Iterative reconstruction using a Monte Carlo based system transfer matrix for dedicated breast positron emission tomography

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To maximize sensitivity, it is desirable that ring Positron Emission Tomography (PET) systems dedicated for imaging the breast have a small bore. Unfortunately, due to parallax error this causes substantial degradation in spatial resolution for objects near the periphery of the breast. In this work, a framework for computing and incorporating an accurate system matrix into iterative reconstruction is presented in an effort to reduce spatial resolution degradation towards the periphery of the breast. The GATE Monte Carlo Simulation software was utilized to accurately model the system matrix for a breast PET system. A strategy for increasing the count statistics in the system matrix computation and for reducing the system element storage space was used by calculating only a subset of matrix elements and then estimating the rest of the elements by using the geometric symmetry of the cylindrical scanner. To implement this strategy, polar voxel basis functions were used to represent the object, resulting in a block-circulant system matrix. Simulation studies using a breast PET scanner model with ring geometry demonstrated improved contrast at 45% reduced noise level and 1.5 to 3 times resolution performance improvement when compared to MLEM reconstruction using a simple line-integral model. The GATE based system matrix reconstruction technique promises to improve resolution and noise performance and reduce image distortion at FOV periphery compared to line-integral based system matrix reconstruction.

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I. INTRODUCTION

Over the past several years, it has become clear that PET imaging with fluoro-deoxyglucose (FDG) can play an important role in the detection and diagnosis of breast cancer. Many encouraging studies using whole-body PET systems to image breast cancer have been reported;¹ however, it is now evident that smaller PET systems dedicated to imaging the breast have substantial advantages. A number of different dedicated breast PET systems have been proposed and developed.^{2–11} In order to reduce radiation dose to the patient, as well as minimize acquisition time, high sensitivity of the breast PET system is important. Scanner designs that maximize geometric efficiency of the scanner by placing the PET detectors close to the breast are therefore appealing. In a ring PET geometry, this means using a small bore designed to have a diameter just larger than the maximum diameter breast size to be imaged.

In addition to high sensitivity, another desired goal of breast PET systems is high resolution, necessary for accurate detection of sub-cm lesions. PET spatial resolution is degraded by various detector and radiotracer properties.

Radiotracer properties such as positron range and non-collinear photon emission can cause small position errors in estimating radioisotope locations. In the detector, incident photons can penetrate through the discrete crystals, as well as undergo scatter, causing mis-positioned line-of-response (LORs). This phenomenon is pronounced at the Field of View (FOV) periphery, where due to an increase in probability of

photons being incident on the detectors at oblique angles, the probability of inter-crystal penetration increases.

A number of algorithms have been proposed for reconstructing PET data, including the analytic 3D reprojection (3DRP) algorithm,¹² RAMLA,¹³ and Fourier rebinning¹⁴ (FORE) coupled with ordered-subsets expectation maximization (OSEM). One disadvantage of both the 3DRP and FORE methods is that these algorithms are derived using the assumption that measured data correspond to line integrals through the unknown activity distribution. One of the major advantages of iterative reconstruction methods is that in theory, the physical degradations that limit spatial resolution can be modeled into the system matrix (SM) used by the reconstruction algorithm. Each entry in the SM corresponds to the probability of detecting an emission from a voxel within the object at a specific LOR thus allowing estimation of voxel counts from detected LOR counts (Eq. (1)). The imaging equation is

$$d_i = \sum_{ij} p_{ij} f_j, \quad (1)$$

where f_j is counts in voxel j , d_i is number of detections in LOR i , and p_{ij} (henceforth called SM in this work) is probability of an emission from voxel j being detected in LOR i . Unfortunately, due to the large number of scintillation crystals present in current PET scanners, the SM can be very large. This realization has prohibited the use of Monte Carlo methods or empirical measurements to generate and store

the full SM. Instead, many approaches have been proposed that use some form of approximations to simplify modeling of the SM.

