Published in final edited form as:

IEEE Trans Plasma Sci IEEE Nucl Plasma Sci Soc. 2014 January 1; 42(1): 29–37. doi:10.1109/TPS.2013.2288493.

Simple Correctors for Elimination of High-Order Modes in Corrugated Waveguide Transmission Lines

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Abstract

When using overmoded corrugated waveguide transmission lines for high power applications, it is necessary to control the mode content of the system. Ideally, overmoded corrugated transmission lines operate in the fundamental HE_{11} mode and provide low losses for long distances. Unwanted higher order modes (HOMs), particularly LP_{11} and HE_{12} , are often excited in the experimental systems due to practical misalignments in the transmission line system. This paper discusses how the unwanted modes propagate along with the fundamental mode in the transmission line system by formulating an equation that relates the center of power offset and angle of propagation of a beam (for the HE_{11} and LP_{11} modes) or the waist size and phase front radius of curvature of a beam (for the HE_{11} and HE_{12} modes). By introducing two miter bend correctors into the transmission system—miter bends that have slightly angled or ellipsoidal mirrors—the HOMs can be precisely manipulated in the system. This technique can be used to eliminate small quantities of unwanted modes, thereby creating a nearly pure fundamental mode beam with minimal losses. Examples of these applications are calculated and show the theoretical conversion of up to 10% HOM content into the fundamental HE_{11} mode with minimal losses.

Index Terms

Corrugated waveguide; gyrotron; linearly polarized modes; millimeter waves; miter bend; mode conversion; oversized waveguide; transmission line alignment

I. Introduction

In the electron cyclotron heating (ECH) and current drive systems of tokamaks and stellarators, corrugated overmoded metallic waveguides are used for low loss transmission of high power millimeter waves (at frequencies of about 77–170 GHz). Ideally, only the fundamental HE_{11} mode travels in high power corrugated waveguide transmission lines. However, since these waveguides are over-sized to provide low losses over long distances for the fundamental mode, higher order modes (HOMs) may also propagate in the

transmission line with low losses. A detailed analysis of the linearly polarized modes of corrugated waveguide is presented in [1]. Power conversion to HOMs must be minimized for transmission efficiency. Ohmic and mode conversion loss calculations for corrugated transmission lines presented in [2] and [3] indicate that high purity (95%) of the operating HE_{11} mode of corrugated waveguide should be provided for the transmission line to meet the specifications of operation.

For ECH systems, the power is inserted into the waveguide system via a Gaussian beam from a gyrotron [4], [5]. For example, each transmission line of the ITER ECH system will use 63.5-mm diameter corrugated waveguides and waveguide components to transmit up to 2 MW of continuous power at 170 GHz [2], [6], [7]. When coupling a Gaussian beam into oversized waveguide, HOMs are often excited due to a misalignment in tilt or offset and/or a mismatch of beam waist size or phase front radius. In addition, miter bends and other waveguide components contribute to high-order mode conversion in the system.

Primarily, the LP_{11} and HE_{12} modes are of interest when studying HOMs in overmoded waveguide. Along with the fundamental HE_{11} mode, these two unwanted modes have low losses. The LP_{11} and HE_{12} modes exist in large quantities due to realistic experimental misalignments in the system. A slight tilt or offset of the input beam excites low levels of power in the LP_{11} mode. A mismatch of radius or a non-flat phase front of the Gaussian beam will result in the HE_{12} mode. Diffraction in miter bends present in the system will also increase the HE_{12} mode content. Other HOMs are usually present at much lower levels and/or have significant ohmic loss on the transmission line, such that they are much less important to the operation of the system.

Several experiments have confirmed that most of the HOM content present in oversized transmission line systems is in the LP_{11} and HE_{12} modes [8]. The radiation pattern from a transmission line powered by a gyrotron has been measured in hot test using an IR camera [4], [9] where the mode content was retrieved from intensity data using the phase retrieval code. In addition, experimental mode content has been determined from cold test measurements by scanning the pattern of radiation from the end of an oversized waveguide [10], [11]. Therefore, this paper will focus on a correction based on the assumption that these two modes are the most significant HOMs in the transmission line system.

The conversion of LP_{11} and HE_{12} mode content to achieve higher purity of the HE_{11} mode can be done using phase correctors incorporated in two miter bends already present in the system. In this paper, we present a simple two-mode description of a two-mirror corrector design, first for an HE_{11} and LP_{11} mode superposition, then for an HE_{11} and HE_{12} mode superposition. A small tilt or offset of the input Gaussian beam can be compensated using tilts in two successive miter bend mirrors. As will be discussed, this technique reduces the LP_{11} mode content in the waveguide and increases the fundamental mode content. In addition, a mismatch of the waist radius to the waveguide radius and a finite phase front curvature of the input Gaussian beam can be compensated by two profiled (ellipsoidal or toroidal) mirrors placed in miter bends to reduce the HE_{12} content in the waveguide. For this analysis, the general multimode theory of phase correctors in overmoded waveguides has been applied [12], [13]. Both HOM corrector designs are checked using the multimode

transmission line code that simulates power propagation in overmoded waveguides and waveguide components [3].

II. Excitation of the LP₁₁ Mode

When a Gaussian beam is inserted with a tilt and/or offset into a waveguide a superposition of the HE₁₁ and LP₁₁ modes is excited [1], as shown in Fig. 1. The HE₁₁ (also denoted LP₀₁) and LP₁₁ modes are linearly polarized (LP). For the geometry shown in Fig. 1, the LP₁₁^(e) even mode is generated, which will result in an x -directed offset, x_0 , and angle of propagation, α_x . Both offset x_0 and angle α_x are z -dependent. The same analysis could be applied for a y -directed offset, y_0 , and angle of propagation α_y , resulting in the excitation of the LP₁₁^(o) odd mode in the same polarization. The general case for any tilt angle or offset is treated by combining the results for the x - and y -axis cases. In this example, we assume x -directed polarization; a y -directed set of modes can be corrected with the same analysis.

