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Limited-angle tomography for analyzer-based phase-contrast X-ray imaging

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Abstract

Multiple-Image Radiography (MIR) is an analyzer-based phase-contrast X-ray imaging method (ABI), which is emerging as a potential alternative to conventional radiography. MIR simultaneously generates three planar parametric images containing information about scattering, refraction and attenuation properties of the object. The MIR planar images are linear tomographic projections of the corresponding object properties, which allows reconstruction of volumetric images using computed tomography (CT) methods. However, when acquiring a full range of linear projections around the tissue of interest is not feasible or the scanning time is limited, limited-angle tomography techniques can be used to reconstruct these volumetric images near the *central plane*, which is the plane that contains the pivot point of the tomographic movement. In this work, we use computer simulations to explore the applicability of limited-angle tomography to MIR. We also investigate the accuracy of reconstructions as a function of number of tomographic angles for a fixed total radiation exposure. We use this function to find an optimal range of angles over which data should be acquired for limited-angle tomography MIR (LAT-MIR). Next, we apply the LAT-MIR technique to experimentally acquired MIR projections obtained in a cadaveric human thumb study. We compare the reconstructed slices near the central plane to the same slices reconstructed by CT-MIR using the full angular view around the object. Finally, we perform a task-based evaluation of LAT-MIR performance for different numbers of angular views, and use template matching to detect cartilage in the refraction image near the central plane. We use the signal-to-noise ratio of this test as the detectability metric to investigate an optimum range of tomographic angles for detecting soft tissues in LAT-MIR. Both results show that there is an optimum range of angular view for data acquisition where LAT-MIR yields the best performance, comparable to CT-MIR only if one considers volumetric images near the central plane and not the whole volume.

Keywords

X-ray; phase contrast; tomosynthesis; limited angle tomography

1 Introduction

Multiple image radiography (MIR) ¹ is a planar analyzer-based phase-contrast x-ray imaging method (ABI) ² that improves over conventional radiography as well as over widely used diffraction-enhanced imaging (DEI) ³. MIR is able to simultaneously generate three different parametric images, showing attenuation, refraction and ultra-small angle scattering (USAXS) features of a tissue. The attenuation image shows the absorption and Compton scatter extinction together. The refraction image shows the integrated effect of refractive index variations along the beam path and is suitable for visualizing soft tissues with low absorption contrast, e.g. tendon and cartilage. The USAXS image represents the sub-pixel textural structure of the object and is suitable for observing textural soft tissues such as breast tumors. It should be noted that similar methods to MIR have also been proposed independently by Pagot *et al* ⁴ and Rigon *et al* ⁵. Further background on this and related phase-contrast imaging approaches can found in ^{2, 6, 7}.

The MIR planar images are linear tomographic projections (line integrals) of the corresponding properties of the object ⁸. The linearity of the projections allows reconstruction of 3D volumetric images of the object properties using computed tomography (CT) methods. In Brankov *et al* ⁹, we demonstrated CT-MIR, which is a computed tomography implementation of MIR. CT-MIR uses a set of MIR projections acquired over a full range of views around the object to reconstruct the entire volumetric image. However, in cases where a full range of angular projections cannot be obtained or where scan time is limited, limited-angle tomography methods can be useful. For example, we envision *limited-angle tomography MIR* (LAT-MIR) to be used for imaging of cartilage in the knee ¹⁰ and mammography ¹¹.

For related DEI-CT methodologies see ^{12, 13} and clinical comparisons between conventional X-ray CT methods and ABI-CT see ¹⁴⁻¹⁷.

The work presented here is an extension and unification of preliminary results presented in two short conference papers ^{18, 19}. Specifically in this paper, first use computer simulation to evaluate the applicability of limited-angle tomography techniques to MIR parametric images. We also investigate the accuracy of the reconstructed images as a function of the number of tomographic angles for a fixed total radiation exposure. We use this function to find an optimal range of angles over which data should be acquired for LAT-MIR.

We then apply LAT-MIR technique to real MIR projections of a human thumb, and compare the reconstructed slices near the central plane, a plane that contains the pivoting point of the tomographic movement located between the x-ray tube and detector, to the same slices reconstructed by CT-MIR. Subsequently, we perform a task-based evaluation of the performance of LAT-MIR for different number of angles. As the refraction image could be the most interesting image for studying soft tissue, we use template matching to detect the thumb cartilage in the refraction images near the central plane. Finally, we use the signal-to-noise ratio of this test as the detectability metric to investigate an optimum range of tomographic angles for detecting soft tissues in LAT-MIR.