One approach calculates probabilities “on-the-fly” during the projector and backprojector operations, thus not requiring storage of the SM matrix. Ollinger *et al.*¹⁵ assumed a detector pair is sensitive to photons emitted within a tube of rectangular cross-section connecting the two detectors. The SM is then computed by calculating the intersection between the voxel of interest and a tube connecting the two detectors at the endpoints of the LOR. Johnson *et al.*¹⁶ have used a rectangular tube model but weighted it with a shift-variant factor to approximate the detector response. None of these approaches explicitly model blurring effects caused by photon interactions within the detector. Recent more comprehensive methods of calculating probabilities “on-the-fly” include techniques for modeling detector response and radiotracer properties such as positron range and non-collinearity.^{17–19}

For estimating the SM, both simulation and experimental measurement approaches have been developed. There are trade-offs associated with both approaches. Although measurement methods can accurately model all of the degradations in the transport of photons through the detector, they typically involve very long acquisition times. Because of this, approximations are made where the detector response is measured at some subset of object locations, and interpolation or parametric models are used to fill-in probabilities at other object locations.²⁰

Simulation methods using Monte Carlo software allow for the estimation of the full SM (at all object locations), however, computational load can be an issue, as well as limitations on computer storage. As mentioned previously, the full SM is too large to store in memory, however, certain scanner symmetries can be used so that only a portion of the SM is needed. In addition, the object can be represented using basis functions that result in a block-circulant system matrix. For example, Vandenberghe *et al.* used generalized natural pixels,²¹ and Leroux *et al.* have used polar pixels.²² Qi *et al.*²³ combined a depth-dependent solid angle calculation of the LOR with a Monte Carlo simulation technique for modeling the spatially variant detector response. This technique factors the SM into a number of terms modeling detector sensitivity, detector blur, attenuation, geometric efficiency, and positron range.

Although with advances in modern computer hardware, computation times are being reduced, it is still challenging to estimate the SM with Monte Carlo software. Zhang *et al.*²⁴ have recently reported a fast SM simulation module using `egs_pet`²⁵ that computed a low-noise estimate of the SM for a Philips Gemini GS PET in 5 days using 50 Intel Xeon 2.66 GHz cores.

Lee *et al.*²⁶ have shown the importance of modeling the non-stationary detector response in small-animal imaging, and subsequently developed an approach²⁷ for modeling this blurring into the reconstruction method. Alessio *et al.*²⁸ have developed a method for modeling blurring in both the radial and axial dimensions using Monte Carlo simulation. To make this approach computationally feasible, point response

functions (PRF) were simulated at sparsely sampled locations, and interpolation was then used to generate the PRF at non-simulated locations.

Zhang *et al.*²⁴ recently described an approach for modeling the full SM using Monte Carlo simulation. In order to limit memory requirements, they used symmetrical properties of the PET ring geometry so that only probabilities for LORs with endpoints in one single detector module were stored. They also used a fast Gaussian rotator for the projection and back-projection operation.

Another approach for modeling the SM is to use empirical measurements. For example, Panin *et al.*²⁰ have reported an approach for modeling the SM using point source measurements. To make this approach computationally feasible, the measured point sources are parameterized to approximate responses at non-measured point locations, and projection data are compressed using mashing to reduce the size of the SM needed. Besides the long time required for the acquisition and difficulty of positioning the point source at different locations, the discrete locations of the point source do not produce a spatially continuous system response as required for reconstruction, thus interpolation is required. Furthermore, the edge effect in block detectors may produce inaccurate system element evaluation.

In this work, we estimated the SM of dedicated breast PET using the GATE Monte Carlo simulation software and incorporate this SM into MLEM image reconstruction. The symmetry of the ring PET geometry was taken advantage by using polar voxels to represent the object to be reconstructed. This results in a block-circulant SM where only one block column needs to be stored in memory. In addition, since the SM is sparse, only non-zero values are stored. This SM was then used with an iterative MLEM algorithm using data stored in LOR histogram format.²⁹ For efficient implementation, the projector and backprojector operations used a rotator to take advantage of the polar voxel object representation. Unlike a Gaussian rotator that has been attributed to ringing artifact in images due to its smoothing property,²⁴ the ideal rotator used in this work has no smoothing property thus reducing ring artifact.