The electric field E_x distribution in the HE_{1n} (or LP_{0n}) modes ($n = 1, 2$) is

$$u_{0n}(r) = \frac{1}{\sqrt{\pi}a} \frac{J_0\left(\frac{\nu_{0n}}{a}r\right)}{|J_1(\nu_{0n})|} \quad (1)$$

normalized such that the modal power in the waveguide of radius a is 1. The field is defined in the cylindrical coordinate system with r , the radius from the center of the waveguide, and ϕ , the angle from the x -axis. The zeros of the Bessel function J_0 are $\nu_{01} = 2.405$ (HE₁₁/LP₀₁ mode) and $\nu_{02} = 5.52$ (HE₁₂/LP₀₂ mode). The normalized LP₁₁ mode E_x field distribution is

$$u_{11}(r, \varphi) = \frac{\sqrt{2}}{\sqrt{\pi}a} \frac{J_1\left(\frac{\nu_{11}}{a}r\right)}{|J_0(\nu_{11})|} \cos\varphi. \quad (2)$$

The first zero of the Bessel function J_1 is $\nu_{11} = 3.832$. In (1) and (2), the scalar representation within the quasi-optical approximation is used that assumes $(ka)^2 \gg \nu_{mn}^2$. A superposition of the HE₁₁ and LP₁₁ modes gives

$$E_x(r, \varphi) = \sqrt{P_{01}}u_{01}(r)\exp(j\theta_{01}) + \sqrt{P_{11}}u_{11}(r, \varphi)\exp(j\theta_{11}). \quad (3)$$

The power in the fields is normalized, such that $P_{01} + P_{11} = 1$ and each mode has a phase,

$\theta_{mn}(z) = \theta_{mn}(0) + k_{mn}z$, where $k_{mn} = \sqrt{\omega^2/c^2 - \nu_{mn}^2/a^2}$, $\theta_{mn}(0)$ is the initial phase difference between the two modes, ω is frequency, and c is the speed of light. The superposition of modes results in an energy center offset in the waveguide

$$x_0(z) = 2\sqrt{P_{01}P_{11}}b_{12}\cos(\theta_{01} - \theta_{11}) \quad (4)$$

and a propagation angle of the energy center [1]

$$\alpha_x(z) = 2\sqrt{P_{01}P_{11}}d_{12}\sin(\theta_{01} - \theta_{11}) \quad (5)$$

where

$$b_{12} = \frac{2\sqrt{2}\nu_{01}\nu_{11}a}{(\nu_{11}^2 - \nu_{01}^2)^2} = 0.329a \quad (6)$$

and

$$d_{12} = \frac{\lambda\nu_{01}\nu_{11}}{\sqrt{2}\pi a (\nu_{11}^2 - \nu_{01}^2)} = 0.233\lambda/a \quad (7)$$

and λ is the wavelength. Equations (4) and (5) can be combined and the tilt-offset conservation theorem can be formulated [1]

$$\left(\frac{x_0}{b_{12}}\right)^2 + \left(\frac{\alpha_x}{d_{12}}\right)^2 = 4P_{01}P_{11}. \quad (8)$$

The power in the LP_{11} mode due to an offset (x_0) or tilt (α_x) at the input to the waveguide can be estimated from (8) as

$$P_{11} = 2.3 \left(\frac{x_0}{a}\right)^2 \quad (9)$$

or

$$P_{11} = 4.6 \left(\frac{a\alpha_x}{\lambda}\right)^2. \quad (10)$$

These estimates are in reasonable agreement with previous estimates of LP_{11} mode content excitation [14], [15].

III. Analytical Theory of LP_{11} Mode Correction

We assume that the beam inserted into the waveguide is a well-formed Gaussian beam, but that the insertion has tilt and/or offset. This insertion excites primarily the HE_{11} mode and a small amount of the LP_{11} mode (less than 10% of the power). As discussed in the previous section, tilt and offset at the entrance will generate a microwave beam in the waveguide that oscillates between energy center (radial) displacement, (4), and propagation angle variation, (5). Since these are out of phase, the microwave beam in the waveguide will always have locations (repeated down the waveguide as the phase evolves) where the beam has a maximum tilt and zero radial displacement. To simplify the problem without loss of generality, we choose the entrance of the waveguide to be a location where the beam is tilted without radial displacement, as shown in Fig. 1. The solution of the case with both tilt and offset at the waveguide entrance can be generalized from the special case solved here.

After propagating a distance of L_1 down the waveguide, there will be a miter bend where, in general, the microwave beam has both an offset and a tilt. The mirror of the first miter bend is adjusted with an angle, α_1 , such that there will be only tilt (and no offset) at the second miter bend. Then, the second miter bend is adjusted with an angle, α_2 , to compensate for the

tilt present in the beam at the second miter bend. The first and second miter bend mirrors are separated by a distance L_2 .

It should be noted that this concept of a two mirror corrector works for any value of the distance L_1 . It also works for any value of the distance L_2 , except for the special case when L_2 is a multiple of half-beat wavelengths between the modes HE_{11} and LP_{11} . This exception is proven in the following derivation.

