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Comparison of Parallel MRI Reconstruction Methods for Accelerated 3D Fast Spin-Echo Imaging

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Abstract

Parallel MRI (pMRI) achieves imaging acceleration by partially substituting gradient-encoding steps with spatial information contained in the component coils of the acquisition array. Variable-density subsampling in pMRI was previously shown to yield improved two-dimensional (2D) imaging in comparison to uniform subsampling, but has yet to be used routinely in clinical practice. In an effort to reduce acquisition time for 3D fast spin-echo (3D-FSE) sequences, this work explores a specific nonuniform sampling scheme for 3D imaging, subsampling along two phase-encoding (PE) directions on a rectilinear grid. We use two reconstruction methods—2D-GRAPPA-Operator and 2D-SPACE RIP—and present a comparison between them. We show that high-quality images can be reconstructed using both techniques. To evaluate the proposed sampling method and reconstruction schemes, results via simulation, phantom study, and in vivo 3D human data are shown. We find that fewer artifacts can be seen in the 2D-SPACE RIP reconstructions than in 2D-GRAPPA-Operator reconstructions, with comparable reconstruction times.

Keywords

parallel imaging; nonuniform subsampling; 3D MRI; GRAPPA; SPACE RIP

In the field of MRI, the goal is to observe a medically significant projection of the state of the human body in a reasonable amount of time. In recent years, the concept of parallel imaging has enabled great increases in the speed of MRI. It is based on the principle of using several phased-array coils, each with different RF sensitivity profiles to provide independent spatial information about the image, to reduce the number of traditionally required phase-encoding (PE) steps. Several parallel imaging techniques (1–7) have been

developed, though to date only sensitivity encoding (SENSE) (3) and generalized autocalibrating partially parallel acquisitions (GRAPPA) (7) have seen wide commercial acceptance. In most clinical implementations, these techniques subsample k -space uniformly due to reconstruction efficiency considerations.

Several features make parallel imaging very attractive for three-dimensional (3D) imaging. First, one may sub-sample along both PE directions, resulting in high image acceleration rates distributed across two data dimensions (8). Second, distributing the reduction factor along two PE directions can result in better image quality and improved encoding efficiency when compared to subsampling along only one PE direction (9). Finally, the signal-to-noise ratio (SNR) in 3D acquisitions is considerably higher than in 2D; therefore, the SNR penalty that occurs from applying parallel imaging can be better tolerated (8–10).

The first 3D parallel imaging technique reported was an extension of the Cartesian SENSE method to two PE directions and was termed 2D SENSE (8). It used the SENSE algorithm to unalias pixels overlaid from both PE directions. Subsequently, Breuer et al. (9,11) presented “controlled aliasing in parallel imaging results in higher acceleration” (CAIPIRINHA) to reduce the appearance of aliasing artifact by using either a specially modulated RF pulse or shifted 2D PE patterns. Both 2D SENSE and CAIPIRINHA used integer reduction factors in each PE direction and required a uniform subsampling pattern. They also required the acquisition of an additional low-resolution data set to estimate the sensitivity profile on the object of interest, thereby reducing the net image acceleration factor. In certain imaging scenarios, the coil profiles can change, e.g., due to unwanted movement of the patient. The resulting mismatch in coil sensitivity between the prescan and imaging scan may lead to artifacts.

In contrast, self-referenced reconstruction methods do not need a separate prescan to estimate coil sensitivity profiles. This approach is exemplified by the method of Blaimer et al. (10), who proposed a k -space-based reconstruction of subsampled 3D parallel imaging data using the 2D-GRAPPA-Operator approach. In this method, the missing k -space data points are reconstructed by applying two separate 1D-GRAPPA reconstruction steps along the two PE directions, such that data resulting from the first reconstruction step are used in the second reconstruction step. While this approach is fast and effective, it may yield residual aliasing due to propagation of errors, and could lead to substantial noise amplification. Moreover, GRAPPA is not optimal in the least-squares sense, and typically shows larger high-spatial-frequency errors than comparable SENSE methods (12).

The appearance of artifacts and effective image resolution in reconstructed parallel MR images is directly dependent on the subsampling pattern employed. It was shown in the initial “sensitivity profiles from an array of coils for encoding and reconstruction in parallel” (SPACE RIP) (5) analysis that a nonuniform distribution of k -space lines with dense subsampling in the center of k -space and sparse subsampling in the outer k -space provides less visible artifacts than in uniformly distributed subsampling. This finding was further supported by Tsai et al. (13). A similar finding was also recently reported for 1D non-Cartesian pMRI using a GRAPPA reconstruction approach (14).

When considering nonuniform subsampling in parallel imaging, one must balance two competing concerns. On the one hand, variable-density subsampling allows one to reduce the visible coherence of aliasing artifacts in the reconstructed image due to an ill-defined field of view (FOV). On the other hand, there exists an inherent tradeoff between sampling densely at the center of k -space, in order to capture lines with greater energy concentration and higher SNR, and sampling evenly throughout k -space to provide the best resolution.