II. METHODS

A. System modeling

For this study, a hypothetical breast PET system was modeled, using a full ring geometry encompassing the breast with detector coverage of 360°. Fig. 1 shows an illustration of this hypothetical breast PET system. The scanner was based on 12 detector modules with each module consisting of 32×96 LYSO crystals of size $2 \times 2 \times 20$ mm. Therefore, there were 96 rings in the scanner, with a ring defined as one crystal in the axial direction. In this study, we consider 2D reconstruction, where the system matrix probabilities only describe LORs that fall within the same ring.

By geometrically limiting the acceptable LORs to those that connect a crystal within a module to its opposite seven modules, there are $384 \times 224 = 86\,016$ possible LORs (this geometric constraint would allow for complete coverage of a 16.6 cm diameter breast) (Fig. 2). One-half of these LORs

FIG. 1. Breast PET system with 12 modules transaxially and 96 rings axially. Each detector is a LYSO crystal of 2 mm by 2 mm length in transaxial and axial dimensions and 2 cm depthwise. The system allows a 20 cm axial length and 25 cm transaxial diameter in the FOV.

were redundant, therefore, there were 43 008 possible non-redundant LORs. If the 2D reconstruction matrix is defined as 200×200 with 1 mm voxels, then the SM will have $43\,008 \times 200 \times 200 = 1.72 \times 10^9$ elements. By using a polar voxel representation of the object, the system matrix becomes block circulant. This means that all elements of the SM can be obtained from only one block column, thereby reducing storage by a factor of 12. This one block column represents the probabilities for LORs connecting one detector module to its opposite seven modules.

B. GATE setup

The Monte Carlo simulation software package GATE³⁰ has been widely used to simulate PET acquisition. It can track photon interactions both through the object and within the detector including crystal penetration and scatter. In this

FIG. 2. Figure illustrates that only 7 opposite detector modules that are furthest from the event originating module were used for pairing of coincidence events. The nearest 5 modules were not considered for pairing.

work, GATE has been used to estimate system matrix elements. The system matrix elements represent the probability of an emission at object voxel j being detected by a specific detector pair (LOR) i . These probabilities model the physics of photon transport within the detector, but not object interactions such as photon attenuation and scatter in the object. To compute the SM probability, a cylindrical activity source in air (diameter of 18 cm—large enough to cover all possible expected object locations) was simulated using the above mentioned breast PET model. GATE allows objects to be simulated in air by using the option of back-to-back photon emission (i.e., without actually modeling positron emission). Thus, positron range was not modeled in the system matrix.

In order to minimize random coincidence events, the cylinder was hypothetically filled with a very low-activity ($10\,\mu\text{Ci}$) source with the total number of back-to-back photons simulated controlled by increasing the simulated acquisition time. Each coincidence event in the simulated list-mode data contained information on the LOR i , as well as the polar voxel emission location j (see below for description of the polar voxel coordinate system). To generate the SM, each event in the list-mode data set is examined and the appropriate SM element, defined by the LOR i and emission voxel j , was updated in a linked-list LOR data structure as described below.

The stopping criterion of GATE simulation dictates the resulting count statistics. Zhang *et al.* defined the stopping criterion based on the average counts emitted from individual voxels ($>54.9 \times 10^6$)²⁴ or the average counts detected in individual LOR ($>4.7 \times 10^4$).²⁴ Simulating a high number of events will ensure accuracy (low variance) in estimating SM probabilities. In this work, the average counts detected per LOR were utilized to estimate noise in the SM.

C. Calculation efficiency

As mentioned previously, probabilities for 1.72×10^9 elements in the SM need to be calculated for complete coverage of a 16.6 cm PET FOV. However, most of these probabilities were zero, thereby making the system matrix highly sparse. In other words, for each LOR, a small number of emission points can be feasibly measured. To utilize the sparseness of the SM, and thereby reduce computational burden and storage space, a linked-list LOR data structure was used. This data structure stores probabilities for only non-zero SM elements. The flow chart describing steps involved in linked-list data creation is described in Fig. 3. The LORs in the data structure were defined by opposite crystals (obtained from GATE). For each LOR, voxels that contribute counts to the LOR were identified and added to the corresponding LOR node in the data structure. If the voxel already existed in the LOR node, then the counter of the voxel was incremented by 1. If the voxel was not present in the LOR node, then a new data point was added to the LOR node that contains all the voxel description parameters described below.

