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A finite element implementation for biphasic contact of hydrated porous media under finite deformation and sliding

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Abstract

The study of biphasic soft tissues contact is fundamental to understanding the biomechanical behavior of human diarthrodial joints. However, to date, few biphasic finite element contact analysis for 3D physiological geometries under finite deformation has been developed. The objective of this paper is to develop a hyperelastic biphasic contact implementation for finite deformation and sliding problem. An augmented Lagrangian method was used to enforce the continuity of contact traction and fluid pressure across the contact interface. The finite element implementation was based on a general purpose software, COMSOL Multiphysics. The accuracy of the implementation is verified using example problems, for which solutions are available by alternative analyses. The implementation was proven to be robust and able to handle finite deformation and sliding.

Keywords

biphasic contact; soft tissue; porous media; hyperelastic; finite deformation; augmented Lagrangian method

1. Introduction

The study of soft tissues contact in human diarthrodial joints is critical to understand the biomechanical behavior of the joints, engineer tissue replacements, improve surgical interventions, and develop better diagnostic techniques. *In vitro* experiments have been widely used for this kind of study. However, not all the mechanical components can be measured experimentally. For example, the results of *in vitro* experiments are limited to the surface of the tissues, and *in vitro* experiments cannot show mechanical components through the tissues. Due to the fundamental limitations of experimental measurements on tissues, the numerical solution is essential to obtain a more complete understanding of diarthrodial joint

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Conflict of interest
None

biomechanics. Soft tissues are naturally hydrated¹, and biphasic theory², which considers a soft tissue as a combination of solid phase and interstitial fluid phase, has been widely used to study the biomechanical behavior of the soft tissues. Analytical solutions for the biphasic contact mechanics in axisymmetric joints have been developed^{3–6}, but these solutions apply to fairly idealized problems. In order to analyze the biphasic contact mechanics of physiological joints, where geometry is far more complex, it is necessary to use numerical approximation methods, such as the finite element method. However, numerical computation of the biphasic contact mechanics remains challenging due to the fact that biphasic contact analysis is highly nonlinear, and only a limited number of studies have addressed this type of problems.

Beside the continuity conditions for displacement and contact traction that a single-phase contact problem consists of, there are two additional continuity conditions on relative fluid flow and fluid pressure in the biphasic contact problem^{7, 8}. Spilker and coworkers developed a Lagrange multiplier method for 2D and 3D biphasic contact under small deformations^{9–11}, and a penetration-based approximation method for 2D and 3D biphasic contact under small deformations¹² or large deformations^{13, 14}. Chen et al.¹⁵ provided a Lagrange multiplier method to study the sliding contact mechanics of 2D biphasic cartilage layers under small strain. Ateshian et al.⁸ developed an augmented Lagrangian method for 3D biphasic contact under large deformations and sliding. More recently, we developed an augmented Lagrangian method for 2D and 3D biphasic contact under small deformations^{16–18} or sliding¹⁹, and proved that the finite element implementation is able to model biphasic contact with physiological geometry¹⁷.

ABAQUS is a commonly used commercial finite element software for porous media contact analysis^{20–27}. Though the program provides many powerful features, its biphasic contact implementation has significant limitations^{8, 16}. First, the "drainage-only-flow" boundary condition (i.e., the fluid only flows from the interior to the exterior of the porous media) is inconsistent with the equation of mass conservation across the contact interface^{7, 8}. Second, the software does not automatically enforce the free draining boundary condition outside of the contact area. This limitation needs to be addressed by a user-defined routine²².

In summary, the objective of this paper is to extend our previous finite element implementation of biphasic contact under small deformation^{16, 17, 19} to finite deformation and sliding problems⁸. The accuracy of the new finite element implementation will be verified using several example problems.

2. Methods

2.1 Governing equations of the hyperelastic biphasic theory

Consider two deformable bodies, labeled A and B, with boundaries Γ^A and Γ^B , which are in frictionless contact over portions denoted by γ^A and γ^B , respectively. The mixed displacement-pressure (u-p) formulation of biphasic theory²⁸ is adopted in this study. The governing equations are

$$\nabla \bullet (\sigma_E^s - pI) = 0 \quad (1)$$

$$\nabla \bullet (\nu^s - \kappa \nabla p) = 0 \quad (2)$$

where σ_E^s is the effective (or elastic) stress of the solid matrix, which is completely determined from the deformation of the solid matrix, p is the fluid pressure, I is the identity tensor, $\nu^s = \frac{du}{dt}$ is the solid phase velocity, and κ is the permeability.