The general problem of coupling of a Gaussian beam with tilt and offset into a waveguide has been treated in [14]. For this analysis, we assume that the power is inserted at the center of the waveguide with a small tilt such that there is no initial power offset, $x_0(0) = 0$ and there is a nonzero input angle $\alpha_x(0)$. In the first length of waveguide ($z < L_1$), the offset $x_0(z)$ and propagation angle $\alpha_x(z)$ are determined from (4) and (5)

$$x_0(z) = 2\sqrt{P_{01}P_{11}}b_{12}\sin(\Delta k_{12}z) \quad (11)$$

$$\alpha_x(z) = 2\sqrt{P_{01}P_{11}}d_{12}\cos(\Delta k_{12}z) \quad (12)$$

where

$$\Delta k_{12} = \frac{\nu_{11}^2 - \nu_{01}^2}{2ka^2} \quad (13)$$

and $k = 2\pi/\lambda$ is the wavenumber. From (11) and (12), the initial displacement $x_0(0) = 0$ and the initial propagation angle is

$$\alpha_x(0) = 2\sqrt{P_{01}P_{11}}d_{12}. \quad (14)$$

At the location before entering the first miter bend corrector, with $z = L_1 - 0$, the displacement and angle of propagation are

$$x_0(z=L_1 - 0) = 2\sqrt{P_{01}P_{11}}b_{12}\sin(\Delta k_{12}L_1) \quad (15)$$

$$\alpha_x(z=L_1 - 0) = 2\sqrt{P_{01}P_{11}}d_{12}\cos(\Delta k_{12}L_1). \quad (16)$$

With a tilt angle of α_1 , the first corrector maintains the same power offset across the boundary while changing the propagation angle such that

$$x_0(z=L_1+0) = x_0(z=L_1 - 0) \quad (17)$$

$$\alpha_x(z=L_1+0) = \alpha_x(z=L_1 - 0) + 2\alpha_1. \quad (18)$$

In (17) and (18), we ignore the finite length of the miter bend, $2a$, as compared with L_1 and L_2 . We also ignore small amounts of mode conversion at the miter bend.

Between the two miter bend correctors ($L_1 < z < L_2$) the modal powers are defined as P'_{01} and P'_{11} with Ψ phase difference between the modes. Assuming small tilts for the correctors, only two modes remain in the system and $P'_{01} + P'_{11} = 1$. The offset and angle in this region are

$$x_0(z=L_1+0) = 2 \sqrt{P'_{01} P'_{11}} b_{12} \sin \Psi \quad (19)$$

$$\alpha_x(z=L_1+0) = 2 \sqrt{P'_{01} P'_{11}} d_{12} \cos \Psi. \quad (20)$$

From (15)–(20), we can determine an analytical solution for mode conversion at a mirror tilted at angle α_1

$$P'_{01} P'_{11} = P_{01} P_{11} + 2 \frac{\alpha_1}{d_{12}} \sqrt{P_{01} P_{11}} \cos(\Delta k_{12} L_1) + \frac{\alpha_1^2}{d_{12}^2} \quad (21)$$

$$\tan \Psi = \frac{\sqrt{P_{01} P_{11}} \sin(\Delta k_{12} L_1)}{\sqrt{P_{01} P_{11}} \cos(\Delta k_{12} L_1) + \alpha_1 / d_{12}}. \quad (22)$$

At the location before the second miter bend corrector, $z = L_1 + L_2 - 0$, the displacement and angle of propagation are

$$x_0(z=L_1+L_2-0) = 2 \sqrt{P'_{01} P'_{11}} b_{12} \sin(\Psi + \Delta k_{12} L_2) \quad (23)$$

$$\alpha_x(z=L_1+L_2-0) = 2 \sqrt{P'_{01} P'_{11}} d_{12} \cos(\Psi + \Delta k_{12} L_2). \quad (24)$$

The angles of the correctors are determined such that there is no offset or angle in the modal propagation after the second corrector, $z = L_2 + 0$. To satisfy this condition, the angle of the first corrector is calculated such that the displacement before corrector two will be zero, $x_0(z = L_1 + L_2 - 0) = 0$. From (23)

$$\sin(\Psi + \Delta k_{12} L_2) = 0 \quad (25)$$

which can be written for use with (22) as

$$\tan \Psi = -\tan(\Delta k_{12} L_2). \quad (26)$$

Thus, the tilt required on the first corrector is determined from (21)–(26)

$$\alpha_1 = -\frac{1}{2} \alpha_x(0) \frac{\sin(\Delta k_{12} [L_1 + L_2])}{\sin(\Delta k_{12} L_2)}. \quad (27)$$

The second corrector, with an angle of α_2 , compensates for the propagation angle that remains after the offset is removed with the first corrector, $\alpha_x(z = L_1 + L_2 - 0)$ (24)

$$\alpha_x(z = L_1 + L_2 + 0) = \alpha_x(z = L_1 + L_2 - 0) + 2\alpha_2 = 0. \quad (28)$$

Therefore, from (24), (26), and (28), the necessary angle for the second corrector to compensate for the propagation angle is

$$\alpha_2 = \sqrt{P'_{01} P'_{11} d_{12}} \quad (29)$$

which can be reduced to

$$\alpha_2 = \frac{1}{2} \alpha_x(0) \frac{\sin(\Delta k_{12} L_1)}{\sin(\Delta k_{12} L_2)}. \quad (30)$$

From (27) and (30), the length L_2 between the correctors cannot be a multiple of half-beat-wavelength $\Lambda_{12}/2 = \pi/k_{12}$ to calculate a finite corrector angle.

Fig. 2 shows a specific example of the miter bend correctors' effect on the angle $\alpha_x(z)$ and displacement $x_0(z)$ as the power propagates along a transmission line. The analysis is done for the ITER transmission line ($a = 31.75$ mm) operated at 170 GHz with an initial insertion of a Gaussian beam with a tilt angle $\alpha_x = 0.34^\circ$, corresponding to an input of $P_{01} = 0.94$ (94% HE_{11}) and $P_{11} = 0.06$ (6% LP_{11}). The beam is coupled to the corrugated waveguide at $z = 0$, the first corrector is at $L_1 = 2$ m from the start of the waveguide, and the second corrector is at $L_2 = 4$ m ($z = 6$ m). The angles of the correctors necessary to compensate the initial input angle were calculated from (27) and (30) to be $\alpha_1 = 0.16^\circ$ and $\alpha_2 = -0.10^\circ$. The modal powers in the section between the correctors 1 and 2 are $P'_{01} = 0.98$ and $P'_{11} = 0.02$. Since this is an ideal calculation, an output of 100% HE_{11} was produced; there is no offset or angle of propagation in the output beam.