For the geometry simulated here, the SM was stored using a 32×384 array of pointers (one for each LOR), 32 crystals in module 0, with 384 crystals in the whole PET

FIG. 3. Flow chart describing steps involved for linked-list data creation. The LORs in the data structure were defined by opposite crystals (obtained from GATE). For each LOR, voxels that contribute counts to the LOR were identified and added to the corresponding LOR node in the data structure. If the voxel already exists in the LOR node, then the count counter of the voxel is incremented by 1. If the voxel was not present in the LOR node, then a new data point was added to the LOR node that contains all the voxel description parameters described in Fig. 4.

ring. Each of these LOR pointers points to a linked-list of polar voxel nodes that represent those polar voxels that contribute counts to that LOR. Each of these polar voxel nodes are identified by radial and angular locations (ring and subsector). If previous data have been recorded for the LOR voxel pair (polvxnum), a counter tracking number of emissions from the voxel to the LOR (hitcnt) is incremented, if not then the new voxel is added to the LOR list. The probability that an emission from the voxel is detected in the LOR (probtimes255) and a pointer to the next voxel (next) are also part of the voxel data (Fig. 4). Utilizing this linked list data format reduced the storage space by more than 99% (from 1.72×10^9 to 1.42×10^7).

Previous studies have noted that there is some symmetry in the system matrix for a ring PET system that uses polar voxels^{31–33} or uses smooth rotator functions²⁴ to utilize that symmetry. This is because photons emitted from different object locations may interact similarly with detectors as a consequence of geometrical symmetry. In this study, rotational symmetry of the cylindrical PET scanner has been utilized by representing the object space with polar voxel basis functions, thereby causing the system matrix to be block circulant. This means that there was substantial redundancy in the system matrix, and only elements describing the LOR probabilities from one module (of total 12 modules) need to be computed and stored (these probabilities make up one block column of the system matrix). This is a 12 fold reduction in storage space and 12 fold improvement in calculation speed in comparison to calculating SM with Cartesian voxel representation. In other words, about 12 times less count statistics is required for calculating SM with similar noise

FIG. 4. The figure describes the LOR data structure for a single LOR. There are total 32×224 such LOR data structure each for an unique LOR. Each empty box in the left corresponds to each unique voxel that contribute counts to the LOR. The field “LorVxNodePtr” is a pointer to the next voxel that contributes counts to the LOR. Other fields in the polar voxel node are “ring” and “subsector” that describes the positional index of polar voxel, “polvxnum” that helps in sorting the voxel in the data structure, “hitcnt” is the number of emissions from voxel detected in the LOR and “probtimes255” is the field holding probability of emission from voxel detected in the LOR.

property for polar voxel representation in comparison to Cartesian voxel representation of SM. Another advantage of using polar voxels was that a rotator based projector/back-projector pair can be used without any interpolation errors because a rotation operation can be implemented with a simple shift in the angular direction. This was preferable to implementing a rotator based projector/backprojector pair with voxels on a Cartesian coordinate system because this results in some interpolation error.

Fig. 5 shows a simplified example of how radial symmetry was used to increase the efficiency in computing the non-redundant system matrix elements. In this simplified example, four modules were shown with four crystals in each module (C1, C2, C3, and C4). Rotational symmetry was

FIG. 5. LORs originating at modules other than “0” (1) are rotated to module 0 (1') utilizing rotational symmetry of the cylindrical scanner to improve statistics. LORs originally from module 0 (2) are not rotated.

utilized to compute all system matrix elements for LORs with one endpoint in module 0. Those simulated events with endpoints in other modules were rotated so that one endpoint resides in module 0. The emission location was also rotated, which was simply a shift in angle in polar coordinates. Using this procedure, accurate (low-variance) estimates of the system matrix probabilities can be computed with a lower computational load (by approximately a factor equal to the number of modules), and with less storage space.

As mentioned previously, a polar voxel representation (Fig. 6) has been utilized for defining image space. Polar voxels were defined by 126 concentric rings of radial dimension 0.8 mm each, covering a total radial distance of approximately 100 mm. Furthermore, the first ring (ring 0) was divided into angular subsectors corresponding to an integer multiple of the number of transaxial crystal (384). With increasing radial distance of the ring from the center, the number of subsectors in the ring was increased such that the angular dimensions of each subsector remain constant for all rings (Fig. 6). Thus, all polar voxels had the same area ensuring that the polar voxels have a uniform resolution as a part of the radial dimension.