2 Multiple image radiography

2.1 MIR imaging model

The MIR imaging technique has been described in detail in Wernick *et al*¹, but we review the imaging system here for completeness. MIR uses the same imaging system that is used in DEI. This system, which is known as a Bonse-Hart camera²⁰, is schematically shown in Figure 1.

First, the object is illuminated with a collimated, monochromated X-ray beam obtained using a pair of silicon (Si 333) crystals. While the X-ray beam passes through the object, it will be affected by the object properties, which will change the beam angular intensity profile (AIP), i.e. X-ray intensity as a function of the angular direction of propagation. Next, an analyzer crystal, which is a narrow angular filter, is used to analyze the beam AIP. This angular filter passes only the beam components that are traveling at or near the crystal Bragg angle, θ_b . Since the angular range of this filter is on the order of microradians we can assume that there is no cross talk between adjacent pixels on the detector and the imaging can be considered on a pixel-by-pixel process. By measuring the intensity at different angular positions θ of the analyzer, the AIP of the beam can be effectively determined. The AIP at each pixel can be used to extract information about attenuation, refraction and USAXS parameters.

To define the relationship between the object features and the measured AIP, it is necessary to formulate the beam intensity at the detector when no object is present. Following the treatment in Wernick *et al*¹, and based on a geometrical-optics approximation of the interaction of the beam with the crystals²¹, we can write the total intensity at the detector plane as a function of analyzer setting, θ as

$$g_0(\theta; x, y) = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} I_0(\theta') R_A(\theta - \theta') d\theta', \quad (1)$$

where $I_0(\theta)$ represents the AIP of the beam before passing through the object (after the monochromators) and $R_A(\theta)$ is the reflectivity function of the analyzer. Note that the measured AIP g_0 is independent of the pixel location (x, y) and $I_0(\theta)$ is zero outside a very narrow angular range; hence, we can set the integration region to $(-\infty, \infty)$ and write the equation in convolution form as

$$g_0(\theta; x, y) = \int_{-\infty}^{\infty} I_0(\theta') R_A(\theta - \theta') d\theta' = I_0(\theta) * R_A(\theta) = R(\theta), \quad (2)$$

Where $*$ denotes the one-dimensional convolution with respect to θ . The function $R(\theta)$ is called the *intrinsic rocking curve*, as it is a property of the imaging system and does not depend on the object.

When the object is present, considering the beam propagation through the object as linear in intensity²², the AIP of the beam after passing through the object can be written as

$$I(\theta; x, y) = \int_{-\infty}^{\infty} I_0(\theta') f(\theta - \theta'; x, y) d\theta' = I_0(\theta) * f(\theta; x, y), \quad (3)$$

where $f(\theta; x, y)$ is the angular impulse response function of the object, which represents the AIP of the beam that would be measured by illuminating the object with a perfectly collimated beam, i.e., a beam having the AIP of the form:

$$I_0(\theta) = I_0 \delta(\theta), \quad (4)$$

where $\delta(\cdot)$ denotes the Dirac delta function. Finally by using equation (2) we write the measured AIP on the detector plane as

$$g(\theta; x, y) = I_0(\theta') * R_A(\theta) * f(\theta; x, y) = R(\theta) * f(\theta; x, y). \quad (5)$$

In Khelashvili *et al*⁸, we represented the image formation model for a stratified medium with randomly distributed scattering based on radiative transfer theory where the object is uniquely described by *eight object parameters* (see Eq.(A.3)). Fortunately the approximate simplification of the full image formation model, shown in Appendix A, yields the following Gaussian curve for the object function defined by only three parameters:

$$f(\theta; x, y) = e^{-\alpha(x, y)} \frac{1}{\sqrt{2\pi}\sigma^2(x, y)} \exp\left(-\frac{(\theta - \Delta\theta(x, y))^2}{2\sigma^2(x, y)}\right), \quad (6)$$

where $\alpha(x, y)$, $\Delta\theta(x, y)$ and $\sigma^2(x, y)$ are the tree MIR parameters. The detailed derivation of this approximation is presented in Appendix A. These parameters can be written as

$$\alpha(x, y) = \int_{A(x, y)} \mu(x', y', z') dx' dy' dz', \quad (7)$$

$$\Delta\theta(x, y) = \int_{A(x, y)} \frac{\partial}{\partial y} n(x', y', z') dx' dy' dz', \quad (8)$$

$$\sigma^2(x, y) = \int_{A(x, y)} usaxs(x', y', z') dx' dy' dz', \quad (9)$$

where $\mu(x', y', z')$ is the net linear absorption coefficient, $n(x', y', z')$ is the refractive index and $usaxs(x', y', z')$ can be interpreted as the USAXS parameter of the object. $A(x, y)$ denotes the set of (x', y', z') points in the object coordinates system that lie on the beam path for the detector location (x, y) . A detailed derivation of the parameters in (7)-(9) is presented in Khelashvili *et al*⁸.

