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Analytical Investigation of Elastic Thin-Walled Cylinder and Truncated Cone Shell Intersection Under Internal Pressure

An elastic solution of cylinder-truncated cone shell intersection under internal pressure is presented. The edge solution theory that has been used in this study takes bending moments and shearing forces into account in the thin-walled shell of revolution element. The general solution of the cone equations is based on power series method. The effect of cone apex angle on the stress distribution in conical and cylindrical parts of structure is investigated. In addition, the effect of the intersection and boundary locations on the circumferential and longitudinal stresses is evaluated and it is shown that how quantitatively they are essential. [DOI: 10.1115/1.4027583]

Keywords: edge solution, internal pressure, truncated cone, thin-walled shell

1 Introduction

Shells of revolution are the shells that are produced by rotation of a curve around symmetric axis. In thin-walled shells, the shearing force variation along the thickness is possible to be ignored. For instance, in a cylindrical shell the ratio of the thickness to radius must be under 0.1 so that it accounts as a thin-walled shell [1]. These shells are usually used as components for bearing pressure. For instance, in civil engineering applications, structures like silos, pressure vessels, tanks, cooling towers, and chimneys involve conical shells in their constructions. Large conical tubes have transition cone between two cylinders of different diameters. In such examples conical shells are modeled as shells under internal pressure.

When a conical shell is joined to a cylindrical shell, a slope discontinuity is introduced in the shell meridian, leading to local high bending and circumferential membrane stresses when the shell is pressurized. The strength of cone-cylinder intersections is, therefore, a major concern for the designer. Circular conical shells, in the form of pressure vessels, also appear in various forms in a number of branches of engineering. For example, in civil engineering, such structures take the form of cooling towers and silos, in ocean engineering, submarine pressure hulls, and off-shore drilling rigs. In aerospace engineering, they appear as aircraft fuselages and as spacecraft hulls.

Generally pressure vessels are the variety of the joints between shells of revolution. So, different intersections between the shells are studied. For instance, Ugural [1] studied the intersection of cylindrical and spherical shell. Uddin [2] analyzed the spherical heads of pressure vessels by assuming large deformation for them. The intersection of oblique cylinders has been studied by Chien and Wu [3] and Saal et al. [4] investigated the intersection of a nozzle to the spherical head of a pressure vessel. Moreover, the intersection of a cone and sphere under static and thermal loads has been analyzed by Miksch and Mera [5] just numerically. In

addition, Oreste [6] studied a truncated cone shell as a final lining of a tunnel analytically. He used shell of revolution equations in his analysis. The other researchers such as Belica et al. [7], Rotter et al. [8], and Li et al. [9] have studied the Shells of revolution under internal pressure or compressive stress.

To author's knowledge, the analytical solution of thin-walled truncated cone and cylindrical shell intersection has not been studied yet. As the study of this structure through elasticity in which all of the displacements and stresses are considered, is arduous, a simplified analytical solution named "edge solution" is utilized. Ugural [1] has chosen this approach for solving the intersection of cylindrical and spherical shell. Because of complexity of the cone equations in comparison to cylindrical and spherical shells, study of this kind of shells is more limited. And to top it all off, the intersection between cone and another shell makes analytical solution more ambiguous. In this study the intersection between truncated conical and cylindrical shells under internal pressure are studied by using edge solution.

The definition of thin-walled shell of revolution for corresponding structure has been employed. The easiest method for analysis of shells is the membrane theory that neglects bending and shear resultants in the element of thin-walled shell of revolution. In spite of this simplification, this theory is convenient for farther location from the intersection and boundary locations. But the closer the intersection location is the more deviation from membrane theory takes place. Hence, there is a need for a more precise theory in the vicinity of the intersection and boundaries. So, the edge solution theory, which takes bending moments and shearing forces into account in the thin-walled shell of revolution element, is used.

Consideration of the stresses along the axial direction of cone and cylinder depicts that the stresses approach to membrane stresses. Although the effects of the intersection and boundaries are important up to limited distances from the intersection and boundaries locations, the results show that the stresses in vicinity of the intersection location can be exceeded, even more than several times of the membrane stresses. In addition, being effective of the intersection depends on the apex angle of the cone such that the more it increases, the more the distance under effect of the intersection will be.

