Kalyan Chatterjee

Associate Professor
Department of Electrical Engineering,
Indian School of Mines (ISM),
Dhanbad, Jharkhand 826004, India
e-mail: kalyanbit@yahoo.co.in

Ravi Shankar

JRF/EE
Department of Electrical Engineering,
Indian School of Mines (ISM),
Dhanbad, Jharkhand 826004, India
e-mail: ravi060173@gmail.com

Amit Kumar

JRF/EE
Department of Electrical Engineering,
Indian School of Mines (ISM),
Dhanbad, Jharkhand 826004, India
e-mail: amibits@gmail.com

Fuzzy Logic Based Controller for a Grid-Connected Solid Oxide Fuel Cell Power Plant

This paper describes a mathematical model of a solid oxide fuel cell (SOFC) power plant integrated in a multimachine power system. The utilization factor of a fuel stack maintains steady state by tuning the fuel valve in the fuel processor at a rate proportional to a current drawn from the fuel stack. A suitable fuzzy logic control is used for the overall system, its objective being controlling the current drawn by the power conditioning unit and meet a desirable output power demand. The proposed control scheme is verified through computer simulations. [DOI: 10.1115/1.4027709]

Keywords: SOFC, VSI, fuzzy logic controller (FLC), distributed generation (DG), distribution system error (DSE)

1 Introduction

The energy crisis in the power sector has led to difficulties in meeting the increasing power demand. Distributed generation (DG) is increasingly being used to meet these energy needs. DG uses small electric power generation systems located near consumers and load centers. Distributed generation provides reliable electric power. In addition, it also allows businesses to save on electricity costs by using their units during high peak demand periods when power is most expensive. This technology offers comparative advantages to large and centralized plants in terms of efficiency, reliability, and security. Growing DG can improve the quality of the atmosphere and reduce the greenhouse effect. Hydrogen fuel cells and photovoltaic cells are some of the environmentally friendly generation systems used in DG. DG covers a broad range of technologies, including many renewable sources of energy that provide small-scale power at the sites close to the consumers, such as highly efficient combined heat and power plants (CHP plants), back up, and peak load systems for increasing capacity. These technologies offer new market opportunities and enhanced industrial competitiveness. On site production minimizes the transmission and distribution losses and costs. Hence, DG helps to bypass congestion in the existing transmission grids. As the loads on the small-scale power system vary, the frequency will change unless the load and generation are again balanced. Hence, there is a need to control the power produced so that the frequency attains the desired value. A formulation of the problem as a distributed system load frequency control problem with its own distribution system error (DSE) is presented. The development of a good load frequency controller for distributed generation systems is therefore receiving renewed attention from the researchers all over the world. Many investigations in the area of load frequency control problem of distributed generation systems [1] have been reported in various research journals over the past two decades. Some of the problems that have been reported in the literature are low-frequency oscillations [2] and frequency deviations [3], which, if uncontrolled, could bring the system down. Specifically, Ref. [3] analyzed a system that comprises combustion, wind, steam, and hydro turbine generators and concluded that

the frequency in the distribution system could become unstable depending on the type and number of DG systems. Instability may occur if the number of DG systems increases. Machine time constants, size, and inertia are important factors that affect system dynamics. Sedghisigarchi and Felichi [4] described the impact of fuel cells on load frequency control in power distributions systems. Some literature [5,6] presents a fuzzy and neuro-based frequency-control method for the SOFC plant and utility-connected large photovoltaic cell (PV)–diesel hybrid system considering power system condition and insolation condition. A number of control strategies have been employed in the design of load frequency controllers in order to achieve better dynamic performance. It is generally recognized that a good distribution system error (DSE) profile indicates good control. The profile of DSE depends on the deviation of frequency. A distribution system error that does not deviate much from zero and crosses zero frequently is desirable. The development of an efficient load frequency controller is therefore receiving renewed attention from the researchers all over the world. It is due to the fact that a controller of better design would provide better control. The most widely employed controllers in the interconnected power system for load frequency control is conventional proportional integral (PI) controller [7]. The advantages of a PI controller are that it is simple for implementation and reduce the steady state error to zero by incorporating error feedback mechanism. However, the inherent singular characteristics of the speed-governor system have a great influence on load frequency control (LFC) behavior, which makes it more difficult to maintain the required frequency accuracy. In recent years; modern soft computing based methods such as artificial neural networks (ANN), fuzzy logic (FL), and fuzzy neural network have gained increasing interest for control applications. The salient feature of these techniques is that they provide a model-free description of control systems and may not require model identification. Some literature analyzes the SOFC with a thermo-electrochemical model [8], grid-connected SOFC analysis, and its operational control mechanisms [9,10] in a very efficient way. Interfacing of SOFC with the grid is very challenging part, hence it is more important aspect to the controlling and operation of SOFC and some literatures are shows these analysis with the SOFC connected grid with good demonstrations [11–13].

