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REGULARIZED IMAGE RECONSTRUCTION FOR PS MODEL-BASED CARDIOVASCULAR MRI

Anthony G. Christodoulou, Bo Zhao, and Zhi-Pei Liang

Department of Electrical and Computer Engineering, and Beckman Institute of Advanced Science and Technology, University of Illinois at Urbana-Champaign

Abstract

Real-time cardiovascular MRI is a useful and challenging dynamic imaging application. The partial separability (PS) model enables reconstruction of dynamic cardiac images from highly undersampled $(\mathbf{k}, t)$ -space data. However, the underlying PS model-based reconstruction problem is ill-conditioned, so regularization is often necessary to stabilize its solution. It has been shown that ℓ_1 regularization is useful for finding sparse solutions, and ℓ_2 regularization is widely used to incorporate anatomical constraints. An important practical question is which regularization scheme to use for PS model-based cardiovascular imaging. We address this problem by implementing both schemes and evaluating their performances in terms of reconstruction error, image artifacts, image noise, computation time, and performance characterizability. The ℓ_1 -regularized results exhibit lower reconstruction error, artifact energy, and noise variance, while ℓ_2 regularization is much faster and produces predictable reconstruction results. This study indicates that the ℓ_1 scheme is preferable when image quality is the main concern.

Index Terms

Spatiotemporal modeling; Inverse problems; Regularization; Cardiovascular imaging; Magnetic resonance imaging (MRI)

1. INTRODUCTION

Real-time (ungated) cardiovascular magnetic resonance (CMR) has many potential applications [1]. Advances in fast-scanning pulse sequences and parallel imaging methods have recently made free-breathing, real-time cardiac imaging a realizable goal. There is also growing research into model-based sparse sampling methods for cardiovascular MRI (e.g., UNFOLD [2], PARADIGM [3], k - t BLAST [4], and k - t FOCUSS [5]), which provide a new opportunity to significantly accelerate CMR, enabling real-time 3D applications. Model-based cardiovascular imaging using sparse sampling is the focus of this paper.

For convenience, we focus on Fourier-encoded parallel MRI, in which the measured signal $d_c(\mathbf{k}, t)$ from the c th coil can be expressed as

$$d_c(\mathbf{k}, t) = \int_{-\infty}^{\infty} S_c(\mathbf{r}, t) \rho(\mathbf{r}, t) e^{-i2\pi \mathbf{k} \cdot \mathbf{r}} d\mathbf{r}, \quad (1)$$

where $S_c(\mathbf{r}, t)$ is the corresponding sensitivity encoding function. Direct Fourier reconstruction of $\rho(\mathbf{r}, t)$ from $\{d_c(\mathbf{k}, t)\}$ is subject to both spatial resolution and temporal resolution limitations. The partial separability (PS) model relaxes MR data acquisition requirements, providing an effective approach to achieve sparse sampling of $(\mathbf{k}, t)$ -space.

2. PS MODEL-BASED IMAGING

The L th-order PS model [6] represents the spatiotemporal image function $\rho(\mathbf{r}, t)$ as

$$\rho(\mathbf{r}, t) = \sum_{\ell=1}^L \Psi_{\ell}(\mathbf{r}) \phi_{\ell}(t), \quad (2)$$

where $\psi_{\ell}(\mathbf{r})$ and $\phi_{\ell}(t)$ are the ℓ th spatial and temporal basis functions, respectively. The model is particularly suited to cardiovascular imaging applications where $\rho(\mathbf{r}, t)$, as viewed as a family of temporal functions, is linearly dependent. This linear dependence enables recovery of $\rho(\mathbf{r}, t)$ from sub-Nyquist samples.

2.1. Data Acquisition

Many sparse sampling data acquisition strategies can be used for PS model-based imaging. One such strategy is to densely sample one region of $(\mathbf{k}, t)$ -space in high temporal resolution while also sparsely sampling a region of $(\mathbf{k}, t)$ -space with extended $\mathbf{k}$ -space coverage (i.e., with high spatial resolution). We can then obtain $\{\phi_{\ell}(t)\}$ directly from the densely sampled data through subspace decomposition techniques, and we can determine $\{\psi_{\ell}(\mathbf{r})\}$ by using the PS model to fit the known $\{\phi_{\ell}(t)\}$ to measured data [6]. We refer to this strategy as navigated data acquisition (the dense $(\mathbf{k}, t)$ -space samples can be viewed as a form of temporal navigation data). Two examples of $(\mathbf{k}, t)$ -space sampling patterns employing the navigated strategy are shown in Figs. 1(a) and 1(b).

