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Accelerating Parameter Mapping with a Locally Low Rank Constraint

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Abstract

Purpose—To accelerate MR parameter mapping (MRPM) using a locally low rank (LLR) constraint, and the combination of parallel imaging (T_1) and the LLR constraint.

Theory and Methods—An LLR method is developed for MRPM and compared with a globally low rank (GLR) method in a multi-echo spin-echo T_2 mapping experiment. For acquisition with coil arrays, a combined LLR and PI method is proposed. The proposed method is evaluated in a variable flip angle T_1 mapping experiment and compared with the LLR method and PI alone.

Results—In the multi-echo spin-echo T_2 mapping experiment, the LLR method is more accurate than the GLR method for acceleration factors 2 and 3, especially for tissues with high T_2 values. Variable flip angle T_1 mapping is achieved by acquiring datasets with 10 flip angles, each dataset accelerated by a factor of 6, and reconstructed by the proposed method with a small normalized root mean square error of 0.025.

Conclusion—The LLR method is likely superior to the GLR method for MRPM. The proposed combined LLR and PI method has better performance than the two methods alone, especially with highly accelerated acquisition.

Keywords

parameter mapping; quantitative MRI; low rank; parallel imaging; compressed sensing

Introduction

MR parameter mapping (MRPM) is a promising approach to characterize intrinsic tissue-dependent information. Changes in parameters such as the longitudinal (T_1) relaxation, transverse (T_2) relaxation, and magnetization transfer, may indicate pathological tissue changes (1–3). Quantitative MRPM requires acquisition of multi-part datasets with modulated pulse sequence parameters (e.g., echo time (TE), flip angle (FA) or inversion time (TI)). Parameter estimation can be achieved by fitting the signal evolution in multi-part datasets with a parameter-dependent model on a pixel-wise basis. One limitation of clinical

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application of MRPM is long scan times due to multiple data acquisitions, which can take up to half an hour or more. This also makes MRPM sensitive to subject motion.

Various approaches have been proposed to accelerate MRPM. The first approach is parallel imaging (PI) (4,5). PI can speed up MRPM by accelerating the acquisition of each individual dataset using coil arrays. Depending on the coil array, an acceleration factor of 2–4 is usually achievable. However, this approach is limited by the inherent SNR penalty of PI with high acceleration. The second approach exploits the nature of signal evolution in MRPM. Similar to the k - t approach (5,7) used in dynamic MRI, the MRPM problem can be considered in k - p space, where p represents the modulated acquisition parameter dimension (8–9). The signal evolution in x - p space usually yields a smooth curve, whose first or second order derivative is sparse. This property can be used as a constraint to accelerate MRPM (9). Furthermore, the multi-part dataset has strong correlation in the p dimension. This is reflected as the low rank property in MRPM (8, 10–12), which can also be used to accelerate MRPM. This type of method is known as the globally low rank (GLR) method. MRPM datasets are even more rank deficient when the images are partitioned into local regions, known as the locally low rank (LLR) method, as in dynamic MRI (13). The advantage of using GLR or LLR is that no specific signal model is assumed during the reconstruction of undersampled data. This is helpful in cases where the signal model is too complicated to apply during reconstruction. The parameter estimation is performed separately after reconstruction.

In this work, the LLR method is investigated in MRPM. Then, we propose a novel method to combine LLR and PI. The proposed method takes the advantages of both LLR and PI, and can achieve higher acceleration than each of the two methods alone. To compare the performance of GLR and LLR, T_2 mapping with a multi-echo spin-echo (MESE) pulse sequence is studied. Finally GLR, LLR, PI, the combination of GLR and PI, and the combination of LLR and PI are compared in T_1 mapping from a variable flip angle (VFA) acquisition (also known as DESPOT1) (14–17) with a multi-channel receiver, under different acceleration factors and acceleration scenarios.