This corrector design has a relatively wide bandwidth. The corrective angles of the miter bends must be calculated assuming an ideal frequency; if a frequency variation of the input power away from the design frequency is considered while the same input mode mixture is maintained, the output will have a small percentage of LP_{11} mode remaining. However, for a frequency variation up to $\pm 2\%$ of the design frequency, the LP_{11} mode output power at the output is less than 0.1% of the total output.

IV. Numerical Theory of LP_{11} Mode Correction

The validity of this simple two-mode calculation has been checked using the MMTL code developed for ITER transmission line loss estimates [3]. In the previous example, the modes were restricted to just the HE_{11} and LP_{11} modes and mode conversion in the system was ignored. The MMTL code considers small quantities of other HOMs that arise in the system in addition to the HE_{11} and LP_{11} modes. With the MMTL code, the diffraction in the two miter bends with the corrector mirrors is considered in the simulations. The miter bend is modeled as a junction of two waveguides whose open ends are cut using two symmetrical miter-cuts. The junction is formed such that the waveguides touch in two points. The miter

bend is thus modeled as a partially covered gap between the waveguides. The propagation of the multimode field through the gap is simulated using an fast Fourier transform method [3]. A miter bend with an angle or phase corrector is modeled as a gap with a phase correction in the middle of the gap.

In this simulation of a miter bend, some mode conversion results. However, due to symmetry of the gap model of a miter bend, no conversion to the LP_{11} mode occurs in this calculation. This is in agreement with alternative simulations of the miter bend using HFSS. These simulations indicate that conversion to the LP_{11} mode in an ideal miter bend is highly reduced as the diameter of the bend is increased (compared with the wavelength). Conversion to the LP_{11} mode in the miter bends is negligible under the parameters of this calculation.

The simulations are conducted for the same length between the correctors, $L_1 = 2$ m and $L_2 = 4$ m, that was used in the two-mode calculation (Fig. 2). The ability of the correctors to reduce the LP_{11} mode content does not depend strongly on the value of L_2 , so long as L_2 is not equal to (or almost equal to) an integer number of half wavelengths of the beat wavelength ($\Lambda_{12}/2 = 2.5$ m) between the HE_{11} and LP_{11} modes.

Fig. 3 shows the HE_{11} mode content at the output as a function of the input LP_{11} mode content for two cases: with-and without the angled correctors. For an LP_{11} input power of 10%, the corrected output power of the LP_{11} mode can be reduced to as low as 0.6% and the HE_{11} mode power is 98.4%. The remaining power is distributed among many modes including HE_{12} (0.3%) and LP_{12} (0.1%). In Fig. 3, the intercept on the y-axis is at 99.6%, not 100%, due to 0.4% diffraction losses in the two miter bends. According to [16], the mode conversion loss in a single miter bend is $0.195(\lambda/a)^{3/2} = 0.26\%$, half of which is diffraction loss. The calculated 0.26% total diffraction loss in two miter bends is in reasonable agreement with the 0.4% diffraction loss in the MMTL code. Ohmic loss at the miter bends is omitted in this analysis.

V. Excitation of the HE_{12} Mode

Ideally, the Gaussian input has a flat phase front and is perfectly matched in waist radius to the HE_{11} mode in the waveguide. A superposition of HE_{11} and HE_{12} modes is excited in an overmoded corrugated waveguide transmission line when the Gaussian beam input is mismatched with the wrong beam waist radius or with a finite phase front radius of curvature. This cylindrically symmetric mismatch is typical in the experimental insertion of real Gaussian beams into overmoded waveguide. In the next section, we will show that the HE_{12} mode can also be eliminated using two correcting mirrors with ellipsoidal surfaces.

The linearly polarized field in the corrugated waveguide is represented by the superposition of the HE_{11} and HE_{12} modes (also expressed as LP_{01} and LP_{02})

$$E_x(r) = \sqrt{P_{01}}u_{01}(r)\exp(j\theta_{01}) + \sqrt{P_{02}}u_{02}(r)\exp(j\theta_{02}). \quad (31)$$

with $P_{01} + P_{02} = 1$ and $\theta_{mn} = \theta_{mn}(0) + k_{mn}z$. An x -directed electric field is assumed, but the analysis also applies to a y -directed field. The difference in the propagation constants between these two modes is

$$\Delta k_{12} = \frac{\nu_{02}^2 - \nu_{01}^2}{2ka^2}. \quad (32)$$

The beat wavelength between the modes HE_{11} and HE_{12} is

$$\Lambda_{12} = 2\pi / \Delta k_{12} = 0.509ka^2. \quad (33)$$

For the ITER transmission line, $\Lambda_{12} = 1.83$ m. The amplitude distribution in the waveguide is oscillating as a function of z with the period Λ_{12} .

We consider, as an example, an input, which consists of 95% HE_{11} and 5% HE_{12} ($P_{01} = 0.95$ and $P_{02} = 0.05$). Fig. 4 shows the beam radius and the inverse phase front curvature radius as a function of propagation distance through a transmission line, z . These parameters are also visually shown in Fig. 5. The phase front of the beam is flat ($R_0 = \infty$) at the minimum and maximum of the beam radius; the phase front has the minimum curvature radius at the average beam radius ($w/a = 0.66$). In this example, we choose $z = 0$ to be the location of the minimum waist size.