Even when $(\mathbf{k}, t)$ -space data are acquired sparsely without a dense subset for temporal navigation, it is often still possible to simultaneously determine $\{\psi_{\ell}(\mathbf{r})\}$ and $\{\phi_{\ell}(t)\}$ using matrix recovery techniques [7, 8]. A typical $(\mathbf{k}, t)$ -space sampling pattern for matrix recovery image reconstruction is pictured in Fig. 1(c). Each of the sampling patterns in Fig. 1 is simple to implement on most modern MR scanners. Because navigated data acquisition allows direct estimation of $\{\phi_{\ell}(t)\}$, we will assume this acquisition strategy in the remainder of this paper.

2.2. Image Reconstruction

With a pre-computed $\{\phi_{\ell}(t)\}$ and measured data denoted as $\{d_c(\mathbf{k}_m, t_n)\}$ (represented in vector form as $\mathbf{d}$), the PS model assumption is expressed as

$$\mathbf{d} = \mathbf{E}\Psi + \xi, \quad (3)$$

where Ψ is a vector which contains all of the spatial basis functions $\{\psi_{\ell}(\mathbf{r})\}$, and ξ represents measurement noise. The forward operator $\mathbf{E}$ is defined as $\mathbf{E} = \mathbf{U} \mathbf{F} \mathbf{S} \mathbf{\Phi}$, where $\mathbf{\Phi}$ is the

matrix constructed from $\{\varphi_\ell(t)\}$ which transforms Ψ into $\rho(\mathbf{r}, t)$, $\mathbf{S}$ applies the coil sensitivity functions $\{S_c(\mathbf{r}, t)\}$ to $\rho(\mathbf{r}, t)$, $\mathcal{F}_t$ is the Fourier operator that transforms $\{S_c(\mathbf{r}, t)\rho(\mathbf{r}, t)\}$ into $\{d_c(\mathbf{k}, t)\}$, and $\mathcal{U}$ retains only the $(\mathbf{k}, t)$ -space data measured in $\{d_c(\mathbf{k}_m, t_n)\}$ [9]. Note that $\mathbf{S} = \mathbf{I}$ in the single-channel case.

When the model is correct and when ξ is white Gaussian noise, the least-squares fit according to (3) has a statistical interpretation as the maximum-likelihood estimator of Ψ . However, the undersampling performed by $\mathcal{U}$ often causes the data fitting problem to be ill-conditioned, resulting in noise amplification and potential image artifacts. It is therefore useful to regularize the model by incorporating additional penalties. In this work, we investigate two regularization methods. The first method employs an ℓ_1 penalty to enforce spatial-spectral sparsity; the second method employs an ℓ_2 penalty to impose anatomical constraints. We will compare reconstruction error, noise properties, image artifacts, computation time, and the overall characterizability of both regularization methods at multiple noise levels.

3. REGULARIZED PS MODEL-BASED RECONSTRUCTION

3.1. ℓ_1 Regularization

The ℓ_1 penalty function is widely used when seeking solutions which are sparse in some domain [10]. CMR image functions generally have compact spatial-spectral support [2, 3], so it is very effective to assume that $\mathcal{F}_t\Phi\Psi$ is sparse (where $\mathcal{F}_t$ is the Fourier transform over the temporal dimension) [5, 11]. Using the ℓ_1 penalty, the spatial basis functions of the PS model are given as

$$\hat{\Psi}^{(1)} = \arg \min_{\Psi} \|\mathbf{d} - \mathbf{E}\Psi\|_2^2 + \lambda \|\mathbf{H}\Psi\|_1, \quad (4)$$

where $\mathbf{H}$ is a linear transformation matrix and λ is the regularization constant [11]. In the remainder of this paper, we will use $\mathbf{H} = \mathcal{F}_t\Phi$ to enforce spatial-spectral sparsity.

The nonlinear optimization problem in (4) can be solved using a number of optimization algorithms. In this paper, we solve it by combining a half-quadratic optimization algorithm with a continuation procedure [11, 12]. By replacing the nondifferentiable ℓ_1 norm of the $M \times 1$ vector $\mathbf{H}\Psi$ with $\|\mathbf{H}\Psi\|_1 \approx \sum_{m=1}^M g[|(\mathbf{H}\Psi)_m|]$, where $g(x)$ is the differentiable Huber function

$$g(x) = \begin{cases} x^2/2\alpha, & |x| \leq \alpha \\ |x| - \alpha/2, & |x| > \alpha \end{cases}, \quad (5)$$

we can minimize an approximation of the cost function in (4) using a half-quadratic optimization algorithm. We then decrease the Huber threshold α in each stage of a continuation procedure until the ℓ_1 penalty is well approximated and the cost function in (4) is minimized [11, 12].