2.2 Deriving the parameters from projections

In practice, measurements of $g(\theta; x, y)$ are made on a digital detector for different discrete angular positions of the analyzer crystal so the expected value of the measured AIP can be written as

$$E[g_{m,n}[l]] = g(\theta_l; x_m, y_n), \quad (10)$$

where m and n are the indices of detector pixels at location (x_m, y_n) , $l=1,2,\dots,L$ represents the index of analyzer angle θ_l , and $E[\cdot]$ denotes the expected value. As described in Brankov *et al*⁹, we can estimate the MIR parameters from the sampled $g_{m,n}[l]$ by the following procedure:

First, we define the normalized AIP for each detector pixel as

$$G_{m,n}[l] = \frac{g_{m,n}[l]}{\sum_{l=1}^L g_{m,n}[l]}. \quad (11)$$

The total intensity that would be measured in the absence of the object can be calculated by

$$I_0 = \sum_{l=1}^L R[l], \quad (12)$$

where $R[l]$ is the measurement of the rocking curve (detector intensity in absence of the object) at the analyzer position θ_l so that

$$E[R[l]] = R(\theta_l). \quad (13)$$

Next, we define the AIP shift of the imaging system as

$$\Delta R_\theta = \frac{1}{I_0} \sum_{l=1}^L \left(l - \frac{L+1}{2} \right) R[l] \Delta, \quad (14)$$

where Δ is the angular spacing between the measurements. Now, we can estimate the three MIR parameters at each pixel as follows:

$$\hat{\alpha}(x_m, y_n) = -\ln \frac{\sum_{l=1}^L g_{m,n}[l]}{I_0}, \quad (15)$$

$$\Delta \hat{\theta}(x_m, y_n) = \sum_{l=1}^L \left(l - \frac{L+1}{2} \right) G_{m,n}[l] \Delta - \Delta R_\theta, \quad (16)$$

$$\hat{\sigma}^2(x_m, y_n) = \sum_{l=1}^L \left[\left(l - \frac{L+1}{2} \right) \Delta - \Delta \hat{\theta}(x_m, y_n) \right]^2 G_{m,n}[l] - \frac{1}{I_0} \sum_{l=1}^L \left[\left(l - \frac{L+1}{2} \right) \Delta - \Delta R_\theta \right]^2 R[l], \quad (17)$$

The MIR parametric images described above are planar images. To enable tomography one should acquire these planar images from different angular views φ and use the estimated images for each angular view to reconstruct the volumetric images.

3 Simulation of LAT-MIR

In order to evaluate the applicability of limited-angle tomography to MIR parametric images and performing a quantitative comparison, we created a 125x125x125 phantom consisting of a set of ellipsoids. The phantom is shown in Figure 2 at four different slices perpendicular to the beam direction (i.e. in the $x' - y'$ plane when $\varphi = 0$).

We assigned a uniform value for the absorption coefficient and the USAXS parameter defined in (7)-(9) to each ellipsoid. For the refraction parameter we assigned a uniform refractive index $n(x', y', z')$ to each ellipsoid and analytically calculated the corresponding refractive index gradient at each voxel. These values were chosen to match the data from the cadaveric human thumb study that we will use later in this paper. From cadaveric study we determined that the sample absorption was around $I / I_0 = 0.75$; maximum refraction $\pm 1 \mu\text{rad}$ and maximum USAXS $0.35 \mu\text{rad}^2$. We then simulated the MIR projections by using the image formation model given in Eq (5) and the same scanning geometry as in Brankov *et al*⁹.

3.1 Simulation of MIR projections

To simulate the MIR projections for each angular view we first calculated the MIR parameters described in (7)-(9) for the beam path corresponding to that view by using the *radon* function in MATLAB. We then used these parameters to form the object model presented in (6). In²³⁻²⁵ the authors have determined theoretically and experimentally that the rocking curve of an ABI system can be modeled using a Pearson type VII function as:

$$R(\theta) = \left(1 + \left(\frac{\theta}{ma^2} \right)^2 \right)^{-m}, \quad (18)$$

where, in our experiments, using three Si (333) crystals, and for the incident photon energy of 40 keV, $a = 0.7146$, and $m = 2.3737$. Now we can calculate the AIP on the detector plane $g(\theta, x, y)$ by using the convolution in (5). We then sampled this AIP at 25 analyzer positions ranging from -4.8 to $4.8 \mu\text{rad}$ with $0.4 \mu\text{rad}$ increments to be consistent with the sampling pattern used to acquire the cadaveric human thumb images.

