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Identifying the interaction between influenza and pneumococcal pneumonia using incidence data

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Abstract

The association between influenza virus and the bacterium *Streptococcus pneumoniae* (pneumococcus) has been proposed as a polymicrobial system, whereby transmission and pathogenicity of one pathogen (bacterium) are affected by interactions with the other (virus).

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Supplementary Materials

Supplementary Materials include the following:

Supplementary Text, section S-1. Additional results

Supplementary Text, section S-2. A brief literature survey on influenza-pneumococcal interaction

Supplementary Text, section S-3. Sensitivity analyses.

Supplementary Figure, Fig. S1. Population of Illinois.

Supplementary Figure, Fig. S2. Likelihood profiles for carriage-related hazard and influenza reporting ratio.

Supplementary Figure, Fig. S3. Simulations from the models fit to Dataset I.

Supplementary Figure, Fig. S4. Simulations from the models fit to Dataset II..

Supplementary Figure, Fig. S5. Comparisons between predictions and the data.

Supplementary Figure, Fig. S6. Inference on the timing of the interaction.

Supplementary Figure, Fig. S7. Schematic representation of the alternative model with vaccine.

Supplementary Figure, Fig. S8. The inference for influenza-pneumococcal pneumonia interaction using the alternative vaccine model.

Supplementary Figure, Fig. S9. Estimate of enhancement on various subsets of Dataset II.

Supplementary Figure, Fig. S10. The effect of timing of influenza peaks on detectability.

Supplementary Table, Table S1. Parameters used in inference algorithm with their ranges.

Supplementary Table, Table S2. Model parameters and their estimated range for epidemiology of pneumococcal pneumonia.

However, studies focusing on different scales of resolution have painted an inconsistent picture: individual-scale animal experiments have unequivocally demonstrated an association, while epidemiological support in human populations is, at best, inconclusive. Here, we integrate weekly incidence reports and a mechanistic transmission model within a likelihood-based inference framework to characterize the nature, timing and magnitude of this interaction. We find support for a strong but short-lived interaction, with influenza infection increasing susceptibility to pneumococcal pneumonia ~100-fold. We infer modest population-level impacts arising from strong processes at the level of an individual, thereby resolving the dichotomy in seemingly inconsistent observations across scales. An accurate characterization of the influenza-pneumococcal interaction can form a basis for more effective clinical care and public health measures for pneumococcal pneumonia.

Introduction

It is increasingly clear that many pathogens interact. Infection with one pathogen can affect the severity, infectivity or susceptibility to subsequent infection with other pathogens and these effects can have profound clinical, epidemiological, and evolutionary implications (1-8). An important example is thought to be the association between the influenza virus and the bacterium *Streptococcus pneumoniae* (pneumococcus). Suspicions concerning a possible interaction among the two date back over two centuries, with the observation that prevalence of pneumonia increased during influenza epidemics (9). Histo-pathological examinations have demonstrated that at least 24% of fatalities during the 1918 Spanish influenza pandemic had evidence of concomitant pneumococcal respiratory infection (10). More recently, an atypical increase in pneumococcal hospitalizations coincided with the occurrence of the A/H1N1 influenza pandemic in fall 2009 (11). During non-pandemic periods, the epidemiology of these pathogens is characterized by peaks during the winter months in temperate countries (12-14), as can be seen from long-term weekly epidemiological records for the State of Illinois, USA, before and after the introduction of pneumococcal conjugate vaccines (PCV) in 2000 [26]. [See Fig. 1]

There is a discrepancy in our understanding of this interaction, arising from the different scales at which it has been studied. The evidence garnered at the level of the individual is both consistent and strong. In humans, histo-pathological examinations have implicated secondary bacterial infections in lethal (10, 15-18) and severe influenza cases (11). In animal models, challenge experiments have shown that prior influenza infection enhances the severity (19-21) and susceptibility (22, 23) of subsequent pneumococcal infection. At the population level, however, comparative analyses of seasonal patterns of influenza and pneumococcal pneumonia incidence have revealed either a modest association (24, 25) or fail to expose any signature of interaction (26). [See SM, section S-2 for more discussion.] One possible explanation for these contrasting findings is that conclusions drawn from animal challenge experiments are not directly relevant to human pathogen systems. Alternatively, the mechanisms identified in animal models may be in operation, but their dynamical footprints in epidemiological data may be too subtle to detect using traditional methodologies. To resolve this dichotomy, here we confronted a mechanistic transmission model with incidence reports of influenza and pneumococcal pneumonia within a

likelihood-based statistical inference framework. This approach permitted not only the quantification of central parameters, but also the rigorous testing of alternative hypotheses.