The effective beam radius w is defined as

$$w^2(z) = 4\pi \int_0^a |E|^2 r^3 dr \quad (34)$$

which can be expressed as

$$w^2 = w_0^2 + 4B_{12} \sqrt{P_{01}P_{02}} \cos(\theta_{01} - \theta_{02}) \quad (35)$$

where the initial beam waist radius at the input of the waveguide is

$$w_0^2 = 2B_{11}P_{01} + 2B_{22}P_{02} \quad (36)$$

and B_{11} , B_{22} , and B_{12} are defined for the HE_{11} and HE_{12} mode superposition as

$$B_{11} = 2\pi \int_0^a u_{01}^2 r^3 dr = \frac{\nu_{01}^2 - 2}{3\nu_{01}^2} a^2 = 0.218a^2 \quad (37)$$

$$B_{22} = 2\pi \int_0^a u_{02}^2 r^3 dr = \frac{\nu_{02}^2 - 2}{3\nu_{02}^2} a^2 = 0.311a^2 \quad (38)$$

$$B_{12} = 2\pi \int_0^a u_{01}u_{02}r^3 dr = \frac{-8\nu_{01}\nu_{02}}{(\nu_{01} - \nu_{02})^2} a^2 = -0.17a^2. \quad (39)$$

From (35)–(39), we obtain a ratio of waist radius to radius of the waveguide

$$\frac{w^2}{a^2} = 0.436P_{01} + 0.622P_{02} - 0.696 \sqrt{P_{01}P_{02}} \cos(\theta_{01} - \theta_{02}). \quad (40)$$

The average inverse curvature radius is defined as

$$\frac{1}{R(z)} = \frac{4\pi}{kw^2} \int_0^a r^2 \frac{\partial \Phi}{\partial r} |E|^2 dr \quad (41)$$

where Φ is the phase distribution. From (41), we derive

$$\frac{kw^2}{R} = 2D_{12} \sqrt{P_{01}P_{02}} \sin(\theta_{01} - \theta_{02}) \quad (42)$$

where

$$D_{12} = 2\pi \int_0^a r^2 \left(\frac{du_{01}}{dr} u_{02} - \frac{du_{02}}{dr} u_{01} \right) dr \quad (43)$$

which can be reduced to

$$D_{12} = -\frac{4\nu_{01}\nu_{02}}{\nu_{01}^2 - \nu_{02}^2} = 2.151. \quad (44)$$

The phase front curvature radius R can be expressed as

$$\frac{R}{ka^2} = \frac{0.436P_{01} + 0.622P_{02} - 0.696 \sqrt{P_{01}P_{02}} \cos(\theta_{01} - \theta_{02})}{4.302 \sqrt{P_{01}P_{02}} \sin(\theta_{01} - \theta_{02})}. \quad (45)$$

From (35) and (42), we derive the conservation theorem, which relates the waist size $w(z)$ and phase front curvature radius $R(z)$ at any point in the waveguide to a constant dependent on the power distribution between the two modes in the waveguide

$$\left(\frac{w^2 - w_0^2}{2B_{12}} \right)^2 + \left(\frac{kw^2/R}{D_{12}} \right)^2 = 4P_{01}P_{02}. \quad (46)$$

This relationship shows that the power distribution between the two modes conserves the ratio between the radius of phase front curvature and beam waist radius as they propagate in the waveguide; in the same way, (8) dictates the relationship between the offset and tilt for the LP_{11} mode excitation case.

Equation (46) may be used to derive expressions for the excitation of power in the HE_{12} mode, P_{02} , due to errors in the size of the beam waist, w , or the curvature radius, R , of the Gaussian beam injected into the waveguide. If the injected beam waist, w , differs from the ideal value of $w_0 = 0.66a$ by a small amount

$$\Delta w = w - w_0 \quad (47)$$

then the power excited in the HE_{12} mode calculated from (46) will be

$$P_{02}=1.57(\Delta w/w_0)^2. \quad (48)$$

From (48), we observe that to restrict the excitation of the HE_{12} mode to $P_{02} < 1\%$, the error in the waist size must be $w/w_0 < 0.08$. For a waveguide of diameter $2a = 63.5$ mm at a frequency of 170 GHz, the waist size w_0 is 21 mm and the allowed error in the waist size for less than 1% excitation of the HE_{12} mode is $|w| < 2$ mm (i.e. the waist size must be between 19 and 23 mm).

Equation (46) may also be used to set a limitation on the phase front curvature, R , of the Gaussian beam at the entrance to the transmission line waveguide. Ideally, the phase front at the entrance to the guide, located at $z = 0$, should be a flat phase front, with $1/R = 0$. From (46), the excitation of power in the HE_{12} mode, P_{02} , due to a finite (nonzero) curvature radius R is given by

$$P_{02}=0.40\left(\frac{a^2}{\lambda R}\right)^2. \quad (49)$$

Equation (49) requires that for an excitation of less than 1% of the power in the HE_{12} mode, the phase front curvature radius for the parameters discussed above must be greater than 3.6 m.

We can obtain the following results for the special case of a TEM_{00} Gaussian beam, which is focused to the correct waist size but at the wrong location on the z -axis; that is, the waist is not formed at $z = 0$. The microwave beam will then have a finite curvature radius (positive or negative) at the entrance to the transmission line waveguide. For a Gaussian beam in the TEM_{00} mode, we have

$$w^2(z)=w_0^2\left[1+(z/z_0)^2\right] \quad (50)$$

$$R(z)=z\left[1+(z_0/z)^2\right] \quad (51)$$

with

$$z_0=\pi w_0^2/\lambda. \quad (52)$$

The quantity z_0 is called the Rayleigh range. For a beam waist of $w_0 = 21$ mm at a frequency of 170 GHz, $z_0 = 0.79$ m. Let us assume that the waist is formed at a location in z that is z distant from the ideal location, where $(z/z_0)^2 \ll 1$. [Note that since the ideal location of the waist using (50) is at $z = 0$, z is the same as z]. Then, $1/R(z)$ is given by

$$1/R(z) \approx \Delta z/z_0^2. \quad (53)$$

Let us also assume that the waist w_0 is equal to the ideal value of $0.66a$; however, the results will not depend strongly on this assumption. Equation (49) will simplify to

$$P_{02} \approx 0.21(\Delta z/z_0)^2. \quad (54)$$

The formation of the beam waist at an offset location z also leads to a larger beam waist, w , as given in (51). The larger waist leads to the generation of power in the HE_{12} mode, as given by (48). However, this effect goes as $(z/z_0)^4$, and is negligible compared with the effect given by (54) under our assumption that $(z/z_0)^2 \ll 1$. For parameters discussed above, (54) requires that, for an excitation of less than 1% of the power in the HE_{12} mode, the ideal beam waist must be formed within 17 cm of the entrance to the transmission line.