In this work we propose a nonuniform subsampling scheme applicable to 3D imaging, and apply it to the 3D fast spin echo (3D-FSE) pulse sequence. This sampling scheme is amenable to reconstruction using either GRAPPA-class or SENSE-class algorithms. FSE sequences are attractive for clinical imaging. However, 3D-FSE is not routinely used, primarily due to the long scan time needed, and although multislabs 3D-FSE pulse sequences are generally provided on commercial scanners, they are often contaminated by artifacts at the junctions between slabs (15). Reducing the acquisition time of 3D-FSE without negatively impacting image quality would certainly be highly relevant for many clinical applications. In this work we show that a carefully chosen nonuniform sampling pattern along both PE directions can yield artifact-free accelerated 3D-FSE images. Since both GRAPPA and SPACE RIP can use nonuniform subsampling along both PE directions, we conducted reconstructions with both techniques using the same acquisition paradigm and performed comparison studies to evaluate their performance. Results via simulation, a phantom study, and in vivo 3D human experiments are presented.

THEORY

2D Parallel Imaging

The concept of parallel imaging is based on using multiple receiver coils, with each element providing independent information about the image. The MR signal received in each coil, l , can be described by the following imaging equation:

$$s_l(k) = \int_V W_l(r) \rho(r) e^{j2\pi k \cdot r} dr \quad [1]$$

Where $\rho(r)$ denotes the excited volume image proton density, r is a vector describing the spatial position within the FOV, $W_l(r)$ is the sensitivity profile of coil l at position r , and k represents a reciprocal spatial term determined by the employed encoding gradients.

Nonuniform Sampling

In contrast to uniform sampling trajectories, the distance between measured adjacent k -space lines in nonuniform sampling trajectories is no longer constant. Most typically, nonuniform sampling trajectories sample the central part of the trajectory at the Nyquist rate and then monotonically reduce the sampling density in the outer parts of the trajectory. We start here with a simple symmetric exponential function:

$$Z_{2d}(c_y) = e^{\beta |c_y|}, \quad [2]$$

Where β is a function shaping parameter that controls the ascent of the exponential function. The slope of this function varies at different c_y points, that is, the slope near the coordinate

space origin is steeper than points farther away. Working from this exponential function, one can smoothly change between uniform ($\beta=0$) to nonuniform ($\beta \neq 0$) subsampling by varying a single parameter.

With k -space coverage from $-M+1$ to M (where M is the maximum k -space resolution, e.g., 1/2 the full span of k -space points acquired) and a reduction factor, R , we convert the exponential sampling distribution function, Z_{2d} , to a set of k -space line indices for subsampled parallel MR acquisition as follows: Since the nonuniform trajectory is symmetric around DC, only the positive part of the trajectory is discussed here. To generalize the function for all values of β , we modify the density function to ensure that the output is in the range $f_1(c_i) \in [0, M]$ for an output range of $c_i \in [0, 1]$. This yields:

$$f_1(c_i) = M \frac{e^{\beta c_i} - 1}{e^{\beta} - 1} \quad [3]$$

The effect of varying β for $f_1(c_i)/M$ is shown in Fig. 1a, where a plot of the distribution is shown for multiple values of β .

To support self-referenced acquisitions, we further ensure that a Nyquist-rate sampled region of width $2W$ is present at the central part of k -space. With the inclusion of these central k -space lines, the remaining span of k -space is determined by sampling the exponential function f_1 uniformly between the maximum extent of the central window and the highest k -space point desired. Specifically, the c_i point in f_1 that corresponds to the maximal point of the calibration window is determined:

$$c_w = \log \left((W/M)^* (e^{\beta} - 1) + 1 \right) / \beta \quad [4]$$

Then f_1 is sampled uniformly between c_w and 1 to determine the remaining phase-encode points. The distance between each c_i is determined as

$$\delta = (1 - c_w) / (M/R - W) \quad [5]$$

and the remaining k_y points are selected as

$$k_y(i) = f_1(c_w + \delta^*(i - W)), i = \{W+1, W+2, \dots, M/R\} \quad [6]$$

In this fashion, we have created a function to map uniform sampling of our exponential function, Eq. [2], to a set of exponentially weighted phase-encode sampling points. Figure 1b shows an example function and the mapping from uniform to nonuniform sampling for the case of $\beta=1.2$ and $W=4$, at an acceleration factor of $R=4$ across a span of 256 ($M=128$) phase-encodes.

We can extend Eq. [2] to a 2D exponential function as:

$$Z_{3d}(c_y, c_z) = e^{\beta_y |c_y| + \beta_z |c_z|} \quad [7]$$

To facilitate nonuniform subsampling within the particular constraints of the 3D-FSE sequence employed in this study, the c_y and c_z components of this separable 2D exponential

function are treated separately to determine the particular phase-encode points. This allows the number of echoes, or echo train length (ETL), to be the same for each excitation, while maintaining a 2D sampling pattern that is dense near the origin of k -space—to enable self-referenced coil sensitivity estimation and capture greater energy points for reduced visible artifacts—and sparse at the highest sampled frequencies to ensure good resolution. First, a 1D exponentially weighted pattern is calculated along the horizontal dimension, k_z . Then, for each k_z point, a second 1D pattern is calculated along k_y . In our proposed scheme, the sampling sparseness varies in a sinusoidal fashion along the k_y direction at this k_z position. Figure 2 shows such a distribution of sample points for $\beta_z = 1.2$, and