Our aim was to identify the nature, strength and timing of the putative interaction between influenza and pneumococcal pneumonia. We examined three distinct hypotheses. The first (**H1**: transmission impact), proposes that individuals infected with pneumococcal pneumonia, contribute more to pneumococcal transmission if they have been recently infected with influenza. The second (**H2**: susceptibility impact) proposes that individuals infected with influenza are more susceptible to pneumococcal pneumonia. The third (**H3**: pathogenesis impact) proposes that influenza infection only affects the pathogenesis of subsequent pneumococcal pneumonia infection, enhancing the severity of clinical symptoms and thereby increasing the odds of notification.

Results

Model

We formulated these focal hypotheses within an *SIRS* compartmental model of pneumococcal transmission (27,28), using influenza incidence as a covariate. Our pneumococcal transmission model split the host population into 3 compartments according to their pneumococcal infection status: with *S*, *I* and *R* representing susceptible, infectious and recently recovered individuals. Compartments *S* and *I* are further sub-divided to take into account individual status with respect to influenza. Specifically, $S_{\mathcal{F}}$ and $I_{\mathcal{F}}$ represent susceptible and infectious individuals currently infected with influenza, while S_U and I_U have no recent history of influenza. The model is schematically represented in Fig. 2D. We estimated the size of $S_{\mathcal{F}}$, assuming it is proportional to the current influenza incidence,

corrected for under-reporting. That is, at time (*t*), $S_{\mathcal{F}}(t) = \frac{\mathcal{F}(t)}{\rho_{\mathcal{F}} N(t)} S(t)$, where *N*(*t*) is the population size, $\mathcal{F}(t)$ is the number of reported influenza cases, and $\rho_{\mathcal{F}}$ is the reporting probability.

In our model, susceptibles experienced a *per capita* hazard of pneumococcal infection, $\lambda(t)$, that comprised two sources of transmission: (i) those currently experiencing acute pneumococcal pneumonia and (ii) a contribution from long-term bacterial carriage, modeled

as a constant, ω , yielding $\lambda(t) = \beta(t) \left[\frac{I(t)}{N(t)} + \omega \right]$. Here, $\beta(t)$ represents seasonally varying transmission rate. Our pneumococcal transmission model was implemented as a Markov Chain, incorporating both demographic and extra-demographic noise (29), together with a probabilistic reporting process, with reporting probability ρ_p . The complete model is described in Materials and Methods.

Hypotheses formulation

To formulate **H1**—increased transmission of pneumococcus as a result of influenza infection—we distinguished between the transmission contribution of those infected with pneumococcal pneumonia according to their status with respect to influenza. Specifically, we assumed that the pneumococcal transmission rate of individuals recently infected with

influenza is modulated by a factor θ , relative to those uninfected with influenza, such that:

$\lambda(t) = \beta(t) \left[\frac{I_U}{N} + \theta \frac{I_{\mathcal{F}}}{N} + \omega \right]$. Thus, **H1** implies $\theta > 1$, with the null hypothesis given by $\theta = 1$. To formulate **H2**—influenza infection increases susceptibility to pneumococcal pneumonia—we assumed the relative hazard rates of susceptibles in the sub-compartments, S_S , and $S_{\mathcal{F}}$ are given by $\lambda(t)$ and $\phi \lambda(t)$, respectively. The hazard ratio ϕ is then a measure of the susceptibility-impact of influenza infection. Again, **H2** implies $\phi > 1$, with the null hypothesis $\phi = 1$. Finally, to formulate **H3**—increased pneumococcal pneumonia severity due to influenza infection—we hypothesized that individuals coinfecting with influenza only develop more severe symptoms and are, therefore, ξ times more likely to be reported. Hypothesis **H3** implies $\xi > 1$, with $\xi = 1$ being the null. To evaluate the empirical evidence in support of each model, we carried out formal hypothesis-testing using likelihood-based inference (30-32) [See Materials and Methods for details], applied to weekly epidemiological records of influenza and pneumococcal pneumonia hospitalizations from Illinois. Further, we tested the impact of the introduction of the pneumococcus vaccination program on these interactions, by fitting separate models to data from the pre-vaccination [Dataset I, Fig. 1A] and vaccination time periods [Dataset II, Fig. 1B]. For each hypothesis, we constructed likelihood profiles for the focal parameters (θ , ϕ and ξ); that is, we systematically varied the parameter while maximizing the likelihood over all other parameters. This approach not only yields the maximum likelihood estimates (MLEs) but also provides the corresponding 95% confidence intervals (CIs) for the parameter profiled.

