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Variation in Timing, Duration, Intensity, and Direction of Adolescent Growth in the Mandible, Maxilla, and Cranial Base: the Fels Longitudinal Study

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Abstract

There is considerable individual variation in the timing, duration, and intensity of growth that occurs in the craniofacial complex during childhood and adolescence. The purpose of this paper is to describe the extent of this variation between traits and between individuals within the Fels Longitudinal Study. Polynomial multilevel models were used to estimate the ages of onset, peak velocity, and cessation of adolescent growth, the time between these ages, the amount of growth between these ages, and peak velocity. This was done at both the group and individual levels for standard cephalometric measurements of the lengths of the mandible, maxilla, and cranial base, the gonial angle, and the saddle angle. Data are from 293 untreated boys and girls age 4 to 24 years in the Fels Longitudinal Study. The timing of the adolescent growth spurt was, in general, not significantly different between the mandible and the maxilla, with each having an earlier age of onset, later age of peak velocity, and later age of cessation of growth compared to the cranial base length. Compared to lengths, angles had in general later ages of onset, peak velocity, and cessation of growth. Accurate characterization of the ontogenetic trajectories of the traits in the craniofacial complex is critical for both clinicians seeking to optimize treatment timing and anatomists interested in examining heterochrony.

Keywords

craniofacial; puberty; development; mathematical modeling; maturation; multilevel model; orthodontics

INTRODUCTION

There is considerable individual variation in the timing, duration, and intensity of growth that occurs in the craniofacial complex during adolescence. The purpose of this paper is to

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describe the extent of this variation between traits and between individuals within the Fels Longitudinal Study. While comprising a functionally integrated unit, the craniofacial components each have unique growth characteristics and these may be differentially affected in various growth disorders. Accurate characterization of the parameters governing ontogenetic trajectories such as onset and offset of the pubertal growth spurt, as well as the timing and magnitude of peak growth velocities, in normal individuals will lead to a better understanding of the alterations of growth and resultant phenotypes in a variety of pathological conditions (Sherwood et al., 1997). In a broader context, such characterization allows for examination of heterochrony in comparative morphology (Alberch et al., 1979).

It has long been recognized that there is a large amount of individual variation in the pattern of facial growth and that this heterogeneity must be taken into consideration when determining the optimal timing of orthodontic and prosthodontic treatment (Björk, 1969; Walker, 1972). Relative to an individual's adolescent growth spurt, the optimal timing varies across treatment objectives. For example, attempts to modify mandibular growth may be best suited to the period of highest growth velocity (Casutt et al., 2007); maxillary protraction therapy is more effective at younger ages (Kapust et al., 1998); and dental implants are most effective if placed after growth has ceased (Mishra et al., 2013). Growth and/or rotation of the mandible and maxilla after implant placement can result in poor alveolar crest positions relative to the adjacent teeth. Thus, numerous studies have estimated growth velocity using changes in facial traits (increments) between regularly spaced measurements (Tracy and Savara, 1966; Savara and Tracy, 1967; Baughan et al., 1979; Ekström, 1982; Lewis et al., 1982, 1985; Krieg, 1987; Nanda, 1988; Hunter et al., 2007), and some have additionally estimated the age of onset of the adolescent growth spurt (Björk, 1963; Roche and Lewis, 1974; Bishara et al., 1981; Jamison et al., 1982; Zionix Alexander et al., 2009; Ball et al., 2011).

For the purpose of creating clinically applicable reference values for changes in facial dimensions, increments are required in order to capture the actual between-individual variation in change over a specified period of time. However, if the goal is to estimate the age at which peak growth velocity occurs, then increments are not ideal as they are constrained by the coarseness of the measurement times (typically semiannual or annual). A more appropriate method for this purpose is multilevel modeling (MLM) (Goldstein, 1986; Goldstein, 2011). MLM has been used extensively to model growth of anatomical structures, providing a means to estimate not only population-average growth curves, but also the amount of individual variation in the pattern of growth.

Various mathematical models for craniofacial growth have been applied to data from individuals including the logistic (Roche et al., 1977), double logistic (Roche and Lewis, 1976), Gompertz (Maunz and German, 1996), and polynomial (Buschang et al., 1986). These can be fit to each individual, with the results then summarized over individuals. However, since the advent of MLM, these models can be fit to all individuals simultaneously. MLM was first applied to facial growth by Buschang et al. (Buschang et al., 1988a; Buschang et al., 1988b). Since then, polynomial MLM has been used in numerous studies to model facial growth.

For example, Buschang et al. (1989) used a 5th degree polynomial to model growth in sellanathion for six to 15 year old French-Canadian girls. They presented estimates of the individual-level variation in the polynomial coefficients, as well as percentiles for size and yearly growth increments derived from the model. Based on this model, they estimated the average ages of two growth spurts – a childhood spurt at 7.6 years and an adolescent spurt at 12.7 years (where a “spurt” is defined as an age at which velocity peaks). A number of similar analyses, motivated by the desire to understand normal and abnormal growth patterns, in particular during the adolescent years, have been carried out for various other facial measurements, for boys and for girls, and for various age ranges and populations (Buschang et al., 1989; Buschang et al., 1990; van der Beek et al., 1991; Henneberke and Prahl-Andersen, 1994; van der Beek et al., 1996; Buschang et al., 1999; Smith and Buschang, 2002; Chvatal et al., 2005; Bills et al., 2008; van Diepenbeek et al., 2009; Arboleda et al., 2011; Wolfe et al., 2011).

These analyses generally report average values of polynomial coefficients, as well as the amount of variation in the coefficients between individuals. However, polynomial coefficients themselves are difficult to interpret. In this paper, we use data from the Fels Longitudinal Study and polynomial MLM to estimate a set of meaningful growth parameters (that are functions of the polynomial coefficients) in a representative set of measures from the maxilla, mandible, and basicranium. The measures examined are mandibular length (Ar-Me), maxillary length (PNS-PtA), cranial base length (Ba-N), gonial angle (Ar-Go-Me), and saddle angle (N-S-Ba) (see Figure 1). We begin by estimating the ages of onset of adolescent growth, peak growth velocity, and cessation of adolescent growth for each measure, both at the group level (average) and at the individual level (variation between individuals). Subsequently, the duration of time between each of these ages, the amount of growth between these ages, and the magnitude peak growth velocity is calculated. Finally, we report the amount of growth relative to adult size, and relative to adult length of the cranial base (Ba-N).