Theory

Low Rank Property in MR Parameter Mapping

In MRPM, a dataset consists of a series of images, is acquired with different pulse sequence parameters. The signal variation of each image pixel across the dataset is a function of the MR parameter to be estimated. Images collected with different acquisition parameters have similar anatomical structures but different contrast. Data redundancy exists along the p dimension in these images. It can be analyzed by forming the Casorati matrix (18–20), in which each column consists of the image pixels from each of the data subsets. The data redundancy in MRPM datasets can be expressed as the low rank property of the Casorati matrix. In other words, the Casorati matrix can be represented by few dominant singular values and the corresponding singular vectors. The low rank constraint can be used to reconstruct an undersampled MRPM acquisition, an approach referred to as GLR in this work. For simplicity, a 2D MRPM problem with a single-coil acquisition is assumed. Define x_p as the image matrix (size: $n_x \times n_y$) with acquisition parameter p , y_p as the matrix (size: n_x

$\times n_y$) of the acquired k-space data with acquisition parameter p , x as the matrix (size: $n_x \times n_y \times n_p$) of all images with n_p different acquisition parameters, F as the Fourier transform operator, D_p as the undersampling operator with acquisition parameter p , and C as an operator that reformats x into its Casorati matrix (size: $n_x n_y \times n_p$). The GLR problem can be formulated as the following optimization problem:

$$\begin{aligned} & \underset{x}{\text{minimize}} \quad \|Cx\|_* \\ & \text{subject to} \quad D_p F x_p + \varepsilon_p = y_p, p = 1, 2, \dots, n_p \end{aligned} \quad (1)$$

where $\|Cx\|_*$ is the nuclear norm of Cx (the sum of singular values of Cx) (21), and ε_p is the noise in the acquired data y_p .

Recent studies in dynamic MRI have shown that the GLR method can be improved by restricting the Casorati matrix to a local image region (13), an approach referred to as LLR. The acquired 2D images x can be partitioned into a set Ω of small image blocks (size: $b_x \times b_y \times n_p$). Define C_b as the operator that takes image block b from the set Ω and forms its Casorati matrix. The LLR problem can be formulated as:

$$\begin{aligned} & \underset{x}{\text{minimize}} \quad \sum_{b \in \Omega} \|C_b x_b\|_* \\ & \text{subject to} \quad D_p F x_p + \varepsilon_p = y_p, p = 1, 2, \dots, n_p \end{aligned} \quad (2)$$

To demonstrate the GLR and LLR property of MRPM, T_2 mapping with an MESE sequence is shown in Fig. 1(a). Figure 1(b) demonstrates the low rank property of the Casorati matrix from the entire image and a small image block.

Combination of Locally Low Rank and Parallel Imaging

While LLR exploits the data redundancy in the parameter dimension, PI can further accelerate the acquisition of each data subset individually. The combination of LLR and PI (22) can potentially achieve higher acceleration than LLR or PI alone. In this work, SPIRiT (23) is used as the PI method. Assume n_c coils are used for data acquisition. Redefine x_p as the matrix (size: $n_x \times n_y \times n_c$) of images from all coils with acquisition parameter p , y_p as the matrix (size: $n_x \times n_y \times n_c$) of acquired k-space data from all coils with acquisition parameter p , x as the matrix (size: $n_x \times n_y \times n_c \times n_p$) of all images from all coils with n_p different acquisition parameters, C_p as the SPIRiT operator with acquisition parameter p that multiplies the SPIRiT kernels in image space (23), F as the Fourier transform operator applied individually to each coil, D_p as the undersampling operator with acquisition parameter p , Ω as a set of image blocks (size: $b_x \times b_y \times n_c \times n_p$), and C_b as the operator that takes image block b from the set Ω and forms its Casorati matrix (size: $b_x b_y n_c \times n_p$). A joint LLR and SPIRiT (LLR-SPIRiT) method can be formulated as:

$$\begin{aligned}
 & \underset{x}{\text{minimize}} \quad \sum_{b \in \mathcal{B}} \|\mathcal{C}_b x\|_* \\
 & \text{subject to} \quad D_p F x_p + \varepsilon_p = y_p, p = 1, 2, \dots, n_p \quad (3) \\
 & \quad \quad \quad G_p x_p = \kappa_p, p = 1, 2, \dots, n_p
 \end{aligned}$$