Nature, timing and the intensity of the interaction

The results concerning the nature of the interaction between influenza and pneumococcal pneumonia were unequivocal in our study. We found no evidence to support the transmission (**H1**) or severity (**H3**) hypotheses in either dataset [Figs. 2B & 2D, and Figs. 2E & 2G, respectively]. The 95% confidence intervals for θ , (Dataset I: [0.41-4.7]; Dataset II: [0.3-1.9]) and for ξ , (Dataset I: [0.4-3.4]; Dataset II: [0.38-1.4]) include the null expectation of 1. In contrast, we find the susceptibility impact, ϕ , to be considerably larger than one in both datasets (Dataset I: [53-230]; Dataset II: [83-280]). In the absence of support for **H1** and **H3**, and for reasons of parsimony, we re-estimated the susceptibility impact ϕ for each dataset after setting θ and ξ to 1 [insets of Figs. 2 C,F]. The maximum likelihood estimate of ϕ —quantifying the magnitude of the susceptibility impact of influenza on pneumococcal pneumonia—is 115 (CI: [70-230]) in Dataset I and 85 (CI: [27-160]) in Dataset II [MLEs are presented in Table S2 of SM]. We also established the time scale of the interaction by producing a likelihood profile for the susceptibility factor when the window of interaction is extended, with influenza preceding pneumococcus super-infection by up to 3 weeks [See S-1.2 in SM for details]. As shown in Fig. S6, there is no evidence in support of the interaction extending beyond one week. Simply put, our analyses identified a transient but significant (~100 fold) increase in the risk of pneumococcal pneumonia following influenza infection.

Epidemiological impact

We translate our MLEs within an epidemiological context by calculating the etiologic fraction of pneumococcal cases, defined as the proportion of cases that is attributable to the interaction with influenza. [See Materials and Methods for details.] We found substantial variation in this quantity [Fig. 3], with the etiologic fraction accounting for up to 40% of pneumococcal cases during peak influenza season, but is generally low (typically $< 1\%$) otherwise. On an annual basis, the etiologic fraction is 2-10%, consistent with values estimated elsewhere from epidemiological time series methods (11, 24). This translated to an estimated 2172[1567,2891], and 1077[782,1430] cases of pneumococcal hospitalizations that could be attributed to influenza in Datasets I and II, respectively.

As a validation step, we compared simulations arising from the MLE and the null models with the data [Figs. S3, S4, and S5]. We found that while ignoring the impact of influenza infection on pneumococcal epidemiology generates dynamics that reproduce the seasonal patterns in pneumococcal pneumonia cases, these fail to capture the observed interannual variability in peak sizes. Accounting for the interaction with influenza explained some of this variability, as reflected in improved goodness of fit statistics (R^2 increased from 0.687 to 0.739 in Dataset I, and from 0.523 to 0.618 in Dataset II [Fig. S5]). The modest increase in R^2 following the inclusion of influenza-pneumococcal interaction highlights the subtlety of its dynamical signature and perhaps helps to explain its elusiveness in previous epidemiological analyses.

Consequences of detection

Finally, we explored the consequences of the influenza-pneumococcal interaction we have detected for pneumococcal epidemiology. We wished to examine the dynamical impact of variability in seasonal influenza peaks on excess pneumococcal cases. To do this, we first manufactured multiple hypothetical influenza datasets which differed in the interannual variability in their peak sizes by a factor of up to 50 [eg, Fig. 4B-F]. Then, using these synthetic influenza data as a covariate, we simulated our pneumococcal transmission model using MLE parameters. [See Materials and Methods for details on inference using manufactured data.] We found the magnitude of pneumococcal pneumonia peaks to be relatively insensitive to moderate year-to-year variation in the size of influenza outbreaks [Fig. 4A]. For instance, a doubling of influenza peak resulted in less than 25% increase in the magnitude of pneumococcal peak. The influenza peak would need to be ~20-fold larger than baseline in order to generate a doubling of the pneumococcal peak [Fig. 4A].