The constitutive relations for each phase are

$$\sigma^s = \sigma_E^s - \phi^s p I \quad (3)$$

$$\sigma^f = -\phi^f p I \quad (4)$$

where σ^α are Cauchy stress tensor ($\alpha=s$ and f), ϕ^s and ϕ^f are the solid and fluid volume fractions, respectively, for the saturated ($\phi^s + \phi^f = 1$) mixture. They are determined by the solid phase deformation

$$\phi^s = \frac{\phi_0^s}{J}, \phi^f = 1 - \frac{\phi_0^s}{J} \quad (5)$$

where the subscript '0' denotes quantities in the reference configuration and $J = \det(F)$ is the Jacobian determinant of the deformation gradient, F .

In hyperelasticity, the Cauchy stress tensor is computed from the second Piola-Kirchhoff stress tensor, S , which depends on the strain energy density function, Ψ^s , and may be expressed in terms of the right Cauchy-Green deformation tensor, $C = F^T F$, or the Lagrangian (or Green-Lagrangian) strain tensor,

$$E = \frac{1}{2}(C - I) = \frac{1}{2}(\nabla u + (\nabla u)^T + \nabla u(\nabla u)^T) \quad (6)$$

$$S = 2 \frac{\partial \Psi^s}{\partial C} = \frac{\partial \Psi^s}{\partial E} \quad (6)$$

The effective stress of the solid phase, σ_E^s is defined as

$$\sigma_E^s = \frac{1}{J} F \bullet S \bullet F^T = \frac{2}{J} F \bullet \frac{\partial \Psi^s}{\partial C} \bullet F^T = \frac{1}{J} F \bullet \frac{\partial \Psi^s}{\partial E} \bullet F^T \quad (7)$$

Several strain energy density functions have been proposed to study hydrated soft tissues under finite deformation. The function proposed by Holmes and Mow³⁰ is the one most widely used⁸ and is used in the present study.

$$\Psi^s = \alpha_0 \frac{e^{\alpha_1(I_1-3) + \alpha_2(I_2-3)}}{I_3^\beta} \quad (8)$$

where I_1 , I_2 , and I_3 are the invariants of the right Cauchy-Green deformation tensor, C ; the dimensionless nonlinear stiffening coefficient $\beta = \alpha_1 + 2\alpha_2$; and α_0 , α_1 , and α_2 are positive material parameters. Usually, the hyperelastic biphasic material properties of the articular cartilage are given as β and Lamé constant λ_s , μ_s . α_0 , α_1 , and α_2 are related to these three material coefficients as

$$\alpha_0 = \frac{\lambda_s + 2\mu_s}{\beta}, \alpha_1 = \frac{2\mu_s - \lambda_s}{\lambda_s + 2\mu_s} \beta, \alpha_2 = \frac{\lambda_s \beta}{\lambda_s + 2\mu_s} \quad (9)$$

The exponential permeability function proposed by Holmes and Mow³⁰ is used in the present study,

$$k = k_0 \left(\frac{J - \phi_0^s}{1 - \phi_0^s} \right)^\alpha e^{m(J^2 - 1)/2} \quad (10)$$

where the exponents α and m are material parameters, and k_0 is the intrinsic permeability associated with the reference configuration.

The initial and boundary conditions on the non-contacting boundaries of bodies A and B (we drop the superscripts A and B for these equations) are

$$u(t=0) = u_0 \quad \text{and} \quad u = \bar{u} \quad \text{on} \quad \Gamma_u \quad (11)$$

$$\nu(t=0) = \nu_0 \quad \text{and} \quad \nu = \bar{\nu} \quad \text{on} \quad \Gamma_\nu \quad (12)$$

$$p = \bar{p} \quad \text{on} \quad \Gamma_p \quad (13)$$

$$t = \bar{t} \quad \text{on} \quad \Gamma_t \quad (14)$$

$$Q = \bar{Q} \quad \text{on} \quad \Gamma_Q \quad (15)$$

where an overbar indicates a prescribed value of the quantity; the subscript $()_0$ denotes an initial values; total traction is defined as $t = (\sigma^s + \sigma^f) \cdot n$, σ^s and σ^f are solid and fluid stresses; and the relative fluid flow is defined as $Q = -(\kappa \nabla p) \cdot n$. The boundaries $\Gamma_{\beta, \beta=u, v, t, \text{ and } Q}$, correspond to portions on which displacement, velocity, fluid pressure, total traction, and relative flow, respectively, are prescribed.