This problem can be solved using a projection onto convex sets (POCS) algorithm (24,25). The POCS algorithm applies PI and the L1R constraint iteratively with three sequential operations: (I) PI reconstruction, (II) promoting the low rank constraint of the Casorati matrices, and (III) promoting consistency with the data acquisition. To enforce the low rank property of the Casorati matrices in (II), a singular value thresholding algorithm is used (26). Details of the POCS algorithm can be found in the Appendix. Similarly, GLR and SPIRiT can also be combined (GLR-SPIRiT) and solved with POCS algorithm.

Methods

GLR and LLR in MESE T_2 Mapping

To investigate the performance of GLR and LLR reconstruction, a single-coil 2D MESE brain dataset of a 24-year-old healthy female volunteer was chosen. The dataset was acquired on a 1.5T Sonata scanner (Siemens Healthcare, Erlangen, Germany) with the following acquisition parameters: 32 echoes, echo spacing = 10 ms, TR = 3000 ms, field of view (FOV) = 22×22 cm², slice thickness = 5 mm, and matrix = 256×256 . The sequence was based on the widely-used MESE sequence, with nonselective composite refocusing pulses and a crusher gradient scheme (27). The total acquisition time was approximately 26 minutes. The dataset was retrospectively undersampled with a pseudo-random variable density pattern (6 centrally lines fully-sampled) by factors of 2 and 3. The sampling density at each k-space point was inversely proportional to its distance from the k-space center, and the sampling patterns were different for each TE. The undersampled dataset was reconstructed by GLR and LLR, using the proposed POCS algorithm with a cooling method (28). The threshold μ was set proportionally to the largest singular value of the Casorati matrix for proper scaling. With the cooling method (28) μ was initialized with 0.02 of the largest singular value, reduced to 0.01 after 20 iterations, and finally reduced to 0.001 after 40 iterations. The number of iterations was 50 for both GLR and LLR. For LLR, the block size was initialized as the entire image size for the first 20 iterations, and reduced to 8×8 after that. After reconstruction, T_2 mapping was performed with a mono-exponential decay model, for the fully-sampled dataset and different reconstructions. To compare the error of the estimated T_2 maps, the normalized root mean square error (nRMSE) was calculated

using the following formula: $\text{nRMSE} = \frac{1}{\max(T_2) - \min(T_2)} \sqrt{\frac{1}{N_i} \sum_{i=1}^{N_i} (T_2(i) - \tilde{T}_2(i))^2}$, where

T_2 is the T_2 map from the fully-sampled dataset, $\tilde{T}_2$ is the reconstruction by GLR or LLR, and N_i is the number of image pixels.