This observation has implications for the ability to detect influenza-pneumonia interaction in epidemiological data; the identifiability of ϕ depends on variability in the peak influenza incidence. To illustrate, we manufactured 5 sets of weekly influenza case reports spanning a four year period, with different magnitudes of epidemic in year 3, marked as I-V in Fig. 4A. For each influenza dataset [Fig. 4B-F], we constructed pneumococcal pneumonia incidences [Fig. 4G-K] by simulating the MLE-model with the manufactured influenza dataset as a covariate. Then, utilizing these simulated influenza and pneumococcal pneumonia incidence data, we attempted to infer the interaction parameter, ϕ using our likelihood-based inference framework [See Materials and Methods for details]. Even in this optimistic scenario, with

perfect knowledge of pneumococcal epidemiology and host demography, we find that it is not possible to detect any interaction unless the influenza outbreak in year 3 is at least four-fold larger than the seasonal baseline [Figs. 4L-P]. This result is robust to some variability in the timing of the influenza peaks [See Fig. S10]. It is also worth noting that the variability observed in the Illinois seasonal influenza peaks is typically below this level [Fig. 4A]. Therefore, the examination of shorter datasets from time periods exhibiting little inter-annual influenza variability can fail to detect any meaningful interaction between influenza and pneumococcal pneumonia. Thus, the epidemiological signature of this strong, but short-lived interaction appears modest and its detectability in data relies on information provided by differential sizes of seasonal influenza outbreaks.

Discussion

Many microbial pathogens associated with infectious diseases co-circulate in a habitat or a population and often co-occur within individual hosts. Despite the ubiquity, detecting interactions between pathogens that is operational in a natural setting remains a challenge that general statistical approaches can fail to meaningfully address (7, 33, 34). We have focused on two common and relatively well studied human pathogens, the influenza virus and the bacterium *Streptococcus pneumoniae*. Our research has employed a framework for statistical inference using mechanistic models to characterize the interaction between two important infectious diseases with the aim of reconciling the contradictory conclusions of studies conducted at different scales. We have identified the nature of this interaction (enhanced susceptibility to pneumococcal pneumonia following recent infection with influenza), its magnitude and time scale. In the process, we have unpacked the population-level implications of immune-mediated processes shown to occur at the level of the individual. This work highlights the potential power of using likelihood-based inferential approaches in conjunction with high resolution data: in addition to providing results that have a mechanistic interpretation, these methods can often uncover underlying processes that are not necessarily visible to the naked eye.

In the absence of higher resolution information—in particular age-stratified pneumococcal carriage (35, 36) and serotype-specific incidence (37)—our modeling choices have been pragmatic. The role of carriage in transmission and incidence of pneumococcal pneumonia is thought to be important (35, 36), and we find similarly strong role of carriage dependent transmission in our models [See SM S-1.1 for discussion and estimates of carriage-dependent transmission]. Furthermore, influenza has also been suggested to affect nasopharyngeal carriage of pneumococcal pneumonia (38). But in the absence of carriage data and clear understanding of the casual link between carriage and transmission, we are unable to incorporate a dynamic and mechanistic description of carriage in this model and the explore possible effects of influenza on carriage. Given the importance, this should be an avenue for future research. Our results may also have been limited by the use of hospitalization reports to infer and estimate interactions. Hospitalization data by design does not account of severity of reported infections, and only consider infections at the severe end of the spectrum. Hence, the true effect of interaction on the severity, as has been seen in animal experiments (19-21), may have been muted in these data.

Despite this, several of our findings are qualitatively consistent with several findings in animal models. This body of experimental work has shown that, for instance, the influenza-pneumococcal interaction operates over a short window—5-7 days (19-21) and that influenza can enhance susceptibility to pneumococcal infection (22, 23). A number of candidate immunological mechanisms have been identified to explain the increased susceptibility (9, 20). In animal models, increased susceptibility is quantified via the reduction in the pneumococcal infectious dose in subjects recently infected with influenza. The experimental observation that increased susceptibility translates into infectious doses that may be many orders of magnitude smaller (19) provides a mechanistic explanation for the MLE of our phenomenological parameter $\phi \sim 100$.

Our work aims to bridge the understanding of between pathogen interactions at two different scales, at the scales of the host and host populations. Our approach here has been to infer interactions at the scale of the host from data of interaction observed at the scale of host populations. For a relatively well-studied system, like the influenza-pneumococcal system, we argue that a framework that couples mechanistic models with statistical inference, can achieve such a goal. In doing so, we also posit a possible mechanism by which potentially strong interactions between pathogens can be masked in nature.