In practical implementation of MIR using the conventional x-ray tubes, flux will be the performance constraint. For this reason we assume that the data is photon-limited and Poisson noise will be the dominant noise source in MIR images¹. So after sampling the AIP of the beam at each pixel, we should generate Poisson distributed values around the samples to simulate the noise model, i.e. $g_{m,n}[l] \sim \text{Poisson}(g(\theta_l; x_m, y_n))$ where $\sim$ denotes distributed according. Now we can use the estimator described in section 2.2 to estimate the three MIR parameters from simulated noisy samples.

3.2 Tomographic reconstruction from limited-angle data

To reconstruct 3D volumetric images from the projections we used the simultaneous iterative reconstruction technique (SIRT)²⁶, which is a modification of algebraic reconstruction technique (ART)²⁷ that uses the data from all the projections simultaneously to update the estimation at that iteration and is more robust to noise than ART. Here, we

review the SIRT algorithm using the attenuation volume as an example but the same procedure was applied to the other two parametric images (i.e. refraction and USAXS).

Let $\mu(x'_i, y'_n, z'_k)$ denote the voxelized volumetric attenuation coefficient per pixel where (x', y', z') denotes the coordinates in the object domain and i, n and k are the voxel indices. Now we can describe the forward imaging model by

$$\sigma(x_m, y_n; \varphi_p) = \sum_i \sum_k W(i, k, m, p) \mu(x'_i, y'_n, z'_k), \quad (19)$$

where φ_p is the tomographic projection angle, and $W(i, k, m, p)$ is a weighting function that describes the influence of the voxels in $\mu(x'_i, y'_n, z'_k)$ on the measured pixels in $\sigma(x_m, y_n; \varphi_p)$. Because the beam direction is perpendicular to the y -axis the weightings $W(i, k, m, p)$ are independent of n .

Like many iterative algorithms, SIRT starts with an initial estimate $\hat{\mu}^0(x'_i, y'_n, z'_k)$, which will be iteratively updated until a stopping criterion is satisfied. At q^{th} iteration

$$\hat{\alpha}^q(x_m, y_n; \varphi_p) = \sum_i \sum_k W(i, k, m, p) \hat{\mu}^q(x'_i, y'_n, z'_k). \quad (20)$$

Next, we calculate the following error in the projection domain:

$$err^q(x_m, y_n; \varphi_p) = \sigma(x_m, y_n; \varphi_p) - \hat{\alpha}^q(x_m, y_n; \varphi_p), \quad (21)$$

which can be used to update the current estimation as

$$\hat{\mu}^{q+1}(x'_i, y'_n, z'_k) = \hat{\mu}^q(x'_i, y'_n, z'_k) + \sum_p \sum_m \frac{W(i, k, m, p) err^q(x_m, y_n; \varphi_p)}{P \sum_i \sum_k W(i, k, m, p)} \quad (22)$$

where P is the total number of projection angles. The iterations continue until the following stopping criterion is satisfied:

$$\|err^q(x_m, y_n; \varphi_p)\|_2 - \|\hat{err}^{q+1}(x_m, y_n; \varphi_p)\|_2 < \beta \quad (23)$$

where $\|\cdot\|_2$ denotes L2-norm, and β is the threshold. Since attenuation and USAXS values can be only positive we enforce a non-negativity constraint for the attenuation and USAXS values at each iteration. Note that the refraction image is proportional to the derivative of the index of refraction (see Eq (8)) and as such may have positive as well as negative values. Therefore we could not enforce a positivity constraint on refraction. In addition integration of the refraction image, using the current scanning geometry, in order to reconstruct volumetric distribution of the index of refraction, is not a trivial task and out of the scope of this manuscript. For estimation of the density image (which is proportional to the index of refraction) see ²⁸ or alternative scanning geometries in ²⁹.