VI. Analytical Theory of HE_{12} Mode Content Correction

To correct the HE_{12} mode, two ellipsoidal mirror correctors are placed in miter bends, in a system similar to Fig. 1. At the waveguide entrance, the phase difference $(\theta_{01}-\theta_{02})$ is, in general, arbitrary. For convenience, we assign the location z_0 so that $(\theta_{01}-\theta_{02}) = (n + 1/2)\pi$ with $n = 0, 1, 2, \dots$. In that case, the beam has a finite curvature radius, R_0 , but a matched waist radius $w_0 = w_{\text{opt}}$ at the waveguide entrance. The finite radius of curvature at the input primarily excites the HE_{12} mode along with the HE_{11} mode. From (36), we calculate for $P_{01} = 1$ that $w_{\text{opt}} = 0.66a$, in agreement with the optimal Gaussian beam radius when the beam is truncated by the waveguide aperture. Let us note that without considering the truncation of the Gaussian beam at the input to the waveguide, the optimum waist radius is calculated to be $w_{\text{opt}} = 0.645a$ with an infinite phase front curvature radius at the input [14]. These values are in good agreement, and $w_{\text{opt}} = 0.66a$ will be used in this analysis for consistency.

The system can be understood as two lenses in the system that act as phase correctors with focal lengths of $F_{1,2}$ and separated by a distance L_2 . It is possible to correct for a finite initial phase front radius, R_0 , or for a beam radius that differs from the ideal radius w_{opt} . The focal lengths of the correctors $F_{1,2}$ are related to the curvature radii of the ellipsoidal mirror correctors at the miter bends of the transmission line: in the plane of the bend, the mirror curvature radius $R_{\parallel} = 2\sqrt{2}F$, and in the perpendicular plane, $R_{\perp} = \sqrt{2}F$.

We assume that the power in the HE_{12} mode $P_{02} \ll 1$, such that $P_{01}P_{02}$ is negligible, but $\sqrt{P_{01}P_{02}}$ must still be considered. Therefore, (36) reduces to $w_0^2 = 2B_{11}$ and $w_0 = w_{\text{opt}} = 0.66a$.

As follows from (35) and (42), the effective parameters w^2 and kw^2/R before the first corrector at $z = L_1 - 0$ are

$$w^2(z=L_1 - 0) = w_0^2 + 4B_{12} \sqrt{P_{01}P_{02}} \sin(\Delta k_{12}L_1) \quad (55)$$

$$\left. \frac{kw^2}{R} \right|_{z=L_1-0} = 2D_{12} \sqrt{P_{01}P_{02}} \cos(\Delta k_{12}L_1). \quad (56)$$

The first corrector at L_1 with a focal length F_1 changes the phase front curvature radius, R , such that

$$\frac{1}{R(z=L_1+0)} = \frac{1}{R(z=L_1-0)} + \frac{1}{F_1} \quad (57)$$

and keeps the effective radius of the beam unchanged

$$w(z=L_1+0) = w(z=L_1-0). \quad (58)$$

The modal powers change after the beam is transmitted through the first corrector to P'_{01} and P'_{02} (with $P'_{01} + P'_{02} = 1$) such that

$$w^2(z=L_1+0) = w_0^2 + 4B_{12} \sqrt{P'_{01}P'_{02}} \sin \Psi \quad (59)$$

$$\left. \frac{kw^2}{R} \right|_{z=L_1+0} = 2D_{12} \sqrt{P'_{01}P'_{02}} \cos \Psi. \quad (60)$$

Using (57)–(60), the phase difference Ψ between the HE_{12} and HE_{11} modes is found to be

$$\tan \Psi = \frac{\sqrt{P'_{01}P'_{02}} \sin(\Delta k_{12}L_1)}{\sqrt{P'_{01}P'_{02}} [\cos(\Delta k_{12}L_1) + \frac{2kB_{12}}{D_{12}F_1} \sin(\Delta k_{12}L_1)] + \frac{kB_{11}}{D_{12}F_1}}. \quad (61)$$

Thus, using (55)–(61), we determine the modal powers as

$$P'_{01}P'_{02} \simeq P_{01}P_{02} + 2\sqrt{P_{01}P_{02}} \frac{kB_{11}}{D_{12}F_1} \cos(\Delta k_{12}L_1) + \left(\frac{kB_{11}}{D_{12}F_1} \right)^2. \quad (62)$$

Equations (61) and (62) characterize the mode conversion of the HE_{11} and HE_{12} mode mixture after a phase corrector of a focal length F_1 . At $z = L_1 + L_2 - 0$, just before the second corrector

$$w^2(z=L_1+L_2-0) = w_0^2 + 4B_{12} \sqrt{P'_{01}P'_{02}} \times \sin(\Psi + \Delta k_{12}L_2) \quad (63)$$

$$\left. \frac{kw^2}{R} \right|_{z=L_1+L_2-0} = 2D_{12} \sqrt{P'_{01}P'_{02}} \times \cos(\Psi + \Delta k_{12}L_2). \quad (64)$$

At the second corrector, the waist radius w must be matched to the HE_{11} mode of the waveguide. From (63), $\sin(\Psi + \Delta k_{12}L_2) = 0$ meets this condition. Therefore, we impose that

$$\tan \Psi = -\tan(\Delta k_{12}L_2). \quad (65)$$

The necessary focal length from the first corrector is found from (61) and (65), we thus determine the focal length of the first corrector, F_1

$$\frac{1}{F_1} = \frac{-1}{R_0 + \frac{2kB_{12}}{D_{12}} \sin(\Delta k_{12} L_1)} \frac{\sin(\Delta k_{12} [L_1 + L_2])}{\sin(\Delta k_{12} L_2)} \quad (66)$$

where we consider that

$$\frac{kw_0^2}{R_0} = 2D_{12} \sqrt{P_{01}P_{02}}. \quad (67)$$

The second corrector compensates for the curvature radius present before it, $R(z = L_1 + L_2 - 0) = R_{\text{out}}$. Therefore, the focal length F_2 is

$$\frac{1}{F_2} = -\frac{1}{R_{\text{out}}} = \frac{1}{R_0} \frac{\sin(\Delta k_{12} L_1)}{\sin(\Delta k_{12} L_2)}. \quad (68)$$

We observe from (68) that the distance L_2 must not be a multiple of the half-beat-wavelength $\Lambda_{12}/2 = 0.915$ m.