$\beta_y(k_z) = 1.1 \times \sin\left(\frac{\pi \times (k_z + N/2)}{N}\right) + 0.1$, $k_z \in [-N/2, N/2]$ where N is the total number of PE lines along the k_z direction. The modulation of β_y by a sine function in this way enables a sampling pattern that varies between a nearly flat sampling pattern ($\beta_y(\pm N/2) = 0.1$) at the high-frequency k -space coordinates and an exponentially weighted pattern ($\beta_y(0) = 1.2$) at low-frequency k -space coordinates. The figure shows acceleration factors of $2.0\times$ and $2.5\times$ along the k_y and k_z directions, respectively, with a central region of 16×16 sample points at the low frequency of k -space. This box of $U \times V$ sample points at the low frequency of k -space is used for coil sensitivity estimation in 2D-SPACE RIP and for autocalibration signal (ACS) data in 2D-GRAPPA-Operator. We note that this sampling pattern effectively enables self-referenced coil sensitivity estimates, eliminating the need for a separate prescan step.

Image Reconstruction Techniques

2D-SPACE RIP Reconstruction in Hybrid Space—One can take a 1D Fourier transform of Eq. [1] along the frequency-encoded direction, x , when phase encoding along k_y and k_z directions to recast the equation as:

$$s_l(k_y, k_z, x) = \iint \rho(r) W_l(r) e^{j2\pi(k_y y + k_z z)} dy dz \quad [8]$$

which is the phase-modulated projection of the sensitivity-weighted volume signal onto the x -axis. Equation [8] can be discretized along both y and z directions and written as follows:

$$s_l(k_y, k_z, x) = \sum_{n=1}^N \sum_{m=1}^M \rho(n, m, x) W_l(n, m, x) e^{j2\pi(k_y n + k_z m)} \quad [9]$$

By concatenating Eq. [9] for each respective coil, l , the SPACE RIP linear system can be recast in a compact matrix form (16):

$$\begin{pmatrix} \text{vec}\{s_1(:, :, x)\} \\ \text{vec}\{s_2(:, :, x)\} \\ \vdots \\ \text{vec}\{s_L(:, :, x)\} \end{pmatrix} = \begin{pmatrix} (Q^H \otimes P^H) \text{diag}\{\text{vec}\{W_1(:, :, x)\}\} \\ (Q^H \otimes P^H) \text{diag}\{\text{vec}\{W_2(:, :, x)\}\} \\ \vdots \\ (Q^H \otimes P^H) \text{diag}\{\text{vec}\{W_L(:, :, x)\}\} \end{pmatrix} \cdot \text{vec}\{\rho(:, :, x)\} \quad [10]$$

where $\otimes$ refers to the Kronecker product, $\text{vec}\{\square\}$ denotes the reshaping of a matrix to a column vector, and $\text{diag}\{\square\}$ creates a diagonal matrix. If the number of phase-encodes used

along the y direction is F , the number of phase-encodes used along the z direction is G , then the matrix sizes of P^H and Q^H are $F \times N$ and $G \times M$. The rows of P^H and Q^H correspond to the exponential terms in Eq. [9], $P(n,k) = e^{-j2\pi k y n}$, $Q(m,y) = e^{-j2\pi k z m}$. The term on the left side of Eq. [10] is an $L \times F \times G$ element vector containing the $F \times G$ PE values for all coils. The middle term in Eq. [10] is a matrix with $L \times F \times G$ rows and $N \times M$ columns, which are constructed from the coil sensitivity estimates and phase-encodes used. The right term represents a reshaped image of size $N \times M$, representing a plane in the volume of the FOV at position x along the frequency-encoded direction. Solving Eq. [10] for each position along the x -axis yields a plane-by-plane reconstruction of the volume, whereby each plane in the volume is reconstructed separately. That is, one must solve J linear systems in order to reconstruct J planes of the volume. Image quality depends on acquiring a sufficient number of phase-encode lines. However, as one increases the number of phase-encodes, a larger system matrix results that imposes a higher computational cost.

To reconstruct an image, the signal model in Eq. [10] can be formed as a linear system of equations:

$$s = \mathbf{P} \cdot \rho \quad [11]$$

As with any linear system, one can alternatively recast the reconstruction as a minimization problem:

$$\rho = \arg \min_{\rho} \left\{ \|\mathbf{P}\rho - s\|_2^2 \right\} \quad [12]$$

with the possibility of using regularization methods:

$$\rho = \arg \min_{\rho} \left\{ \|\mathbf{P}\rho - s\|_2^2 + \lambda^2 \|\rho\|_2^2 \right\} \quad (13)$$

This minimization perspective is consistent with iterative reconstruction techniques, such as conjugate gradient least-squares (CGLS) or least square QR-factorization (LSQR), which eliminates the need to explicitly invert the system matrix given in Eq. [10]. Furthermore, as the encoding model employs Fourier transforms, illustrated by P and Q in Eq. [10], one can rapidly compute the matrix-vector product described by Eq. [10] without explicitly forming the encoding matrix operator (17,18).