METHODS

Participants

Data used in this analysis come from the Fels Longitudinal Study (FLS). Initiated in 1929, the FLS is currently the world’s longest running study of human growth and body composition change over the lifespan (Roche, 1992). Participants were not selected for any specific feature or trait as the FLS was designed to be a study of normal variation. Most FLS participants live in or near southwest Ohio and neighboring states. Typically, participant examinations were scheduled at 1, 3, 6, 9, and 12 months of age, then every six months until age 18 years, then every two years during adulthood. Visits at these “target ages” included individuals at slightly younger and older ages (e.g., the “age 4 years” visit included individuals from ages 3.75 years to 4.24 years). Lateral cephalographs were part of the FLS protocol from 1936 to 1982, however from age 7 to 18 years they were for the most part collected only yearly (near the child’s birthday) rather than every six months.

The focus of this analysis is on variation in growth during the ages surrounding and including adolescence. As such, we restricted attention to radiographs taken at ages from

3.75 to < 25 years (target ages 4 to 24 years). We included only participants with at least two radiographs both before and after age 13.5 years (boys) or 11.5 years (girls), and who had not had any kind of orthodontic treatment prior to age 25 years. This historical sample contains almost no non-white individuals, and the sample for this analysis was restricted to non-Hispanic whites. This resulted in a set of 3,106 radiographs from 148 boys and 145 girls, born 1929 to 1969, each with from 4 to 19 serial radiographs (median=10).

Measurements

Lengths were measured as follows: mandibular length = distance (mm) between the cephalometric points articulare (Ar) and menton (Me); maxillary length = distance (mm) between the posterior nasal spine (PNS) and point A (PtA); cranial base length = distance (mm) between the points basion (Ba) and nasion (N). The gonial angle was measured as the angle formed by the points articulare (Ar), gonion (Go), and menton (Me). The saddle angle was measured as the angle formed by the points nasion (N), sella (S), and basion (Ba). The location of these cephalometric points is shown in Figure 1. All measurements were made using the commercially available program Nemoceph (CDIImaging) as part of a statistical genetic study, and the details are described elsewhere (Sherwood et al., 2011). All protocols and procedures were approved by the Wright State University Institutional Review Board.

Statistical Analysis

Multilevel modeling—Hoeksma and van der Beek (1991) have provided a clear, concise summary of MLM written for orthodontic researchers. Briefly, an n^{th} order polynomial MLM for a trait Y is of the form

$$Y_{ij} = \beta_{i0} + \beta_{i1}(t_{ij} - t_0) + \beta_{i2}(t_{ij} - t_0)^2 + \beta_{i3}(t_{ij} - t_0)^3 + \cdots + \beta_{in}(t_{ij} - t_0)^n + \varepsilon_{ij}$$

where Y_{ij} is the measurement of the i^{th} child at age t_{ij} (the ages of measurement need not be equally spaced, or the same for different children, nor do children need to have the same number of measurements), t_0 is the “centering” age, and ε_{ij} is measurement error (assumed to be normally distributed). We set $t_0 = 13.5$ years for boys and 11.5 years for girls. The centering results in the intercept term (β_{i0}) being interpretable as the average value of the trait at the centering age. Each polynomial coefficient β_{ik} is actually made up of two terms: a population-average value β_k and the individual’s deviation from that value b_{ik} . Thus, the k^{th} coefficient can be rewritten as $\beta_{ik} = \beta_k + b_{ik}$. These individual-level coefficients are referred to as “random effects” and are assumed to follow a multivariate normal distribution.

In fitting a polynomial model, two choices that must be made are the order (e.g., quadratic, cubic, etc.) and the age range over which to fit the data. The age range of the data can impact the estimated growth curve, and the choice of optimal age range is not always obvious. We considered polynomials of order 1 to 5, and all combinations of age ranges given a younger age of 4.0, 4.5, 5.0, 5.5, 6.0, 6.5, or 7.0 years and an older age of 17, 18, 20, 22, or 24 years (i.e., 35 possible age ranges). Also, we considered all possible combinations of random effects. The best fitting model may not necessarily include individual-level variation for every polynomial coefficient, nor correlation between every random effect.

For each possible age range, we found the best fitting polynomial order and random effects structure using Akaike's Information Criterion (AIC) (Akaike, 1974). After this step, for each trait, by sex, there was more than one age range that resulted in an adequately fitting model (details on the criteria for "adequate" are provided in Supplementary Material). The resulting sets of candidate models varied in number from 3 to 13 across sexes and traits. Choosing a single best model at this point would result in an artificial overstatement of precision due to ignoring the uncertainty in model selection. Therefore, all candidate models were kept and results are presented in two ways. First, we present the model-averaged parameter estimates, and their standard errors, (computed using the bootstrap). Second, we present the range of estimated growth parameters across the candidate models. In this way, our presentation accounts for both within-model variation and the variation resulting from uncertainty in the choice of the optimal age range.

Growth patterns—Scammon (1930; Molinari and Gasser, 2004) suggested a taxonomy of growth curves, including the lymphoid, neural, general, and genital types. Traits of the "lymphoid" type experience a spurt in velocity during mid-childhood and reach a peak size during adolescence, after which they decrease in size. Traits of the other three types all increase in size monotonically until a final adult size is reached, with the three types differing in the timing and tempo of growth. Genital traits experience little growth until puberty, after which they grow to their adult size rapidly. Traits that follow the "neural" growth pattern (e.g., head circumference) are characterized by rapid infant growth, with velocity decreasing with age, with a large proportion of adult size achieved in childhood, and no adolescent growth spurt. As with the neural pattern, traits that follow the "general" pattern, such as stature, are characterized by rapid infant growth with decreasing velocity. However, unlike the neural pattern, these traits exhibit an acceleration of growth velocity that defines the onset of the adolescent growth spurt (Nanda, 1955; Baughan et al., 1979; Lewis et al., 1982; Baume et al., 1983). Some children also experience a noticeable pre-adolescent ("juvenile", "mid-growth", or "childhood") growth spurt (Ekström, 1982; Buschang et al., 1989), with velocity increasing and then decreasing again prior to the adolescent spurt. During the adolescent growth spurt, velocity peaks and then decreases, approaching zero in young adulthood, although some growth may continue into adulthood (West and McNamara Jr, 1999). The craniofacial traits to be analyzed in this paper all follow the "general" pattern of growth, and all discussion below is restricted to this pattern. Note that the cranial base length has characteristics of both the neural and general patterns in that it achieves a large proportion of its adult size during childhood, yet still exhibits an adolescent growth spurt.