Limited-angle tomography does not contain the full range of data required for a perfect reconstruction. For this reason the reconstructed images suffer from several artifacts such as truncation artifacts at the edges, ringing artifacts, and blurry artifacts caused by out-of-plane structures. To investigate these artifacts for different MIR parameters, we simulated the noiseless MIR projections from our phantom by choosing 91 number of tomographic angles ($P = 91$) with one degree increments in the range of $\phi_p \in [-45^\circ, 45^\circ]$. The reconstructed phantom is shown in Figure 3 for the same slices presented in Figure 2.

We can observe that the slices near the central plane are effectively reconstructed and are good estimations of the original phantom. However, the refraction images are suffering from artifacts due to the fact that we do not impose any constraint during the reconstruction of refraction volume.

3.3 Performance evaluation of LAT-MIR

In the case where the noise levels of all projections are the same, i.e. projections are acquired with the same imaging time and energy regardless of the number of angles used, it is obvious that the more tomographic angles we use in LAT-MIR, the more close our reconstruction would be to the phantom. In order to evaluate the performance of LAT-MIR, we considered an imaging situation that the radiation dose delivered to the object should be fixed regardless of number of angles P . In this case, the accuracy of the reconstructions is a trade-off between the noise level and P .

To be consistent with the imaging system of the human thumb study that we will use later, we simulated the projections with an angular distance of $\phi = 0.9^\circ$. We vary the range of the tomographic angles from $\phi_p \in [-4.5^\circ, 4.5^\circ]$ to $\phi_p \in [-90^\circ, 90^\circ]$ which results in the number of projections to go from $P=11$ to $P=201$. The total number of photons striking the object was fixed to 5500 photons for each analyzer position, i.e. $I_0 = 5500/P$. For this I_0 , the noisy MIR projections for $P=201$ would be simulated by maximum count of 25 photons at each analyzer position which corresponds to 114 photons/pixel. Next, as a metric for accuracy of the reconstructions in the region of interest (ROI) near the central plane we used the following mean square error function:

$$MSE(P) = \frac{1}{C} \sum_i \sum_n \sum_{k=-K_{ROI}}^{K_{ROI}} \left[\mu(x'_i, y'_n, z'_k) - \hat{\mu}(x'_i, y'_n, z'_k) \right]^2, \quad (24)$$

where C is the total number of voxels in the ROI, and $K_{ROI} = 20$. Note that this ROI includes all pixels in the xy plane, and in 40 planes (out of 125 in total) for which $z' \in [-20, 20]$, including the central plane at $z' = 0$. Empirically we found that the results do not change qualitatively as we choose a smaller range than proposed $K_{ROI} = 20$. The mean square error versus the number of tomographic angles used for each parametric image is shown in Figure 4. According to this graph the optimum number of angular views is $P=139$ for attenuation and USAXS images, and $P=123$ for refraction image. However, one can observe that there is a range for the number of angular views over which the data acquisition can be considered optimum with a minimum number of angular views around $P = 101$.

The reconstructed images by LAT-MIR using 139 tomographic angular views (which is almost optimal for all of the parameters) and iterative CT-MIR using 201 views (reconstructed by SIRT) are shown in **Figure 5**, Figure 6 and Figure 7. One can observe that the images for $P=11$ and in optimal range, at $P=139$, are comparable to the CT-MIR only near the central plane, especially for the refraction image. One can also note that the LAT-MIR images have strong horizontal streak artifacts that are not present in the CT-MIR images; however, the CT-MIR images are noisier than the LAT-MIR images. Not surprisingly, the LAT-MIR images far from the central plane suffer from significant distortion and artifacts. This set of images demonstrates the limitations and potential benefits of the LAT-MIR methodology.

4 LAT-MIR on experimental data

In previous sections we evaluated the applicability of limited-angle tomography using simulated phantom data. In this section, we test the LAT-MIR method on actual, experimental MIR images measured in a cadaveric human thumb study. First we compare the LAT-MIR images with the CT-MIR reconstructed images. This is followed by a task-based evaluation of LAT-MIR and comparison with simulated results in section 3.3. The task is defined as detecting the cartilage in the refraction image using template matching.

4.1 Comparing LAT-MIR to CT-MIR

The experimental images are acquired using a synchrotron light source at Brookhaven National Laboratory with beam energy of 40keV at each of the 25 analyzer positions ranging from -4.8 to 4.8 μrad with 0.4 μrad increments. We acquired 800 evenly spaced tomographic projections covering $[-180^\circ, 180^\circ]$ interval with $\phi = 0.45^\circ$. The detector pixel size was 50 μm . Note that these data have finer tomographic sampling with $\phi = 0.45^\circ$ than the simulated data in previous section with $\phi = 0.9^\circ$.