For the final visual display of the reconstructed image, the image in polar voxels needs to be interpolated to Cartesian coordinate voxels. Zhang *et al.*²⁴ had implemented the transformation of polar to Cartesian coordinates by calculating the area of intersection between polar and Cartesian voxels utilizing the convex hull algorithm.³⁴ This approach could introduce inaccuracy due to difficulty in calculating the area of an irregular shape. In our approach, the location of the emitted photons reported by the GATE simulation software allows for the ability to relate the number of emission counts located in intersecting polar and Cartesian voxels. This relationship provides a Monte Carlo based solution to the surface integral problem of interpolating from polar to Cartesian voxels.

D. Estimating probability of system matrix

In forming the system matrix, simulated events producing LORs in a single module (module 0) were obtained by

FIG. 6. Partial ring, subsector structure for polar voxel image space definition. Subsectors are obtained by dividing radial rings in equal angular area.

rotating the simulated LORs measured in other modules in such a way that one end of the LOR always resided in module 0 (Fig. 5). Upon completion of assigning all LORs in the event list post-rotation, the SM probabilities represented the probability for a photon emitted from voxel j and detected at LOR i in module 0. However, the desired SM element values were the probability of a photon emitted from voxel j and detected at LOR i in any module (not just module 0), i.e., pre-rotation of LORs to module 0. Thus, an adjustment was implemented by using the following expression:

$$p_{ij} = \frac{\text{Counts emitted from voxel } j \text{ to LOR } i \text{ post-rotation}}{\text{Counts emitted from voxel } j \text{ post-rotation}} \times \frac{\text{Counts to Module 0 from voxel } j \text{ pre-rotation}}{\text{Total counts from voxel } j \text{ pre-rotation}}. \quad (2)$$

E. The MLEM reconstruction

Reconstruction was performed using the MLEM algorithm by incorporating the system matrix (p_{ij} in Eq. (2)) computed as described above. The measured data were binned into LOR histogram format prior to implementing reconstruction, where any LOR bins with zero events were not stored. The back projection contribution of a single LOR on all voxels was computed at one time in a single run through the LOR. The calculation burden was thus reduced by reducing the number of times the forward projection matrix was calculated for a LOR. It also allows usage of an efficient LOR access technique for the linked list LOR data structure. The reconstruction algorithm is follows:

$$\lambda_j^{m+1} = \frac{\lambda_j^m}{\sum_{i=1}^I p_{ij}} \sum_{k=1}^{N'} p_{ikj} \frac{n_k}{\sum_{b=1}^{J'} p_{ikb} \lambda_b^m}, \quad (3)$$

where m is the iteration number, λ_j^m is the intensity in voxel j at iteration m , p_{ij} is $p(\text{detected in LOR } i \mid \text{emitted in voxel } j)$, N' is the number of distinct LORs, J' is the total number of voxels lying in LOR i_k , and n_k is the number of events detected in LOR i_k .

F. Reconstruction performance

To evaluate the tomographic spatial resolution of MLEM reconstruction with different methods of modeling the SM, point sources were simulated at varying locations within a breast sized object. Point sources were simulated at radial offsets of 0, 15, 25, 35, 45, 55, and 65 mm from the center in both the x and y directions (Fig. 7, left).

Two reconstruction methods were compared: (1) MLEM using Siddon's³⁵ ray-tracing algorithm to model the SM, and (2) MLEM using a Monte Carlo based SM computed as described above. A 2D Gaussian fit was applied to evaluate the FWHM of point sources.