Determination of timing of onset, peak, and cessation of adolescent growth—

During childhood, growth velocity is positive but non-increasing (acceleration = 0). The onset of adolescent growth is defined by an increase in velocity (positive acceleration). This increase in velocity is what is meant by the term "growth spurt." Due to the fact that polynomials do not extrapolate well, it is possible that a fitted polynomial curve will have a period of negative velocity (implausibly implying that size is decreasing), with the velocity then increasing and crossing zero at the age of onset. For the purpose of this analysis, we

operationally define the age of onset of adolescent growth as the oldest age before the age of peak velocity at which either the estimated velocity or acceleration was zero.

At the age of cessation of adolescent growth, acceleration changes from negative to zero and velocity changes from positive to zero. Polynomials, however, do not plateau and so the fitted curve will continue to fluctuate. We operationally define the age of cessation of adolescent growth as the youngest age after the age of peak velocity at which the estimated velocity or acceleration was zero. Our definition of “cessation” of growth therefore includes the possibility of continued growth into adulthood. By “cessation”, we mean cessation of the adolescent growth spurt, not necessarily the cessation of growth altogether.

Estimated ages of onset or cessation outside of the range of the data were not allowed as they require extrapolation and, as previously stated, polynomial functions do not extrapolate well. Given these ages, we also calculated the duration and intensity of adolescent growth as the time and amount of growth between these three ages.

Analysis of angles: The gonial and saddle angles, in general, decrease with age during adolescence. However, as shown in our results, they do experience spurts during which the magnitude of their (negative) velocity increases. In contrast to the lengths, which are increasing, “growth” and “peak velocity” for these angles are negative.

Relative growth—In addition to absolute growth, we estimated growth of each trait relative to size at the age of cessation of adolescent growth for that trait, and also relative to final cranial base length.

Peak velocity—The estimate of peak growth velocity from an MLM is an estimate of maximum *instantaneous* velocity; actual average yearly increments would be smaller.

Comparison of growth parameters between sexes and traits—Hypothesis tests of differences in the timing of the adolescent spurt between traits and sexes were computed assuming normality with standard errors estimated using the bootstrap (Efron and Tibshirani, 1993). The hypothesis tests were not adjusted for multiple testing as they are exploratory in nature and intended only to reveal possible patterns of differences across traits and between sexes that could be tested in confirmatory analyses using data from other collections. All data analysis was carried out with R v2.13.1 (r-project.org) (R Development Core Team, 2011), with MLM models fit using the lme (linear mixed effects) function (Pinheiro et al., 2011).

Individual variation—For each growth parameter, we estimated the range within which 95% of individuals fall. This demonstrates the extent of individual variation in the timing, duration, and intensity of craniofacial growth.

RESULTS

Table 1 provides descriptive statistics for the sample.

Figure 2 illustrates the estimated model-averaged mean curves by trait and sex. A trait that follows the “general” pattern will have ages of onset, peak velocity, and cessation. Mathematically, for a polynomial to have this pattern, it must be of at least 4th degree. Based on the AIC criteria, all the candidate models were 5th degree polynomials with the following exceptions: for girls’ Ar-Go-Me, four of the nine candidate models were 3rd degree; for boys’ N-S-Ba, all three candidate models were 4th degree; and for girls’ N-S-Ba, two of the three candidate models were 3rd degree. Thus, according to the statistical criteria, these traits clearly follow the general growth pattern, with the possible exception of N-S-Ba.

Due to the nature of polynomials, the estimated curves after the estimated age of cessation will not plateau. Thus, in Figure 2, the shapes of these curves after the ages of cessation (see Tables 2 and 3 below) are simply an artifact of the modeling method and do not represent the actual growth trend.

Tables 2 and 3 present, for boys and girls, respectively, the number of candidate models, the model-averaged root mean-squared error (RMSE), and the model-averaged means and standard errors for the growth curve parameters. Table 4 presents the results of hypothesis tests comparing model-averaged mean timing and duration between traits and sexes. Figure 3 illustrates the means and standard errors for the ages of onset, peak velocity and cessation, by sex and trait. Specific results from these tables and Figure are presented below.

Timing

As shown in Figure 3 and Tables 2 and 3, within each sex the mean ages of onset, peak velocity, and cessation were very similar for the lengths of the mandible (Ar-Me) and the maxilla (PNS-PtA), with the only significant difference (Table 4) being an earlier age of cessation of growth of the maxilla in girls. In both sexes, the lengths of the mandible and maxilla had earlier ages of onset and later ages of peak velocity and cessation compared to cranial base length (Ba-N) (all significantly different with the exception of girls’ peak velocity). Note that this is referring specifically to the timing of *adolescent* growth; with respect to growth from birth to adulthood, the cranial base length achieves most of its growth prior to the adolescent growth spurt.

While the gonial and saddle angles do change during adolescence, relative to measurement error they do not change nearly as much as the lengths. As a result, their estimated ages of onset, peak, and cessation were more difficult to estimate, as evidenced by their larger standard errors (Tables 2 and 3). In Table 3, the column for N-S-Ba has many blank rows. Only three models met the criteria for adequacy of fit for the saddle angle for girls, and only one of these was of sufficient order (5th degree) to allow estimation of all three ages. Given the variability in the ages across models for the other traits, and the fact that the curve for the saddle angle was relatively flat, we did not consider this single model reliable enough to warrant reporting of an age of onset or peak velocity, and only report the age of cessation (averaged over the two 3rd degree models and one 5th degree model).

The angles were found to have, in general, later ages of onset, peak velocity, and cessation compared to the lengths. Despite the larger standard errors, the timing of onset, peak, and

cessation of growth for the gonial angle were significantly later compared to the lengths (with the exception of age of cessation of the maxilla in boys) (Table 4).

The ages of onset, peak, and cessation were, with the exception of the age of onset in the gonial angle, all earlier for girls than for boys (with almost all differences being statistically significant) (Table 4).

Duration & Growth

As shown in Tables 2 and 3, for all traits and for both sexes, the estimated post-peak duration (time from age at peak velocity to the cessation of growth) was longer than the pre-peak duration (time from onset to peak velocity). However, this did not necessarily correspond to more post-peak growth. For boys' mandibles and maxillae, and for girls' maxillae, the mean post-peak growth was less than the mean pre-peak growth, indicating that the intensity of growth was greater before the age of peak velocity than after it. At the individual level, 98% of boys (but only 19% of girls) had less mandibular growth post-peak; 74% of boys and 56% of girls had less maxillary growth post-peak.