Materials and Methods

Data

Datasets I and II consisted of weekly hospitalization data from the state of Illinois, which we obtained from the State Inpatient Databases (SID) of the Healthcare Cost and Utilization Project (HCUP)(www.hcup-us.ahrq.gov/db/state/siddbdocumentation.jsp), maintained by the Agency for Healthcare Research and Quality (AHRQ), through an active collaboration between AHRQ and NIH. This database contains all hospital discharge records from community hospitals in the state. HCUP databases bring together the data collection efforts of State data organizations, hospital associations, private data organizations, and the Federal government to create a national information resource of patient-level health care data (39). Cases were identified by the presence of the relevant diagnostic codes listed anywhere in the patients record, including pneumococcal pneumonia (International Classification of Diseases revision 9, code 481), influenza (487-488), or all-cause pneumonia, excluding influenza (480-486). Weekly time series were created for each disease outcome. Mid-year population size estimates (Fig. S1) for the state were obtained from the United States Census Bureau.

The weekly reports of influenza and pneumococcal pneumonia cases in Datasets I & II, span periods respectively before and after the introduction of pneumococcal conjugate vaccine. Dataset I, as shown in Fig.1A, spans from the middle of 1989 to the end of 1997, consisting of 442 weeks of data. Dataset II, as shown in Fig.1B, spans from the beginning of 2000 to the end of 2009, consisting of 520 weeks of data. Data are presented as incidences, based on the population of Illinois, which are shown in Fig. S1.

Models

We present the complete model that was introduced in the Results section and was used for the inference of interaction. We proceed by first describing the deterministic skeleton of the model. Subsequently we explain how seasonality and demography are incorporated, how the stochastic analog of the model is constructed, and finally how the observation process is modeled.

Pneumococcal transmission and interaction with influenza—The equations below describe the deterministic skeleton of the pneumococcal transmission model, and its interaction with influenza.

$$\frac{dS(t)}{dt} = \mu(N(t) - S(t)) - \lambda S_U(t) - \phi \lambda S_{\mathcal{F}}(t) + \epsilon R(t)$$

$$\frac{dI(t)}{dt} = \lambda(t) S_U(t) + \phi \lambda(t) S_{\mathcal{F}}(t) - \gamma I(t) - \mu I(t)$$

$$\frac{dR(t)}{dt} = \gamma I(t) - \mu R(t) - \epsilon R(t)$$

$$S_{\mathcal{F}}(t) = \frac{\mathcal{F}(t)}{\rho_{\mathcal{F}} N(t)} S(t)$$

$$S_U(t) = S(t) - S_{\mathcal{F}}(t) = S(t) - \frac{\mathcal{F}(t)}{\rho_{\mathcal{F}} N(t)} S(t)$$

$$\frac{dI_{\mathcal{F}}(t)}{dt} = \phi \lambda(t) S_{\mathcal{F}}(t) - \gamma I_{\mathcal{F}}(t) - \mu I_{\mathcal{F}}(t)$$

$$\lambda(t) = \beta(t) \left[\frac{I_U(t)}{N(t)} + \theta \frac{I_{\mathcal{F}}(t)}{N(t)} + \omega \right]$$

Incorporation of seasonality and demography—The pneumococcal transmission is modeled to be seasonal. Seasonality is implemented using 6 cubic b-spline functions. We pay careful attention to estimating the shape of the seasonality accurately. This is reflected in the choice of flexible b-spline functions, with 6 basis. Since we are fitting data at high resolution in time, it is important to estimate the shape of the seasonality function

accurately. Within the pomp framework, this can be called using `periodic.bspline.basis`, and supplying time periods, degree and the number of basis.

We also incorporate the population of Illinois, which are shown in Fig.S1. The population data is treated as a covariate in these models, and the change in population is treated as excess birth that directly enter the susceptible compartment.

