Science and Culture: Hunting fractals in the music of J. S. Bach

Stephen Ornes

Science Writer

Mathematician Benoit Mandelbrot coined the term “fractal” in a 1975 book on the subject, and his seminal 1982 book *The Fractal Geometry of Nature* (1), which catalogs the ubiquity of the geometric patterns, is widely credited for bringing them to the masses.

Mathematicians disagree on a precise definition, but a fractal is typically described as exhibiting self-similarity, which means identical (or nearly identical) patterns appear, whether the shape is viewed from up close or far away. That is, the part looks like the whole, and the whole looks like a part. Perhaps not surprisingly, researchers who study music—the mathematical underpinnings of which are well documented—have sought to analyze such patterns in compositions.

In one of the earliest such investigations (2), University of California physicists Richard F. Voss and John Clarke analyzed audio recordings of music, including Scott Joplin’s piano rags and Bach’s first *Brandenburg Concerto*, as well as the output from a variety of radio stations. Voss and Clarke identified scaling behaviors—the word “fractal” wasn’t yet in common use—in the fluctuations of volume in the recordings, and in the melody fluctuations of popular music on the radio.

Bach has similarly attracted the attention of other fractal hunters, including Harlan Brothers, a jazz guitarist, composer, and mathematician in Branford, Connecticut. For more than a decade, Brothers has been mapping fractals in music. In a study published in *Fractals* in 2007 (3), Brothers reported on

fractal structure in the phrasing of notes Bach used to compose his *Cello Suite No. 3*. Within that composition, patterns of long and short notes within measures reappeared as patterns of long and short phrases at larger scales. The suite’s self-similarity, Brothers noted, bore a striking resemblance to the Cantor Comb, a visualization of a historical fractal called the Cantor Set (which was described by Georg Cantor in 1883, a century before Mandelbrot first introduced the term “fractal”).

Identifying fractals in music requires a different approach than seeing them in an image. “Unlike a picture, which is all laid out so that you can instantly see the structure, music is fundamentally a serial phenomenon,” Brothers says. “With music, the whole piece takes shape in your mind. This makes it more challenging to identify the self-similarity.”

Mandelbrot, who died in 2010, long suspected that music contained hidden fractal patterns but didn’t have time to devote to the subject, says Brothers. Brothers contributed a chapter giving an overview of fractal music to *Benoit Mandelbrot: A Life in Many Dimensions*, a biography of the mathematician scheduled to be published next year (4).

“Benoit was excited to see the fractal analysis of music further developed in a rigorous fashion,” Brothers says.

The top of this drawing shows a Cantor comb, which depicts self-similar patterns repeating at different scales on different lines. The lower diagram depicts the distribution of note durations in a 16-measure excerpt from a cello suite by Bach. The two patterns are similar. [Reprinted with permission from ref. 3 (Copyright 2007, World Scientific).]

1 Mandelbrot B (1982) *The Fractal Geometry of Nature* (W. H. Freeman and Company, San Francisco).

2 Voss RF, Clarke J (1975) ‘1/f noise’ in music and speech. *Nature* 258(5533):317–318.

3 Brothers HJ (2007) Structural scaling in Bach’s Cello Suite no. 3. *Fractals* 15(1):89.

4 Frame M, ed (2015) *Benoit Mandelbrot: A Life in Many Dimensions* (World Scientific Publishing Company, Singapore), in press.