An example of these correctors is shown in Fig. 6 with $a = 31.75$ mm and the frequency of 170 GHz. Fig. 6 shows the normalized effective beam radius w/a and inverse phase front curvature radius ka^2/R as functions of z calculated using this two-mode approach. The input curvature radius $R_0 = 2.1$ m, the waveguide section lengths $L_1 = 1$ and $L_2 = 2$ m. The input power was composed of 97% HE_{11} and 3% HE_{12} . To correct the modes in the waveguide, focal lengths of the correctors were calculated to be $F_1 = 1.6$ and $F_2 = -4$ m. For the ideal calculations, these focal lengths result in an output with 100% HE_{11} mode, a waist radius of $w = 0.66a$, and an infinite phase front radius.

VII. Numerical Theory of HE_{12} Mode Correction

The two-mode analytical approach has been checked using the MMTL code [3]. The MMTL code was used to propagate the initial field distribution in the form of a superposition of the HE_{11} and HE_{12} modes with a phase difference of $\pi/2$ at the entrance to the transmission line. This superposition corresponds to an input with an optimal effective waist radius $w_0 = 0.66a$ and a finite effective phase front curvature radius R_0 for various HE_{12} input powers. In this simulation, $L_1 = 2$ m and $L_2 = 4$ m to match the two-mode analysis. The HE_{11} mode output power as a function of the input HE_{12} mode power is shown in Fig. 7 for two cases: with- and without the correctors. The simulations indicate that even an input HE_{12} mode content of 10% can be greatly reduced with the correctors. For this case, the output consists of 1.3% HE_{12} , 97.3% HE_{11} , 0.8% HE_{13} , and the remaining power is spread among several other HOMs. As opposed to the two-mode analysis, the output power for the MMTL code cannot reach 100% HE_{11} mode content due to the inherent truncation loss and mode conversion at the miter bends.

VIII. Conclusion

Realistically, an overmoded waveguide system has many challenges when being implemented. One of the largest challenges is the coupling of a Gaussian beam into the

transmission line with limited excitation of HOMs. In addition to causing high losses in the system, HOMs can cause problems coupling the power out of the waveguide system, resulting in distorted beams and misdirected radiation.

A perfectly matched Gaussian beam, with $w_0 = w_{\text{opt}} = 0.66a$ and a flat phase front, $1/R_0 = 0$, will have 1% truncation loss when coupled into a waveguide of radius a and 1% loss due to excitation of HOMs. Particularly, the power coupled into the HE_{12} mode is 0.3% and HE_{13} mode is 0.2%. In the experimental implementation, additional coupling errors from a mismatched Gaussian beam contribute to larger quantities of HOMs. The LP_{11} mode is excited due to a tilt or offset of the Gaussian beam at the input; the HE_{12} mode is excited due to waist or phase front radius mismatch between the Gaussian beam and the waveguide aperture. The HE_{12} and HE_{13} modes are excited if the waist radius is larger than w_{opt} , but only the HE_{12} mode is excited if the waist is smaller than w_{opt} . Therefore, when the waist radius mismatch is significant and the HE_{13} and HOMs are excited, the technique proposed in Section IV is less effective. Table I summarizes the types of errors that occur when coupling into the waveguide along with the primary mode that each error excites. The percentage of power that couples into those modes is shown for the two-mode theory in the third column. Table II lists the HE_{11} mode content in the waveguide as calculated by the projection of a Gaussian beam with a mismatch to the set of waveguide modes. The HE_{11} mode loss due to the Gaussian beam tilt and offset are in agreement with [14].

Previously, it was shown that the LP_{11} mode develops due to a tilt and/or offset of the Gaussian beam input. The behavior of the LP_{11} mode with the HE_{11} mode is highly predictable, with a conservation of motion theorem relating the observable tilt and offset of the two-mode beam [1]. This paper has shown how the HE_{11} and LP_{11} mode superposition can be corrected to be primarily the HE_{11} mode using two tilted miter bend mirrors.

An azimuthally symmetric mode, the HE_{12} mode develops due to a Gaussian beam, which is inserted with a mismatched beam waist or radius of phase front curvature. This paper has developed a second conservation of motion theorem for azimuthally symmetric modes, which relates the beam waist size and radius of phase front curvature to a conservation of motion theorem. In addition, this paper has shown that a superposition of HE_{11} and HE_{12} modes can be corrected by using two curved miter bend mirrors to manipulate the phase front curvatures.

In addition to the correction of HOMs in the system, the technique for mode correction discussed in this paper can be used to systematically induce HOMs at the output of the waveguide system. This technique could be used to implement remote steering, where the beam can be directed through the systematic introduction of small amounts of HOMs. In this paper, we have only treated miter bend mirrors of simple shape, considering only tilted flat mirrors or ellipsoidal mirrors. In a real application, mirrors with a more general phase correction to treat an arbitrary mixture of HOMs might have to be considered. However, in many cases, where only the LP_{11} and/or HE_{12} modes are excited, the present approach may be expected to prove successful.

Acknowledgments

This work was supported in part by the U.S. ITER Project managed by Battelle/Oak Ridge National Laboratory and in part by the NIH and NIBIB under Contract R01-EB004866 and Contract R01-EB001965.