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Fig. 1 Shell of revolution element with symmetric load (membrane theory)

2 General Theory of Shell of Revolution Under Symmetric Loading

In the membrane theory, bending resultants are not considered. In addition, because the loading is symmetric, the stress resultant $N_{\phi\theta}$ is omitted. Figure 1 shows a shell element accompanied with stress resultants required for membrane theory under symmetric loading. As the element of the shell is the static determined problem, equilibrium equations are adequate. Equations (1a) and (1b) show the equilibrium equations in y and z directions, respectively,

$$y : \frac{d}{d\phi} r_0 N_\phi - N_\theta r_1 \cos \phi + Y r_0 r_1 = 0 \quad (1a)$$

$$z : N_\phi r_0 + N_\theta r_1 \sin \phi + Z r_0 r_1 = 0 \quad (1b)$$

where the stress resultants, defined radii and the coordinates have been shown in Fig. 1. Y and Z are the components of applied force per unit area in y and z directions, respectively. The solutions of these equations for cylindrical and conical shells are completely straightforward. In following two subsections, the general solutions of cylindrical and conical shells are formulated, respectively, by Timoshenko and Woinowsky-Krieger [10].

2.1 General Solution of the Cylinder. In this theory, both the moments and shear forces have been considered. The shell element with applied forces and moments has been shown in Fig. 2. Considering the symmetry, we have

$$X = 0, Y, Z \neq 0$$

$$N_\theta = \text{cte.} \quad N_{\phi\theta} = N_{\theta\phi}$$

$$M_\theta = \text{cte.} \quad M_{\phi\theta} = M_{\theta\phi}$$

$$Q_\phi \neq 0 \quad Q_\theta = 0$$

Fig. 2 Shell of revolution element with symmetric load (general theory)

where the stress resultants and the coordinates have been shown in Fig. 2. Having written the equilibrium equations, stress-strain relationships and curvature-deformation, a set with 11 equations and 11 variables is driven. Introducing two new variables U and V with following definitions:

$$U = r_2 Q_\phi \quad (2a)$$

$$V = \frac{1}{r_1} \left(v + \frac{d\omega}{d\phi} \right) \quad (2b)$$

where v and w are displacements in y and z directions, respectively, that set of equations is transferred to following simplified equations:

$$\begin{cases} L(U) + \frac{v}{r_1} U = EhV \\ L(V) + \frac{v}{r_1} V = \frac{-U}{D} \end{cases} \quad (3)$$

The operator L is also defined as

$$L(\dots) = \frac{1}{r_1} \left[\frac{d}{d\phi} \left(\frac{r_2}{r_1} \right) + \frac{r_2}{r_1} \cot \phi \right] \frac{d(\dots)}{d\phi} + \frac{r_2}{r_1^2} \frac{d^2(\dots)}{d\phi^2} - \frac{r_1}{r_2 r_1} \cot^2 \phi (\dots) \quad (4)$$

In cylindrical shell, we have

$$U = RQ \quad dx = r_1 d\phi \quad r_2 = r_0 = R \quad r_1 = \infty$$

where R is the radius of the cylinder. Hence the operator L is reduced to

$$L(\dots) = \frac{R d^2(\dots)}{dx^2} + 0 + 0 \quad (5)$$

Similarly, Eq. (3) is simplified as

$$\begin{cases} L(U) = \frac{Rd^2(RQ)}{dx^2} + 0 = EhV \\ \frac{Rd^2(V)}{dx^2} - 0 = \frac{-RQ}{D} \end{cases} \quad (6) \quad \text{where}$$

where E , h , Q , and D are module of elasticity, thickness of the shell, shear resultant, and bending stiffness, respectively. Removing V from Eq. (6), following ordinary differential equation obtains:

$$\frac{d^4 Q}{dx^4} + \frac{EhQ}{R^2 D} = 0 \quad (7)$$

The general solution of Eq. (7) is

$$Q = e^{\lambda x} (A_1 \cos \lambda x + A_2 \sin \lambda x) + e^{-\lambda x} (A_3 \cos \lambda x + A_4 \sin \lambda x) \quad (8)$$