To evaluate contrast and noise resulting with the different reconstruction methods as a function of spatial location, spheres were positioned at various locations within a uniform cylindrical background (see Fig. 7, right). Three spheres of 8 mm diameter were positioned at 2, 4, and 6 cm

FIG. 7. Left: Resolution phantom showing the point sources are located in both the X and Y directions. The locations of point sources vary from center to 65 mm away from the center in steps of 15 mm in both X and Y dimensions. The concentration ratio between point source and cylindrical background is 10:1. Right: Contrast estimation phantom showing hot spheres (contrast ratio with background: 8:1) of 8 mm diameter at positions of 1.7 cm, 3.7 cm, and 5.7 cm from center in the X directions. Background counts were obtained from ROIs drawn on the cylindrical background, which are of same size and positioned diagonally opposite to the sphere locations.

from the center. Each sphere was modeled with 8:1 signal to background ratio. Monte Carlo simulation was used to generate an event list modeling a 1 min of acquisition. The signal (S) was defined as the mean number of counts in an ROI within each sphere, whereas the background (B) was defined in the same size ROI located opposite to the sphere (see Fig. 7, right). The contrast recovery coefficient (CRC) was calculated as follows:

$$CRC(\%) = S/B \times 100. \quad (4)$$

Noise was calculated based on the expression,

$$Noise(\%) = \sigma_B/B \times 100, \quad (5)$$

where S represents mean counts within the sphere, B represents the mean counts within the background, and σ_B is the standard deviation of background counts.

To compare contrast recovery and noise performance of SM and Siddon line-integral MLEM as a function of sphere size, multiple spheres at varying locations in a cylinder background were simulated. Spheres of 3.5, 4.5, 5.5, 6.5, 7.5, and 8.5 mm diameter at the same radial location of 4.5 cm from the center of the PET FOV (Fig. 8) were simulated. The background was modeled as a 14 cm diameter cylinder and the sphere:background concentration ratio was 8:1. Background counts were estimated by drawing a circular ROI of 8.5 mm diameter at the center of the cylinder, while sphere counts were estimated by drawing an ROI over the reconstructed sphere image (see Fig. 9). CRC was evaluated as a function of noise by reconstructing at various iterations (1–40).

III. RESULTS

A. Simulation statistics

To compute the SM, a uniform cylindrical phantom positioned within the breast PET scanner was simulated using the GATE program, with a run-time of 9.5 h to

produce 2.52×10^9 total coincidences. The SM was thus computed only once and implemented for multiple object reconstructions. By utilizing rotation of all LORs to a single module (module 0) as previously discussed, statistics can be increased by a factor of 12. This allowed average detection of counts per LOR as 6.4×10^4 (noise is 0.4%), which is comparable to the literature reported value (4.7×10^4).²⁴

B. Resolution performance

The tomographic spatial resolution as a function of x-axis location was shown in Fig. 9, and as a function of

FIG. 8. Contrast estimation phantom showing hot spheres (contrast ratio with background: 8:1) of 3.5, 4.5, 5.5, 6.5, 7.5, and 8.5 mm diameter at radial positions of 45 mm from center of cylindrical background. Background counts were obtained from the ROI drawn at the center of the cylindrical background of diameter 8.5 mm.

FIG. 9. Comparison of resolution of point source as function of location offset from FOV center for x-axis locations.

y-axis location in Fig. 10. The graph illustrates the overall improved resolution by the SM based MLEM method as compared to MLEM using the Siddon line-integral approach. Improvement in resolution increased as a function of distance offset from the center of the FOV with the smallest improvement at 15 mm (1.5 times improvement) and the largest improvement at 65 mm (3 times improvement) from the center. Overall, the SM based MLEM reconstruction substantially reduced the spatial variability of tomographic resolution.

C. Contrast performance

The CRCs of the hot spheres (8:1) were all similar with the SM based reconstruction, independent of sphere locations (Fig. 12), while becoming gradually worse with increasing radial distance from the center for the Siddon based reconstruction (Fig. 11). Each point in the graph corresponds to reconstruction results at a particular iteration with noise increasing with iteration (1–60 iterations). Fig. 12 shows the same graph for SM-MLEM reconstruction. Noise was less for SM-MLEM compared to Siddon-MLEM at the

FIG. 10. Comparison of resolution of point source as function of location offset from FOV center for y-axis locations.

FIG. 11. CRC vs Noise for spheres at various locations from center of PET FOV (given in legend) from Siddon line-integral based MLEM.

same iteration (maximum noise for SM-MLEM was observed at less than 10%, while it was 55% for Siddon-MLEM at around 60 iterations as shown in Figs. 11 and 12). Thus, the graphs illustrate that improvement of CRC with iterations was less expensive in terms of noise increment by SM based MLEM reconstruction compared to Siddon's line-integral based MLEM.