Using LAT-MIR we reconstructed the central plane slices by using $P=101$ equally spaced projections (from 800 available) in the range of $\phi_p \in [-45^\circ, 45^\circ]$ and $\phi = 0.9^\circ$. This number of projection angles corresponds to a point in **Figure 4** where the MSE value obtains a relative flat minimum for the absorption and refraction. Three reconstructed slices at 1.2 mm, 0 mm and -1.2 mm distance from the central plane along with the same slices reconstructed by CT-MIR are shown in Figure 8 to Figure 10. One can observe that the central plane images obtained by LAT-MIR from $P=101$ projections (only 12.5% of what used in CT-MIR), are still capable of revealing the information present in the images obtained by CT-MIR. For example in the refraction images, soft tissues such as articular cartilage (bright arrows), and particularly their tissue borders, can still be highlighted.

4.2 Task-based evaluation of LAT-MIR

The data used in section 4.1 are almost noiseless, and all the projections were acquired with the same imaging time (same radiation exposure dose). Therefore in the study presented in this section we use these essentially noiseless experimental data as the mean for a simulation study. Note that the image formation model is not used here. We used experimental data to simulate noisier acquisitions. We hope to show similar results as in a phantom study; i.e., the

LAT-MIR method does not need all tomographic angles to reconstruct images of good quality, near the central plane, comparable to those obtained by CT-MIR. Therefore, we fixed the total number of photons to $I_0 = 5500/P$ as in the previous simulation study, and sought to find an efficient angular range for the LAT-MIR method.

Here, we performed a task-based evaluation for the refraction image as it could be the most interesting image for using MIR. Specifically, we used template matching to detect the cartilage near the central plane and calculated the signal-to-noise ratio (SNR) in this test as a detectability metric. We used two volumes between the bones, one with the cartilage (ROI_1) and one containing the background only (ROI_0). Both ROIs are near the central plane of the system and defined as follows:

$$\begin{cases} ROI_1 \triangleq \left\{ (x'_i, y'_n, z'_k) \mid I_1 < i \leq I_2, N_1 < n \leq N_2, K_1 < k \leq K_2 \right\} \\ ROI_0 \triangleq \left\{ (x'_i, y'_{n-\Delta n}, z'_k) \mid I_1 < i \leq I_2, N_1 < n \leq N_2, K_1 < k \leq K_2 \right\} \end{cases}, \quad (25)$$

where n is the displacement between the regions in y' direction. We selected both regions on the same position on x' and z' to have the same artifacts in their reconstructions. These ROIs for one slice are shown in Figure 11.

The desired template voxel values in ROI_1 were obtained from the noiseless CT-MIR data. We also formed two testing regions from the noisy LAT-MIR reconstructed volumes using ROI_1 and ROI_0 . The subsets are defined as

$$\begin{cases} T_{CT}(i', n', k') = b_{CT}(x'_{i+I_1}, y'_{n'+N_1}, z'_{k'+K_1}) \\ \hat{T}_{1,P}(i', n', k') = \hat{b}_{CT}^P(x'_{i+I_1}, y'_{n'+N_1}, z'_{k'+K_1}) \\ \hat{T}_{0,P}(i', n', k') = \hat{b}_{CT}^P(x'_{i+I_1}, y'_{n'+N_1-\Delta n}, z'_{k'+K_1}) \end{cases}, \quad (26)$$

where b_{CT} is the refraction volume reconstructed from noiseless data using CT-MIR, $\hat{b}_{CT}^P$ is the refraction volume reconstructed by LAT-MIR using P noisy projections, $i' = 1, 2, \dots, I_2 - I_1$, $n' = 1, 2, \dots, N_2 - N_1$ and $k' = 1, 2, \dots, K_2 - K_1$. Now, we define our template vector as

$$\mathbf{r} = Lex \{T_{CT}\}, \quad (27)$$

where $Lex \{\cdot\}$ denotes the lexicographical order of the matrix. The image vectors in each of the reconstructed volumes are also defined by

$$\begin{cases} \hat{\mathbf{r}}_{1,P} = Lex \{\hat{T}_{1,P}\} \\ \hat{\mathbf{r}}_{0,P} = Lex \{\hat{T}_{0,P}\} \end{cases}. \quad (28)$$

By applying the template in (27) to the image vectors in (28) for different number of views, we can measure two template matching values for each P , i.e.

$$\begin{cases} \hat{t}_{1,P} = \mathbf{r}^T \hat{\mathbf{r}}_{1,P} \\ \hat{t}_{0,P} = \mathbf{r}^T \hat{\mathbf{r}}_{0,P} \end{cases} \quad (29)$$