Stochastic analog—To model the presence of stochasticity, we translate the system of ordinary differential equations defined above into a stochastic process model. We do this by considering each flux between compartments to be a random process. In particular, we assume that, over a small time interval of duration t , the *per capita* rates are constant and that the fluxes out of each compartment are independent, multinomial random variables. Thus for example, if we focus on the R compartment, there are two ways of exiting: loss of immunity, and death. By assumption, the *per capita* probability of exit in the interval $[t, t + t)$ is constant and, letting $E_R(t)$ denote the number that actually do exit R in this interval, we have $E_R \sim \text{Binomial}(R, 1 - \exp(-(\epsilon + \mu)t))$. Among those that exit, the numbers of hosts respectively losing immunity and dying over this time interval are distributed as

$$\text{Multinomial}\left(R, \frac{\epsilon}{\epsilon + \mu}, \frac{\mu}{\epsilon + \mu}\right).$$

Finally, to model extra-demographic stochasticity, we include a gamma-distributed multiplicative white noise, dW/dt , in the transmission process (29,40). The standard deviation of this noise, β_{sd} is also fit along with the parameters. The force of infection is given by:

$$\lambda(t) = \beta(t) \left[\frac{I_U(t)}{N(t)} + \theta \frac{I_{\mathcal{F}}(t)}{N(t)} + \omega \right] \frac{dW}{dt}.$$

Measurement model—Pneumococcal pneumonia cases in all our data are reported at weekly interval. We model the observation process to match this frequency of data reporting. We define $H(t)$ to be the total number of new recoveries in week t , of which $H_{\mathcal{F}}(t)$ are recoveries coinfecting with influenza, and the remaining $H_U(t)$ are not. We assume that the weekly case reports, C_P , are normally distributed. ie,

$$C_P \sim \mathcal{N}(\rho_p H, \sigma),$$

where, ρ_p is the reporting ratio, and σ is the standard deviation. We assume that the variance scales linearly with the mean. i.e. $\sigma^2 = c^2 \rho_p C_P$. The parameter related to the scaling of the standard deviation, c , is also fit along with other parameters.

To allow for differences in reporting in pneumococcal pneumonia due to presence of influenza, we assume that the cases with influenza are ξ times more likely to be reported compared to the cases without influenza. This results in the following expansion of the above equation:

$$C_p \sim \mathcal{N}(\rho_p(H_{\mathcal{F}} + \xi H_U), \sigma),$$

Likelihood inference framework

For likelihood based inference, we utilize the framework of partially observed Markov processes (7, 30, 31, 41), that is implemented in a freely available software package pomp (32). This framework consists of following four components:

- i. **Data.** These are weekly case reports of pneumococcal pneumonia between 1989 to 1997 for Dataset I and between 2000 to 2009 for Dataset II.
- ii. **Covariates.** We use weekly case reports of influenza matching the time period spanned by the data, and the population of Illinois as covariates.
- iii. **The process model.** The process model is proposed to describe the underlying epidemiological and demographic processes. This is described in Models portion of the Materials and Methods.
- iv. **The measurement model.** The measurement model is proposed to describe the process by which the data are reported. This is described in Measurement Model portion of the Materials and Methods.

For a dataset Y , consisting of observations of $y(t_j)$, $j = 1, \dots, n$ at n points in time, we calculate the likelihood that a chosen parameter set θ explains the complete data (within the confines of the process and observation models). This likelihood function $\mathcal{L}(\theta)$ is a product of conditional likelihoods, $\mathcal{L}_{t_j}(\theta)$, calculated at each time t_j for all n data points in time. If $f(y | t, \theta)$ is the probability of observing the data $y(t)$ at time t , given parameters θ (measurement model), then the likelihood and log-likelihood functions are defined as follows:

$$\begin{aligned}\mathcal{L}(\theta) &= f(y(t_1), y(t_2), \dots, y(t_n) | \theta) \\ &= \prod_{j=1}^n f_{\theta}(y(t_j) | y(t_{j-1}), y(t_{j-2}), \dots, y(t_1)) \\ &= \prod_{j=1}^n \mathcal{L}_{t_j}(\theta).\end{aligned}$$

$$\log \mathcal{L}(\theta) = \sum_{j=1}^n \log \mathcal{L}_{t_j}(\theta).$$

The Markov property of the model allows one to calculate the conditional likelihood $\mathcal{L}_{t_j}(\theta)$ sequentially starting from $t_j = t_1$. For each t_j , the likelihood function is:

$$\begin{aligned}\mathcal{L}_{t_j}(\theta) &= f_{\theta}(y(t_j) | y(t_{1:j-1})) \\ &= f_{\theta}(y(t_j) | y(t_{j-1})) \\ &= \int \int f_{\theta}(y(t_j) | x(t_j)) f_{\theta}(x(t_j) | x(t_{j-1})) f_{\theta}(x(t_{j-1}) | y(t_{1:j-1})) dx(t_j) dx(t_{j-1})\end{aligned}$$