2D-GRAPPA-Operator Reconstruction in k -Space

The conventional 1D-GRAPPA (7) algorithm performs an individual reconstruction for each coil in the array, and the final image is constructed by combining the separate coil data using optimal SNR approaches (e.g., sum of squares). The foundation of GRAPPA is to estimate the 2D convolution that occurs in the Fourier domain between the coil sensitivity and excited spin distribution (19). This is accomplished by using multiple k -space lines from multiple coils to fit a set of k -space points to designated ACS, which are typically located at the low-frequency coordinates of k -space in each component coil. For a reduction factor of R

using an $N_x \times N_y$ kernel, the linear equation to determine the coil weights can be represented as:

$$s_j^{ACS}(i) = \sum_{l=1}^L \sum_{b=0}^{N_y-1} \sum_{c=-(N_x-1)/2}^{(N_x-1)/2} d(l, b, c, j) s_l(c+k_x, \text{ceil}(R(b - (N_y - 1)/2) + (N_y \% 2)/2) + k_y) \quad (14)$$

Here the summation is performed over each of the coils, l , using N_x points along k_x and N_y points along k_y . The elements of the vector s_j^{ACS} are taken from data points within the ACS lines. Vector d is the coil weights of the target coil j , $\text{ceil}()$ means rounding symbolic toward positive infinity, and $\%2$ is modulo 2. Once coil weights are calculated, these parameters are employed in a similar reconstruction equation to estimate the missing k -space data, which have a k -space shift of $m \ k_y$ within each coil:

$$s_j(:, k_y + m\Delta k_y) = \sum_{l=1}^L d_{l,j} s_l(:, k_y) \quad [15]$$

The 2D-GRAPPA-Operator method (10) used in 2D parallel imaging reconstruction is an extension of 1D-GRAPPA to 3D imaging. It uses the GRAPPA operator formalism (20) to divide the 2D GRAPPA reconstruction into two separate 1D reconstructions and performs GRAPPA reconstruction along two PE directions sequentially in order to result in improved image quality over 2D-GRAPPA. That is, after reconstruction of the missing data in the k_x - k_y plane at each acquired k_z position, it performs 1D-GRAPPA to reconstruct the missing data in k_x - k_z plane at each k_y position:

$$\begin{aligned} \text{for } k_z = 1, 2 \dots N_z, \bar{s}_j(:, k_y + m\Delta k_y, k_z) &= \sum_{l=1}^L d_{l,j} s_l(:, k_y, k_z) \\ \text{for } k_y = 1, 2 \dots N_y, \bar{s}_j(:, k_y, k_z + n\Delta k_z) &= \sum_{l=1}^L d_{l,j} s_l(:, k_y, k_z) \end{aligned} \quad [16]$$

Here, N_z is the number of acquired PE lines along k_z , and N_y is the number of total PE lines along k_y .

MATERIALS AND METHODS

Implementation Details

The PE strategy for 3D-FSE is with in-plane PE (k_y) in the inner loop of the echo train and through-plane PE (k_z) in the outer loop between the echo trains. The profile order of the in-plane PE is center-out ordering, which means that low values of k_y lines are acquired in the early echo train echoes and high values of k_y lines are acquired in the later echo train echoes. Full-FOV 3D uniform-phantom and T_2 -weighted 3D fast recovery fast spin-echo sequence (3D FRFSE) whole-brain acquisitions both with and without subsampling were performed on the head of a 25-year-old male volunteer in a GE Signa EXCITE 3.0T scanner (GE Medical Systems, Milwaukee, WI, USA) equipped with a standard eight-channel head array coil. Written informed consent, according to the guidelines of the hospital internal review board, was obtained from the volunteer before the study. With the volunteer lying supine in

the scanner, the frequency-encoding direction was superior–inferior (X) and the two PE directions were anterior–posterior (Y) and left–right (Z). In order to minimize echo spacing and reduce image blurring, hard refocusing RF pulses were applied (21). During the accelerated scan, a block of fully sampled data points, as specified previously, near the origin of k -space were obtained to compute self-referenced coil sensitivity profiles.

Doped water phantom and in vivo whole-brain experiments were performed with PE along Y and Z , $FE \times PE1 \times PE2 = 256 \times 192 \times 160$, $FOV = 24.0 \times 24.0 \times 20.8 \text{ cm}^3$, receiver bandwidth = 31.25 kHz, echo spacing = 7.6ms, number of excitations (NEX) = 1. The acquisition parameters of the acquired data were: (1) full-FOV nonaccelerated phantom scan: TR/TE = 2000/35 ms, ETL = 48, acquisition time (min:s) = 21:24; (2) accelerated phantom scans with reduction factor $R = R_y \times R_z = 2 \times 2.5$, 2.5×2.5 , and 3×3 : TR/TE = 2000/35 ms. Each scan was repeated 18 times with the same acquisition parameters to enable SNR calculations; (3) T_2 -weighted sagittal full-FOV nonaccelerated whole-brain scan: TR/TE = 2600/200 ms, ETL = 48, acquisition time (min:s) = 27:49; (4) T_2 -weighted sagittal full-FOV accelerated whole-brain scan: TR/TE = 2600/200 ms, reduction factors along PE1 (R_y) = 2, reduction factor along PE2 (R_z) = 2.5, ETL = 48, acquisition time (min:s) = 5:38.

All raw data were saved and parallel image reconstructions were performed offline in Matlab (The Mathworks, Natick, MA, USA) on a laptop computer (Intel Core Duo Processor T2400, 1.83GHz, 2GB RAM). Simulated accelerated acquisitions were performed by subsampling the full 3D phantom data and T_2 -weighted unaccelerated 3D full data set using the nonuniform sampling along two PE directions paradigm described above.