where

$$\lambda^4 = \frac{3(1-\nu^2)}{R^2 h^2}$$

ν in above equation is the Poisson ratio. With the help of following relationship, displacement formula, w , in general form can be extracted from Eq. (8)

$$Q = -D \frac{d^3 w}{dx^3} \quad (9)$$

So, displacement equals

$$w = e^{\lambda x} (C_1 \cos \lambda x + C_2 \sin \lambda x) + e^{-\lambda x} (C_3 \cos \lambda x + C_4 \sin \lambda x) \quad (10)$$

Bending moments in cylinder is also calculated by

$$M_x = -D \frac{d^2 w}{dx^2} \quad (11)$$

2.2 General Solution of the Cone. In order to solve Eq. (3) for the conical shell, another variable, y , is defined such that measures the distance from apex point of the cone (Fig. 3). The operator L for cone and in terms of y is

$$L(\dots) = \tan \alpha \left[y \frac{d^2(\dots)}{dy^2} + \frac{d(\dots)}{dy} - \frac{1}{y}(\dots) \right] \quad (12)$$

After rewriting Eq. (3), using Eq. (1), following complex differential equation is appeared:

Fig. 3 Defined parameters of the conical shell

$$\tan \alpha \left(y \frac{d^2 U}{dy^2} + \frac{dU}{dy} - \frac{U}{y} \right) \pm i \mu^2 U = 0 \quad (13)$$

$$U = r_2 Q_\varphi = y \tan \alpha Q_y$$

Having substituted $\eta = 2\lambda\sqrt{iy}$ in Eq. (13), it yields

$$y \frac{d^2(yQ_y)}{d\eta^2} + \frac{1}{\eta} \frac{d(yQ_y)}{d\eta} + \left(1 - \frac{4}{\eta^2} \right) (yQ_y) = 0 \quad (14)$$

Solving Eq. (14), it obtains

$$yQ_y = c_1 \left[\psi_1(\xi) + \frac{2}{\xi} \psi_2'(\xi) \right] + c_2 \left[\psi_2(\xi) - \frac{2}{\xi} \psi_1'(\xi) \right] + c_3 \left[\psi_3(\xi) + \frac{2}{\xi} \psi_4'(\xi) \right] + c_4 \left[\psi_4(\xi) - \frac{2}{\xi} \psi_3'(\xi) \right] \quad (15)$$

where

$$\psi_1(\xi) = 1 - \frac{\xi^4}{(2 \times 4)^2} + \frac{\xi^8}{(2 \times 4 \times 6 \times 8)^2} - \dots \quad (16a)$$

$$\psi_2(\xi) = -\frac{\xi^2}{(2)^2} + \frac{\xi^6}{(2 \times 4 \times 6)^2} - \frac{\xi^{10}}{(2 \times 4 \times 6 \times 8 \times 10)^2} + \dots \quad (16b)$$

$$\psi_3(\xi) = \frac{1}{2} \psi_1(\xi) - \frac{2}{\pi} \left(R_1 + \psi_2(\xi) \log \frac{\beta \xi}{2} \right) \quad (16c)$$

$$\psi_4(\xi) = \frac{1}{2} \psi_4(\xi) + \frac{2}{\pi} \left(R_2 + \psi_1(\xi) \log \frac{\beta \xi}{2} \right) \quad (16d)$$