Qualitative evaluation showed a reduction in sphere and background noise and improvement of sphere contrast when using the SM based reconstruction compared to the Siddon's line-integral based MLEM reconstruction (Fig. 13). CRC was less for the Siddon based MLEM reconstruction compared to the SM based reconstruction for all size spheres (small, medium, and large). CRC performance with respect to noise was depicted in Figs. 14, 15, and 16, respectively). CRC was 2.9 and 3.3 for the smallest sphere (3.5 mm diameter), 5.4 and 6.2 for the medium sphere (9.5 mm diameter), and 7.1 and 7.8 for the largest sphere (16 mm diameter) at 30 iterations for Siddon's line-integral based MLEM and GATE SM based MLEM algorithm, respectively. Better modeling of detector response reduces noise for SM based MLEM

FIG. 12. CRC vs Noise for spheres at various locations from center of PET FOV (given in legend) from system matrix based MLEM reconstruction.

FIG. 13. Comparison of reconstruction by SM (upper) and Siddon line-integral (lower) based MLEM techniques at various iterations.

reconstruction compared to Siddon's line integral technique. The SM-MLEM provides better resolution than Siddon's line-integral based MLEM reconstruction for the smallest sphere (Fig. 17).

IV. DISCUSSION

In order to improve sensitivity of breast PET, it is desired to image the patient with detectors closer to the body. However, this may increase oblique photon incidence to detectors thereby increasing crystal penetration, scatter and the depth of interaction effect. The resulting resolution degradation at the FOV periphery may distort the reconstructed image. The proposed method addresses the image distortion problem at the FOV periphery by accurately modeling the PET system response.

A technique for software modeling the PET detector response into the system transfer matrix using the GATE Monte Carlo simulation has been developed. By using polar voxel basis functions, the SM for a ring PET system becomes block-circulant. This results in a number of advantages, including a reduction in the Monte Carlo run time required to achieve low variance estimates of the system matrix elements. Another advantage was that the SM requires reduced storage space. The SM was observed to be very sparse, and thus a linked-list format was used for an additional reduction

FIG. 14. CRC with respect to noise for 3.5 mm sphere is illustrated for SM based and Siddon line-integral based MLEM reconstruction; the CRC values are depicted with respect to the gold standard of CRC values for this particular simulation (contrast ratio is 8:1).

FIG. 15. CRC with respect to noise for 9.5 mm sphere is illustrated for SM based and Siddon line-integral based MLEM reconstruction; the CRC values are depicted with respect to the gold standard of CRC values for this particular simulation (contrast ratio is 8:1).

of computer storage by approximately 99%. The resultant total storage space for the SM was 230 Kbytes.

This SM was incorporated into MLEM reconstruction algorithm using data binned into LOR histograms. Once the solution was obtained, interpolation from polar voxels to cubic voxels was performed using Monte Carlo based interpolation. It was observed that this approach improved various parameters of image quality such as spatial resolution, contrast, and noise compared to MLEM using ray-tracing. Another approach for estimating the SM is to use empirical measurements of point sources positioned in the object space. However, these measurements require long acquisition times, and thus it is generally not possible to measure the point response at all possible object locations. Thus, these methods typically involve interpolating the response between measured point source locations. This may produce inaccuracy at detector block corners where the system response may be different than at the non-corner regions of blocks.

FIG. 16. CRC with respect to noise for 16 mm sphere is illustrated for SM based and Siddon line-integral based MLEM reconstruction; the CRC values are depicted with respect to the gold standard of CRC values for this particular simulation (contrast ratio is 8:1).

FIG. 17. (a) Reconstruction of various size spheres, (b) Reconstruction of various size sphere with Siddon line-integral based MLEM, and (c) profile through smallest sphere by SM based MLEM and Siddon line-integral based MLEM.

In this work, the effect of light propagation in scintillators on position estimate was not modeled. Including this degradation in the SM model will degrade resolution somewhat, and would produce deviations in comparison to a measured SM.