Accelerating Variable Flip Angle T_1 Mapping

To investigate the performance of GLR, LLR, SPIRiT, GLR-SPIRiT and LLR-SPIRiT, a 3D 8-channel VFA brain dataset of a 26-year-old male volunteer was chosen. The dataset was acquired on a 3T MR750 scanner (GE Healthcare, Waukesha, WI, USA) with the following acquisition parameters: TE = 3.0 ms, TR = 7.3 ms, FOV = $22 \times 22 \times 18$ cm³, slice thickness = 3 mm, matrix = $256 \times 256 \times 60$, FAs = (2°, 3°, 4°, 5°, 6°, 7°, 9°, 11°, 16°). The spoiling gradient was increased to the maximum achievable area for the TR used, as recommended (29). Ten FAs were used to achieve good estimation accuracy over a large T_1 range (30). The total acquisition time was approximately 20 minutes. The fully-sampled dataset was retrospectively undersampled by factors of 2, 3, 4, 5, and 6 with variable density Poisson disk patterns (31) including an autocalibration region of 24×24 in the k_y - k_z plane. The sampling density at each k-space point was inversely proportional to its distance from the k-space center, and the sampling patterns were different for each FA. The undersampled datasets were reconstructed by GLR, LLR, SPIRiT, GLR-SPIRiT and LLR-SPIRiT. A 5×7 SPIRiT kernel was used for SPIRiT, GLR-SPIRiT and LLR-SPIRiT. The same reconstruction parameters from the previous T_2 mapping experiment were used for GLR and LLR. For GLR-SPIRiT and LLR-SPIRiT, the threshold μ_t was initialized as 0.02 of the largest singular value, reduced to 0.01 after 10 iterations, and finally reduced to 0.005 after 20 iterations. The number of iterations was 30 for SPIRiT, GLR-SPIRiT and LLR-SPIRiT, and the block size was reduced from the entire image size to 8×8 after 10 iterations for LLR-SPIRiT. The undersampled datasets were inverse Fourier transformed along the readout direction into (x, k_y, k_z) space, where reconstructions were performed separately at each x location. Following reconstruction, T_1 estimation was performed on a pixel-wise basis using the linearized VFA signal equation and a weighted linear-least-squares method that takes into account the relative signal intensity of x_p (30). The nRMSE was calculated for each reconstruction in this experiment.

In a second experiment, two undersampling approaches were compared using the VFA data: (I) reduce the number of FAs and keep each dataset fully-sampled, and (II) maintain the same number of FAs (10 FAs) and undersample each acquisition in the same fashion as in the previous experiment. For the first approach, the selected FAs were (2°, 3°, 11°, 16°) for 4 FAs, (2°, 11°, 16°) for 3 FAs and (3°, 16°) for 2 FAs, based on literature recommendations (16, 30, 32). The total acceleration factors were equivalent to 2.5, 3.3 and 5 respectively. Variable density Poisson disk patterns with identical acceleration factors were applied to the 10-FA dataset in the second approach. The undersampled datasets in the second approach were reconstructed by LLR-SPIRiT. T_1 maps of both approaches were estimated, and the nRMSEs were calculated relative to the fully-sampled data.

In this work, all reconstructions and parameter estimation were performed in MATLAB (The MathWorks, Natick, MA, USA) on a 24-core 2.0 GHz Intel Xeon E5-2620 PC. The total reconstruction time was approximately 80 seconds and 85 minutes for single-slice T_2 mapping with LLR and 3D T_1 mapping with LLR-SPIRiT respectively. A demonstration of VFA T_1 mapping with LLR-SPIRiT can be found at <http://www.stanford.edu/~tzhang08/software.html>. All studies were approved by the Institutional Review Board, and informed consents were obtained.

Results

GLR and LLR in MESE T_2 Mapping

T_2 maps were estimated from the fully-sampled dataset, and GLR and LLR reconstructions with $R = 2$ and 3 (Fig. 2). The nRMSEs from CLR were 0.018 for $R = 2$ and 0.027 for $R = 3$, while those from LLR were 0.011 and 0.016 respectively. For the same acceleration factor, LLR had smaller nRMSE than CLR. LLR was more accurate at tissue boundaries between cerebrospinal fluid and gray matter near the ventricles, better seen from the difference T_2 maps. This shows that a low rank method can be used to accelerate MRPM and that LLR performs better than GLR for the same undersampled dataset in MESE T_2 mapping.