2.2 Hyperelastic biphasic contact modeling

Contact boundary conditions defined on the boundaries γ^A and γ^B are^{7, 8}

$$\nu^{sA} \bullet n + \nu^{sB} \bullet n = 0 \quad (16)$$

$$\sigma_E^{sA} \bullet n + \sigma_E^{sB} \bullet n = 0 \quad (17)$$

$$Q^A + Q^B = 0 \quad (18)$$

$$p^A - p^B = 0 \quad (19)$$

These equations correspond to the continuities of location of points, Eq. (16), effective stress of the solid phase, Eq. (17), the relative fluid flow, Eq. (18), and the fluid pressure, Eq. (19), on the contact boundary.

To enforce the contact constraint based on augmented Lagrangian method, the normal component of the contact stress is defined as

$$\tau_n = \begin{cases} \eta_n g & g < 0 \\ 0 & g \geq 0 \end{cases} \quad (20)$$

where g is the gap distance from the destination boundary γ^B to the source boundary γ^A in the direction normal to the destination surface, and η_n is the normal penalty factor. As the penalty factor goes to infinity, the augmented Lagrangian method ensures that the contact boundaries overlap by an acceptably negligible amount g .

The augmented Lagrangian framework for single-phase contact problem developed by Simo and Laursen³¹ is adapted to the current biphasic contact framework (Table 1^{16, 17, 19}). An augmented component is introduced for the normal component of the contact stress τ_n , and an additional iteration level was added. The contact stress is solved separately from the solid displacement and fluid pressure variables.

The hyperelastic biphasic contact were implemented in general purpose finite element software (COMSOL Multiphysics 4.2a[®], COMSOL Inc., Burlington, MA). Solid mechanics in the Structural Mechanics Module and Darcy's Law in the Earth Science Module were used. The strain energy function, Eq. (8) was inputted as user-defined strain energy function³². The Contact Pair feature was used to enforce contact constraint for the solid phase and the Identity Pair feature was used to enforce fluid continuity constraint for the fluid phase. The search distance of the contact pair was set to 0.001 mm, and the destination boundary was meshed finer than the source boundary to get the best results. The penalty factor was set as $E/h_m * c$, where E is the elastic modulus of the materials, h_m is the mesh size, and c is a user-defined constant with typical range of 0.1 to 10.

3. Example problems

To validate the accuracy of the finite element implementation developed in the present study, several example problems were evaluated. The first step was to validate the accuracy

of the hyperelastic biphasic implementation, and the second step was to validate the accuracy of the hyperelastic biphasic contact method.

3.1 Validation of the hyperelastic biphasic implementation

3.1.1 Equilibrium stress-strain relation—To illustrate the accuracy of the strain energy function implemented in COMSOL, the steady state solid stress in confined compression test was measured as a function of the strain for both bovine and human articular cartilage, and it was compared to the analytical solution reported by Holms and Mow³⁰. Specifically, a 2D confined compression creep test model was created (Figure 1). The articular cartilage was modeled as a square with width and thickness of 1 mm. The confining chamber and porous plate were not explicitly modeled; instead, they were represented by appropriate boundary conditions: the bottom boundary of the articular cartilage was impermeable and fixed; a free draining boundary condition was applied to the top boundary of the articular cartilage; and impermeable roller boundary conditions were applied to the left and right boundaries of the articular cartilage. Different forces were used to produce different steady state strains, and they were applied linearly in 10s to the top surface of the articular cartilage and held as constant thereafter. To achieve steady state conditions, the finite element models were computed to a time point when no further change in the solution was observed over time. The material properties of the bovine articular cartilage were $\lambda_s=0$ MPa, $\mu_s=0.165$ MPa, and $\beta=0.761$ ³⁰. The material properties of the human articular cartilage were $\lambda_s=0$ MPa, $\mu_s=0.2035$ MPa, and $\beta=1.105$ ³⁰. Since the permeability does not affect the steady state behavior of the articular cartilage, it is not given here.

