tip}} \quad (26)$$

The simultaneous measure of absolute tip velocity, v_{tip} , and short-circuit current, i_{sc} , enables measurement of ϕ . Fig. 12 summarizes measurements on the PP-electrode device, with spring tip velocity plotted using the left vertical axis and measured current plotted using the vertical axis at right. ϕ may be extracted from data at any frequency in Fig. 12. Use of the peak values at resonance ensures high signal to noise ratio. For the PP-beam data in Fig. 12, $\phi = 1.220 \times 10^{-10} \text{A} / 1.394 \times 10^{-5} \text{m/s} = 8.754 \times 10^{-6} \text{C/m}$. The resonance frequency and Q of the case study device are 2.775 kHz and 925 as can be extracted from the data presented in Fig. 12. Similar measurements were made for the IDT electrode device and $\phi = 1.270 \times 10^{-6} \text{C/m}$ is obtained. Table III summarizes measured data for both springs. The ratio of measured ϕ values, $\phi_{PP}/\phi_{IDT} = 6.89$, is in close agreement (within 3% difference) of the ratio predicted by theory as summarized in Section 2.4.

Table IV summarizes the ratio of measured ϕ values and measured capacitance values for the IDT and PP electrode springs, and the corresponding ratios predicted by theory as summarized in Section 2.4. Excellent agreement between theory and measurement is obtained.

4. Discussion

With theory confirmed by experimentation, it is meaningful to use the theory to explore potential advantages of one electrode configuration compared to the other for sensing, energy harvesting, and actuator applications.

4.1 Energy Coupling Coefficient Comparison

Energy coupling coefficients may be compared by directly comparing electrical energy stored in IDT vs. PP electrode configurations under open-circuit conditions. The comparison may be made on a unit-cell basis or using complete electrodes of equal length and width. Each approach yields identical results. Using the Norton equivalent circuits in Figs. 2(c) and 3(c),

$$\frac{k_{IDT}^2}{k_{PP}^2} = \frac{q_{IDT}^2}{q_{PP}^2} \frac{C_{PP}}{C_{IDT}} = \left(\frac{e_{33f}}{e_{31f}} \right)^2 \frac{\varepsilon_{33f,PP}}{\varepsilon_{33f,IDT}} \Gamma, \quad (27)$$

In the limit of $\Gamma \rightarrow 1$ (achieved with vanishing small IDT trace widths), and using material properties of PZT-5H, the ratio is computed as 1.84. In practical implementations of IDT electrodes, Γ is less than one. For the case study device presented, $\Gamma = 0.5$ and the ratio computed using Eq. (27) is 0.92. It is interesting to note that absolute dimensions of the IDT cells (e.g., w, a, b , and g in Fig. 2(a)) do not appear in Eq. (27). In addition to material properties, the fill factor is the only parameter appearing in the ratio.

For vibration energy harvesters operating at resonance, the theoretical maximum amount of power available for capture is determined purely by the mechanical system parameters (resonance frequency, mass, and Q of resonance) and not properties of the transducer [24]. There exists, however, a minimum transducer coupling coefficient required to enable capture of the maximum theoretical power [25]. With these criteria in mind, IDT configurations offer the possibility for a slight advantage over PP-electrode configurations. As noted by Kim et al., IDT configurations have the additional advantage of higher open-circuit output voltages [19].

4.2 Sensor Signal-to-Noise Ratio Comparison

A fundamental noise limit in piezoelectric sensors is Johnson-Nyquist noise (i.e., thermal noise) induced by dielectric loss in the piezoelectric material itself. This loss is commonly characterized as the ratio of real to imaginary electrical impedance of the film, referred to as the loss tangent or $\tan \delta$ of the material where “ δ ” is the angle formed between the real and imaginary impedance when plotted in the complex plane. For dielectric-loss noise limited sensors, it has been shown that SNR is proportional to $\sqrt{k^2}$ where k^2 is the coupling coefficient. Derivation of this relationship is possible using a Norton equivalent representation of output charge and capacitance, as in Fig. 2(c) for example for the case of the IDT. The model may be modified by adding a resistor in series with the capacitance with value $R_L = \tan \delta / \omega C_{IDT}$. The thermal voltage-noise generator induced by R_L ,

$v_n = \sqrt{4k_b T R_L}$, also appears in series with the device capacitance. A source transformation of the series combination of C_{IDT} and v_n enables the noise to be represented in terms of

charge, $q_{n,IDT}$, in parallel with the device capacitance. $q_{n,IDT}$ is proportional to $\sqrt{C_{IDT}}$, and SNR for a dielectric-loss-noise limited piezoelectric sensor is therefore proportional to