Accelerating Variable Flip Angle T_1 Mapping

Figure 3 shows a sagittal slice of the T_1 maps from the fully-sampled dataset and reconstructions with different acceleration factors. T_1 maps from GLR and LLR were very similar with increased nRMSE (0.021, 0.028, 0.034, 0.039 and 0.043 for GLR with $R = 2, 3, 4, 5$ and 6 respectively, and 0.021, 0.027, 0.033, 0.038 and 0.043 for LLR), and became more blurry as the acceleration increased, while T_1 maps from SPIRiT became more noisy, also with increased nRMSE (0.018, 0.017, 0.022, 0.025 and 0.029 for $R = 2, 3, 4, 5$ and 6 respectively). GLR and LLR suffered from blurring and reduced image contrast for high accelerations. This is similar to compressed sensing artifact, and was not due to reconstruction parameter selection. LLR-SPIRiT combined the advantages of LLR and SPIRiT, and its T_1 maps were closest to the reference. Similarly, GLR-SPIRiT also had better performance than GLR or SPIRiT alone. In this experiment, the CLR-SPIRiT and LLR-SPIRiT results were very similar. For identical acceleration factors, T_1 maps from LLR-SPIRiT were sharper than those from LLR, and had no obvious P1 artifacts compared to SPIRiT alone.

The estimated T_1 maps from the reference 10-FA dataset and different acceleration scenarios are shown in Fig. 4. With fewer FAs, the T_1 map became more noisy. The nRMSE was 0.019 for the 4-FA case, 0.021 for the 3-FA case, and 0.048 for the 2-FA case. The error increased dramatically from 3 FAs to 2 FAs. For the second approach, the nRMSEs for $R = 2.5$ and 3.3 were very similar to the first approach (0.019 and 0.021 respectively). For $R = 5$, the second approach performed better (nRMSE = 0.028) than the first approach, as can be seen from Fig. 4(b).

Discussion

The low rank property of MRPM can be used to reconstruct undersampled datasets. LLR is likely to perform better than GLR by restricting the low rank property to a small image region, where neighboring tissues are more likely to have similar MF parameters. Smaller imaging regions are likely to result in a Casorati matrix with a lower rank. At the same time, one needs to ensure that the Casorati matrix is rank-deficient in the p dimension instead of the x dimension (by imposing $b_x \times b_y > n_p$). In general, consistent results were observed with block sizes ranging from 8×8 to 16×16 . The superiority of LLR over CLR may vary for different experiments. The assumption for LLR and GLR is that the series of parameter

mapping images are low rank. In the MESE T_2 mapping experiment, 32 echoes were acquired. There was a lot of data redundancy in these images, reflected as the low rank property. As shown in Fig. 15, LLR is clearly a better constraint than GLR in T_2 mapping with more rapidly decreasing singular values. However, LLR was not dramatically superior to GLR in T_1 mapping, as shown in Fig. 5. In general, SVD of the fully sampled MRPM datasets may indicate the performance of LLR and GLR.

To help speed up the convergence of the POCS algorithm, the image block can be initialized with the entire image and reduced to a smaller region after several iterations. The cooling method can help speed up the convergence by gradually reducing the threshold for the LLR constraint. The final threshold is dependent of the SNR of the acquisition, and was empirically chosen in this manuscript. From our experience, LLR-SPIRiT needs fewer iterations to converge than LLR alone because the SPIRiT reconstruction performed within each iteration also helps the convergence.

In the MESE T_2 mapping experiment, both GLR and LLR had little error in gray matter and white matter. LLR performed better than GLR in regions with high T_2 value (e.g., ventricles). The majority of brain tissues in our data are gray and white matter, therefore GLR would enforce the entire reconstructed image to have a T_2 value in that range. This reduces the accuracy of the T_2 estimation for high T_2 values. By restricting the region of interest to a local image region, LLR can achieve better accuracy for a larger T_2 range than GLR.