R_1 and R_2 in Eqs. (16c) and (16d) are

$$R_1 = \left(\frac{\xi}{2} \right)^2 - \frac{S(3)}{(3 \times 2)^2} \left(\frac{\xi}{2} \right)^6 + \frac{S(3)}{(5 \times 4 \times 3 \times 2)^2} \left(\frac{\xi}{2} \right)^{10} - \dots$$

$$R_2 = \frac{S(2)}{(2)^3} \left(\frac{\xi}{2} \right)^4 - \frac{S(4)}{(4 \times 3 \times 2)^2} \left(\frac{\xi}{2} \right)^8 + \dots$$

$$S(n) = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}$$

$$\log \beta = 0.57722$$

In order to calculate c_1 , c_2 , c_3 , and c_4 in Eq. (15) for applying the desired boundary conditions, it is inevitable to derive deflection, moment, and slope relationships in terms of yQ_y . In Eqs. (17)–(20), the relations for slope of the cone (V), the moment resultants in y and θ directions, and the deflection (δ) have been formulated

$$v = \frac{1}{Eh} L(U) = \frac{\tan^2 \alpha}{Eh} \left(y \frac{d^2(yQ_y)}{dy^2} + \frac{d(yQ_y)}{dy} - Q_y \right) \quad (17)$$

$$M_y = -D \left(\frac{dv}{dy} + \frac{v}{y} \right) \quad (18)$$

$$M_\theta = -D \left(\frac{v}{y} + v \frac{dv}{dy} \right) \quad (19)$$

$$\delta = \frac{y \sin \alpha \tan \alpha}{Eh} \left(-\frac{d(yQ_y)}{dy} + v Q_y \right) \quad (20)$$

Fig. 4 A flowchart indicating the steps for performing the necessary calculations

Fig. 5 Geometric parameters of the analyzed structure along with corresponding coordinates

We can also derive the in-plane stress resultants (N_y and N_θ)

$$N_y = -Q_y \tan \alpha \quad (21)$$

$$N_\theta = -\frac{dU}{dy} = -\frac{d(r_2 Q_y)}{dy} = -\frac{d(y Q_y)}{dy} \tan \alpha \quad (22)$$

3 Edge Solution of the Structure

In Fig. 4, the overall steps for performing necessary calculations in the edge solution are depicted. In order to calculate the stresses in each part of the structure (cylinder and cone) that

Fig. 6 Equilibrium condition between two parts of the structure

has been depicted in Fig. 5, the authors exploited the edge solution approach. For achieving the edge solution, the boundary conditions should be applied so that the constants of Eqs. (10) and (15) are calculated. Having assumed clamped in both ends of the structure and taken shear forces, H_1 and H_2 , are unknown for cylinder and cone-part, respectively, at the intersection location, and unknown moment at the intersection location, M , which are shown in Fig. 6, the constants C_i for the cylinder and c_i for the cone will be obtained in terms of H_1 , H_2 , and M . Note that the membrane deflections at clamped locations must be added.

Then, equilibrium and compatibility conditions should be satisfied. In order to compute the stresses distribution, the shear force and bending moment must be obtained. The compatibility conditions include:

- (1) the deflection of cone and cylinder at intersection location are equal to one another,
- (2) the slopes of the cone and cylinder at intersection location are the same.

In addition, the equilibrium conditions (equality of forces and moments in intersection location) must be satisfied. Of course, having considered equal and opposite-direction moments in intersection location, the equilibrium of moments are spontaneously satisfied. The equilibrium of forces and compatibility conditions are also written as follows:

$$H_1 + H_2 = 0 \quad (23)$$

$$\delta_{m_{\text{cone}}} + \delta_{b_{\text{cone}}} = \delta_{m_{\text{cylinder}}} + \delta_{b_{\text{cylinder}}} \quad (24)$$

$$v_{m_{\text{cone}}} + v_{b_{\text{cone}}} = v_{m_{\text{cylinder}}} + v_{b_{\text{cylinder}}} \quad (25)$$

Note that the membrane deflections at clamped locations must be added to Eq. (23). Equation (26) calculates the membrane displacement for cylinder, $\delta_{m_{\text{cylinder}}}$

$$\delta_{m_{\text{cylinder}}} = R \varepsilon_\theta = \frac{R}{Eh} (N_{\theta m} - v N_{xm}) = \frac{R}{Eh} \left[PR - v \frac{PR}{2} \right] = \frac{PR^2}{Eh} \left(1 - \frac{v}{2} \right) \quad (26)$$

where $N_{\theta m}$ and N_{xm} are circumferential and axial membrane stresses, respectively. These two quantities are easily calculated by Eqs. (1a) and (1b). In order to calculate the membrane displacement of the cone, Eq. (26) can be used again. But, N_{ym} (the membrane stress resultant along with the cone slant) must be replaced with N_{xm} and the following equations that are corresponded to the cone are used:

$$N_{\theta m} = Py \tan(\alpha) \quad (27)$$

$$N_{ym} = P \tan(\alpha) \frac{y^2 - l_1^2}{y} \quad (28)$$

where l_1 is the distance between the cone apex and the clamped end (Fig. 5). Hence the membrane displacement of the cone is available by Eq. (29)