In this work a fuel cell model and control concept will be derived for the fuel cell system for optimal operation. First the actual mathematical model [14] of the fuel cell stack will be

Contributed by the Advanced Energy Systems Division of ASME for publication in the JOURNAL OF FUEL CELL SCIENCE AND TECHNOLOGY. Manuscript received December 10, 2013; final manuscript received April 17, 2014; published online June 10, 2014. Assoc. Editor: Umberto Desideri.

represented. This model is used to find out slow dynamics associated with the gas flow and the fuel processor operation. Then, suitable control architecture will be presented to the overall system. The main objective of this controller is to regulate the input fuel flow in order to meet a desirable output power demand, simultaneously meeting a near optimal operation of the fuel cells. During a change in the operating power level of the fuel cell (FC), the utilization factor will deviate from its steady-state value U_s . The steady-state value of U_s is maintained through modulation index (m) and phase angle (δ_f) which is being associated with the voltage source inverter (VSI) in the power conditioning system. To ensure the FC operation within the safe operating area, current control reference, I_{fc-ref} , is limited. In this paper I_{fc-ref} is controlled by a proposed fuzzy logic controller. The total mathematical model of the fuel cell is simulated using MATLAB/SIMULINK and a power system toolbox.

2 Fuel-Cell Dynamic Modeling

A power generation fuel cell system has the following three main parts:

- (a) Fuel processor: The fuel processor converts fuels such as natural gas to hydrogen and byproduct gases.
- (b) Power section (fuel cells): The power section generates the electricity. There are numerous individual electrochemical fuel cells in the power section.
- (c) Power conditioner: The power conditioner converts dc power to ac power output and includes current, voltage, and frequency control.

Power generation by an SOFC is the conversion of chemical energy in hydrogen (H_2) and oxygen (O_2) into electric energy through the following basic reaction at two nodes of the SOFC:

DC current I_{fc} is generated from the releasing of electrons ($2e^-$) at the anode due to the above chemical reaction. A dynamic model of an SOFC power plant suitable for the use in power system studies is shown by Fig. 1 [2,4,13]. In Fig. 1, electrical dynamic describes the chemical reaction to restore the charge that has been quickly drained by the load. This is modeled by a first-order transfer function. To ensure the FC operation within the safe operating area, current control reference, I_{fc-ref} , is limited by the following boundaries:

$$I_{fc-ref-max} = \left(\frac{U_{max}}{2K_r} \right) q_{H_2-in}, \quad I_{fc-ref-min} = \left(\frac{U_{min}}{2K_r} \right) q_{H_2-in} \quad \text{and} \quad K_r = \left(\frac{N_0}{4F} \right) \quad (3)$$

where U_{max} and U_{min} are the maximum and minimum fuel unitization, respectively; N_0 is the number of cells in series in the FC stack; q_{H_2-in} the hydrogen input flow rate; and F the Faraday constant. The fuel processor depicts the dynamic of fuel supply, which is a process to reform the natural gas supply to the hydrogen-rich fuel. This can be simply represented by a first-order lag model of time constant T_f , where U_{opt} is the optimal fuel utilization. For fuel-cell stack protection, cell utilization has to be kept between the $0.7 < U < 0.9$; otherwise, the following conditions will occur:

- (i) Underused fuel $U < 70\%$ (fuel cell voltage would rise rapidly). The current of the fuel cell is not allowed to be below a minimum stack current.
- (ii) Overused fuel $U > 90\%$ (fuel starvation and permanent damage to the cells). The current of the fuel cell is not allowed beyond a maximum stack current.