Reconstruction Details

2D-SPACE RIP

Coil sensitivity estimation: 2D-SPACE RIP requires an estimate of the coil sensitivity profiles used to encode spatial position. In our work, sensitivity profiles are computed using a self-calibrated estimation method (22) in which a central region of k -space is sampled at the Nyquist rate to supply the coil calibration data needed for the image reconstruction process. We filtered the fully sampled low-frequency k -space data and used them to reconstruct the final high-resolution image. In this case, data within a $U \times V$ box covering the lowest frequency coordinates in k -space were used to estimate the coil sensitivity maps. A Gaussian envelope was applied to these data to limit Gibbs ringing associated with a truncated Fourier series. The coil sets were then normalized in the spatial domain.

CGLS: Following the estimation of coil sensitivity, the image reconstruction used the CGLS iterative method (23). This algorithm solves the least-squares estimate by iteratively solving the normal equation form of Eq. [11]:

$$\mathbf{P}_s^H = \mathbf{P}^H \mathbf{P} \rho, \quad [17]$$

where $(.)^H$ indicates a complex-conjugate (a.k.a. Hermitian) transpose. The CGLS algorithm can be described as solving the projected, or normal equation, form of Eq. [13]:

$$\rho = \arg \min_{\rho} \left\{ \left\| \left(\mathbf{P}^H \mathbf{P} + \lambda_{CGLS} I \right) \rho - \mathbf{P}^H s \right\|_2 \right\} \quad [18]$$

where λ_{CGLS} is a regularization term. When λ_{CGLS} is 0, CGLS is identical to Hestenes and Stiefel's CG algorithm for least-squares problems (24). One sees from Eq. [18] that as λ_{CGLS} is increased, $\|\rho\|_2$ decreases and the residual error $\|r\|_2 = \|s - \mathbf{P}\rho\|_2$ increases. In each CGLS loop, the most time-consuming matrix-vector multiplication was replaced with a sequence of more efficient operations through the use of the FFT (18). When solving Eq. [18], one may restrict the number of iterations or specify a threshold ε for CGLS to halt. The iteration numbers in CG effectively act as a regularization parameter (25). Figure 3 shows a plot of the residual norm vs. iteration numbers in our experiments. It shows a typical convergence of the reconstruction and demonstrates that 15 iterations provide a reasonable loop count. We implemented our reconstruction algorithm with a fixed scalar λ_{CGLS} for the regularization parameter in Eq. [18] such that the residual error was 0.1% after 15 iterations. Alternatively, one can use the LSQR-Hybrid-based method to efficiently generate a number of solutions for a range of λ_{CGLS} values and determine a good regularization parameter value using L-curve or discrepancy techniques (26).

2D-GRAPPA-Operator—To evaluate a k -space reconstruction method for our proposed subsampling approach, the 2D-GRAPPA-Operator was chosen due to ease of implementation and a reconstruction time comparable to the 2D-SPACE RIP method described previously. The Nyquist-rate sampled data in the center block of k -space supplied the ACS data needed to calculate the GRAPPA reconstruction parameters. Because of nonuniform subsampling, the local acceleration between acquired k -space lines varied at different locations in k -space. We identified a separate set of GRAPPA reconstruction parameters for each local 1D acceleration factor in the sampling pattern. In this way, the missing k -space points were reconstructed in between two acquired source points until k -space fulfilled the Nyquist criterion everywhere. A number of kernel sizes were evaluated, and below we present reconstructions using a 2×3 kernel for computing coil weights in each of the two separate 1D-GRAPPA reconstruction steps. We found that this kernel size provides good images within a reasonable reconstruction time. Alternate kernels, e.g., a 2×5 , gave only slightly better image reconstructions but came at a cost of nearly triple the reconstruction time.

Quantification

Artifact Power—To quantify the performance of our proposed method, the artifact power (27) was calculated for different nonuniform subsampling trajectories and reconstruction approaches. It is calculated based on the simulated 2D pMRI images and the fully sampled reference images by the following equation:

$$\text{Artifact Power (AP)} = \frac{\sum_{j=1}^N \|I_j^{\text{reference}}\| - \|I_j^{\text{recon}}\|^2}{\sum_{j=1}^N \|I_j^{\text{reference}}\|^2} \quad [19]$$

where j is the pixel index, N is the number of voxels, $I^{reference}$ is the reference image, and I^{recon} is the reconstructed image. A higher artifact power (AP) shows increased aliasing artifacts and reduced image quality.

SNR

Because the proposed nonuniform subsampling trajectories will reduce scan time but also reduce the SNR in the reconstructed images, we next present an SNR analysis of each reconstruction method. The SNR measurement followed the dual-acquisition method in Ref. 28. We acquired a series of 18 consecutive scans using the same acquisition parameters, and then reconstructed images using both 2DGRAPPA-Operator and 2D-SPACE RIP. We used neighboring images in the sequence to form 17 matched pairs. For each pair, the average of the signal intensity between two images and the standard deviation of the difference image were used to calculate SNR in five 30×30 pixel regions of interest (ROIs).