Table 1 Specified geometrical and material quantities for the structure

Magnitude	Quantity
0.29	Poisson's ratio, ν
0.003	Thickness, h (m)
210,000	Module of elasticity, E (MPa)
0.5	Length of cylinder, l (m)
0.5	Ratio of minimum to maximum radius of cone-part (r/R)
0.6	Internal pressure, p (MPa)
0.3	Cylinder radius (m)

$$\delta_{m_{\text{cone}}} = R\epsilon_{\theta} = \frac{R}{Eh}(N_{\theta m} - \nu N_{ym}) = \frac{PR}{Eh}\tan(\alpha)\left(y - \nu\left(\frac{y^2 - l_1^2}{y}\right)\right) \quad (29)$$

If y is replaced by l_2 (the distance between apex of the cone and the intersection location) in Eqs. (17) and (20), the magnitudes of Bending terms in Eqs. (24) and (25) are obtained. The Bending terms corresponding to the cylinder are obtained as following relationships:

$$\delta_{b_{\text{cylinder}}} = w(x=0) \quad (30)$$

$$V_{b_{\text{cylinder}}} = -D\left(\frac{dw(x=0)}{dx}\right) \quad (31)$$

Note that

$$V_{m_{\text{cylinder}}} = V_{m_{\text{cone}}} = 0 \quad (32)$$

Having rewritten Eqs. (23)–(25), H_1 , H_2 , and M are calculated.

4 Stress Calculation in Cylinder and Cone

In next step, according to shear forces and the moment which have been obtained in Sec. 3, calculation of stresses in both

cylinder-part and cone-part is feasible. Because the governing differential equations of the cylinder and the cone are considered to be homogeneous, the membrane stresses must be added to the general solution as a private solution. Furthermore, two relationships may be derived for inner and outer surfaces because the moment with the specified direction causes tensile and compressible stresses in inner and outer surfaces, respectively. Hence axial stresses for cylinder are

$$\sigma_{x_{\text{internal}}} = \frac{PR}{2h} + \frac{6\nu D}{h^2} \frac{d^2 w}{dx^2} \quad (33a)$$

$$\sigma_{x_{\text{external}}} = \frac{PR}{2h} - \frac{6\nu D}{h^2} \frac{d^2 w}{dx^2} \quad (33b)$$

First expression in equations above is indicative of membrane stresses and second one leads to bending moment of M_x . Circumferential stresses are also calculated by Eqs. (34a) and (34b) for inner and outer surfaces

$$\sigma_{\theta_{\text{internal}}} = \frac{PR}{h} - \frac{Ew}{R} + \frac{6\nu D}{h^2} \frac{d^2 w}{dx^2} \quad (34a)$$

$$\sigma_{\theta_{\text{external}}} = \frac{PR}{h} - \frac{Ew}{R} - \frac{6\nu D}{h^2} \frac{d^2 w}{dx^2} \quad (34b)$$

First expression in equations above is indicative of membrane stresses and second one leads to N_{θ} and third expression calculates stresses caused by M_x .