Fig. 1 Block diagram of fuel cell dynamic modeling

Fig. 2 Block diagram of a fuel cell and grid interface

The chemical process of fuel reaction inside the FC stack is described by three first-order transfer functions for hydrogen, oxygen, and water, respectively, where r_{H_2O} is the ratio of hydrogen to oxygen, K_{H_2} , K_{O_2} , and K_{H_2O} the valve molar constant, T_{H_2} , T_{O_2} , and T_{H_2O} the time constant, and p_{H_2} , p_{O_2} , and p_{H_2O} the partial pressure, for hydrogen, oxygen, and water flow, respectively. The internal electromagnetic force (EMF) generated by the SOFC stack is calculated by the well-known Nernst equation where E_0 is the ideal standard potential, R universal gas constant, T absolute temperature, and r the ohmic loss.

Since the FC stack operates at a low dc voltage range, the power conditioning system (PCS) must boost the dc voltage and invert it to the ac grid frequency (50 Hz) for grid-connected operation, the above two stage configuration of power electronics interface as shown in Fig. 1. In addition, it is desirable for the PCS to provide galvanic isolation between the FC terminals and the grid. Hence, the PCS may include an inverter at the output of the FC, followed by a 50 Hz transformer (ac-ac voltage gain) or a switch-mode dc-dc converter at the FC (dc-dc voltage gain) followed by the grid-connected inverter.

Regarding the dc/dc converter, the full-bridge converter topology is selected, being a well-developed and proven technology, presenting good efficiency and switch utilization. Applying a pulse width-modulation (PWM) technique to a gate turn off (GTO)-based voltage-source converter (VSC) allows representation of the fuel cell with a voltage at the fundamental frequency. The model, shown in Fig. 2, is used for simulation in this paper.

3 Fuel-Cell Control Loops

In a fuel cell, control of power can be achieved through the control of the input fuel flow (called, primary control loop) or by adjusting the firing angle of the inverter (secondary control loop) and are already described in Ref. [2].

- (a) Primary control loop: The primary control loop avoids the underused or overused fuel; the dc/dc converter can be

controlled to ensure a constant current output from the fuel cell stack.

- (b) Secondary control loop: The objective of the secondary control loop is to regulate the system frequency by adjusting the firing angle of the fuel-cell inverter. Fuel-cell generators are connected to the grid by a power-conditioning unit (PCU). Applying a pulse width-modulation (PWM) technique to a GTO-based voltage-source converter (VSC) allows representation of the fuel cell with a voltage at the fundamental frequency. The secondary loop controller is designed for the fuel cell to correct the frequency deviations by tuning the inverter firing angle. Therefore, this controller measures the system frequency and adjusts the firing angle of the inverter to regulate the frequency deviations.

4 Fuel Cell Control Mechanism

In the DG system under study, there are two variables that can be used to control the generation and transfer real and reactive powers. These are control of fuel input to the stack, i.e., called the primary control loop and control of a power conditioning unit by controlling the modulation index m and phase shift δ_f associate with the VSI (secondary control loop). Figure 3 shows a schematic diagram of the primary and secondary control loops. The design of these two control loops is presented below after the SOFC dynamic model is presented.

4.1 Control of Fuel Input to the Stack (Primary Control Loop). The real output current is proportional to the fuel input of the FC stack with a constant utilization factor in the steady state. The equation shows that, $N_f = (K_f I_{fc} / U_s)$ where U_s is the utilization factor in steady state. The relationship between a small change of stack current ΔI_{fc} and a small change in hydrogen input q_{H_2-in} fed to the FC stack can be derived as

$$I_{fc} = \left(\frac{u \times q_{H_2-in}}{2K_r} \right) \quad (4)$$

4.2 Control of Power Conditioning Unit (Secondary Control Loop). The output rms voltage of the inverter is equal to

$$V_{rated} = 0.5 m_{rated} \left(\frac{V_{dc,rated}}{\sqrt{2}} \right) \quad (5)$$

where $V_{dc,rated}$ is the rated stack terminal voltage in volts and m_{rated} is the modulation index of the inverter under rated operating condition. If V_{rated} consider as a base value, then the inverter output voltage can be expressed in per unit form as $V_f = m V_{dc}$ where m is the per-unit value of the variable modulation index with respect to the base value of $V_{dc,rated}$. The fundamental component