For both bovine articular cartilage and human articular cartilage, the finite element model successfully reproduced the equilibrium stress-strain curve of the analytical solution (Figure 2). The excellent agreement between the present study and the analytical solution demonstrated that the strain energy density function implementation of the present study is accurate.

3.1.2 Confined compression creep test—The confined compression creep model developed in the last section was used to validate the accuracy of the time-dependent behavior of the hyperelastic biphasic implementation. According to the analytical solution of linear biphasic theory developed by Mow et al.² and extended by Soltz and Ateshian³³, the vertical displacement of the articular cartilage under infinitesimal deformation in confined compression creep test is given by

$$u(y, t) = -\frac{\sigma_0}{H_A} \left[y - \frac{2h}{\pi^2} \sum_{n=0}^{\infty} \frac{(-1)^n}{(n+0.5)^2} \sin \left((n+0.5) \frac{\pi y}{h} \right) e^{-(n+0.5)^2 \frac{\pi^2 H_A \kappa t}{h^2}} \right] \quad (21)$$

where y is the vertical coordinate, t is time, σ_0 is the stress applied on the articular cartilage; h is the thickness of the articular cartilage, $H_A = \lambda_s + 2\mu_s$ is the aggregate modulus of the articular cartilage, and κ is the permeability. Please note that this analytical solution is only suitable for cases with infinitesimal deformation.

The material properties of the articular cartilage used in this model were $\lambda_s=0$ MPa, $\mu_s=0.2035$ MPa, $\beta=1.105$, $k_0=2.519 \times 10^{-15}$ m⁴/Ns, $\alpha=0$, and $m=0$. Since constant permeability was used in the analytical solution, for this specific analysis, constant permeability was also used in the finite element models. To produce cases with different equilibrium strain, several different stresses were applied linearly in 1 s to the top boundary of the articular cartilage and held as constant until 3000 s. Two kinds of comparisons were made: (1) the vertical displacements of the top boundary of the articular cartilage under small loads (0.000407 MPa, 0.00407 MPa, 0.02035 MPa, and 0.0407 MPa) predicted by the present study were compared to the FEBio solutions and the linear biphasic analytical solutions^{2, 33}; (2) the vertical displacements of the top boundary of the articular cartilage under large loads (0.0814 MPa, 0.1221 MPa, 0.1684 MPa, and 0.2442 MPa) were compared between the present study and the FEBio solutions. These two kinds of comparisons were chosen due to the fact that the linear biphasic analytical solution^{2, 33} is limited to cases with infinitesimal deformation. FEBio^{8, 34} and the present study used hyperelastic biphasic theory.

For the cases with equilibrium strain of 0.1% (Figure 3a) and 1% (Figure 3b), the results of both FEBio and present study were in good agreement with the linear biphasic analytical solutions. For cases with equilibrium strain of 5% and 10% in analytical solutions (Figure 3c and d), the FEBio and the present were in good agreement with the analytical solutions in the first 500s, but they diverged thereafter; at steady state, the results of the displacement predicted by the FEBio and the present study were smaller than those predicted by the analytical solution. For cases with large load (Figure 4), the results of the present study matched well with those of the FEBio. In summary, the results of the present study were in good agreement with the analytical solution in small deformation range (i.e. 0.1% strain and 1% strain); in both small deformation range and large deformation range, the results of the present study matched well with those of the FEBio. All these results demonstrated that the hyperelastic biphasic implementation of the present study is accurate.

3.2 Validation of the hyperelastic biphasic contact method

3.2.1 Patch test using unconfined compression stress relaxation test—The unconfined compression stress relaxation test was used as a patch test for the hyperelastic biphasic contact implementation. A contact model and a no-contact model were developed respectively (Figure 5). In the contact model, two flat articular cartilage layers with thickness of 1 mm and radius of 3 mm were in contact. In the no-contact model, a flat articular cartilage layer of 2 mm and radius of 3 mm were modeled. The nonporous plates were assumed to be adhesive and were modeled as impermeable boundaries with no motion in the radial direction. The top boundary of the articular cartilage was subjected to a displacement of 0.5 mm applied in a ramp time of 1 s and then held. A free draining boundary condition was applied to the peripheral boundary of the articular cartilage. Material properties of the articular cartilage were $\lambda_s=0$ MPa, $\mu_s=0.2035$ MPa, $\beta=1.105$, $k_0=2.519 \times 10^{-15}$ m⁴/Ns, $\alpha=0.0848$, and $m=4.638$ ³⁰.