The stresses along the cone slant are derived by Eqs. (35a) and (35b) for inner and outer surfaces, respectively,

$$\sigma_{y_{\text{internal}}} = \frac{N_{ym}}{h} + \frac{N_y}{h} - \frac{6}{h^2} M_y \quad (35a)$$

$$\sigma_{y_{\text{external}}} = \frac{N_{ym}}{h} + \frac{N_y}{h} + \frac{6}{h^2} M_y \quad (35b)$$

where N_{ym} , N_y , and M_y can be obtained by Eqs. (28), (21), and (18), respectively. Circumferential stresses are also calculated by

Fig. 7 Comparison between analytical and FEM results

Fig. 8 Circumferential stress in cylinder-part in different angles (a) inner surface and (b) outer surface

$$\sigma_{\theta_{\text{internal}}} = \frac{N_{\theta_m}}{h} + \frac{N_{\theta}}{h} - \frac{6}{h^2} M_{\theta} \quad (36a)$$

$$\sigma_{\theta_{\text{external}}} = \frac{N_{\theta_m}}{h} + \frac{N_{\theta}}{h} + \frac{6}{h^2} M_{\theta} \quad (36b)$$

where N_{θ_m} , N_{θ} , and M_{θ} can be obtained by Eqs. (27), (22), and (19).

5 Result and Discussion

In order to solve a typical problem, material and geometrical properties of the structure have been assumed according to Table 1. The angles of the cone-part were specified so that the effect of angle on stresses magnitudes can be investigated. The ratio of minimum to maximum radius of cone-part is assumed to be constant, therefore, the height of the cone-part changes with apex angle variation (in rad).

To verify the results by numerical approach, Ansys, as commercial finite element software, is exploited. Accordingly, element shell208 that is suitable for modeling of axisymmetric shells is

used to build an FE element. Boundary conditions at both ends of the model are exerted based on the prescribed ones described in Sec. 4. Figure 7 shows the comparison between analytical method results and FEM results. This figure shows some discrepancies exist between the analytical and numerical results in edge solution (less than 20%). It is discussed by Timoshenko and Woinowsky-Krieger [10] that the general solution for conical shell has some simplification and may be inaccurate near the boundaries. This inaccuracy affected both end of conical shell and one end of cylindrical shell near the intersection. However, the edge solution yields to higher stresses and the design by using this method is in safe side.

Figures 8(a) and 8(b) show the change of the circumferential stresses along the cylinder length in four apex angles. As being further from the intersection and boundary locations, the stress approaches to membrane one, i.e., pR/h . The absolutely maximum circumferential and longitudinal stresses occur in the vicinity of intersection or boundary location. For instance, the absolutely maximum circumferential and longitudinal stresses coincide the intersection location exactly in $\alpha=0.5$ or a point very near to intersection location in $\alpha=0.3$. For $\alpha=0.1$, the effects of the

Fig. 9 Longitudinal stress in cylinder-part in different angles (a) inner surface and (b) outer surface

Fig. 10 Circumferential stress in cone-part in different angles (a) inner surface and (b) outer surface

Fig. 11 Longitudinal stress in cone-part in different angles (a) inner surface and (b) outer surface

boundary and intersection locations on the absolutely maximum circumferential and longitudinal stresses are the same and both the vicinity of the intersection and boundary locations are critical. Generally, the intersection location is more important for larger apex angle in inner and outer surfaces of cylinder-part. The reverse of this matter is true for smaller apex angles.

The longitudinal stresses in cylinder-part have been shown in Figs. 9(a) and 9(b) for four apex angles. Approaching to membrane stress, i.e., $pR/2h$ can be seen by being further from the intersection and boundary locations. Other results about the maximum circumferential stresses are acceptable for longitudinal ones.

The results of the cone-part have been shown in Figs. 10 and 11 for those specified apex angles. In addition to a linear region as the membrane stresses in the cone results, two nonlinear regions that lead to intersection and boundary location is distinguishable. In contrast to linear regions in inner and outer surfaces, nonlinear regions are different in both inner and outer surfaces. The circumferential stresses in inner and outer surfaces of the cone-part along the cone radius variation can be seen in Figs. 10(a) and 10(b), respectively. Absolutely maximum inner circumferential stress is

located in the intersection location ($\alpha = 0.3, 0.5$) or close to the intersection location ($\alpha = 0.1, 0.2$). But in outer surface, it always comes up in the vicinity of intersection location. Figure 11 shows the longitudinal stresses in inner and outer surfaces of the cone-part along the cone radius variation. It can be seen that the absolutely maximum circumferential stress magnitudes occur in boundary ($\alpha = 0.1, 0.2, 0.3$) or in the vicinity of the intersection location ($\alpha = 0.5$). The absolutely maximum outer longitudinal stress is always in the vicinity of the intersection location. Note that the abscissa axes in Figs. 10 and 11 have been measured in terms of cross-section radius changes.