For VFA T_1 mapping with high accuracy over a large T_1 range, or to allow characterization of complex relaxation behavior, many flip angles are preferred (30,33). In our experiment, 10 FAs were chosen. There were two approaches to accelerate this acquisition: fewer FAs with full sampling, and undersampling each acquisition with 10 FAs. The experiment showed that the second approach performed similarly to the first for $R = 2.5$ and 3.3 , and better for $R = 5$. Because pseudo-random sampling patterns were used in the second approach, the acceleration factor is more flexible than the first approach and does not need to be an integer. Robust T_1 estimation can be achieved by LLR-SPIRiT for $R = 6$ with 10 FAs, equivalent to acquiring only 1.6 FAs, reducing the total acquisition time to about 3 minutes. Since a 3D Cartesian acquisition was used in VFA T_1 mapping, the data can be first inverse Fourier transformed along the readout direction. Then the reconstruction can be performed separately at each x location, and it is easy to apply parallel computing with separable problems for faster reconstruction. Note that separate reconstructions at different x locations already partly promote a locally local rank constraint at each spatial location. This type of GLR reconstruction sometimes can achieve similar results to LLR. The locally low rank constraint could also be promoted within a 3D image cube instead of 2D image block at each x location. Similar performance would be expected, although parallel computing might not be easy to directly apply to the complete 3D reconstruction. VFA T_1 mapping was based on a traditional SPGR sequence. Prospective pseudo-random undersampling might introduce bigger eddy current effects, but with a careful planning of the order of undersampling view points, similar to existing dynamic MRI studies (22,34), we do not expect the performance of prospective undersampling to be significantly different than retrospective undersampling in VFA T_1 mapping.

The proposed LLR and LLR-SPIRiT methods, and others like k - t PCA, all exploit the low rank property of MRPM. Approaches like k - t PCA (7,10) restrict the reconstruction to the selected principle components. Therefore, the rank of the MRPM datasets has to be known based on prior knowledge. LLR and LLR-SPIRiT exploit the low rank as a constraint, and the exact number of principle components is not fixed in the optimization problem. Moreover, LLR and LLR-SPIRiT restrict the low rank property to a local image region and are more flexible than approaches that enforce a GLR property.

The proposed method can also be combined with an additional image sparsity constraint commonly used in compressed sensing reconstructions (35). However, tuning reconstruction parameters with multiple constraints needs to be further studied. Non-optimal reconstruction can lead to artifacts such as loss of image contrast, image blurring, or incorrect parameter estimation. The benefits of using multiple constraints also need to be investigated.

Conclusion

In this work, the low rank property has been exploited in MRPM. LLR methods can potentially improve the reconstruction compared to GLR. An LLR-SPIRiT method has been proposed that combines PI and LLR. The proposed method can achieve better performance for highly accelerated MRPM by taking advantages of both PI and LLR. LLR-SPIRiT has been evaluated in VFA T_1 mapping with superior performance over PI or LLR alone.

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Appendix A POCS algorithm for LLR-SPiRiT

The convex optimization problem in Eq. 3 can be solved using a POCS algorithm, in which SPiRiT, the LLR constraint, and consistency with the data acquisition are enforced sequentially. The SPiRiT kernel is first calculated from the fully-sampled central k-space before the POCS algorithm is carried out. Details of this calibration step can be found in reference (23). The algorithm is shown in the following table.

ITERATIVE POCS ALGORITHM FOR LLR-SPiRiT

INPUTS:

y_p - k-space measurements with acquisition parameter p
 D_p - subsampling operator selecting acquired k-space with acquisition parameter p
 G_p - SPiRiT operator with acquisition parameter p
 μ - a threshold in the singular value thresholding operation of the Co-sensit matrices
 b - a pre-defined image partition of Ω

OPTIONAL PARAMETERS:

MaxIter - stopping criteria by number of iterations (default = 30)
 TolDiff - stopping criteria by reconstruction difference between two iterations (default = 10^{-7})

OUTPUTS:

x - the approximate solution to Eq. 3

% Initialization

$k = 0; x_p^{(k)} = F^{-1} D_p^T y_p, k = 1, 2, \dots, n_p$

% Iterations

while ($k < \text{MaxIter}$ and $\|x^{(k)} - x^{(k-1)}\|_2^2 / \|x^{(0)}\|_2^2 > \text{TolDiff}$) {