$q_{IDT} / \sqrt{C_{IDT}}$. An identical relationship exists for analysis of the PP device or any generic

piezoelectric transducer that may be reduced to a Norton circuit of the types presented in Figs. 2 and 3. The SNR comparison follows:

$$\frac{\text{SNR}_{\text{IDT}}}{\text{SNR}_{\text{PP}}} = \frac{q_{\text{IDT}} \sqrt{C_{\text{PP}}}}{q_{\text{PP}} \sqrt{C_{\text{IDT}}}} = \frac{\frac{e_{33f}}{e_{31f}} \left(\frac{b}{a}\right) \Gamma}{\sqrt{\frac{\epsilon_{33f, \text{IDT}}}{\epsilon_{33f, \text{PP}}} \frac{b^2}{a^2} \Gamma}} = \frac{e_{33f}}{e_{31f}} \sqrt{\frac{\epsilon_{33f, \text{PP}}}{\epsilon_{33f, \text{IDT}}}} \sqrt{\Gamma} \quad (28)$$

In the limit $\Gamma \rightarrow 1$ and using material properties of PZT-5H, the ratio is computed as 1.36 (or 2.67 dB). The largest SNR advantage an IDT-electrode configuration may have compared to a PP-electrode configuration of identical size is therefore 2.67 dB. For the case-study device presented, $\Gamma = 0.5$ and the ratio computed using Eq. (28) is 0.96. In summary, for the case study structure fabricated in this work with $\Gamma = 0.5$, and the PP-electrode configuration has a slight SNR advantage over the IDT configuration. From Eqs. (27) and (28), the two configurations produce equivalent k^2 and SNR for $\Gamma = 0.544$. In practice, Γ is limited by the electric potential that can be generated during poling processes and by how thin IDT traces can be reliably fabricated.

5. ϕ COMPUTATION FOR BENDING BEAMS

Referencing Figs. 2(c) and 3(c), the comparative analysis between IDT and PP configurations does not require values for l , the electrode elongation at the beam surface, since it is the same for both configurations compared. The goal to quantitatively compare the two configurations is therefore complete without further analysis. A theoretical model incorporating computation of l is useful to enable computation of absolute charge outputs in the transducer design stage and is therefore provided.

5.1 ϕ COMPUTATION FOR THIN FILMS

For bending beams with thickness h as labeled in Fig. 1(a) and deflection profile, $u(x)$, strain at the beam surface is $S_{\text{beam}} = (h/2)u''$, and elongation of the beam surface is

$$\Delta l = \int_{L_s}^{L_e} (h/2)u'' dx = \frac{h}{2} [u'(L_e) - u'(L_s)] \quad (29)$$

where L_e and L_s are the “starting” and “ending” location of the electrode, respectively. Eq. (29) is a general expression that may be used for any combination of beam boundary conditions for electrodes positioned anywhere along the beam surface. Eq. (29) in combination with Eqs. (7) and (12) enable simple analytical computation of charge output for IDT- and PP-electrode configurations on top of bending beam structures. In many cases it is desirable to compute the charge output per beam tip displacement, defined as ϕ in Eq. (20), in which case u' should be related to z_{tip} . Such relationships are easily obtainable from classical mechanics [26]. For beams of length “ L ” as labeled in Fig. 1(a) with clamped/guided, or “S” shaped curvature as in the case of first mode deformations of the case study device,

$$u = z_{tip} \left(\frac{3x^2}{L^2} - \frac{2x^3}{L^3} \right) \text{ and } u' = z_{tip} \left(\frac{6x}{L^2} - \frac{6x^2}{L^3} \right) \quad (30)$$

Eqs. (10), (29), and (30), for example, enable computation of ϕ_{PP} for parallel plate electrode beams. Owing to the reciprocal nature of piezoelectric transducers as elaborated on in more detail elsewhere [27], ϕ_{PP} is also the constant relating clamped beam tip force per applied voltage to the device, with units [N/V]. The simple analytical expressions presented are therefore also useful for actuator applications. As a final illustration of the utility of ϕ computation, when solved backwards, a measured ϕ_{PP} may be used to compute the e_{31f} material property. As summarized in Table III, $\phi_{PP} = 8.75 \times 10^{-6}$ N/V. For the geometrical parameters of the device, an e_{31f} of 9.57 C/m² is implied for the sol-gel deposited PZT, which is consistent with values previously obtained through independent impedance spectroscopy techniques [23].

5.2 Extension of ϕ computation to Thicker Films

Thus far the piezoelectric material has been assumed thin compared to total beam thickness, which is why computation of total film elongation, l in Eq. (29) uses the beam strain only at the beam surface, which resides a distance $z = h/2$ from the central or neutral bending axis of the beam. Computation of ϕ and charge output can be generalized for thicker films by computing a film elongation that is averaged across the film's thickness:

$\Delta l_{avg} = \frac{1}{2}(\Delta l_{top} + \Delta l_{bottom})$ where l_{top} and l_{bottom} are the elongations of the piezoelectric film at its top and bottom surface, respectively. These two quantities can be computed using Eq. (29), but with h replaced by the z distance of the top and bottom film surface from the neutral bending axis of the beam, respectively. The legitimacy of this extension can be argued using superposition, whereby a film of finite thickness is viewed as a composition of many thin layers stacked in the z -direction and therefore connected electrically in parallel, with each layer contributing charge according to its Norton representation following Figs. 2(c) and/or 3(c).

