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Multi-Echo Acquisition of O-Space Data

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Abstract

Purpose—Nonlinear gradient encoding methods, such as O-Space imaging, have been shown to provide good images from very few echoes. Acquiring data in a train of spin or gradient echoes is a very flexible way to further speed acquisition time. However, combining these strategies presents significant challenges, both in terms of the contrast and artifacts. We present strategies in both pulse sequence design and image processing to mitigate these effects.

Theory and Methods—The three strategies include a new echo ordering that takes into account the unique way that O-Space samples the k-space of the image; a new post-processing filter that allows tuning of T2-weighting by emphasizing the contribution of low frequency spatial information at selectable points in space and time; and an offset between linear and nonlinear gradients that makes the central point of each echo unique.

Results—Simulations indicate that these strategies mitigate artifacts introduced by T2 (or T2*) decay and produce contrast that reflects relaxation at a given TE.

Conclusion—Turbo spin echo O-Space is theoretically feasible, and the greater undersampling should make it possible to use shorter echo trains for a given scan time.

Keywords

O-Space; turbo spin echo; fast spin echo; RARE; EPI; parallel imaging; accelerated imaging

Introduction

In a standard turbo spin echo acquisition, the T2 decay imposes a magnitude weighting (approximately $e^{-TE/T2}$) on each line of k-space depending on the TE at which that line of data was acquired. (1–3) By simply interleaving the phase encoding order, these coefficients can be made to resemble a smooth exponential decay in the k-space of the object. The Fourier transform of a decaying exponential function is the well known Lorentzian shape, and thus multiplying the data by this decay function is equivalent to convolving the image with the Lorentzian which is the source of the blurring in such sequences.

The intensity of objects in the image depends on the value of $e^{-TE/T2}$ for the TE at which we acquire the low frequency spatial information. Thus, the relative intensity of objects, and

consequently the contrast, can be straightforwardly controlled by choosing the echo time at which we collect those central lines of k-space.

These concepts do not easily translate to non-Cartesian trajectories, such as radial or spiral imaging, and there are still further complications for data acquired with nonlinear read out gradients, as in O-Space imaging. (4–7) In both non-Cartesian and O-Space acquisitions, a multi-echo sequence collects the center of k-space with each echo and thus with variable T2 weightings. In addition, with O-Space imaging, every echo and even every timepoint contains some low frequency spatial information.

O-Space imaging adds an additional radially varying gradient, like that shown in Figure 1a, to the acquisition readout. (8,9) Each echo is acquired under an off-center radially varying field like that shown in Figure 1b. Thus, the Fourier transform of this echo, shown below, indicates the spin density along each ring of isochromats. More specifically the field is $B_z = z^2 - 1/2((x-x_i)^2 + (y-y_i)^2)$, and it has a relatively flat region surrounded by an increasingly steep field gradient. For a full dataset, echoes are collected for a series of (x_i, y_i) (like those shown in panels b–e of Figure 1) that trace out a circle in the field of view.

Because of the spatially varying steepness of the field, different parts of the field of view experience different gradients, and these then change from echo to echo. Thus each spatial region has its own local k-space function and the low frequency information is collected at different echo times (10). This can easily lead to disjoint magnitude weightings in k-space, and the conflicting values create both artifacts and ill-defined T2 contrast unless other solutions are implemented. (4)

For example, the center point of every O-Space echo refocuses the signal and thus collects the exact center of k-space. In a train of echoes, each middle timepoint will be weighted by a sum of exponentials evaluated at the appropriate TE. (As a function of TE, the coefficient is $\alpha(TE) = \sum \rho_i e^{-TE/T2_i}$, where ρ_i is the spin density in the field of view of the i^{th} spin species with decay constant $T2_i$.) O-Space is reconstructed via Algebraic Reconstruction Techniques that seek consistency between three aspects of the acquisition: the encoding functions, the estimated object, and the measured data. (8,11–13) However, the data will appear fundamentally inconsistent when the same encoding (e.g., $\mathbf{k}=0$) produces conflicting data points (because of different TEs), and this creates artifacts in the reconstruction.