$k = k + 1$

 % SPiRiT operation

$x_p^{(k)} = G_p x_p^{(k-1)}, p = 1, 2, \dots, n_p$

 % Enforcing LLR

ITERATIVE POCS ALGORITHM FOR LLR-SPIRiT

$$[u_b^{(k)}, s_b^{(k)}, v_b^{(k)}] = \text{SVT} \{C_b x^{(k)}\}, b \in \Omega$$

$$x^{(k)} = \bigcup_{b \in \Omega} \{C_b^T [u_b^{(k)}, s_b^{(k)}, v_b^{(k)}]\}$$

% Gamma consistency

$$x^{(k)} = \mu^{-1} [(I - D_p^T D_p)(F x^{(k-1)} + D_p^T y_p)], n = 1, 2, \dots, n_p$$

The soft-thresholding operation is defined as $S_\mu(a) = \frac{1}{|a|} \max(0, |a| - \mu a)$, where $a \in \mathbb{R}$.

To speed up convergence, μ is gradually reduced as the iteration number k increases. Note that the image partition is different for each iteration to avoid sharp transitions between different image blocks.

Figure 1
Low rank property of MR parameter mapping. (a) Illustration of T_2 mapping with an MESE sequence (RF pulses are shown as red arrows). Images are acquired at different echo times ($p=TE$). (b) The Casorati matrices from the entire image (blue) and local image region (green) are low rank. Singular values of GLR and LLR are shown in the middle plot. The Casorati matrix from LLR is more rank-efficient than from GLR. In other words, there is more data redundancy in LLR.

Figure 2
 MESE T_2 mapping with GLR and LLR. T_2 maps of the fully-sampled dataset, and from the GLR and LLR reconstructions with $P = 2$ and 3 are shown in the top row. The corresponding difference ($\times 10$) of the T_2 maps between fully-sampled dataset and the four reconstructions are shown in the bottom row. The nRMSE of each reconstruction is listed under each T_2 map. LLR has smaller nRMSE than GLR for the same acceleration factor. Both GLR and LLR have good accuracy of T_2 estimation for gray matter and white matter. T_2 estimation of LLR is more accurate for tissue with high T_2 values (e.g. cerebrospinal fluid). Note that the horizontal line in the middle of the T_2 maps is a flow artifact.

Figure 3
VFA T_1 mapping with GLR, LLR, SPIRiT, GLR-SPIRiT and LLR-SPIRiT. T_1 maps of one sagittal slice with GLR, LLR, SPIRiT, GLR-SPIRiT and LLR-SPIRiT reconstructions at different acceleration factors are shown in the second, third, fourth, fifth and last column respectively. The nRMSE is shown on top of each T_1 map. When the acceleration factor increases, T_1 map from the GLR and LLR reconstruction becomes more blurry while that from the SPIRiT reconstruction becomes more noisy. T_1 maps from GLR-SPIRiT and LLR-SPIRiT are similar, and closest to the reference among all the reconstructions.

Figure 4
Accelerated VFA T_1 mapping with two different approaches: (I) reducing the number of FAs and (II) keeping the same number of FAs and undersampling each FA. (a) T_1 maps of one sagittal slice with the first approach are shown in the top row and T_1 maps with the second approach are shown in the bottom row. Each column has the same total acceleration factor. (b) T_1 map differences ($\times 20$) between each reconstruction and the original 10-FA acquisition. The corresponding nRMSEs are shown under each T_1 difference map. Both approaches have similar performance for total acceleration factors of 2.5 and 3.3. For a total acceleration factor of 5, the second approach has superior performance than the first.

Figure 5
Comparison of singular values from GLR and LLR in VFA T_1 mapping. The singular values (normalized) decrease slightly faster in LLR (with a 8×8 image block) than GLR. However, the superiority of LLR over GLR is not as strong as that in MESE T_2 mapping as shown in Fig. 1b.