During adolescence, the mandible grows more than the maxilla, both in absolute terms as well as in relative terms (growth as a proportion of size at cessation of adolescent growth). Also, boys experience more growth during adolescence than girls, again both absolutely and relative to their size. That is, prior to the onset of puberty, girls have achieved a greater proportion of their growth than have boys.

While there is a growth spurt, the cranial base grows less during adolescence relative to its final size than does the mandible or maxilla, as it completes a large portion of growth prenatally and during infancy and childhood. The angles change even less relative to their final size, with the saddle angle changing the least.

Between-model variability

Tables 5 and 6 present, for boys and girls, respectively, the ranges of the mean growth curve parameters across the candidate models, demonstrating the extent of between-model variability. Some trait's parameter estimates were robust to the choice of age range. For example, across the eight candidate models for boys' PNS-PtA, the ages of onset, peak velocity, and cessation differed by at most 0.28 years, 0.33 years and 0.49 years, respectively. For other traits, the parameter estimates were quite variable. For example, there were 13 candidate models for Ar-Me and the ages of onset, peak velocity, and cessation differed by up to 1.15 years, 0.55 years, and 0.86 years, respectively.

Individual variation in growth parameters

At the individual level there is substantial variation in the estimated growth parameters. Tables 7 and 8 present the L=2.5th percentile, M=median, and U=97.5th percentile of the distribution of individual-level model-averaged growth curve parameters. Thus, for each parameter, 95% of participants had an estimated value of that parameter between L and U. For example, for boys, 95% of individuals' ages of onset of adolescent growth in the mandible fall between 7.0 and 9.1 years.

DISCUSSION

In this study, we have used polynomial multilevel models to estimate growth curves for mandibular length, maxillary length, cranial base length, the gonial angle, and the saddle angle. Rather than choose a single “best” combination of polynomial degree and age range for each sex $\times$ trait, we averaged over a set of adequately fitting candidate models. For each sex $\times$ trait, we estimate the ages of onset, peak velocity, and cessation of adolescent growth, size at each of these ages, and the amount of time and growth between these ages. We estimated both the means of these growth curve parameters, as well as their between-individual and between-model variation.

In general, fifth degree polynomials adequately fit the data, although third and fourth degree models were among the final candidate models for the gonial and saddle angles. Relative to measurement error, angles change much less during adolescence than do lengths. As a result, their estimated growth curve parameters are much more variable and statistical comparisons between them and other traits are therefore less powerful.

The timing of adolescent growth was similar between the mandible and the maxilla, with each having an earlier age of onset, later age of peak velocity, and later age of cessation of growth compared to the cranial base length. Again, this is referring only to the adolescent growth spurt, not to overall growth; the cranial base achieves a large proportion of its total growth prior to adolescence. Compared to lengths, angles had in general later ages of onset, peak velocity, and cessation of growth.

Relevance of results to treatment timing

In this sample the timing of the adolescent growth spurt for the gonial and saddle angles was later than for the lengths. On average, onset, peak velocity, and the cessation of change in the angles occurred at older ages. The cause for the delay in growth parameters of angles versus lengths is not clear. The three points defining each of these angles define a triangle (see Figure 1). In the present analysis, we have only considered one angle and one side of each of the triangles corresponding to the gonial and saddle angles. Future work will examine additional measures, both linear and angular, and better assess the relationship between growth parameters and type of measurement.

If the general trend of differential timing of angular measures holds true, this potentially impacts treatment timing. Efforts to modify angular relationships may be more successful if initiated later, and may succeed even in later adolescence, while length of the individual bones may be addressed earlier. These differences in timing may also indicate that length and shape are under different control mechanisms.

Importance of the extent of individual variation

There is considerable variation in the pattern of craniofacial growth during adolescence. In particular, the timing of the adolescent growth spurt can vary greatly between individuals. This is highly relevant from a clinical perspective. While it has generally been recognized that the amount of individual variation is large, this paper is the first to explicitly present estimates of the extent of variation in the ages of onset, peak velocity, and cessation of

adolescent growth. Clinical decisions based on a patient's maturational status relative to their adolescent growth spurt should take into account more than the mean ages of onset, peak, or cessation in a population.

Orthodontists are taught that traits closer to the cranium stop growing sooner. The "cephalocaudal growth gradient" theory (CCGG) suggests that the maxilla completes its growth before the mandible (Kingsbury, 1924; Proffit et al., 2012). Looking only at the means, it appears that for boys in this sample both the mandible and maxilla cease growth at approximately age 20 years. On the individual-level, however, the distribution of ages of cessation for the maxilla was skewed towards younger ages and 65% of boys had an estimated age of cessation that was indeed earlier for the maxilla than for the mandible. For girls, 92% had an earlier estimated age of cessation in the maxilla (a significantly greater proportion than for boys, $= 28.5$, $p < 0.01$). These results therefore support the conclusion that while the CCGG theory holds for most children, it may not be universally applicable across all children, and may be less applicable in boys than in girls. The timing of cessation of maxillary and mandibular growth may be closer in some individuals than clinicians assume.

Importance of choice of age range in growth studies

The choice of age range over which to measure individuals has an impact on the estimated growth curve. It is best to consider as wide a range as possible and then fit models to a variety of age ranges in order to investigate the extent of variation in results between models with different age ranges. Given a set of adequately fitting candidate models, it is better to average results over those models than to report results from a single best model and ignore the uncertainty in model selection.

Comparisons with other studies

There is quite a bit of variation in estimated ages of onset and peak velocity across studies. Table 9 compares our model-averaged estimated mean ages of onset, peak velocity, and cessation to the range of estimates found in other studies (Björk, 1963; Tracy and Savara, 1966; Savara and Tracy, 1967; Pileski et al., 1973; Roche and Lewis, 1974; Roche et al., 1977; Baughan et al., 1979; Bishara et al., 1981; Jamison et al., 1982; Lewis et al., 1985; Buschang et al., 1988b; Buschang et al., 1989; Buschang et al., 1999; Chvatal et al., 2005; Hunter et al., 2007; Zionix Alexander et al., 2009; Arboleda et al., 2011; Ball et al., 2011). Note that these studies used a variety of measures of length, not necessarily Ar-Me, PNS-PtA, and Ba-N, but while different specific cephalometric points were used, they were roughly equivalent. Differences between studies can arise due to differences between study populations, as well as differences in analytical methods. These studies include data on boys and girls from the United States, Canada, Italy, Denmark, and Columbia. Most newer studies used polynomial MLM while older studies compared growth increments at different ages. Studies using polynomial MLM differed in the degree used (from 2nd to 6th degree polynomials). Additionally, these studies differed in their participants' age range and each used a single age range (as opposed to the model averaging technique employed in our analysis).