For timepoints other than the center of each echo, the timepoints in an O-Space acquisition contain multiple Fourier components (Figure 1f) and so do not represent individual points in k-space. Instead, each timepoint is a weighted sum of several different k-space points. As described in reference (14), each O-Space timepoint collects a 2D patch of k-space, also known as a stamp. More specifically, data acquired with center placements like those shown in Figure 1g roughly correspond to collecting the color-matched k-space regions shown in Figure 1h. Though this schematic does not represent actual k-space coverage, which is far more overlapping, complex and difficult to capture in a single image, this simplified picture is sufficient to motivate the observed T2 artifacts as well as their solutions. Readers interested in more detailed analysis of the k-space coverage of O-Space may consult Reference [14], especially Figures 8–10 and their accompanying text.

Just as traditional trajectories need to create smooth T2 weightings for adjacent k-space points (1), the O-Space trajectory must create smooth weighting between adjacent and overlapping k-space stamps. Especially at the center of k-space, center placements acquired with different T2 weightings create a highly discontinuous weighting function. This leads to significant artifacts in the final reconstruction.

Smooth T2 weighting is important not only at the center but also at the edges of k-space, where adjacent center placements acquire overlapping stamps. In addition, center placements opposite each other (e.g., the dashed and red center placements in 1g) sample the same regions of k-space but with different phase. Thus, those echoes should also be acquired with similar T2 weightings to minimize data inconsistency.

Finally, a fundamental issue unique to O-Space is that each echo contains a mix of both high and low spatial frequencies. Consider a timepoint like that shown in Figure 1f collected at the edge of echo. Though the encoding applied at this time primarily contains high spatial frequencies, the region near the center placement contains low frequency intensity/contrast information. Collecting this kind of information over a range of T2 weightings can again lead to nonspecific T2 contrast as well as artifacts.

Despite these challenges, multi-echo O-Space would combine a very efficient encoding method that needs less data than Cartesian imaging with an acquisition that obtains data very quickly, potentially enabling very fast acquisitions. As shown in Ref [8], undersampled O-Space images are expected to fare far better than undersampled SENSE, as demonstrated in Ref [9], and produce less structured artifacts than radial trajectories. The advantages over undersampled radial can be seen in Figure 2. Because artifacts are more distributed, certain features of a lesion phantom that become unidentifiable under highly accelerated radial acquisition remain visible in O-Space images. Note that for this 8-channel acquisition, 8 and 16-fold undersampled Cartesian images would be either dominated by noise amplification or not definable. Thus, since O-Space can reconstruct images from very few total echoes, some of the challenges of implementing a TSE acquisition are actually made more manageable.

Here we report three modifications that together mitigate the difficulties of multi-echo O-Space imaging. We present a modified trajectory that generates relatively smooth weightings in the k-space of the reconstructed object. We also present a new weighting method that can be used to selectively suppress low frequency regions of the image, except those acquired at or near the desired TE. And finally, we introduce a small modification to echo centering that controls our collection of the center of k-space.

Theory

Smoother Trajectory

Normally, the center placements in an O-Space trajectory are simply acquired in order of increasing θ , i.e., sequential ordering. This is illustrated in panels 3a and 3c for a two shot, 48 center placement acquisition. However, this sequential ordering leads to conflicting TE weightings on neighboring center placements, which cover overlapping samples of k-space. (Figure 1g–h) A better acquisition order would create a smooth TE weighting for all

neighboring center placements. In addition, the ideal acquisition order would also create smoothly varying weightings between opposite center placements, since these too address the same regions of k-space.

To address both these requirements, we propose an acquisition order like that shown in Figure 3b and 3d. For an acquisition divided over n TRs, this order can be found by identifying n anchor center placements spaced $360/n$ degrees apart. We refer to these anchor center placements as θ_i , and the separation between nearest neighbor center placements (360 divided by the total number of echoes) will be referred to as θ . The first echo train collects $\{\theta_1; \theta_1 + \theta; \theta_1 - \theta; \theta_1 + 2\theta; \theta_1 - 2\theta; \dots\}$. In this way the collected center placement moves back and forth from the left to the right side of the first anchor, θ_1 , until all (total number of echoes)/ n echoes in the train have been collected. The next TR begins at θ_2 , where θ_2 is the next anchor point $360/n$ degrees away, but otherwise it follows the same pattern.

This pattern generalizes to any even number of TRs greater than one, but acquiring an entire dataset in a single TR is a special case that requires a small modification. For a single echo sequence, the equivalent weighting pattern would be achieved by interleaving the acquisition order prescribed for a two-echo sequence. In other words, the order would be: $\{\theta_1; \theta_2; \theta_1 + \theta; \theta_2 + \theta; \theta_1 - \theta; \theta_2 - \theta; \theta_1 + 2\theta; \theta_2 + 2\theta; \dots\}$. For an odd number of TRs greater than one, this scheme would create conflicting TEs in opposite center placements, so the formula is designed to use an even number of shots.