For the contact model, distributions of the fluid pressure were symmetric with respect to the mid-height of the articular cartilage (Figure 6). High fluid pressure occurred near the central

part of the **top** and bottom surface and decreased toward the peripheral edges and the middle of the articular cartilage. The continuity of fluid pressure across the interface was clearly satisfied. The plot of the fluid pressure distributions at the contact interface similarly showed that the fluid pressure distributions were identical on the contact boundaries at each and every time points shown here (Figure 7a). The distributions of the total normal stress and the maximum principal shear stress were also identical on the contact boundaries at all time points shown here (Figure 7b and c). The changes of displacements over time at the most peripheral point of the contact interfaces showed identical displacements on the nodes of the contact boundaries (Figure 7d). All these results demonstrated that the continuity conditions were accurately satisfied for both primary parameters (displacement and fluid pressure) and derived parameters (total normal stress and maximum principal shear stress).

The contact model and the no-contact model predicted identical distributions of the fluid pressure on the articular cartilage (Figure 6). In addition, the distributions of the fluid pressure, the total normal stress, and the maximum principal stress on the contact interfaces in the contact model and the corresponding lines of the no-contact model were identical at each and every time points (Figure 7a–c). The contact model and the nocontact model also predicted identical displacement relaxation behaviors (Figure 7d). Therefore, the hyperelastic biphasic contact implementation was proven to be accurate for this unconfined compression stress relaxation analysis.

3.2.2 Biphasic contact of a semicylindrical articular cartilage and a flat articular cartilage layer—

Unlike the unconfined compression test, contact of a semicylindrical articular cartilage and a flat articular cartilage layer involves evolving contact boundary (Figure 8). Radius of the semicylindrical articular cartilage was 2mm, and the width and thickness of the flat articular cartilage were 4mm and 2 mm, respectively. Because of the symmetry with respect to the central axis, only half of the model is considered in the finite element representation. A displacement of 0.5 mm was applied linearly in 10s on the top boundary of the cylindrical articular cartilage and held as constant thereafter. The top boundary of the cylindrical articular cartilage was impermeable. The bottom boundary of the flat articular cartilage was impermeable and fixed. Free draining boundary condition was applied to the peripheral boundary of the flat articular cartilage and the articular cartilage surface outside of the contact area. The material properties of the articular cartilage used this model were $\lambda_s=0$ MPa, $\mu_s=0.2035$ MPa, $\beta=1.105$, $k_0=2.519 \times 10^{-15}$ m4/Ns, $\alpha=0.0848$, and $m=4.638^{30}$.

Distributions of the fluid pressure and the total normal stress on the articular cartilage at different time points are shown in Figure 9 and are in good agreement with FEBio results. These results clearly demonstrated continuity of the fluid pressure and the total normal stress across the contact interface at different time points. Distributions of the normal tractions along the top boundary of the flat articular cartilage layer (Figure 10) demonstrated that the free draining boundary condition was accurately applied on the articular cartilage surface outside the contact area. Consistent with a previous unconfined compression analysis³⁵, the flat articular cartilage layer underwent radial displacement relaxation; the fluid phase carried most of the total load at early time; as the fluid flow diminished over time, the fluid pressure decreased, and the total load was increasingly carried by the solid phase. Stress

concentration occurred at the upper corner of the semicylindrical articular cartilage. This is probably an artifact caused by the sharp corner of the finite element model, which has also been found in previous biphasic contact studies^{28, 36}. In summary, this analysis demonstrated the ability of the hyperelastic biphasic contact implementation to handle large deformation.