While our estimates of age at peak velocity are all within the range of estimates found in other studies, our estimates of the age of onset are earlier. The other studies that used MLM all used data ranging from age 6 years to age 15, 16, or 17 years. To check to see if our earlier age of onset was due to our use of data over a wider age range, we refit MLMs to our data using the same polynomial degree and age range as the studies found in the literature. The estimated ages of onset were still earlier for the FLS, indicating that this difference from previous research is not due to differences in methodology.

In general, other studies either used increments and did not estimate an age of cessation, or they used MLM but the estimated curve did not reach zero velocity within the age range used. The single study for which an age of cessation was estimable (Arboleda et al., 2011) used MLM with a third degree polynomial for antero-posterior mandibular length (Gn-Op), resulting in an age of cessation of 15.3 years for girls, compared to our estimate of 17.2 years. When fitting the Fels data using a 3rd degree polynomial over the age range used in that study (6 to 17 years), the estimated age of cessation is 16.7 years, still much larger than their estimate (although the difference may not be clinically important). The cause of these differences between our sample (from southwestern Ohio) and theirs (mestizos in private schools in Medellin, Columbia) is not clear. As Arboleda and colleagues point out, ethnic-specific craniofacial growth norms are currently unavailable.

Alternative mathematical models

Alternatives to the use of polynomial models include non-linear models such as the double-logistic (Bock et al., 1973; Roche and Lewis, 1976) or Preece-Baines (Preece and Baines, 1978), but these also are sensitive to the age range chosen (Hauspie and Molinari, 2004), and have the added problem of being more difficult to fit with multiple random effects than are linear (in the model parameters) models such as polynomials; that is, including all the model parameters as random is more likely to lead to failure to converge. Since in these non-linear models certain model parameters correspond exactly to the growth parameters of interest (e.g., age of onset, age of peak velocity, etc.), they have to be specified as random effects in order to estimate individual variability. With polynomial models, the growth parameters are functions of multiple model parameters and therefore one can estimate individual variability in growth parameters even if not all the model parameters are random.

An extension of MLM is multivariate MLM (van der Beek et al., 1996), with which one can more effectively estimate correlations between growth parameters for pairs of traits. Also, multivariate MLM could be used to estimate correlations between growth parameters and stature in order to estimate the probability that a child is younger than their age of peak facial growth velocity given their stature, or given a series of stature measurements.

Supplementary Material

Refer to Web version on PubMed Central for supplementary material.

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Fig. 1.
Cephalometric points identified on each lateral cephalograph.

Fig. 2.
Model-averaged mean growth curves, by sex and trait.

Fig. 3. Model-averaged mean ages of onset, peak velocity, and cessation of adolescent growth, by sex and trait. Vertical gray lines indicate 95% confidence intervals for the means.

Table 1

Descriptive statistics

	Sex	N	Mean	SD	Range
Age	Boys	148	11.93	5.47	(3.93, 24.94)
	Girls	145	11.20	4.98	(3.99, 24.52)
Birth year	Boys	148	1947.37	10.14	(1929, 1966)
	Girls	145	1946.32	11.85	(1929, 1969)
Ar-Me	Boys	148	94.87	11.97	(67.57, 124.77)
	Girls	145	89.31	9.43	(65.32, 112.88)
PNS-P/A	Boys	148	45.96	4.98	(33.71, 60.61)
	Girls	145	43.49	3.76	(32.25, 57.11)
Ba-N	Boys	148	99.88	8.04	(80.85, 117.83)
	Girls	145	94.68	6.51	(76.74, 112.42)
Ar-Co-Me	Boys	148	128.65	6.75	(106.1, 149.1)
	Girls	145	129.75	5.47	(112.9, 147.9)
N-S-Ba	Boys	148	131.82	5.00	(115.7, 148.3)
	Girls	145	131.71	5.31	(116.1, 152.3)

Table 2

Model-averaged estimated mean growth parameters (bootstrap SE) (boys)

	Ar-Me		PNS-PtA		Ba-N		Ar-Go-Me		N-S-Ba	
# Candidate models	13		8		8		5		3	
RMSE (mm)	1.78		1.54		1.67		1.83		1.57	
Timing (y)										
Onset	8.37 (0.06)		8.17 (0.10)		8.88 (0.11)		9.46 (0.26)		9.16 (1.34)	
Peak velocity	13.41 (0.04)		13.29 (0.09)		12.57 (0.11)		14.23 (0.21)		14.28 (1.62)	
Cessation	19.85 (0.08)		20.04 (0.22)		18.49 (0.13)		20.63 (0.34)		19.57 (0.86)	
Duration (y)										
Onset to Peak	5.05 (0.04)		5.12 (0.09)		3.69 (0.12)		4.77 (0.24)		5.12 (0.97)	
Peak to Cessation	6.44 (0.09)		6.75 (0.22)		5.91 (0.19)		6.39 (0.41)		5.29 (1.39)	
Total	11.48 (0.11)		11.87 (0.24)		9.61 (0.16)		11.16 (0.45)		10.41 (1.26)	
Size (mm) or Angle (°)										
at Onset	88.16 (0.16)		43.63 (0.12)		97.19 (0.22)		130.26 (0.30)		132.50 (0.42)	
at Peak Velocity	99.39 (0.13)		47.58 (0.10)		102.29 (0.19)		127.07 (0.28)		131.63 (0.41)	
at Cessation	109.82 (0.16)		51.27 (0.12)		107.86 (0.16)		124.08 (0.30)		130.93 (0.37)	
Growth (mm or °)										
Onset to Peak	11.23 (0.09)		3.95 (0.07)		5.10 (0.17)		-3.19 (0.17)		-0.87 (0.18)	
Peak to Cessation	10.43 (0.10)		3.69 (0.07)		5.57 (0.14)		-3.00 (0.16)		-0.70 (0.24)	
Total	21.66 (0.18)		7.63 (0.13)		10.67 (0.20)		-6.18 (0.29)		-1.57 (0.31)	
Growth relative to size at cessation (%)										
Onset to Peak	10.23 (0.08)		7.70 (0.14)		4.73 (0.16)		-2.57 (0.14)		-0.66 (0.15)	
Peak to Cessation	9.49 (0.08)		7.19 (0.14)		5.16 (0.13)		-2.41 (0.14)		-0.53 (0.18)	
Total	19.72 (0.15)		14.89 (0.24)		9.89 (0.18)		-4.98 (0.24)		-1.20 (0.23)	
Growth relative to adult Ba-N (%)										
Onset to Peak	10.41 (0.09)		3.66 (0.07)		4.73 (0.16)		--		--	
Peak to Cessation	9.67 (0.09)		3.42 (0.07)		5.16 (0.13)		--		--	
Total	20.08 (0.17)		7.08 (0.12)		9.89 (0.18)		--		--	
Peak Velocity (mm/y)										
	2.71 (0.02)		0.96 (0.02)		1.56 (0.02)		-0.82 (0.03)		-0.19 (0.03)	