References

1. Kowalski EJ, Tax DS, Shapiro MA, Sirigiri JR, Temkin RJ, Bigelow TS, et al. Linearly polarized modes of a corrugated metallic waveguide. *IEEE Trans. Microw. Theory Tech.* 2010 Nov.58(11): 2772–2780.
2. Doane JL. Design of circular corrugated waveguides to transmit millimeter waves at ITER. *Fusion Sci. Tech.* 2008 Jan.53:159–173.
3. Shapiro MA, Kowalski EJ, Sirigiri JR, Tax DS, Temkin RJ, Bigelow TS, et al. Loss estimate for ITER ECH transmission line including multimode propagation. *Fusion Sci. Tech.* 2010 Apr. 57:196–207.
4. Shimozuma T, Kubo S, Yoshimura Y, Igami H, Takahashi H, Takita Y, et al. Handling technology of mega-watt millimeter-waves for optimized heating of fusion plasmas. *J. Microw. Power Electromagn. Energy.* 2009; 43(1):60–70. [PubMed: 21384725]
5. Lohr J, Cengher M, Doane JL, Ellis RA, Gorelov YA, Kolemen E, et al. Performance, diagnostics, controls and plans for the gyrotron system on the DIII-D tokamak. *Proc. EPJ Web Conf.* 2012; 32:02009–02015.
6. Gandini F, Bigelow TS, Becket B, Caughman JB, Cox D, Darbos C, et al. The EC H&CD transmission line for ITER. *Fusion Sci., Tech.* 2011; 59(4):709–717.
7. Takahashi K, Kajiura K, Oda Y, Kasugai A, Kobayashi N, Sakamoto K, et al. High power millimeter wave experiment of ITER relevant electron cyclotron heating and current drive system. *Rev. Sci. Instrum.* 2011; 82(6):063506-1–063506-2. [PubMed: 21721690]
8. Lohr J, Cengher M, Doane JL. The multiple Gyrotron system on the DIII-D tokamak. *J. Infr. Millim. Terahz Waves.* 2011; 32(3):253–273.
9. Oda Y, Kajiura K, Takahashi K, Kasugai A, Shapiro MA, Temkin RJ, et al. Measurement of RF transmission mode in ITER relevant EC H&CD transmission line. *J. Infr. Millim. Terahz Waves.* 2010; 31(8):949–957.
10. Idei H, Shapiro M, Temkin RJ, Shimozuma T, Kubo S. Mode content analysis in circular corrugated waveguide using radiated field. *Proc. Joint 32nd Int. Conf. IR MM Waves 15th Int. Conf. Terahertz Electron.* 2007; 1–2:69–70.
11. Jawa SK, Nanni EA, Shapiro MA, Woskov PP, Temkin RJ. Mode content determination of terahertz corrugated waveguides using experimentally measured radiated field pattern. *IEEE Trans. Plasma Sci.* 2012 Jun.40(6):1530–1537.
12. Katsenelenbaum BZ. To theory of quasioptical elements in wide waveguides. *Radiotekh. Elektron.* 1971 Jan.16:1797–1804.
13. Vaganov RB. Trajectory of energy center of wave beam. *Radiotekh. Elektron.* 1970 Jan.15:2016–2023.
14. Ohkubo K, Kubo S, Idei H, Sato M, Shimozuma T, Takita Y. Coupling of tilting Gaussian beam with hybrid mode in the corrugated waveguide. *Int. J. Infr. Millim. Waves.* 1997; 18(1):23–41.
15. Thumm MK, Kasparek W. Passive high-power microwave components. *IEEE Trans. Plasma Sci.* 2002 Jun.30(3):755–789.
16. Doane JL, Moeller CP. HE_{11} mitre bends and gaps in a circular corrugated waveguide. *Int. J. Electron.* 1994; 77(4):489–509.
17. Siegman AE. Defining the effective radius of curvature for a nonideal optical beam. *IEEE J. Quantum Electr.* 1991 May; 27(5):1146–1148.

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Fig. 1. Schematic of compensation of the input Gaussian beam tilt through additional tilts in the two miter bend mirrors.

Fig. 2.
Example of correctors for the case of the ITER transmission line with a tilt of 0.34° at the entrance.

Fig. 3.

Result of multimode analysis of the mode content correction. Output HE₁₁ mode content versus input LP₁₁ mode content without and with correction.

Fig. 4. Normalized beam radius (w/a) and inverse curvature radius of the phase front (ka^2/R) as a function of distance in the waveguide, z , for a mode superposition of 95% HE_{11} and 5% HE_{12} , at 170 GHz with $a = 31.75$ mm.

Fig. 5. Illustration of the beam radius and phase front curvature as the HE_{11} and HE_{12} mode mixture propagates through the waveguide.

Fig. 6.

Example of correctors for the case of the ITER transmission line with input power composed of 97% HE_{11} and 3% HE_{12} . (a) Beam effective radius and (b) inverse phase front curvature radius through the transmission line system with two mirror correctors.

Fig. 7.

As calculated with MMTL code, the output HE11 mode content as a function of the input HE12 mode content without and with miter bend correctors.

TABLE I

HOM Generation Due to Waveguide Input Coupling Errors With a Two-Mode Calculation

Mismatch	HOM	$P_{\text{HOM(two-mode)}}$
Tilt, α_x	LP ₁₁	$4.6 (a\alpha_x / \lambda)^2$
Offset, x_0	LP ₁₁	$2.3 (x_0/a)^2$
Waist, w	HE ₁₂	$3.67 (w/a)^2$
Curvature, R	HE ₁₂	$0.4 [a^2/(\lambda R)]^2$

TABLE II

HE₁₁ Power Due to Waveguide Input Coupling Errors With a Gaussian Beam Insertion

Mismatch	HE ₁₁ Power, P_{01}
Tilt, α_x	$0.99 - 3.9 (a\alpha_x/\lambda)^2$
Offset, x_0	$0.99 - 2.3(x_0/a)^2$
Waist, w	$0.99 - 2 (w/a)^2 + 8 (w/a)^3$
Curvature, R	$0.99 - 0.3 (a^2/[\lambda R])^2$