3.2.3 Sliding contact of a rigid impermeable cylinder with a flat articular cartilage layer

—Few studies have investigated the sliding contact of the hydrated soft tissues. Ateshian et al.⁴ developed a semi-analytical solution of steady-state sliding contact of a rigid impermeable cylinder with a flat articular cartilage layer under small deformations. More recently, Ateshian et al.⁸ and Guo et al.¹⁹ respectively developed finite element solutions of the problem. Pawaskar et al.³⁷ used finite element method to study the effect of sliding on fluid load support in the cartilage and its implications to frictional and lubricating characteristics. To verify the accuracy of the hyperelastic biphasic contact finite element implementation of the present study, the same problem was modeled here. Specifically, a sliding contact analysis was performed between a rigid impermeable cylindrical indenter of radius $R_{ind}=100$ mm and a flat articular cartilage layer of thickness $h=1$ mm and width $w=60$ mm, attached to rigid impermeable subchondral bone (Figure 11). The subchondral bone was represented by a fixed, impermeable boundary. The material properties of the articular cartilage were $\lambda_s=0$ MPa, $\mu_s=0.5$ MPa, $\beta=0$, $k_0=1 \times 10^{-12}$ m⁴/Ns, $\alpha=0$, and $m=0$. The indenter tip was modeled as linear elastic material, with Young's modulus $E=10$ GPa, and Poisson's ratio $\nu=0.3$. The rigid indenter imparted a sliding velocity $V=0.01$ mm/s, 1 mm/s, or 100 mm/s, and the loading was 1 N/mm⁴.

Finite element models for all three cases successfully converged. Steady state response of the fluid pressure and total normal stress were computed and compared to the previous solutions (Figure 12 and 13). Distributions of the fluid pressure and total normal stress (Figure 12) were in good agreement with the semi-analytical solutions⁴ and previous finite element solutions¹⁹. When the sliding velocity is small (Figure 12a and 13a), the interstitial fluid had sufficient time to flow in and out of the solid matrix and fluid pressure was low; the normal deformation of the solid phase was very large and most of the load was carried by the solid phase. When the sliding velocity is large (Figure 12c and 13c), little fluid transport occurred except near the free surface of the articular cartilage layer, normal deformation of the solid phase was small, and most of the load was carried by the fluid phase. When the sliding velocity is medium (Figure 12b and 13b), the two effects described above competed with each other and incomplete recovery was observed at the trailing edge of the indenter. The distributions of the fluid pressure and total normal stress along the top surface of the articular cartilage layer were in good agreement between the present study and the semi-analytical solution (Figure 13). In summary, this sliding contact analysis demonstrated the ability of the hyperelastic biphasic contact implementation to model sliding contact.

4. Discussions

Biphasic finite element analysis of the joint is critical to understand the biomechanical behavior of the joints, engineer tissue replacements, improve surgical interventions, and

develop better diagnostic techniques. As daily joint activity imposes large deformation on soft tissues, there is a significant need for the hyperelastic biphasic contact implementation of the hydrated soft tissues, yet few studies have addressed this type of problems. The objective of this paper was to extend the augmented Lagrangian biphasic contact implementation for small deformation developed in our previous studies to finite deformation and sliding problem.

Several biphasic contact algorithms have been developed, yet most of them are limited to small deformation problem^{9–11, 15}. The penetration-based biphasic contact approximation algorithm developed by Spilker and co-workers^{12–14} is able to model finite deformation^{13, 14}, and it represents a simplified approach by deriving approximate time-dependent contact boundary conditions from experimental data and biphasic material laws, and applying those within a finite element scheme. This approximation algorithm avoids the nonlinearity associated with the biphasic contact analysis for finite deformation. Yet, the approximation algorithm needs kinematic information describing an *in vitro* joint articulation, measured while the cartilage is deformed under physiological loads. In addition, it is not a true biphasic contact algorithm. Therefore, the application of the penetration-based biphasic contact approximation method is limited. To the best of our knowledge, only Ateshian et al.⁸ developed a biphasic contact finite element implementation for finite deformation and sliding using an augmented Lagrangian method. The augmented Lagrangian method incorporates strong features from the penalty and Lagrange multiplier methods, and is more robust than either individual method³¹. The augmented Lagrangian algorithm used in the present study differed from that used by Ateshian et al.⁸. First, in the present study the contact constraints condition of the fluid pressure was the gap distance between the source boundary and the destination boundary; Ateshian et al.⁸ used contact traction. Second, in the biphasic contact framework of the present study, the augmentation component was only introduced for the contact stress; in the study of Ateshian et al.⁸, augmentation components were introduced for both the contact stress and fluid pressure. Therefore, compared to the finite element implementation developed by Ateshian et al.⁸, our biphasic contact finite element implementation avoids an additional iteration level for the fluid pressure.