Modified Reconstruction

The next issue is that “high frequency” datapoints, that is, those datapoints collected at the edge of the acquisition window, still contain low frequency intensity information about some region of the image. We address this by applying a filter in the Kaczmarz reconstruction algorithm. This approach does not filter out experimental data, and thus should not significantly lower signal to noise ratio. Rather, it uses a filtered basis to reconstruct the image, while using the original basis functions to encode the image.

In a Kaczmarz reconstruction, each datapoint corresponds to an encoding, and each of those encodings serves as a basis function used to reconstruct the image. (15–18) For each of these encodings, the algorithm performs the following two steps. First, using the previous estimate of the image and the corresponding experimental datapoint, the algorithm estimates how much¹ that basis function is believed to contribute to the final image. This answer is captured in the β coefficient for that i^{th} encoding function, calculated by the formula:

$$\beta_i = \text{Data}_i - A_i - \text{Image}_{i-1} \quad [1]$$

Here A_i is the i^{th} encoding function, i.e., a normalized row of the encoding matrix, Data_i is the experimental value associated with that encoding, and Image_{i-1} is the previous image estimate. When a projection of our image onto the i^{th} basis function yields a coefficient that

¹To be precise, the algorithm calculates how much *more* the basis function contributes to the final image, taking into account the extent to which that basis function was already represented in the previous estimate of the image, either from other encodings or from previous iterations.

is congruent with our data, β_i approaches zero, and our estimate of the image converges to a final and relatively stable answer.

Otherwise, β_i will be a non-negligible number, and the image will be significantly modified in the update step, which is:

$$\text{Image}_i(x, y) = \text{Image}_{i-1}(x, y) + \lambda \beta_i * A_i(x, y) \quad [2]$$

The procedure is then repeated for every datapoint/encoding pair, which iteratively minimizes $\|A * \text{Image} - \text{Data}\|$. Cycling through the entire dataset multiple times can further refine the estimate. In the above equation, λ is simply the Kaczmarz relaxation parameter that controls the convergence properties of the algorithm and is typically set for strongly under-relaxed reconstructions for gradual convergence.

Our goal is to modify the reconstruction to suppress low spatial frequency information anywhere within the image except when the low frequency information for that region is acquired near the desired TE. This is accomplished by modifying the basis functions used to reconstruct the image. More specifically, we mask out and dampen the less modulated regions (i.e. the less modulated region of Figure 1f) of the basis function.

In our multi-echo reconstruction, we calculate the contribution of each encoding function (β_i) in the usual way. However, rather than updating the image with the full encoding function, we use a modified version of our encoding function in the update step. In essence, we make λ a function of x & y , as illustrated in Figure 4. We can further control the timing at which low frequency information contributes to the final image by making λ a function of x, y , and also $\mathbf{k}=(k_x, k_y, k_z)$ and time. At the target echo time, we can allow less modulated regions of the encoding function (i.e. analogs to the central region of k -space) to contribute so that the low frequency components of the image are weighted for that echo time.

Center Echoing

Finally, we have also modified the echo centering to avoid collecting conflicting readings for the $\mathbf{k}=(0,0,0)$ point of k -space. In this strategy, the linear gradients always refocus at the center of the echo, but the nonlinear gradient refocuses at a different time, i.e. the time marked by the dashed line in Figure 5. Thus, the center of the first echo collects an encoding that corresponds to pure nonlinear spin encoding, rather than an unmodulated encoding. We then progressively reduce the offset until $\text{TE}=\text{target TE}$. At that time, the refocusing of the linear gradients coincides with the refocusing of the nonlinear gradient, and a fully refocused $\mathbf{k}=(0,0,0)$ datapoint is collected. For subsequent echoes, the offset moves in the opposite direction, now producing pure nonlinear spin encoding of opposite phase at the center of each echo.