Here, $x(t_j)$ represents the state of the complete system at time t_j . Consequently, $f_\theta(y(t_j)|x(t_j))$ is the measurement model, $f_\theta(x(t_j)|x(t_{j-1}))$ is the process model, and $f_\theta(x(t_{j-1})|y(t_{1:j-1}))$ is the filtering distribution. Furthermore, the calculation of the filtering distribution is simplified identifying a recursive relation by applying Bayes' Theorem.

$$\begin{aligned} f_\theta(x(t_{j-1})|y(t_{1:j-1})) &= \frac{f_\theta(y(t_{j-1})|x(t_{j-1}))f_\theta(x(t_{j-1})|y(t_{1:j-2}))}{\int f_\theta(y(t_{j-1})|x(t_{j-1}))f_\theta(x(t_{j-1})|t(t_{1:j-2}))dx(t_{j-1})} \\ &= \frac{f_\theta(y(t_{j-1})|x(t_{j-1}))f_\theta(x(t_{j-1})|y(t_{1:j-2}))}{\mathcal{L}_{t_{j-1}}(\theta)} \end{aligned}$$

The density functions $f_\theta(\cdot|\cdot)$ are calculated via Sequential Monte Carlo particle filtering method (41, 42).

The likelihood functions are optimized using mif algorithm, which is also implemented in pomp (32) package. For details pertaining to the methods and implementation of the algorithm please refer to (30-32). The range of algorithm parameters used in the inference work are shown in Table S1.

Influenza attributable etiological fraction of pneumococcal pneumonia cases

Influenza attributable etiological fraction of pneumococcal pneumonia cases in any time interval, is taken to be the ratio of pneumococcal cases as a result of influenza to the total pneumococcal cases in the given time interval. Consider that $H(t)$ is the total number of new recoveries in week t , of which $H_{\mathcal{F}}(t)$ are recoveries coinfecting with influenza. With the normal reporting process, with the reporting ratio, ρ_p , and the standard deviation σ , the total reported cases are $C_P \sim \mathcal{N}(\rho_p, H_{\mathcal{F}}, \sigma)$, and total reported cases of pneumococcal infections with influenza are $C_P^{\mathcal{F}} \sim \mathcal{N}(\xi \rho_p H_{\mathcal{F}}, \sigma)$. Recall, ξ is the hypothesized measure of severity impact. The influenza attributable etiological fraction of pneumococcal pneumonia cases during week t , $E_{\mathcal{F}}(t)$ is given by:

$$E_{\mathcal{F}}(t) = \frac{C_P^{\mathcal{F}}(t)}{C_P(t)}.$$

For results presented in Fig. 3, we estimate C_P , and $C_P^{\mathcal{F}}$, using separate MLE models for Dataset I and Dataset II. Note that $\xi = 1$ in these MLEs. We use 1000 simulations of the MLE models to find the mean and the 95% confidence intervals for these estimates.

Detectability experiments in manufactured data

To study the effect of variation in influenza on pneumococcal pneumonia incidence, and further on the ability to detect the enhancement effect, we manufacture several sets of influenza data, each taken to be weekly case reports of the same length (208 weeks, ~ 4 years). In 3 out of 4 years, we set the influenza reports to the same level as seen in the 2000-2001 season in Dataset II. The remaining third season is varied between datasets. This is to reflect one central component of interannual variability in influenza, the height of the peak.

We then take the MLE-model (corresponding to Dataset II, which predicts a 85 fold susceptibility enhancement), and generate simulated datasets of pneumococcal pneumonia cases for each of the influenza datasets. The population is taken to match the first four years in Illinois in Dataset II. Now, on the basis of pneumococcal pneumonia and influenza, we ask whether the same likelihood based framework is able to detect susceptibility enhancement in each of the datasets. We attempt to infer ϕ , given we have perfect knowledge of all of the other parameters. The likelihood profiles of ϕ from inference tests on five of such datasets are shown in Fig. 4.

Supplementary Material

Refer to Web version on PubMed Central for supplementary material.

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Fig.1. Weekly incidences of influenza, and pneumococcal-pneumonia in Illinois, (A; Dataset I) before and (B; Dataset II) after the introduction of pneumococcal conjugate vaccines (PCV). Incidences are the weekly hospitalization case reports as a fraction of the total population. [See Materials and Methods of details.]