Finally, the signal to noise ratio for each P is defined as

$$SNR(P) = \frac{|\bar{t}_{1,P} - \bar{t}_{0,P}|}{\sqrt{\sigma_{t_{1,P}}^2 + \sigma_{t_{0,P}}^2}}, \quad (30)$$

where $\bar{t}_{j,P} = E[\hat{t}_{j,P}]$ and $\sigma_{t_{j,P}}^2$ is the variance of the estimation for $j = 0, 1$. We can use the values in (29) in a hypothesis test for detecting the cartilage and calculating the area under the receiver operating characteristic (ROC) curve in order to find an efficient range for LAT-MIR. However, on tested data, the SNR values were too high so that the area under the curve (AUC) was equal to 1 for every P . Hence we will report SNR values as in (30) for different number of tomographic angles. SNR(P) was evaluated for 12 different number of tomographic angles varying from 11 projections in $\varphi_p \in [-4.5^\circ, 4.5^\circ]$ and $\varphi = 0.9^\circ$ to 187 projections in $\varphi_p \in [-83.7^\circ, 83.7^\circ]$ and $\varphi = 0.9^\circ$. We also evaluated the performance of iterative CT-MIR using 201 views (reconstructed by SIRT) in $\varphi_p \in [-90^\circ, 90^\circ]$ and $\varphi = 0.9^\circ$, simulated with the same exposure dose used in LAT-MIR. Due to time constraints, we

only simulated 10 noise realizations for each P in order to estimate $\bar{t}_{j,P}$ and $\sigma_{t_{j,P}}^2$ in (30). The estimated SNR for each P along with its error bar is shown in Figure 12. We see that for the task we defined, the optimal tomographic range in terms of maximum SNR for detecting the cartilage near the central plane, exists from $P=107$ to $P=155$ which is close to the results we had previously in our simulations for refraction image (Figure 4).

5 Conclusion

Here we explored the implementation of limited-angle tomography in MIR. We suggest that in the cases in which a full range of angles cannot be obtained or the scan time is limited, limited-angle tomography MIR (LAT-MIR) can be used to effectively reconstruct the 3D object near the central plane. The applicability of LAT-MIR was first shown with computer simulation using simultaneous iterative reconstruction technique (SIRT). Next, the performance of LAT-MIR was evaluated in cases that the scan time is limited, such as in a clinical setting, and the total radiation dose to the object should be fixed regardless of number of projections. The accuracy of the reconstructions was measured using the mean square error (MSE) between the object and the reconstructions in the region of interest (ROI) near the central plane. The results showed that an optimal range for data acquisition exists for which LAT-MIR shows the best performance even comparing to iterative CT-MIR. Not surprisingly, LAT-MIR images far from the central plane suffer from significant distortion and artifacts.

We also tested the proposed approach on MIR projection of a cadaveric thumb. We showed that the central plane images obtained by LAT-MIR using only one eight of projections required by non-iterative CT-MIR (FBP) reconstruction have almost the same visual contents. A task-based evaluation was also performed for LAT-MIR with the same imaging

conditions used in phantom simulation. The task was detecting a cartilage in the central plane by using template matching. The results showed that an optimal tomographic range for achieving the maximum signal to noise ratio (SNR) exists and is consistent with the results obtained in phantom simulation.

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Appendix A. Approximation of the object function

We first review the model proposed in Khelashvili *et al*⁸. The image formation model is based on radiative transfer theory and the object is modeled as a stratified medium where

each stratum is characterized by a linear absorption coefficient $\mu(\vec{r}) = \mu(z)$ and by a

spatially varying refractive index $n(\vec{r}) = n_0 + n_y(z)$ where $\vec{r} = (x, y, z)^T$ is the spatial coordinate of the system where z is the direction of propagation. Furthermore, the object is assumed to have identical scattering centers distributed in the medium with density