Point Spread Function

It is generally accepted that nonuniform subsampling trades a decrease in resolution for a reduction in visible artifacts. One method to evaluate the reconstruction performance is to visualize/analyze the point spread function (PSF). The reconstructed image can be interpreted as a convolution of the (true) object and the PSF. We use 2D-SPACE RIP to do the reconstruction and evaluate the PSFs to understand the performance of our proposed nonuniform subsampling trajectory.

Robson et al. (29) suggested that the PSF of SENSE reconstructions could be estimated using an explicit calculation of the forward model pseudo-inverse. This is impractical in 3D imaging, as the linear system associated with the reconstruction problem is quite large (e.g., $24,5760 \times 30,720$ here). Thus, we employed an alternate approach to estimate the effective PSF produced by the acquisition/reconstruction process. We first estimated coil sensitivity profiles from a doped water phantom data set. Then we constructed a phantom image object with a unity value at the center pixel and zero values at all other pixels. A multicoil k -space data set was then generated by Fourier-transforming the element-by-element product between the estimated coil sensitivities with this delta-point object. Accelerated acquisitions were then simulated by subsampling these k -space data at an acceleration factor of $R = R_y \times R_z = 3 \times 2$. We shaped the subsampling pattern by varying the values of β from a uniform distribution ($\beta = 0$) to two different exponential distributions ($\beta = 1.2$ and $\beta = 2.4$). Each exponential weighting subsampling had a central box of 16×16 . These subsampled data were then reconstructed using the 2D-SPACE RIP technique, which employed CGLS for 15 iterations. The resulting images were then compared with the original delta-point image to produce a PSF estimate.

RESULTS

Simulations

In order to establish that enhanced reconstructions can be obtained in 3D parallel imaging when nonuniform sub-sampling along two PE directions is performed, our first results simulated exponentially weighted subsampling along two directions from the full k -space

3D data sets. The subsampled data were subsequently reconstructed using both the 2D-GRAPPA-Operator and 2D-SPACE RIP algorithms as described above. Various subsampling patterns (see Table 1) with different acceleration factors and center box reference data were simulated in the 3D k -space. Simulation results from both doped water phantom images and brain images are presented.

Figure 4 shows one slice of the phantom along the readout direction and reconstructed images simulated from the five different accelerated acquisition schemes in Table 1 by using 2D-SPACE RIP and 2D-GRAPPA-Operator reconstruction strategies. Images shown from left to right are reconstructed images from down-sampling schemes A–E, respectively. The difference between each reconstructed image and the reference image is shown below the reconstructed image using the same grayscale window. These images show that the 2D-SPACE RIP reconstruction method produces images with less artifact and noise amplification than the 2D-GRAPPA-Operator method. The AP is shown in Fig. 5. This figure illustrates that 2D-SPACE RIP yields less residual reconstruction errors than 2D-GRAPPA-Operator when using the same subsampling pattern, even when comparing a high reduction factor in 2D-SPACE RIP with a low reduction factor in 2D-GRAPPA-Operator.

The results from a healthy volunteer study are shown in Fig. 6. The reference image (axial direction frame 100) from the complete data is shown in Fig. 6a. Images shown from left to right are reconstructed images from down-sampling schemes A–E of Table 1, respectively. Below the reconstructed images are the difference images between the images reconstructed from subsampled data and the reference image. Consistent with the phantom simulation, the image reconstructed by 2D-SPACE RIP has improved image quality (reduced noise and less aliasing artifact) relative to the images reconstructed using the 2DGRAPPA-Operator implementation.

To quantitatively estimate reconstruction error, the AP is shown in Fig. 7. The result from this data set shows that the 2D-SPACE RIP reconstruction method outperforms the 2D-GRAPPA-Operator method, consistent with the water phantom experiment, although the difference between the measured AP is not as great as for the homogeneous phantom.

Figure 8 shows the ROIs (I–V) and noise maps of two consecutive images reconstructed by both 2D-GRAPPA-Operator and 2D-SPACE RIP. The final SNR for each of the ROIs shown in Table 2 was calculated from the average of the 17 pairs. Although the calculated SNR varies in the different ROIs, 2D-SPACE RIP consistently shows better SNR than the 2D-GRAPPA-Operator reconstruction method.

Figure 9 shows PSFs corresponding to different values of β . Both a 2D view of the PSF and two 1D central PSF profiles of each PE plane are shown. It can be observed that the artifact level decreases as the β increases, but as is apparent from Table 3, the full width at half maximum (FWHM) increases, which implies a loss in resolution. This demonstrates that the value of β can be used to balance the trade-off between artifact level and image resolution. In our brain image simulation experiment, the resolution is highest for $\beta_y = \beta_z = 0$, but it also demonstrates unacceptable aliasing artifact. With increasing β , the aliasing artifacts diminished at the expense of resolution. So this PSF analysis matches our simulation

experiments. We note that $\beta = 1.2$ offers a good balance between aliasing artifact suppression, with a minimal increase in the width of the PSF as measured by the FWHM.