Methods

Data was simulated for a 48 echo 8-channel O-Space acquisition acquired in a two shot TSE pulse sequence (echo train length =24) and used to reconstruct a 128^2 matrix. All encoding

functions were simulated on a 512^2 grid before each 4×4 block is summed to create a 128^2 resolution for reconstruction. All simulations were also performed simulating a 5% noise level unless otherwise noted. We investigated the effect of artifacts by performing simulations on brain phantoms. For these images, a global T2 decay was simulated both within the echo ($.005 \times T2$) and between echoes (5%T2, 7%T2 and 10%T2). Another way to interpret 5%T2, 7%T2, and 10%T2 is to regard these as echo spacing of 5, 7, and 10ms, respectively, for spins with a T2 of 100ms. However, it is only the ratio of TE and T2 that defines signal loss, as well as the ensuing artifacts and contrast. Thus, it would be equally valid to regard these results as representing echo spacings of 2.5, 3.5, and 5ms for spins with a T2 of 50ms. To emphasize this generality of the simulations, we report our results in these more general terms.

Contrast sensitivity was interrogated by performing reconstructions of a 4 compartment phantom with differing T2 that corresponded to 0.6, 0.9, 1.1 and 1.4 times the brain T2. If we assume the brain images are simulated with a T2 of 100ms and the aforementioned echo spacings, our contrast phantom would then correspond to solutions with T2s of 60, 90, 110 and 140ms. But again, it is only the ratios of T2 and TE that set relative brightness and contrast. All calculations were performed in Matlab, and reconstruction was done with a Kaczmarz algorithm.

Cartesian images were simulated using the same procedure, with encoding functions simulated on a 512^2 grid, and then summed across each 4×4 block to generate a 128^2 matrix for reconstruction. TE weighting was based on an interleaved acquisition, with the most central k-space line in the train being acquired at the middle echo ($N/2$). As with the nonlinear case, these images were reconstructed via a Kaczmarz algorithm. For an overall comparison, we simulated both Cartesian and O-Space images for 32 echoes collected over 2 shots, as well as for a fully sampled Cartesian image with 128 echoes collected over 2 shots.

We present simulation results for two reconstruction filters used to suppress the low frequency region of certain basis functions: a static filter and a time-varying filter. Both are based on a Gaussian shaped suppression with origin at the center placement, and we defined our spatially varying lambda as:

$$\lambda(x, y, \text{Echo}) = 1 - e^{-(|r - \mathbf{CP}(\text{Echo})|)^2 / \sigma_0^2} \quad [3]$$

where $\mathbf{r} = (x, y)$, $\mathbf{CP}(\text{Echo})$ gives the coordinates of the gradient center placement for that echo, and σ_0 is a parameter used to control the width of the filter. For the static filter, we set σ_0 to a constant value of 36 pixels. This suppresses encoding around a fixed radius surrounding each center placement.

For our time-varying filter, we use $\lambda(x, y, \text{Echo})$ as defined in Equation 3 but replace σ_0 with the following expression.

$$\sigma(t) = \sigma_0 * \frac{\sqrt{k_{x,max}^2 + k_{y,max}^2 + \left(\frac{FOV}{2}\right) k_{z2,max}^2}}{\sqrt{k_x^2(t) + k_y^2(t) + \left(\frac{FOV}{2}\right) k_{z2}^2(t)}} \quad [4]$$

except when t falls within our target echo (target for generating the appropriate T2 contrast), in which case $\sigma(t)$ defaults to an infinitesimally small number. In this case, the exponential term in Eqn 3 approaches $e^{-\infty}$, i.e. zero, so λ is one everywhere and nothing is suppressed. For non-target echoes, when the spins are not very modulated (i.e. near the echo peak), $\sigma(t)$ is large, λ is close to zero over a large area, and the filter suppresses a relatively large circle around the center placement. In fact at the exact center of this echo, the standard deviation goes to infinity and the entire FOV is suppressed. At the very edge of the echo, $\sigma(t)$ approaches its smallest value of σ_0 (36 pixels in our simulations), and a modest radius around the center placement is suppressed.

As we approach maximum spin modulation (i.e. near the beginning or end of each echo), the filter shrinks to its initial value. This suppresses encoding information around a relatively small circle around the center placement. At all times except the target echo, this scheme suppresses the unmodulated part of the encoding function, the part associated with low frequency k-space and potentially conflicting data that doesn't have the correct T2 weighting. Filter weighted images were also simulated at various filter widths (3.6, 36, and 360) and noise levels (5, 10, 20, and 40%) to study the impact of the filter on SNR.