$\rho_n(\vec{r}) = \rho_n(z)$. Each scatter center is characterized by its extinction cross section, which is defined as the sum of the scattering cross section σ_s and the absorption cross section σ_a . Next, we use a quantity called *albedo*⁸, defined as

$$W_0(z) \triangleq \frac{\sigma_s}{\sigma_s + \sigma_a}, \quad (\text{A.1})$$

and for each scattering center we define:

$$\alpha_p = \left[4\delta^2 \ln \left(\frac{2}{\delta} + 1 \right) \right]^{-1}, \quad (\text{A.2})$$

where α_p is the broadening due to a single scattering center³⁰ δ is defined as the difference between the refractive indices of the medium and the scatters. Now, by using the equation presented in Khelashvili *et al*⁸, we can write the object function for this stratum as

$$f(\theta) = e^{-\int_0^Z \bar{\mu}(\vec{r}) dz} \sum_{k=0}^{\infty} \left[\frac{(\tau(Z) W_0)^k}{k!} e^{-\tau(Z) W_0} \sqrt{\frac{\alpha_p}{k\pi}} e^{\left(\frac{\alpha_p}{k}(\theta - \Delta\theta)^2\right)} \right], \quad (\text{A.3})$$

where $\bar{\mu}(z) \triangleq \rho_n(z) \sigma_a + \mu(z)$, and we have:

$$\tau(Z) = \int_0^Z (\rho_n(z) \sigma_{ext} + \mu(z)) dz, \quad (\text{A.4})$$

$$\Delta\theta(\vec{r}) = \int_0^Z \frac{\partial}{\partial y} \ln(n(x, y, z)) dz = \int_0^Z \frac{n_y(z)}{n_0} dz, \quad (\text{A.5})$$

where Z is the medium thickness. We can relate the terms in the summation of Eq. (A.3) to an expectation of a Gaussian curve with a variance of k/α_p , over a Poisson distribution of k .

For large values of $\tau(Z)W_0$, we can accurately approximate this object function by its dominant term, which is the Gaussian curve with $k = \tau(Z)W_0$.

Finally, we can rewrite (A.3) as

$$f(\theta) = \exp(-\alpha(x, y)) \frac{1}{2\pi\sigma^2(x, y)} \exp\left(-\frac{(\theta - \Delta\theta(x, y))^2}{2\sigma^2(x, y)}\right), \quad (\text{A.6})$$

where we have:

$$\alpha(x, y) = \int_0^Z \bar{\mu}(\vec{r}) dz, \quad (\text{A.7})$$

$$\Delta\theta(x, y) = \int_0^Z \frac{n_y(z)}{n_0} dz, \quad (\text{A.8})$$

$$\sigma^2(x, y) = \frac{1}{2} \frac{\tau(Z)W_0}{\alpha_p} = \frac{1}{2} \int_0^Z \frac{\rho_n(z)\sigma_s}{\alpha_p} dz. \quad (\text{A.9})$$

and $\alpha(x, y)$ describes the net linear absorption of the medium (including absorption in scattering centers), $\Delta\theta(x, y)$ is the scaled refractive index gradient, and $\sigma^2(x, y)$ can be interpreted as USAXS parameter of the medium.

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Figure 1. Schematic diagram of the MIR imaging system (not drawn to scale). (x, y, z) are the coordinates of the imaging system, and (x', y', z') are the coordinates of the object. For tomography, the projections are acquired by rotating the object by ϕ° with respect to the imaging system.

Figure 2.

Different slices from the experimental phantom showing three MIR parameters. These values of each ellipsoid is chosen to match the bone and cartilage data from the cadaveric human thumb study that we will use later in this paper.

Figure 3. LAT-MIR slices reconstructed by SIRT using 91 projections in the range of $\phi_p \in [-45^\circ, 45^\circ]$.

Figure 4.
Mean square error in ROI for different number of angles used for LAT-MIR

Figure 5.
Reconstructed slices by LAT-MIR for $P=11$ and $\varphi_p \in [-4.5^\circ, 4.5^\circ]$

Figure 6.
Reconstructed slices by LAT-MIR for $P=139$ and $\phi_p \in [-69^\circ, 69^\circ]$

Figure 7.
Reconstructed slices using SIRT in CT-MIR with 201 projections and $\phi_p \in [-90^\circ, 90^\circ]$

Figure 8. Attenuation image for CT-MIR and LAT-MIR using 101 equally spaced projections in the range of $\phi_p \in [-45^\circ, 45^\circ]$. **Slice distances are 1.2 mm.**

Figure 9. Refraction image for CT-MIR and LAT-MIR using 101 equally spaced projections in the range of $\phi_p \in [-45^\circ, 45^\circ]$. **Soft tissue features are shown with an arrow.**

Figure 10. USAXS image for CT-MIR and LAT-MIR using 101 equally spaced projections in the range of $\phi_p \in [-45^\circ, 45^\circ]$.

Figure 11.

ROI₁ and ROI₀ (cartilage and background) used for template matching

Figure 12.
SNR defined in (25) for different number of angles P .