In Vivo Studies

The advantages of nonuniform subsampling along two PE directions demonstrated in the simulated experiments were tested experimentally with an in vivo study. A product 3D-FSE General Electric (GE) pulse sequence was modified in order to allow for flexible subsampling along both PE directions. Some user-defined inputs, e.g., R_y , β_y , R_z , β_z and center window width, were added that resulted in the selection of the phase-encodes acquired along both the y and z directions according to Fig. 2. Data were acquired at an acceleration of $5\times$ with the parameters $R_y = 2$, $R_z = 2.5$, $\beta_y = \beta_z = 1.2$, 16×16 total ACS lines, on a 3T clinical scanner (GE Medical Systems, Milwaukee, WI, USA) equipped with the standard eight-channel head array coil. Figure 10 shows the in vivo 2D parallel imaging results. As can be seen, comparable image quality to the reference images was obtained. As expected, the SNR in the final reconstructed images was reduced due to decreased scan time. It is clear from the figure that the remaining foldover artifacts are better suppressed in the 2D-SPACE RIP image than in the 2D-GRAPPA-Operator reconstructions.

DISCUSSION AND CONCLUSIONS

The advantages of parallel imaging in MRI have been widely appreciated in the last decade, as demonstrated by the wide deployment of techniques such as SENSE and GRAPPA. Commonly, these techniques employ a uniform subsampling of k -space due to image reconstruction speed considerations. Nonuniform sampling along one dimension has been shown to yield better-quality images than uniform sampling in 2D parallel imaging. This is due to the fact that variable-density sampling reduces the coherence of aliasing artifacts, and that a dense sampling of the center of k -space increases SNR. Recent advances in image reconstruction, coupled with ever-improving computer hardware, enable nonuniform subsampling to be considered in clinical imaging.

pMRI is particularly attractive for 3D acquisitions, where the abundance of signal, along with the existence of two PE directions, allow for considerably higher image acceleration rates. In this work, we have presented an extension of our nonuniform subsampling strategy to 3D-FSE imaging, where the distribution of acquired k -space lines in both PE directions is parameterized to an exponential function. The choice of this parameterization was motivated by its simplicity and ease in varying the sampling distributions between uniform to centrally weighted nonuniform, while providing the acquisition of central Nyquist rate k -space lines for coil sensitivity calibration. Our formulation could potentially be used to determine an optimal k -space distribution along both PE directions. However, finding an optimized sampling pattern is a difficult optimization task in a very high-dimensional space, and thus the study of optimal nonuniform subsampling in 2D parallel imaging is beyond the scope of this work.

Our implementation included the modification of a GE 3D-FSE product pulse sequence to allow for the subsampling described along both PE directions. Accelerated doped water phantom data and in vivo brain data were acquired, and reconstructed using both 2D-

GRAPPA-Operator and 2D-SPACE RIP reconstruction methods. The results of the two techniques were subsequently analyzed using standard image quality metrics and compared.

Our comparison of 2D-GRAPPA-Operator and 2D-SPACE RIP reconstructions of the nonuniformly sub-sampled 3D-FSE data, at accelerations varying between $4\times$ and $7.5\times$, consistently showed less AP in 2D-SPACE RIP images both in phantom and in vivo, though the difference was less accentuated in vivo. These results demonstrate that the use of 2D-SPACE RIP yields better image quality than 2D-GRAPPA-Operator in the reconstruction of the subsampled 3D-FSE data. We believe this is due to the fact that GRAPPA is nonoptimal in the least-squares sense, which appears to be critical at higher acceleration factors. In contrast, one has greater control to suppress noise with the regularization CGLS iterative algorithm, as demonstrated by our results.

Nonuniform sampling schemes generally carry a heavier computational burden than uniform ones, due to a larger linear system of equations in SENSE and SPACE RIP, and a need for a greater number of reconstruction patterns in GRAPPA. GRAPPA is often available on clinical scanners and is currently used with acceptable image reconstruction speed. SPACE RIP's clinical use, on the other hand, has been hampered by its initially slow computation times. In our study, we found that the time necessary to estimate the coil sensitivity in 2D-SPACE RIP is 0.3 s using CGLS. Calculating the coil weights in 2D-GRAPPA-Operator takes 4.3 s in two separate consecutive 1D-GRAPPA reconstructions with a block of 16×16 ACS lines and a 2×3 kernel. The image reconstruction time of 2D-SPACE RIP for one slice is 3.5 s with 15 iterations. However, the computation time to reconstruct missing data using 2DGRAPPA-Operator is 4.6 s for the two separate consecutive 1D-GRAPPA reconstructions. These measurements were calculated on a 1.83Ghz INTEL processor using MATLAB and can be accelerated considerably using higher-quality hardware and an improved coding implementation. None-theless, these numbers provide a relative indication of performance between the two reconstruction techniques. Practically, it shows that both methods can be considered comparable in terms of computation speed.

To summarize, we have demonstrated an effective approach to significantly reduce the acquisition time required for T_2 -weighted 3D-FSE images using parallel imaging. Our proposed subsampling strategy is amenable to both SPACE RIP- and GRAPPA-based reconstruction methods. Our results demonstrate better image quality with SPACE RIP, in a time comparable to that of the well-accepted GRAPPA reconstruction approach.

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FIG. 1.

a: Illustration of the function f_1/M for various values of β . **b:** Nonuniform sampling pattern (shown on the vertical axis) for $\beta = 1.2$ found through regular subsampling of the function f_1 .

FIG. 2.