3.2. ℓ_2 Regularization

In biomedical imaging, the ℓ_2 penalty function is often used to impose anatomical constraints on the smoothness of the reconstruction [13]; this is the method we will evaluate in this work. The general ℓ_2 Tikhonov-regularized solution $\hat{\Psi}^{(2)}$ to the model fitting problem in (3) is given as

$$\hat{\Psi}^{(2)} = \arg \min_{\Psi} \|\mathbf{d} - \mathbf{E}\Psi\|_2^2 + \lambda \|\mathbf{B}\Psi\|_2^2, \quad (6)$$

where $\mathbf{B}$ is a weighting matrix and λ is the regularization constant [9]. We can impose anatomical constraints on the smoothness of the image by using $\mathbf{B} = \mathbf{W}\mathbf{G}\Phi$, where $\mathbf{G}$ is a spatial gradient operator and $\mathbf{W}$ is a weighting matrix which imposes the statistical prior [13]. In dynamic applications such as CMR, the large amount of data collected and represented in $\{d_c(\mathbf{k}, t)\}$ can be used to generate the statistical prior necessary to define $\mathbf{W}$ [14]. In the remainder of this paper, we will use $\mathbf{B}$ as defined above.

Equation (6) has a closed-form solution

$$\hat{\Psi}^{(2)} = (\mathbf{E}^H \mathbf{E} + \lambda \mathbf{B}^H \mathbf{B})^{-1} \mathbf{E}^H \mathbf{d}, \quad (7)$$

which can quickly be evaluated using a conjugate gradient algorithm.

4. RESULTS AND DISCUSSION

4.1. Monte Carlo Reconstruction Analysis

Nonlinear reconstruction methods do not allow for direct separation of image artifacts and image noise in the resulting reconstructions. In this work, we perform Monte Carlo simulation to perform this separation. This involves representing the reconstructed image vector $\hat{\rho}^{(n)}$ as a sum of the true image vector ρ , an image artifact vector $\mathbf{a} = E[\hat{\rho} - \rho]$ (where E denotes expectation), and an image noise vector $\eta^{(n)}$ (i.e., $\hat{\rho}^{(n)} = \rho + \mathbf{a} + \eta^{(n)}$). We consider $\mathbf{a}$ to be the deterministic error component which represents the systematic reconstruction error (i.e., the spatiotemporally localized error bias) for a specific input noise distribution and η to be the error component whose elements are realizations of zero-mean random variables. We perform this analysis for each input noise distribution in our simulations.

4.2. Results

To demonstrate both the ℓ_1 and ℓ_2 regularization methods using a realistic gold standard, we include simulation results using a single-channel human cardiac phantom. The phantom was created from retrospectively ECG- and respiratory-gated experimental data; both variable-rate breathing and cardiac arrhythmia were simulated through a combination of temporal warping and spatial deformations. The phantom image sequence features one 200×256 frame every 3 ms over a duration of 31 s.

Simulations were conducted by sparsely sampling $(\mathbf{k}, t)$ -space data from the numerical phantom. In accordance with time-sequential sampling theory, only one $(\mathbf{k}, t)$ -space readout line was collected every 3 ms (the effective T_R). Data acquisition was alternated between sparse and dense $(\mathbf{k}, t)$ -space sampling patterns over the full duration of the image sequence. In the ℓ_1 case, the order of phase encodings for the sparse subset was permuted to simulate a uniform random sampling pattern. In the ℓ_2 case, the sparse subset was collected using sequential phase encoding (from one end of $\mathbf{k}$ -space to the other). In both cases, the dense subset was acquired by repeatedly measuring data from seven lines at the center of $\mathbf{k}$ -space.

Three noise levels were considered for each regularization scheme, specifically the noiseless case and two noisy cases. For the noisy cases, complex Gaussian noise ξ was added to the $(\mathbf{k}, t)$ -space samples. The artifact and noise components of the resulting noisy reconstructions were separated through Monte Carlo simulation, using a sample size of 10 for each regularization scheme at each noise level. Each λ was set according to the noise level in the data. All reconstructions were performed with model order $L = 16$.