A k_{z2} offset was simulated by shifting echoes by 4 points per echo. For a 24 echo train, this amounted to the first echo arriving 44 points early, the second arriving 40 points early, and the last echo arriving 48 points late.

Results

The solutions presented in the theory section address different aspects of the problems with naïve multiecho O-Space. Echo ordering most improves continuity at the high spatial frequencies, but rapid (if smooth) changes remain near the center of k-space. Our reconstruction filter ensures the correct T2-weighting for the lower spatial frequencies, and the gradient offset reduces redundancy in our data by not repeatedly collecting the same point of k-space, allowing us to keep more encoding while maintaining consistency. Thus, we examine our proposed modifications both individually and also in tandem with the other modifications.

We first compare the effect of reordering the echoes to produce a relatively smooth weighting in k-space. The first row of Figure 6 shows reconstructions simulated for sequential echo ordering, i.e. the echo ordering described in Figure 3a. The second row follows the echo ordering described in Figure 3b. As can be seen, the smoother trajectory does reduce the prevalence of artifacts, though the intensity features in these images are still obscured, especially at the larger echo spacings.

Next we look at the effect of the spatial filter described above. In the first row of Figure 7, we apply only a time-independent filter (using the unmodified trajectory), while the second

row shows the use of the time varying filter described above. The time varied filter greatly improves the intensities in the image, though significant higher frequency artifacts remain until we combine both the filter and the smoother trajectory. Those reconstructions are shown in the third row of Figure 7, and the improvement is especially notable at larger echo spacings.

Since the filtering appears to ignore some encoding, it might be expected to reduce SNR even though it uses the entire dataset. While a full analysis akin to a g-factor map would be an ideal way to characterize the SNR, such maps are far more difficult to calculate for nonlinear projection encoding methods. G-factor maps assume constant and uniform spatial resolution, and with nonlinear gradients for spatial encoding, the spatial resolution is variable across the field of view. A g-factor equivalent measure for nonlinear imaging has not yet been developed, and such analyses are therefore beyond the scope of this paper. Instead, we qualitatively characterize the SNR effects by applying filters of different widths (Gaussians with a σ_0 of 3.6, 36, and 360 pixels) to data simulated without any confounding T2 decay. Those images (Figure 8) show that at reasonable filter widths, these small perturbations to the information added by each encoding function do not noticeably impact the SNR.

At very large filter widths, when the basis functions used for reconstruction become dramatically different from those used for encoding, image quality does begin to suffer, possibly due to difficulties with the algorithm converging. The primary impact of this filter is that certain frequency components are not fully represented across the entire field of view, leading to slightly greater artifacts in the impulse response function, shown in the second row. However, these are still preferable to the gross artifacts caused by conflicting data in the presence of T2 weighting.

To separate noise from noise-like artifacts, we also generated images with each filter width at noise percentages of 0, 5, 10, 20, and 40. Using a difference image between noisy and noise-free images at each filter width, we calculated the standard deviation of the remaining difference image, and this was the denominator dividing the mean signal in the noisy image. In both cases we restricted our metric to the coil encoded region of the image. A plot of that metric is also shown in Figure 8, and shows that filtering out certain frequency components actually provides some denoising, though at the price of greater artifacts.

Finally, we consider the effect of adding an offset between the refocusing time of the linear and nonlinear gradients. The first row of Figure 9 shows images simulated with a smooth trajectory and a time varied filter, but no k_{z2} offset. The first two images in the top row show that O-space can lead to subtle ring-like artifacts even in the absence of T2 decay, but the contrast is more pronounced in the presence of T2 decay. The second row is reconstructed with a k_{z2} offset of 4 points per echo. Adding the offset improves reconstructions in both cases, though the improvement is more salient in T2 weighted data. To quantify this artifact, we calculated the mean square error of these images, masking out the high intensity regions that would otherwise dominate this metric, and those are shown above each reconstruction.

In summary, Figure 10 shows our simulations of the cumulative effect of the presented modifications alongside some comparable Cartesian reconstructions of the brain phantom, the T2 phantom, and a single point phantom. At an equal number of echoes (2 shots each with 16 echoes each of duration $.07 \cdot T_2$ and acquisition with 8 coils), Cartesian shows fewer multi-echo artifacts, but it results in significant unfolding artifacts. These appear as large intersecting arcs at the center of the brain and T2 phantoms, and they appear as the notable unfolding artifact in the single point phantom. The next column shows a two-shot fully sampled Cartesian trajectory using the same echo spacing. In this case, unfolding artifacts are eliminated, but we see a significantly reduced SNR and greater contrast due to the longer TE at which the center of k-space is collected. The image of a single point also makes clear that this image suffers from significant T2 blurring in the phase encode direction.