Fig. 3 Block diagram of fuel cell control loop

of line current I_r per unit (pu) can be expressed in d, q axes as follows:

$$\begin{aligned} I_r &= I_{dr} + jI_{qr} = \left(\frac{V_f \angle \delta_f - V_s \angle 0}{jX_f} \right) \\ &= \left\{ \frac{V_f (\cos \delta_f + j \sin \delta_f) - V_s}{jX_f} \right\} \\ &= \frac{V_f \sin \delta_f}{X_f} + j \left(\frac{V_s - V_f \cos \delta_f}{X_f} \right) \end{aligned} \quad (6)$$

So,

$$I_{dr} = \left(\frac{V_f \sin \delta_f}{X_f} \right) \quad \text{and} \quad I_{qr} = \left(\frac{V_s - V_f \cos \delta_f}{X_f} \right) \quad (7)$$

where V_s is the per unit voltage at the grid bus. The three phase real and reactive powers at the grid bus in steady state can be expressed as

$$\begin{aligned} P_r &= V_s I_{dr} = \left(\frac{m V_{dc} V_s \sin \delta_f}{X_f} \right) \quad \text{and} \quad Q_r = \left(\frac{m V_{dc} V_s \cos \delta_f - V_s^2}{X_f} \right) \\ &= V_s I_{qr} \end{aligned} \quad (8)$$

If power losses in the VSI, transformer, and feeder are neglected, the real power received at the grid bus is equal to the power drawn from the FC stack, i.e.,

$$P_r \Rightarrow P = V_{dc} \times I_{fc} \Rightarrow I_{fc} = \frac{P_r}{V_{dc}} = \frac{V_s m V_{dc} \sin \delta_f}{V_{dc} X_f} = \frac{m V_s \sin \delta_f}{X_f} \quad (9)$$

From the above equation it is shown that the stack current can be controlled by modulation index m and phase shift δ_f . During a change in the operating power level of the FC, the utilization factor will deviate from its steady-state value U_s even when its fuel input is controlled as described at the beginning of Sec. 4., i.e., primary control. This is because of the delay associated with the fuel processor operation. Therefore, it is necessary to make sure that U is within the allowable range during the transient state as well. This can be possible by controlling VSI to draw current from the stack to follow a set reference current $I_{fc,ref}$. The $I_{fc,ref}$ is set within the relation between utilization factor and power deviation. The details are described in the Appendix.

Table 1 Fuzzy rule base for the proposed control scheme for SOFC

Δu	ΔP					
	NB	NS	Z	PS	PB	
NB	Z	Z	PS	PS	PB	
NS	NS	Z	Z	PS	PB	
Z	NS	NS	Z	PS	PS	
PS	NS	NS	Z	PS	PS	
PB	NS	NS	Z	Z	Z	

The relationship with I_{ref} between δ_f and m is given below:

$$\begin{aligned} \tan \delta_f &= \left(\frac{X_f V_s I_{dr}}{V_s^2 + X_f V_s I_{qr}} \right) = \left(\frac{V_s I_{dr}}{\left(\frac{V_s^2}{X_f} + V_s I_{qr} \right)} \right) \\ &= \left(\frac{P_{ref}}{\left(\frac{V_s^2}{X_f} + P_{ref} \tan \Phi \right)} \right) \end{aligned} \quad (10)$$

$$m = \left(\frac{X_f I_{fc,ref}}{V_s \sin \delta_f} \right) \quad \text{and} \quad \delta_f = \tan^{-1} \left(\frac{P_{ref}}{\left(\frac{V_s^2}{X_f} + P_{ref} \tan \Phi \right)} \right) \quad (11)$$

The detailed derivation of the above equation is shown in the Appendix. From the above equation $I_{fc,ref}$ will be continuous adjusted until the FC reaches the desired power P_{ref} .