In case of assuming the same material behavior in compression and tension and using maximum normal stress criterion, the critical point in the structure can be distinguished. In Table 2, the absolutely maximum circumferential and longitudinal stresses in inner and outer surfaces for four prescribed angles have been shown. In cone-part, the outer longitudinal stress is always maximal. But maximum stress in the cylinder-part depends on the apex angle. In addition, a location of cylinder-part always becomes the critical point of the structure in four prescribed angles.

Table 2 Absolutely maximum of stresses (MPa) for different apex angle of the cone

Cone-part				Cylinder-part				Angle
Longitudinal stress		Circumferential stress		Longitudinal stress		Circumferential stress		
Inner	Outer	Inner	Outer	Inner	Outer	Inner	Outer	
59	75	65	70	77	145	70	65	0.1
59	112	65	82	100	145	75	67	0.2
60	144	65	93	135	145	110	185	0.3
77	212	118	130	225	165	220	255	0.5

The distance where the effects of the intersection and boundary locations are noticeable, depends on the geometry with the same material. It has been seen that with increase of the apex angle as a geometrical parameter focused on this study, the effects of the intersection location (deviation from membrane stresses) project in further length of the cylinder- and cone-parts. As a typical example, see Figs. 10 and 11 and compare stress distribution in $\alpha = 0.5$ and $\alpha = 0.1$. As it can be seen, tiny length of the cone is under the effect of the intersection location in $\alpha = 0.1$. But, in $\alpha = 0.5$, significant length of the cone is affected by the intersection and boundary locations. In addition, the distance of boundary location affection in cone-part depends on the apex angle, but it is constant for cylinder-part in different apex angles (Figs. 8 and 9).

6 Conclusions

In this study an analytical solution for elastic thin-walled cylinder-truncated cone shell intersection under internal pressure is presented. Although the different intersection problems had been solved by the various methods, due to complicity of the general theory in cone, this problem has not been considered so far. The authors used edge solution to calculate stresses in circumferential and longitudinal directions. The results show that the stresses in vicinity of the intersection location can be exceeded even more than two times of the membrane stresses in some apex angle of the cone. This emphasizes on the effects of the apex angle of the cone in diversion from the membrane theory prediction.

Nomenclature

D	= flexural stiffness
E	= Young modulus
h	= thickness
H_1	= shear force in interface at cone-segment
H_2	= shear force in interface at cylinder-segment
i	= complex number $\sqrt{-1}$
l_1	= distance between the cone apex and the clamped end
M_x	= bending stress resultant along x
M_θ	= bending stress resultant along θ
M_φ	= bending stress resultant along φ
$M_{\varphi\theta} = M_{\theta\varphi}$	= torsional stress resultants
N_y	= extensional stress resultant along y
N_θ	= extensional stress resultant along θ
N_φ	= extensional stress resultant along φ
$N_{\varphi\theta} = N_{\theta\varphi}$	= inplane shear stress resultants
P	= pressure
Q	= shear force per unit thickness in cylinder
Q_y	= out of plane shear stress resultant along y

Q_θ	= out of plane shear stress resultant along θ
Q_φ	= out of plane shear stress resultant along φ
r_0	= distance of element from symmetric axis
r_1	= curvature radius of the element
R	= cylinder radius
v	= displacement along Y
v_b	= bending gradient slope
v_m	= membrane gradient slope
w	= deflection
x	= cylinder length
X	= components of applied force per unit area in x direction
y	= cone slant
Y	= components of applied force per unit area in y direction
z	= thickness direction
Z	= components of applied force per unit area in z direction
α	= half of apex angle
δ_b	= bending deflection
δ_m	= membrane deflection
σ_x	= stress in x direction (along cylinder length)
σ_y	= stress in y direction (along cone slant)
σ_θ	= stress in θ direction

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