Discussion

The comparison with Cartesian multi-echo acquisition highlights that while O-Space brings inherent challenges for a multi-echo implementation, it also has an important advantage: less echoes need to be collected per TR. Compared to Cartesian acquisitions with equal repetitions, O-Space imaging has fewer unfolding artifacts, and compared to a fully sampled acquisition, O-Space images have fewer TSE artifacts. By more judiciously planning the acquisition order, adding a filter to the reconstruction, and making a slight adjustment to the readout dephaser area, our simulations suggest we can reduce the O-Space T2 artifacts and restore contrast.

While the reconstruction filter seems to ignore some of data and might be expected to reduce SNR, at appropriate widths it primarily changes the convergence properties of the reconstruction. If the filter is absent, convergence is thwarted by conflicting readings on the low frequency parts of k-space, which creates artifacts. Heavily filtering images seems able to provide some denoising, but it can also create a sizable mismatch between the functions we use for encoding and those we use for reconstruction. This can put convergence at risk and can also reduce the frequency components available to reconstruct our image. The ideal filter introduces small perturbations and primarily affects a very densely sampled region of k-space, so neither of these effects markedly affect image quality but T2 artifacts are reduced.

Finally, though this analysis has focused on T2 decay and spin echo trains, each of these ideas can apply to a gradient echo sequence where T_2^* decay is the concern. In practice, though T_2^* is shorter, echo spacings can also typically be shorter, so the ratio of echo spacings and decay rates is still on the order of those studied here. (19) Gradient echo acquisitions do have some additional complications, but it is notable that one of these is the effect of off-resonance spins in the FOV. For this problem, preliminary data has shown that a reconstruction filter similar to the one described above may help, and studies of this effect are in progress. (20) The filter may also make it possible to reconstruct a single dataset at multiple target TEs, a topic of current investigation.

Conclusion

Like radial trajectories, each echo of the O-Space trajectory goes through the center of k-space, thus TSE acquisitions with nonlinear gradients present two problems. Data with very different T2 weightings crowd together in the center of k-space, which can create artifacts and a range of contrast. O-Space imaging, like all nonlinear gradient encoding, is further complicated by the unique feature that most time points contain a mixture of both high and low frequency information. Our simulations show that the strategies presented here improve both the artifacts and the relaxation based contrast observed in TSE O-Space acquisitions. They show it is theoretically possible to merge this important class of sequences with O-Space encoding. Though a multi-echo acquisition of O-Space requires several modifications, the resulting sequence has the advantage of requiring fewer echoes per TR than a Cartesian acquisition. The resulting images lack the prominent undersampling artifacts of undersampled Cartesian as well as the pronounced T2 blurring that comes from a long echo train length.

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Figure 1. O-Space Imaging with T2 Weighting

O-Space imaging is performed by adding a radially varying field such as the one shown in panel (a). By playing this field along with linear gradients, an echo is collected for a field with isocontours like those shown in panel (b). The Fourier transform of that echo (shown below a–e) tells us how many spins exist on each isocontour of the ring shaped field lines. On subsequent echoes, different linear gradients provide projections onto geometrically different isocontours, and this is repeated until the center of the field has moved through a full circle of placements. Encodings at each timepoint of an O-Space echo contain both high and low frequency encoding, as illustrated in panel (f). In k-space, this corresponds to sampling a swatch of k-space with each timepoint. Overlaying the swatches sampled by all the timepoint in an echo, we can look at what parts of k-space are sampled by different echoes. The echoes corresponding to center placements shown in panel (g) roughly correspond to the color coded k-space sampling shown in panel (h). The dashed center placement samples the same region of k-space as the dark red center placement.

Figure 2. Experimental Comparison of O-Space and Radial

Using a lesion phantom, there is seemingly similar performance between undersampled O-Space images (top row) and radial images (bottom row). However, at very high acceleration factors, various features of the phantom become more blurred in the radial images, as highlighted by zooming into the green region, as shown in the last columns for accelerations of 8 and 16. Note acceleration factors are calculated as the number of echoes used for the image divided by the number of echoes needed for a fully sampled Cartesian image.