5 Overall Control Strategy of SOFC Connected Grid Power System

Figure 4 shows the block diagram of overall control strategy.

Fig. 4 Block diagram of overall control strategy

Fig. 5 Two area interconnected power systems integrated with the SOFC power plant

Table 2 Input parameters of interconnected power systems

For SOFC power plant parameters:

$T = 1273$, $F = 96487$, $R = 8.314$, $E_0 = 1.18$, $N_0 = 384$, $K_r = 0.966 \times 10^{-6}$, $U_{\max} = 0.9$, $U_{\min} = 0.8$, $U_{\text{opt}} = 0.85$, $r_{h0} = 1.145$, $C_{dc} = 1$, $K_{h2} = 8.43 \times 10^{-4}$, $K_{h20} = 2.81 \times 10^{-4}$, $K_{02} = 2.52 \times 10^{-3}$, $T_{h2} = 26.1$, $T_{h20} = 78.3$, $T_{02} = 2.91$, $r = 0.126$, $T_f = 5$, $T_e = 0.08$

For generator parameters:

$X_d = 1.8$, $X'_d = 0.25$, $X''_d = 0.3$, $X_q = 1.7$, $X'_q = 0.25$, $X''_q = 0.55$, $X_l = 0.2$, $R_a = 0.0025$, $T'_{do} = 8$ s, $T'_{q0} = 0.4$ s, $T_{d0}'' = 0.03$ s, $T_{q0}'' = 0.05$ s, $A_{\text{sat}} = 0.015$, $B_{\text{sat}} = 9.6$, $\psi_{T1} = 0.9$, $H = 6.5$ (for G_1 & G_2), $H = 6.175$ (for G_3 & G_4), $K_D = 0$

For step-up transformer:

$Z = 0 + j0.15$ pu, 900 MVA, 20/230 KV base

For transmission system:

100 MVA, 230 KV base per unit, $r = 0.0001$ pu/km, $b_c = 0.00175$ pu/km, $x_L = 0.001$ pu/km

For load parameters:

(at bus no. 7) $P_L = 967$ MW, $Q_L = 100$ MVar, $Q_c = 200$ MVar

(at bus no. 9) $P_L = 1767$ MW, $Q_L = 100$ MVar, $Q_c = 350$ MVar

Fig. 6 MATLAB and SIMULINK diagram for simulated fuzzy approached SOFC power plant with two area power system

- (i) Fuel controller: The fuel valve is tuned according to the feedback stack current with the proportional gain $2K_f/u_s$, as described by Eq. (4).
- (ii) u calculation block: From the online measurement of q_{H_2-in} and I_{fc} , u can be determined from $u = 2K_f I_{fc} / q_{H_2-in}$ in real time.
- (iii) Generating reference current: The online calculated u and desired steady-state utilization factor generated an error Δu . According to the Δu and power error ΔP , information is generated by stack current reference signal $I_{fc,ref}$. In this paper, a fuzzy logic controller is used for finding out the $I_{fc,ref}$ for effective operation of fuel stack.
- (iv) VSI controller: During the transient period output changes from it is reference value. During this stage, the equations in (i) and (ii) are used together with the $I_{fc,ref}$ generated from the fuzzy logic to determine the appropriate value of

- modulation index m and phase shift δ_f . m and δ_f are controlled until output power levels reach the desired level P_{ref} .
- (v) Generating reference current by the fuzzy logic controller or proposed control scheme: The basic fundamentals and details of fuzzy logic concepts can be available through various literature; here, only the short and required basic details have been provided. A fuzzy logic controller has been designed to control the $I_{fc,ref}$. The actual inputs to the fuzzy system are scaled versions of active power error ΔP and change of utilization constant ΔU . The current $I_{fc,ref}$ is assigned to the output of the FLC. Five uniformly distributed triangle membership functions are used for the fuzzification of the inputs. Each of the FLC input and output signals are fuzzy variables and are assigned five linguistic variables namely NB, NS, Z, PS, and PB, which stand for negative big, negative small, zero, positive small, and