Proposed exponentially-weighted sampling pattern on a rectilinear grid for 3D parallel imaging. The k_z direction is exponentially weighted, and the k_y direction is both exponentially weighted and modulated with a sine function.

FIG. 3. CGLS reconstruction result of a 2×2.5 accelerated exponentially weighted brain imaging experiment. **a:** Plot of residual norm (normalized) vs. iteration count. **b:** The iteration CGLS reconstruction result, with the iteration count shown in the lower left corner of each image.

FIG. 4.

2D-GRAPPA-Operator and 2D-SPACE RIP reconstruction simulations with five subsampling patterns (listed in Table 1) using a uniform water phantom's 3D fully sampled sagittal MRI acquisition with PE along Y (anterior–posterior) and Z (left–right). The reference image and all reconstructed images are displayed with the same grayscales. The difference images between these reconstructed images and the reference are shown below each reconstructed slice with the same grayscales.

FIG. 5. Graph of AP in phantom images reconstructed by 2D-SPACE RIP and 2D-GRAPPA-Operator simulations with five sub-sampling patterns (see Table 1).

FIG. 6.

2D-GRAPPA-Operator and 2D-SPACE RIP reconstruction simulations with five subsampling patterns (listed in Table 1) using a healthy volunteer's 3D fully sampled sagittal MRI acquisition with PE along Y (anterior–posterior) and Z (left–right). The reference image and all reconstructed images are displayed with the same gray-scales. The difference images between these reconstructed images and the reference are shown below each reconstructed slice with the same gray-scales.

FIG. 7.

Graph of AP in brain images reconstructed by 2D-SPACE RIP and 2D-GRAPPA-Operator simulations with five subsampling patterns (see Table 1).

FIG. 8. 2D-GRAPPA-Operator vs. 2D-SPACE RIP reconstructions for a reduction factor $R = R_y \times R_z = 2 \times 2.5$. ROI placement for SNR calculation is shown in (a) a 2D-GRAPPA-Operator reconstructed image and (b) a 2D-SPACE RIP reconstructed image. The corresponding SNR values are listed in Table 2. Images of noise present in the (c) 2D-GRAPPA-Operator and (d) 2D-SPACE RIP reconstructions, generated by calculating the difference between two subsequent acquisitions, are shown.

FIG. 9.

Comparison of PSFs for three different values of β_y and β_z , as marked on the left side of the figure. **a:** The 2D PSFs in two PE plane. Only the central 30×40 matrix is shown. The central 1D profiles along two independent PE directions are shown in **b** and **c**.

Reconstructed images are shown in **d**. With increasing β , the subsampling data set provides fewer artifacts, but image resolution degrades.

FIG. 10.

Image quality comparison between in vivo $5\times$ accelerated 3D-FSE reconstructions from the axial, coronal, and sagittal views. **a:** Unaccelerated image. **b:** Image from 2D-SPACE RIP reconstruction of 2D exponentially weighted subsampling, $5\times$ acceleration, $R_y = 2$, $R_z = 2.5$, $\beta_y = \beta_z = 1.2$, 16×16 total ACS lines. **c:** Image from 2D-GRAPPA-Operator reconstruction of the same subsampling pattern as in b. **d** and **e:** The magnified anatomy of 2D-SPACE RIP and 2D-GRAPPA-Operator reconstructed brain images. The arrows in the magnified images point to aliasing artifacts.

Table 1

Five Different Subsampling Patterns Used in the Simulated Experiments

	A	B	C	D	E
$R = R_y \times R_z$	2×2	2.5×2	3×2	2.5×2.5	3×2.5
β_y	1.2	1.2	1.2	1.2	1.2
β_z	1.5	1.2	1.2	1.2	1.2
Center box for reference data	32×32	24×24	24×24	24×24	16×16

Table 2

Comparison Between 2D-GRAPPA-Operator and 2D-SPACE RIP Reconstructions *

ROI	$R = R_y \times R_z = 2 \times 2.5, \beta_y = \beta_z = 1.2$		$R = R_y \times R_z = 2.5 \times 2.5, \beta_y = \beta_z = 1.2$		$R = R_y \times R_z = 3 \times 3, \beta_y = \beta_z = 2.4$	
	2D-GRAPPA-Operator	2D-SPACE RIP	2D-GRAPPA-Operator	2D-SPACE RIP	2D-GRAPPA-Operator	2D-SPACE RIP
I	49.8	86.4	42.4	81.9	17.6	54.8
II	45.9	79.4	41.8	76.1	16.6	51.6
III	42.1	98.2	35.9	97.5	12.7	67.3
IV	38.9	128.7	37.8	123.5	12.5	96.9
V	40.1	65.5	37.7	63.0	14.7	46.4

* SNR of different ROIs in reconstructions of data acquired at reduction factors of $R_y \times R_z = 2 \times 2.5$, 2.5×2.5 , and 3×3 .

Table 3

Relative PSF FWHM for Different Subsampling Schemes at $R = R_y \times R_z = 3 \times 2$ *

β	$\beta_y = \beta_z = 0$	$\beta_y = \beta_z = 1.2$	$\beta_y = \beta_z = 3.6$
Relative PSF FWHM in in-plane PE direction	1.007	1.042	1.051
Relative PSF FWHM in through-plane PE direction	1.006	1.021	1.032

* FWHM of the nonaccelerated PSFs was used as the standard.