Table 3

Model-averaged estimated mean growth parameters (bootstrap SE) (girls)

	Ar-Me	PNS-PtA	Ba-N	Ar-Go-Me	N-S-Ba
# Candidate models	5	8	3	9	3
RMSE (mm)	1.60	1.33	1.30	1.81	1.46
Timing (y)					
Onset	7.23 (0.12)	7.16 (0.15)	8.44 (0.27)	9.78 (0.31)	--
Peak velocity	10.84 (0.11)	10.98 (0.11)	10.48 (0.28)	13.18 (0.40)	--
Cessation	17.17 (0.15)	16.26 (0.16)	15.14 (0.25)	19.04 (0.27)	16.06 (1.19)
Duration (y)					
Onset to Peak	3.61 (0.12)	3.82 (0.11)	2.04 (0.29)	3.41 (0.35)	--
Peak to Cessation	6.33 (0.19)	5.28 (0.20)	4.66 (0.45)	5.10 (0.54)	--
Total	9.94 (0.19)	9.10 (0.23)	6.70 (0.36)	8.51 (0.36)	--
Size (mm) or Angle (°)					
at Onset	82.99 (0.32)	41.36 (0.12)	92.82 (0.46)	130.10 (0.28)	--
at Peak Velocity	90.55 (0.26)	44.02 (0.10)	95.50 (0.41)	128.43 (0.29)	--
at Cessation	99.23 (0.24)	46.61 (0.10)	99.28 (0.28)	126.64 (0.19)	131.39 (0.43)
Growth (mm or °)					
Onset to Peak	7.57 (0.24)	2.66 (0.07)	2.69 (0.38)	-1.67 (0.17)	--
Peak to Cessation	8.68 (0.21)	2.59 (0.08)	3.78 (0.36)	-1.72 (0.19)	--
Total	16.25 (0.31)	5.25 (0.12)	6.47 (0.42)	-3.39 (0.20)	--
Growth relative to size at cessation (%)					
Onset to Peak	7.62 (0.24)	5.71 (0.14)	2.71 (0.39)	-1.32 (0.14)	--
Peak to Cessation	8.75 (0.21)	5.55 (0.16)	3.81 (0.36)	-1.36 (0.15)	--
Total	16.37 (0.30)	11.26 (0.25)	6.51 (0.42)	-2.68 (0.16)	--
Growth relative to adult Ba-N (%)					
Onset to Peak	7.62 (0.24)	2.68 (0.07)	2.71 (0.39)	--	--
Peak to Cessation	8.74 (0.21)	2.61 (0.08)	3.81 (0.37)	--	--
Total	16.37 (0.32)	5.29 (0.12)	6.51 (0.42)	--	--
Peak Velocity (mm/y)	2.29 (0.03)	0.85 (0.02)	1.38 (0.04)	-0.55 (0.03)	--

Table 4

Significance ($p < 0.05$) of timing and duration differences between sexes and traits. Significant pairwise comparisons are shaded (e.g., boys' age of onset for Ar-Me = 8.37 years is significantly different from boys' age of onset of Ba-N = 8.88 years).

Boys													
Timing (y)	Ar-Me	vs.			PNS PtA	vs.			Ba N	vs.			vs. ♀
		Ba N	Ar Go Me	N S Ba		Ba N	Ar Go Me	N S Ba		Ar Go Me	N S Ba	N S Ba	
Onset	8.37				8.17				8.88				9.16
Peak velocity	13.41				13.29				12.57				14.28
Cessation	19.85				20.04				18.49				19.57
Duration (y)	Ar-Me	vs.			PNS PtA	vs.			Ba N	vs.			vs. ♀
		Ba N	Ar Go Me	N S Ba		Ba N	Ar Go Me	N S Ba		Ar Go Me	N S Ba	N S Ba	
Onset to Peak	5.05				5.12				3.69				5.12
Peak to Cessation	6.44				6.75				5.91				5.29
Total	11.48				11.87				9.61				10.41
Girls													
Timing (y)	Ar-Me	vs.			PNS PtA	vs.			Ba N	vs.			vs. ♂
		Ba N	Ar Go Me	N S Ba		Ba N	Ar Go Me	N S Ba		Ar Go Me	N S Ba	N S Ba	
Onset	7.23				7.16				8.44				
Peak velocity	10.84				10.98				10.48				
Cessation	17.17				16.26				15.14				16.06
Duration (y)	Ar-Me	vs.			PNS PtA	vs.			Ba N	vs.			vs. ♂
		Ba N	Ar Go Me	N S Ba		Ba N	Ar Go Me	N S Ba		Ar Go Me	N S Ba	N S Ba	
Onset to Peak	3.61				3.82				2.04				
Peak to Cessation	6.33				5.28				4.66				
Total	9.94				9.10				6.70				

Table 5

Range of mean growth parameters across candidate models (boys)