(Experimental parameters: 3T Trio, 8-channel acquisition coil, 128^2 matrix, TR=500ms, TE=9ms, 3mm slice thickness, flip angle 15°)

Figure 3. Echo Ordering in T2w O-Space

(a) O-Space normally acquires center placements in order of increasing angle. But translating this order directly into a multi-echo acquisition produces conflicting data. (c) Both neighboring and opposite center placements address similar regions of k-space, so collecting these at very different echo times can lead to artifacts. The proposed trajectory (b) produces smooth TE weighting between adjacent and also opposite center placements (d).

$$\text{Image}_i = \text{Image}_{i-1} + \lambda \beta_i \underbrace{A_i}_{\text{Encoding Function}}$$

$$\text{Image}_i = \text{Image}_{i-1} + \beta_i \underbrace{\lambda(x,y,t)A_i}_{\text{Weighted Encoding Function}}$$

Figure 4. Filtered Reconstruction To Suppress Low Frequency Encoding

Each step of the Kaczmarz algorithm updates the current image estimate by adding in an appropriate amount of the encoding function related to a certain datapoint. In our modification, shown below, we use the β coefficient. But the encoding function we add in is weighted to suppress the low frequency parts of the function. This weighting can be a constant for all data taken at that center placement, it can evolve as a function of the winding, and/or it can be dropped for target echoes.

Figure 5. Progressive Echo Centering

By progressively offsetting the refocusing time between the linear and nonlinear gradients, we collect the fully refocused signal only at the target TE.

Figure 6. Smoother Echo Ordering Reduces High Frequency Artifacts

Using a 48 echo trajectory that leads to a smoother weighting in k-space, especially in the outer region of k-space, reduces some of the T2 artifacts in a multi-echo acquisition. In our simulations, high frequency features of the image are significantly improved in the lower row, but low frequency intensity features are still quite distorted, most noticeably at the largest ratio of TE/T2.

Figure 7. Filtered Reconstruction Reduces Low Frequency Artifacts

Filtering out the region where the gradient is flat on each echo somewhat improves artifacts (compare to Figure 6), especially with small echo spacings. But a much greater improvement is realized by allowing the area suppressed by the filter to grow as each echo approaches the center of k-space (except for the echo at the target TE). Combining this with a smooth trajectory further reduces high frequency artifacts, especially at larger echo spacings. Note each column is shown at the same grayscale.

Figure 8. SNR and PSF Effects of Reconstruction Filter

To better characterize the properties of the reconstruction filter, we applied the time varying filter to data simulated without any T2 weighting. In the top row, we show reconstructions where the standard deviation of the gaussian suppression reaches a minimum of 3.6, 36, and 360. In each case the width of the gaussian grows to infinity at the center of each echo, excepting at the target echo. Each row is shown at a uniform grayscale. When a very large area is suppressed, the overall signal intensity does suffer, but smaller values seem to correct T2 artifacts while keeping enough frequencies to reconstruct the image. The single point images shown in the second row do not seem to be drastically broadened for small and modest filter widths. Since this method does not discard any data, it is not expected to significantly impact SNR, and filtering out frequency components of the basis functions even seems to have a denoising effect, as shown in the plot to the right.

Figure 9. Gradient Timing Offset Reduces Subtle Ring-like Artifacts

Adding an offset between the refocusing time of the z2 and that of the linear gradients allows us to further suppress low frequency information that comes in at TEs other than the target TE. This allows us to keep more of the encoding from the middle of each echo while still applying the reconstruction filter. This results in a slight reduction in the ring-like artifacts (see online for best contrast) visible at the edge of each image. To quantify this artifact, we calculated the mean square error for each image, masking out the very high intensity artifacts that would otherwise dominate this metric, and this is displayed above each image.

Figure 10. Overall Comparison

Though a naïve implementation of multi-echo O-space has serious artifacts, the presented modifications restore image quality while maintaining T2 contrast. Our simulations show that compared to a Cartesian acquisition with the same number of echoes, O-Space shows less undersampling artifact. We also show a comparison to an acquisition with the same number of TRs but a full complement of echoes. In this case, the longer echo train leads to stronger T2 decay and significant blurring, most notably seen in the single point images. (Note $2 \times 16 = 2 \text{ shots} \times 16 \text{ echoes per shot}$, likewise for 2×64).