Fig.2.

The nature and intensity of influenza-pneumococcal interaction. (A) Schematic representation of the pneumococcal transmission model. Following the SIRS framework, individuals progress along $S \rightarrow I \rightarrow R \rightarrow S$ at per capita rates λ , γ and ϵ , respectively. Progression of individuals recently infected with influenza are tracked separately, via classes S_F and I_F . Pneumococcal-pneumonia case reports are fraction of the infecteds as they recover. Births and deaths are present in the model, but omitted in this illustration for clarity. [See Materials and Methods for the complete model.] We test three hypothesized pathways of influenza-pneumococcal interaction: **H1** ($\theta > 1$): Individuals infected with pneumococcal pneumonia, contribute more to pneumococcal transmission if they have been recently infected with influenza. **H2** ($\phi > 1$): Individuals recently infected with influenza are more susceptible to pneumococcal pneumonia. **H3** ($\xi > 1$): Individuals infected with pneumococcal pneumonia are more likely to be reported, if recently infected with influenza. The nature and intensity of interactions between influenza and pneumococcal-pneumonia, inferred in Illinois from (B,C,D) 1990 to 1997 (Dataset I); and (E,F,G) 2000 to 2009 (Dataset II). Arranged column-wise are the tests for the three hypotheses, **H1**, **H2**, and **H3**. Plotted in each graph are likelihood profiles for the respective parameters—the profiles are created by fitting a smooth line through the log of the arithmetic mean likelihoods (shown in colored filled circles) in 10 repeated likelihood estimates (shown in colored empty circles).

The values within the two dashed black lines are within the estimated 95% confidence interval, and the value marked with dashed colored line represents the maximum likelihood estimate (MLE). The values corresponding to the MLE and the 95% confidence intervals are given on the top margin of the graphs. The 95% confidence interval is taken to be $\chi^2_1(0.95)/2 \approx 1.92$ log-likelihood units below the maximum — univariate confidence limits using the χ^2 distribution. For each of the three parameters, value of 1 represents the null hypothesis. For **H2**, we show the profiles with $\theta=1$, and $\xi=1$ (i.e. after rejecting **H1** and **H3**) in the inset graphs.

Fig.3.

Impact of influenza on pneumococcal epidemiology. (A) Estimates of influenza attributable etiological fraction of pneumococcal pneumonia cases in Illinois in (A,B) Dataset I; and (C,D) Dataset II. The estimates are based on the corresponding MLE models for each dataset. Panels B and D are fractions averaged over annual periods, mid-year to mid-year, for intervals shown in A and C, respectively. Influenza attributable etiological fraction of pneumococcal pneumonia cases in any time interval, is taken to be the ratio of pneumococcal cases as a result of influenza to the total pneumococcal cases in the given time interval. The estimates for Dataset I and Dataset II are based on 1000 replicate simulations of the respective MLE models. [See Materials and Methods for details on the calculation of etiological fractions.]

Fig.4.

Detectability of influenza-pneumococcal interaction in manufactured data. We manufactured multiple hypothetical influenza datasets that differ in their interannual variability in peak sizes. On the basis of the MLE model, we then predicted pneumococcal pneumonia incidences for each influenza dataset. (A) Color coded contours represent the magnitude of annual peaks in pneumococcal pneumonia incidences, when the annual peaks in influenza incidences (plotted on the horizontal axis), and susceptibility impact, ϕ (plotted on the vertical axis) vary. Magnitude of both influenza and pneumococcal pneumonia peaks are presented as fold-increase relative to their respective baseline peaks. Marked in filled circles are where actual annual peaks in the data lie. We then sampled 5 scenarios (I,II,III,IV,V), indicated by five open red circles. Plotted column-wise are inference tests performed on these 5 sets of manufactured data in each of the scenarios. [See Materials and Methods for details on inference using manufactured data.] Figs. (B,C,D,E,F) are manufactured weekly influenza incidences (on a $\log_{10}$ scale); Figs. (G,H,I,J,K) are simulated weekly pneumococcal pneumonia incidences (on a $\log_{10}$ scale) using the MLE-model, and influenza cases as covariates; and Figs. (L,M,N,O,P) are likelihood profiles of the susceptibility impact, ϕ . The dashed red line is the actual value of interaction ($\phi = 85$, MLE-model), and the values of ϕ between the two dashed black lines are within 95% confidence interval.