		Ar-Me		PNS-PtA		Ba-N		Ar-Go-Me		N-S-Ba	
# Candidate models		13		8		8		5		3	
		Min	Max	Min	Max	Min	Max	Min	Max	Min	Max
RMSE (mm)		1.63	1.89	1.50	1.57	1.61	1.75	1.79	1.90	1.51	1.66
Timing (y)	Onset	7.81	8.96	8.02	8.30	8.71	8.98	9.17	9.84	8.92	9.47
	Peak velocity	13.10	13.65	13.09	13.42	12.03	12.80	14.06	14.49	13.85	14.74
	Cessation	19.36	20.22	19.78	20.27	18.06	19.23	20.25	21.04	19.22	19.82
Duration (y)	Onset to Peak	4.69	5.29	4.96	5.21	3.31	3.85	4.65	4.90	4.93	5.27
	Peak to Cessation	5.71	7.08	6.51	6.96	5.26	7.20	5.90	6.88	4.48	5.97
	Total	10.40	12.37	11.48	12.05	9.09	10.52	10.55	11.62	9.75	10.90
Size (mm) or Angle (°)	at Onset	86.87	89.49	43.55	43.72	96.76	97.50	130.02	130.47	132.36	132.57
	at Peak Velocity	98.71	99.95	47.47	47.66	101.42	102.72	126.92	127.17	131.46	131.75
	at Cessation	109.39	110.17	51.12	51.41	107.69	108.04	124.02	124.15	130.83	131.00
Growth (mm or °)	Onset to Peak	10.46	11.84	3.86	4.01	4.65	5.34	-3.30	-3.10	-0.90	-0.83
	Peak to Cessation	9.52	11.31	3.56	3.77	5.01	6.62	-3.08	-2.87	-0.75	-0.63
	Total	19.98	23.15	7.42	7.79	10.23	11.28	-6.37	-5.97	-1.59	-1.53
Growth relative to size at cessation (%)	Onset to Peak	9.56	10.76	7.54	7.81	4.30	4.95	-2.66	-2.50	-0.69	-0.63
	Peak to Cessation	8.69	10.28	6.96	7.34	4.65	6.13	-2.48	-2.31	-0.57	-0.48
	Total	18.25	21.04	14.51	15.15	9.50	10.44	-5.14	-4.81	-1.21	-1.17
Growth relative to adult Ba-N (%)	Onset to Peak	9.70	10.98	3.58	3.72	4.31	4.95	--	--	--	--
	Peak to Cessation	8.82	10.49	3.30	3.50	4.65	6.14	--	--	--	--
	Total	18.53	21.46	6.88	7.22	9.49	10.46	--	--	--	--
Peak Velocity (mm/y)		2.63	2.82	0.93	1.00	1.46	1.62	-0.86	-0.78	-0.20	-0.18

Table 6

Range of mean growth parameters across candidate models (girls)

# Candidate models	Ar-Me		PNS-PtA		Ba-N		Ar-Go-Me		N-S-Ba	
	Min	Max	Min	Max	Min	Max	Min	Max	Min	Max
	5		8		3		9		3	
RMSE (mm)	1.56	1.65	1.31	1.35	1.28	1.34	1.79	1.83	1.43	1.49
Timing (y)										
Onset	7.12	7.33	6.67	7.46	7.77	8.83	9.44	10.01	--	--
Peak velocity	10.80	10.89	10.83	11.16	9.92	10.84	12.87	13.40	--	--
Cessation	17.08	17.22	15.88	17.30	14.87	15.55	17.81	20.20	15.26	16.67
Duration (y)										
Onset to Peak	3.57	3.68	3.57	4.16	1.97	2.15	3.32	3.50	--	--
Peak to Cessation	6.21	6.40	4.87	6.47	4.03	5.63	4.58	5.79	--	--
Total	9.84	10.06	8.44	10.64	6.03	7.78	8.08	9.23	--	--
Size (mm) or Angle (°)										
at Onset	82.82	83.16	41.04	41.52	91.82	93.45	130.00	130.27	--	--
at Peak Velocity	90.45	90.66	43.93	44.17	94.74	96.02	128.33	128.56	--	--
at Cessation	99.02	99.36	46.49	46.80	99.26	99.32	126.51	126.83	131.22	131.59
Growth (mm or °)										
Onset to Peak	7.51	7.63	2.53	2.89	2.56	2.92	-1.71	-1.64	--	--
Peak to Cessation	8.43	8.85	2.44	2.87	3.30	4.53	-1.85	-1.59	--	--
Total	16.03	16.42	4.97	5.76	5.87	7.45	-3.56	-3.29	--	--
Growth relative to size at cessation (%)										
Onset to Peak	7.56	7.70	5.45	6.18	2.58	2.95	-1.35	-1.29	--	--
Peak to Cessation	8.52	8.91	5.25	6.13	3.32	4.56	-1.46	-1.25	--	--
Total	16.19	16.53	10.70	12.31	5.91	7.51	-2.81	-2.59	--	--
Growth relative to adult Ba-N (%)										
Onset to Peak	7.56	7.69	2.55	2.92	2.58	2.94	--	--	--	--
Peak to Cessation	8.49	8.91	2.46	2.89	3.32	4.56	--	--	--	--
Total	16.14	16.54	5.01	5.80	5.91	7.51	--	--	--	--
Peak Velocity (mm/y)	2.27	2.32	0.81	0.89	1.36	1.41	-0.59	-0.54	--	--

Table 7

Individual variation in model-averaged growth curve parameters (L=2.5th percentile, M=median, U=97.5th percentile) (boys)