6. Conclusion

A direct 1:1 comparison has been made between two common types of electrode configurations for piezoelectric MEMS: IDT (3-3 mode) vs. PP (3-1 mode). To facilitate the most direct comparison possible, two different electrode configurations were fabricated and attached to the same moving structure. Theoretical analysis has been used to predict the relative capacitances and charge output capabilities of the two configurations. The theory has been experimentally validated to a high degree of accuracy (within 3% difference) with measurements on the case study device. The Norton equivalent framework used to study the model is general and, once validated, may be used to assess the figures of merit for many applications. For sensing applications where the dielectric loss induced noise floor is achieved, an IDT structure has the potential to achieve only modest advantages over PP structures (less than 2.7-dB improvement).

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Highlights

- We compare two common modes of piezoelectric transduction for MEMS: 3-1 vs. 3-3
- Analysis is based on a Norton equivalent circuit representation
- Analysis experimentally validated via measurements on a microfabricated structure
- 3-3 mode offers potential for modestly larger (less than 2×) energy coupling coefficient
- 3-3 mode offers potential for modestly higher (less than 3dB) signal-to-noise ratio

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3-3 mode: E-field in lateral direction
due to horizontal strain

Fig. 1. Schematics of (a) piezoelectric elements with a passive bending structure, (b) 3-1 mode operation, and (c) 3-3 mode operation

Fig. 2.

(a) A unit-cell schematic of IDT-electrode 3-3 mode piezoelectric element, and circuit models of (b) 3-3 mode unit cell and (c) 3-3 mode IDT electrode (multiple unit cells)

Fig. 3.
 (a) A unit-cell schematic of PP-electrode 3-1 mode piezoelectric element, and circuit models
 of (b) 3-3 mode unit cell and (c) 3-3 mode IDT electrode (multiple unit cells)

Fig. 4.

(a) A model geometry of a IDT unit cell for FEA and electric field and potential simulation results for unit cells with (b) 1.2- μm - and (c) 4.0- μm -thick PZT-5H layer (for both case, a 8.0- μm -thick air and a 4.0- μm -thick SiO_2 layer reside atop and beneath the PZT-5H layer respectively).

Fig. 5.
Capacitance of an IDT cell vs. film thickness

Fig. 6.
Photorealistic images of (a) the entire sensor, the magnified views of (b) one of springs with PP electrodes (3-1 mode) and (c) with IDT electrode (3-3 mode), and (d) details of IDT electrodes

Fig. 7. SEM images of (a) the device cross section, and (b) the magnified view of the PZT film cross section

Fig. 8.

Micrographs of (a) IDT-type 3-3 mode piezoelectric element on a spring, (b) the magnified view of IDT electrodes, and (c) PP-type 3-1 mode piezoelectric element on a spring, of the tested structure

Fig. 9.
Circuit diagram of the TIA used for output charge measurements

Fig. 10.

V_{in} and V_{out} ; traces from admittance spectroscopy for 6-pF test capacitor and IDT and PP electrodes of the test structure

Fig. 11.
Schematic of the experimental setup for output charge measurements

Fig. 12.
Measured tip velocity and short-circuit current at the device resonance frequency for PP-electrode device

TABLE I

Comparison of effective material properties for different types of PZT

Material	e_{31f} [C/m ²]	e_{33f} [C/m ²]	e_{33f}/e_{31f}	$\epsilon_{33f/PP}$ [F/m]	$\epsilon_{33f/PDT}$ [F/m]	$\epsilon_{33f/DT}/\epsilon_{33f/PP}$
PZT-5H	-23.42	27.62	1.179	1.726×10^{-8}	1.301×10^{-8}	0.753
PZT-5A	-16.04	19.15	1.194	9.571×10^{-9}	7.561×10^{-9}	0.790
PZT-4	-14.96	17.88	1.195	7.862×10^{-9}	6.074×10^{-9}	0.773

TABLE II

Summary of blocked capacitance measurements from IDT and PP electrodes

	V_{in} [V]	V_{out} [V]	C_{eb} [F]
6-pF capacitor	2.94×10^{-4}	5.70×10^{-6}	6.16×10^{-12}
IDT electrode	2.94×10^{-4}	3.34×10^{-5}	36.13×10^{-12}
PP electrode	1.47×10^{-6}	7.70×10^{-6}	1665.06×10^{-12}

TABLE III

Summary of measured data from IDT and PP electrodes

	i_{sc} [A]	v_{tip} [m/s]	ϕ [C/m]
IDT electrode	1.740×10^{-11}	1.371×10^{-5}	1.270×10^{-6}
PP electrode	1.220×10^{-10}	1.394×10^{-5}	8.754×10^{-6}

TABLE IVSummary of predicted and measured φ and C_{eb} from IDT and PP electrodes

	Theoretical	Measured	Difference (%)
$\varphi_{PP}/\varphi_{IDT}$	7.08	6.89	2.76
$C_{eb,PP}/C_{eb,IDT}$	46.08	46.08	0.01