Each ℓ_1 optimization problem took ~ 20 minutes to solve on a dual quad-core Intel Xeon 3.16 GHz workstation with 48 GB RAM. Each ℓ_2 optimization problem took ~ 1.5 minutes to solve on a quad-core Intel Xeon 3.60 GHz workstation with 8 GB RAM.

Figure 2 shows reconstructed frames over the 63×63 cardiac region. The top row shows the gold standard plus the appropriate level of noise. The top right image is from the noiseless gold standard ρ against which all reconstructions were compared. The second row shows least-squares PS reconstructions (i.e., PS without regularization). The bottom two rows show PS reconstructions with ℓ_1 and ℓ_2 regularization. Figure 3 shows spatial maps of the separated artifact (bias) and noise terms at the high input noise level. Each spatial map depicts RMS values across time. Table 1 gives normalized RMS values for the reconstruction error, image artifacts, and image noise. The ℓ_1 scheme resulted in overall lower reconstruction error energy, image artifact energy, and image noise variance in all cases.

4.3. Discussion

Because the ℓ_2 regularization scheme imposes a spatially-weighted smoothness penalty, the resulting image noise is predictably focused in the areas which correspond to low smoothness penalties. The spatial location of the image noise in the ℓ_1 -regularized reconstructions exhibits less structure: that is to say, the image noise is more evenly distributed spatially. Reconstructions from the ℓ_2 scheme shows significantly higher error bias over most spatial regions when compared to reconstructions from the ℓ_1 scheme.

The spatial maps of artifacts and error give a general idea as to the utility of each regularization scheme for different biomedical applications. Although there are some spatial regions of the reconstructions over which the ℓ_2 solution exhibits both smaller error bias and lower noise, the cardiac region is generally better represented by the ℓ_1 solution.

Overall, the ℓ_1 solution is consistently the most accurate in terms of reconstruction error, image artifacts, and image noise. The ℓ_2 solution is much quicker to compute, and the

closed-form linear solution to the ℓ_2 optimization problem has the benefit of easier characterizability as compared to the nonlinear ℓ_1 optimization problem. When computation time and resources are freely available, the ℓ_1 method will be preferable in most situations. The ℓ_2 method may be preferable when computational efficiency is desired.

5. CONCLUSION

We have evaluated two regularization methods for PS model-based image reconstruction for highly undersampled $(\mathbf{k}, t)$ -space data. A Monte Carlo method to separate image artifacts from image noise has been described and implemented for the purposes of this study. The ℓ_1 -regularized results generally show smaller reconstruction error, image artifacts, and image noise at all noise levels. The ℓ_2 regularization problem is faster to compute and has the benefit of a linear (and therefore easily characterizable) optimization problem. When sufficient computational resources are available, the ℓ_1 method is generally preferable; when computational resources are limited, the ℓ_2 method may serve as a simpler substitute. Both methods are useful for PS model-based reconstruction from highly undersampled data.

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Fig. 1.

(a) Navigated data acquisition with dense $(\mathbf{k}, t)$ -space sampling over a region limited to three $\mathbf{k}$ -space lines and with sequential sparse sampling over the remainder of $(\mathbf{k}, t)$ -space. (b) Navigated data acquisition with dense sampling over one $\mathbf{k}$ -space line and random sparse sampling according to a uniform distribution. (c) Sparse data acquisition using random sampling with Gaussian density.

Fig. 2.

Reconstructions over the 63×63 cardiac region. The top row depicts the gold standard plus noise with appropriate variance. The second row depicts least-squares PS reconstructions. The bottom two rows depict the PS reconstructions with ℓ_1 and ℓ_2 regularization.

Fig. 3. Spatial maps of artifacts and noise (i.e., RMS values of the bias and noise across time) for the regularized PS reconstructions corresponding to the high input noise level. For clarity, the contrast window is 8 times brighter here than the contrast window in Fig. 2

Table 1

Normalized RMS values for total reconstruction error, image artifacts, and image noise

		Total Error	Artifacts	Noise
Noiseless	ℓ_1	3.34%	3.34%	n/a
	ℓ_2	3.91%	3.91%	n/a
Low Noise	ℓ_1	3.58%	3.38%	1.17%
	ℓ_2	4.36%	3.96%	1.82%
High Noise	ℓ_1	4.98%	3.68%	3.35%
	ℓ_2	6.09%	5.04%	3.42%