	Ar-Me			PNS-PTA			Ba-N			Ar-Go-Me			N-S-Ba			
	L	M	U	L	M	U	L	M	U	L	M	U	L	M	U	
Timing (y)	Onset	7.01	8.42	9.08	7.23	8.13	9.63	7.38	8.87	9.89	7.88	9.59	11.65	5.46	10.11	14.96
	Peak velocity	12.15	13.49	14.67	11.95	13.31	14.39	11.46	12.66	13.87	13.07	14.27	15.25	9.60	15.28	17.31
	Cessation	18.38	19.87	21.21	17.76	19.74	20.88	17.61	18.49	19.19	17.69	20.54	21.62	14.39	19.03	19.82
Duration (y)	Onset to Peak	4.28	5.14	5.87	2.72	5.21	6.64	2.42	3.88	4.92	1.66	4.73	6.76	2.00	4.18	7.56
	Peak to Cessation	5.60	6.45	7.20	5.16	6.35	7.28	5.06	5.85	6.43	3.76	6.32	7.30	1.73	3.65	8.39
	Total	10.21	11.58	12.80	8.46	11.62	13.29	8.55	9.67	10.76	6.30	10.91	13.48	3.83	8.45	13.52
Size (mm)	at Onset	81.12	87.86	94.55	39.59	43.66	48.14	90.13	96.59	103.81	121.00	129.96	140.76	125.26	132.46	142.34
	at Peak Velocity	92.10	99.28	107.39	43.24	47.53	52.05	94.67	101.93	108.30	117.79	126.71	137.51	124.14	131.77	141.03
	at Cessation	101.21	109.52	121.02	46.31	51.01	56.84	99.49	107.69	115.74	112.94	124.00	135.16	122.53	130.91	139.04
Growth (mm)	Onset to Peak	7.62	11.32	15.32	1.58	3.91	5.73	2.81	5.18	8.27	-7.63	-3.06	-0.42	-2.67	-0.75	-0.06
	Peak to Cessation	6.99	10.67	14.71	2.14	3.76	5.39	3.55	5.55	8.21	-6.88	-3.21	-0.53	-3.81	-0.64	-0.07
	Total	14.52	21.94	29.79	3.82	7.67	11.12	6.75	10.65	16.23	-14.35	-6.34	-1.07	-6.25	-1.42	-0.17
Growth relative to size at cessation (%)	Onset to Peak	7.40	10.41	13.17	3.18	7.76	10.59	2.64	4.80	7.46	-6.63	-2.47	-0.32	-2.08	-0.58	-0.07
	Peak to Cessation	6.80	9.74	12.67	4.48	7.37	9.81	3.45	5.18	7.22	-5.92	-2.59	-0.41	-2.96	-0.49	-0.05
	Total	14.16	20.16	25.52	7.69	15.04	20.20	6.49	9.98	14.53	-12.52	-5.12	-0.83	-4.86	-1.09	-0.13
Growth relative to adult	Onset to Peak	7.07	10.59	13.82	1.45	3.64	5.35	2.64	4.80	7.46	--	--	--	--	--	--
	Peak to Cessation	6.52	9.99	13.29	1.98	3.49	4.97	3.45	5.18	7.22	--	--	--	--	--	--
	Total	13.51	20.55	26.79	3.45	7.13	10.20	6.49	9.99	14.53	--	--	--	--	--	--
Ba-N (%)																
Peak Velocity (mm/y)		2.04	2.74	3.48	0.59	0.98	1.30	1.06	1.56	2.21	-1.74	-0.86	-0.22	-0.98	-0.26	-0.03

Table 8

Individual variation in model-averaged growth curve parameters (L=2.5th percentile, M=median, U=97.5th percentile) (girls)

	Ar-Me			PNS-PTA			Ba-N			Ar-Go-Me			N-S-Ba		
	L	M	U	L	M	U	L	M	U	L	M	U	L	M	U
Timing (y)	Onset	5.81	7.20	8.20	6.68	7.16	7.59	6.66	8.43	9.43	9.17	9.87	11.42	--	--
	Peak velocity	9.35	10.88	12.41	9.90	10.99	11.94	9.44	10.62	12.15	12.12	13.16	14.20	--	--
	Cessation	15.60	17.09	18.23	14.66	16.05	16.88	13.59	15.04	16.22	15.52	18.95	20.25	13.61	17.85
Duration (y)	Onset to Peak	1.88	3.78	5.18	2.60	3.84	4.96	0.91	2.42	3.49	1.51	3.30	4.58	--	--
	Peak to Cessation	5.37	6.23	6.92	4.13	5.01	5.76	3.16	4.38	5.10	2.61	5.11	6.93	--	--
	Total	8.42	9.98	11.23	7.37	9.02	9.60	5.11	6.81	7.92	5.47	8.34	9.87	--	--
Size (mm)	at Onset	74.67	82.88	89.55	37.78	41.35	45.06	85.23	92.33	98.50	122.74	129.07	140.64	--	--
	at Peak Velocity	82.48	90.62	96.91	40.33	44.10	47.98	88.06	95.67	101.77	120.82	127.86	139.36	--	--
	at Cessation	90.31	99.53	106.77	42.46	46.71	50.92	90.52	99.60	106.67	118.08	125.71	137.74	121.18	141.79
Growth (mm)	Onset to Peak	3.76	7.64	11.23	1.84	2.62	3.58	1.08	2.99	5.16	-2.65	-1.58	-0.29	--	--
	Peak to Cessation	5.56	8.79	12.00	1.66	2.65	3.72	1.86	3.94	5.98	-4.14	-1.76	-0.28	--	--
	Total	10.06	16.48	22.28	3.50	5.30	7.23	3.53	6.85	10.49	-6.11	-3.43	-0.58	--	--
Growth relative to size at cessation (%)	Onset to Peak	3.95	7.82	11.05	4.11	5.62	7.36	1.08	3.04	5.12	-2.18	-1.25	-0.23	--	--
	Peak to Cessation	5.99	8.87	11.56	3.81	5.67	7.59	1.94	3.96	5.70	-3.37	-1.37	-0.22	--	--
	Total	10.69	16.71	21.80	8.05	11.27	14.91	3.64	6.87	10.06	-5.03	-2.68	-0.46	--	--
Growth relative to adult	Onset to Peak	3.77	7.72	11.33	1.89	2.64	3.57	1.08	3.04	5.11	--	--	--	--	--
	Peak to Cessation	5.62	8.85	12.21	1.69	2.67	3.74	1.94	3.96	5.71	--	--	--	--	--
	Total	10.16	16.66	22.82	3.63	5.31	7.26	3.65	6.87	10.06	--	--	--	--	--
Ba-N (%)															
Peak Velocity (mm/y)															
		1.59	2.34	3.00	0.65	0.87	1.09	0.86	1.44	2.12	-0.98	-0.57	-0.16	--	--

Table 9
Comparison of model-averaged estimated mean ages of onset, peak velocity, and cessation with estimates from other studies

		Mandibular length		Maxillary length		Cranial base length	
		Fels	Range in other studies	Fels	Range in other studies	Fels	Range in other studies
Boys							
Timing (y)	Onset	8.37	9.2 to 12.2	8.17	8.6 to 11.2	8.88	11.5
	Peak velocity	13.41	13.1 to 14.5	13.29	11.4 to 14.6	12.57	12.8 to 13.9
	Cessation	19.85	--	20.04	--	18.49	--
Girls							
Timing (y)	Onset	7.23	8.0 to 10.8	7.16	8.5	8.44	--
	Peak velocity	10.84	9.6 to 12.9	10.98	10.5	10.48	10.3 to 12.6
	Cessation	17.17	15.32	16.26	--	15.14	--