Fig. 7 (a) Partial pressure of hydrogen ($p(H_2)$), (b) partial pressure of water ($p(H_2O)$), (c) partial pressure of oxygen ($p(O_2)$), (d) stack terminal current (I_{DC}), (e) stack terminal voltage (V_{DC}), and (f) real and reactive power of fuel cell (PQ_{FC}) when P_{ref} is 0.85 pu

Fig. 8 (a) Partial pressure of oxygen ($p(\text{O}_2)$), (b) partial pressure of hydrogen ($p(\text{H}_2)$), (c) partial pressure of water ($p(\text{H}_2\text{O})$), (d) stack terminal current (I_{dc}), (e) stack terminal voltage (V_{dc}), and (f) real and reactive power of fuel cell (PQ_{FC}) when P_{ref} is 0.95 pu

positive big, respectively. The fuzzy rule bas has been shown in Table 1, which is used for the fuzzy logic controller incorporated to the SOFC connected multimachine power system in the MATLAB and simulation environment.

6 Results, Simulation, and Discussion

For the need of proposed controller, a four machine two-area single line diagram of the power system integrated with an SOFC power plant is shown in Fig. 5. The parameters of the interconnected power system [15] and the steady-state data for this fuel cell are given in Table 2. This proposed control technique is simulated in MATLAB and SIMULINK toolbox. The MATLAB and SIMULINK diagram for the simulated fuzzy approach SOFC power plant with two area power system has been shown in Fig. 6 for the load change, i.e., P_{ref} changes from 0.85 pu, 0.95 pu and 1 pu, respectively. The power change is attempted to be met by $I_{fc,ref}$ with the applied strategy of control using fuzzy logic controller (i.e.,

artificial intelligent controller). The utilization factor u and P_{ref} is used as the input parameter to the applied control strategy, which generates the required controlling approach current, i.e., I_{fc-ref} , which forms the whole basis for secondary control of the interconnected power system. The simulation figure from Figs. 7 to 9 shows the partial pressure of hydrogen, oxygen, and water as well as I_{fc} , V_{dc} , and PQ_{fc} with the time, which is responsible for generating the desired output power of the SOFC power plant when it is exposed to the different per unit load change for the simulated power system. The power conditioning unit (PCU), which has been shown in the Fig. 4, is accomplished through voltage sourced inverter (VSI) topology concepts, through which it interfaces the SOFC with the grid. This topology is wholly controlled by modulation index and phase shift, which is overall controlled by proposing a fuzzy controller to maintain the desired output of the SOFC. The simulation figures support as well as demonstrate the effectiveness of the proposed control technique.

Fig. 9 (a) Partial pressure of hydrogen ($p(\text{H}_2)$), (b) partial pressure of oxygen ($p(\text{O}_2)$), (c) partial pressure of water ($p(\text{H}_2\text{O})$), (d) stack terminal voltage (V_{DC}), (e) stack terminal current (I_{DC}) and (f) real and reactive power of fuel cell (PQ_{FC}) when P_{ref} is 1 pu

7 Conclusion

The main contribution of this paper is to change the output power level of an SOFC power plant connected to an ac grid through a VSI topology. The natural gas input to the plant was controlled proportionally with the current to operate the stack with a constant utilization factor in steady state, which finally reduces the terminal voltage variations. The phase shift and modulation index of the VSI topology were utilized to regulate the current stack. The fuzzy logic control is used to regulate the input fuel flow in order to meet a desirable output power demand. It is also demonstrated that the fuel cells are capable of providing the load change in the power system.

Acknowledgment

The authors sincerely thank the Indian School of Mines (ISM) Dhanbad, India, for the financial support provided for carrying out the present research work.