In summary, an augmented Lagrangian biphasic contact implementation for finite deformation and sliding was developed. The mixed u-p formulation of hyperelastic biphasic theory was adopted. An augmented Lagrangian method was used to enforce the continuity of contact traction and fluid pressure across the contact interface. The finite element implementation was based on a general purpose software, COMSOL Multiphysics, and its accuracy was validated by several example problems. The biphasic finite element implementation was proven to be robust and able to handle large deformation and sliding.

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Figure 1. schematic diagram of the confined compression test. A articular cartilage disc is placed in a rigid confining chamber. A force or displacement is applied to the articular cartilage disc via a porous plate.

Figure 2.

The equilibrium solid stress-strain relation for bovine articular cartilage (a) and human articular cartilage (b) in confined compression test predicted by the analytical solution³⁰ and the present study.

Figure 3.

Comparisons of results predicted by the linear biphasic analytical solution^{2, 33}, the FEBio, and the present study: the displacement of the top boundary of the articular cartilage under small stresses, σ_0 , in confined compression creep test.

Figure 4.

Comparisons of results predicted by the FEBio and the present study: the displacement of the top boundary of the articular cartilage under large stresses, σ_0 , in confined compression creep test.

Figure 5.

A schematic diagram of the unconfined compression test in a 2D axisymmetric analysis. Results on the mid-line of the non-contact model (dash line) were output and compared with those on the contact boundaries of the contact model.

Figure 6.
Distributions of fluid pressure on the articular cartilage at $t=1$ s in the unconfined compression stress relaxation test. Black lines are the initial positions.

Figure 7.

Distribution of the fluid pressure (a), total normal stress (b), and maximum principal stress (c) on the nodes of the contact interfaces in the contact model and the corresponding nodes of the no-contact model at different time points in the unconfined compression stress relaxation analysis. (d) Changes of radial displacement over time at the most peripheral point of the contact interfaces in the contact model (see figure 5 for the location of the point) and corresponding nodes of the no-contact model.

Figure 8.

A schematic diagram of the biphasic contact of a semicylindrical articular cartilage and a flat articular cartilage layer in a 2D plane strain analysis.

Figure 9.

Distributions of fluid pressure, p , (a, in kPa) and total normal stress, σ_y , (b, in kPa) on the articular cartilage at different time points in the biphasic contact analysis of a semicylindrical articular cartilage and a flat articular cartilage layer. Black lines are the initial positions of the articular cartilage.

Figure 10.

Distributions of the normal tractions along the top boundary of the flat articular cartilage layer at 10 s in the biphasic contact analysis of a semicylindrical articular cartilage and a flat articular cartilage layer.

Figure 11.

A schematic diagram of the sliding contact of a rigid impermeable indenter with a flat articular cartilage layer.

Figure 12.

Distributions of the fluid pressure and total normal stress at steady state in the cases with different sliding velocity: (a) 0.01mm/s, (b) 1mm/s, and (c) 100 mm/s. Red arrows are the fluid flow vector. Black lines are the initial positions of the models. Only interested part of the model is shown here.

Figure 13.

Steady state response of the total normal stress σ_y , and fluid pressure p , along the top surface of the cartilage layer in the cases with different sliding velocity: (a) 0.01mm/s, (b) 1mm/s, and (c) 100 mm/s. $x=0$ corresponds to the geometric midpoint of the contact area. The contact area moves toward the positive direction of the x axis. Lines are results of the present study, and symbols are results of semi-analytical solutions⁴.

Table 1

Augmented Lagrangian algorithm for hyperelastic biphasic contact of hydrated porous media

1.	Initialization
	Set $k=0$
	Set $\lambda_n^{(k)} = \lambda_n$ from last time step
2.	Solve step
	Set $t_n = \begin{cases} \lambda_n^{(k)} + \eta_n g & g < 0 \\ 0 & g \geq 0 \end{cases}$
	Set $\begin{cases} p^A - p^B = 0 & g < 0 \\ p^A = p^B = 0 & g \geq 0 \end{cases}$
	Solve for u and p
3.	Check for constraint satisfaction
	If $ g(X^B) \leq GTOL^*$ for all $X^B \in \gamma^B$
	Converge. Exit
	Else
	Augment:
	$\lambda_n^{(k+1)} = \begin{cases} \lambda_n^{(k)} + \eta_n g & g < 0 \\ 0 & g \geq 0 \end{cases}$
	$k \leftarrow k + 1$
	Goto 2.
	EndIf

* GTOL is the tolerance for gap distance