Previous studies using a Gaussian rotator in the projector/backprojector operations (with cubic voxels) suggested that there was Gibbs artifacts (ringing) present at image boundaries and at lesion boundaries caused by Gaussian based rotator utilized in transferring the Cartesian voxel location to base-symmetry voxel location.³⁶ It was reported that these ring artifacts subside when utilizing an ideal rotator with polar voxels. In this work, rotational symmetry has been exploited by using a polar voxelization scheme for representing image space. This simplifies the rotation operation to a simple shift in the projector/backprojector pair, and interpolation was not needed. Thus, a blurring rotator function was not required.

One disadvantage of using Siddon projection based OSEM reconstruction in Cartesian coordinate grid is unevenly placed LORs with respect to projection angles requiring interpolation to adjust to evenly placed LORs. The adjustment has been attributed to reduced noise-resolution tradeoff caused by interpolation error. In a previous work, the rotational-based projector²⁹ was used to map the Cartesian image matrix to unevenly spaced LORs. The better noise-resolution tradeoff compared to standard OSEM was attributed to reduction in interpolation error in forming projections from LORs. As the present work and the comparable Siddon technique use LOR binned and not projection binned histogram, the interpolation of LORs was not implemented thus circumventing any interpolation error. Moreover, the rotational-based projector using Siddon technique may not model physical degradation factors such as crystal penetration and scatter thus degrading imaging performance compared to the present method.

It has been suggested that reducing the size of the SM kernel by decreasing the number of modeled resolution degradation factors reduces the Gibbs artifact in the image.³⁷ It has also been reported that though the Gibbs artifact increases with broader SM kernel due to blurring of the system response, it will also improve contrast recovery and

resolution in the presence of noise.³⁸ We observed reduced Gibbs artifact without sacrificing contrast recovery (maximum contrast recovery for the 16 mm sphere was 97.5% at 30 iterations) illustrating limited over-blurring of SM.

The effect of photon acolinearity and positron range was not modeled in the system matrix. By not modeling these effects, the Monte Carlo simulation efficiency can be improved by a factor of four.²⁴ Due to the smaller bore of the PET scanner, photon acolinearity would probably have less impact on the reconstruction results. Moreover, positron range can be ignored for lower energy positrons (F-18) without compromising image quality.²⁴

To apply the SM for reconstructions universally, the SM should be independent of object related degradation factor such as scatter that may produce inaccurate estimate of SM probabilities. In order to make the SM independent of acquisition scenario, only true events were included and scatter events were neglected for making the SM.

Object attenuation may also add complexity in SM estimation. Both attenuation and scatter are object dependent and as such not rotationally symmetric. Thus, incorporating attenuation and scatter effect in the SM that is rotationally symmetric may add complexity in SM calculation. To make the SM calculation simple and efficient by exploiting its rotational symmetric nature, the attenuation and scatter effect are not incorporated in SM calculation. To make the SM calculation independent of object attenuation, an air background was used as the object material for simulation. Thus, the SM based correction can only be applied for LOR data pre-corrected for attenuation and scatter.

The main advantage of using SM based reconstruction was increased resolution without the increased noise as generally observed at higher iterations of MLEM reconstruction. A similar reduction of noise in the sphere contrast estimate was previously observed with SM based reconstruction techniques.²⁰ Iterative reconstruction in conjunction with depth-dependent resolution recovery has similarly improved image quality in full-time and half-time Single Photon Emission Computed Tomography (SPECT) imaging compared to non-iterative reconstruction technique without resolution recovery modeling.³⁹ One reason for the reduced noise may be accurate modeling of the detector response leading to reduced mis-

positioning of counts in reconstruction space. By reducing number of iterations, Siddon based MLEM reconstruction can also provide reduced noise at the cost of reduced contrast. With SM based MLEM reconstruction, the increase in noise was not that severe with increased iteration (Fig. 13). One possible advantage of reduced noise is reduced dose to the patient.

V. CONCLUSIONS

Quantitative and qualitative evaluation illustrated the improved contrast and resolution performance of SM based MLEM reconstruction compared to Siddon line-integral based MLEM reconstruction. Such improvement from the SM method was observed at reduced noise compared to ray-tracing. The technique promises to reduce reconstruction distortion at the FOV periphery for smaller bore PET systems by improved modeling of the PET system response.

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