Appendix

$$P_r = V_s I_{dr} = \left(\frac{m V_{\text{dc}} V_s \sin \delta_f}{X_f} \right) \quad (\text{A1})$$

$$\sin \delta_f = \left(\frac{X_f V_s I_{dr}}{m V_{\text{dc}} V_s} \right) \quad (\text{A2})$$

$$Q_r = \left(\frac{m V_{\text{dc}} V_s \cos \delta_f - V_s^2}{X_f} \right) = V_s I_{qr} \quad (\text{A3})$$

$$m V_{\text{dc}} V_s \cos \delta_f - V_s^2 = X_f V_s I_{qr} \quad (\text{A4})$$

$$\cos \delta_f = \left(\frac{V_s^2 + X_f V_s I_{qr}}{m V_{\text{dc}} V_s} \right) \quad (\text{A5})$$

Again,

$$P_{\text{ref}} = V_s I \cos \Phi \quad (\text{A6})$$

$$I_{dr} + I_{qr} = I \angle \Phi = I(\cos \Phi + j \sin \Phi) \quad (\text{A7})$$

Hence, from Eq. (A7), we get

$$I_{dr} = I \cos \Phi \quad (\text{A8})$$

$$I_{qr} = I \sin \Phi \quad (\text{A9})$$

$$V_s I_{dr} = V_s I \cos \Phi \quad (\text{A10})$$

$$V_s I_{qr} = V_s I \sin \Phi = \left(\frac{V_s I \cos \Phi \sin \Phi}{\cos \Phi} \right) = P_{\text{ref}} \tan \Phi \quad (\text{A11})$$

From Eqs. (A2) and (A5)

$$\begin{aligned} \tan \delta_f &= \left(\frac{X_f V_s I_{dr}}{V_s^2 + X_f V_s I_{qr}} \right) = \left(\frac{V_s I_{dr}}{\left(\frac{V_s^2}{X_f} + V_s I_{qr} \right)} \right) \\ &= \left(\frac{P_{\text{ref}}}{\left(\frac{V_s^2}{X_f} + P_{\text{ref}} \tan \Phi \right)} \right) \end{aligned} \quad (\text{A12})$$

$$\delta_f = \tan^{-1} \left\{ \frac{P_{\text{ref}}}{\left(\frac{V_s^2}{X_f} + P_{\text{ref}} \tan \Phi \right)} \right\} \quad (\text{A13})$$

References

- [1] Chiu, Y. L., Diong, B., and Gemmen, R. S., 2004, "An Improved Small-Signal Model of the Dynamic Behavior of PEM Fuel Cells," *IEEE Trans. Ind. Appl.*, **40**(4), pp. 970–977.
- [2] Sedghisigarchi, K., and Feliachi, A., 2006, "Impact of Fuel Cell on Load Frequency Control in Power Distribution System," *IEEE Trans. Energy Convers.*, **21**(1), pp. 50–256.
- [3] Llic, M., Galianca, F., and Fink, L., 1998, *Power System Restructuring, Engineering & Economics*, Kluwer, Norwell, MA, Ch. 12.
- [4] Sedghisigarchi, K., and Felichi, A., 2002, "Dynamic Model of a Grid-Connected Solid Oxide Fuel Cell (SOFC)," 34th North American Power Symposium, Tempe, AZ, October 13–15, pp. 107–113.
- [5] Jurado, F., and Valverde, M., 2005, "Genetic Fuzzy Control Applied to the SOFC for Power Quality Improvement," *Elec. Power Syst. Res.*, **76**(1-3), pp. 93–105.
- [6] Jurado, F., 2003, "Power Supply Improvement With a SOFC Plant by Neural-Network Based Control," *J. Power Sources*, **117**(1-2), pp. 75–83.
- [7] Chatterjee, K., 2011, "Design of Dual Mode PI Controller for Load Frequency Control," *Int. J. Emerging Elec. Power Syst.*, **11**(4).
- [8] Petruzzi, L., Cocch, L., and Fineschi, F., 2003, "A Global Thermo-Electrochemical Model for SOFC Systems Design and Engineering," *J. Power Sources*, **118**(1-2), pp. 96–107.
- [9] Li, Y. H., Rajakaruna, S., and Choi, S. S., 2009, "Control of a Solid Oxide Fuel Cell Power Plant in a Grid-Connected System," *IEEE Tran. Energy Convers.*, **22**(2), pp. 405–412.
- [10] Hatziaadoniu, C., Lobo, A. A., and Pourboghra, F., 2002, "A Simplified Dynamic Model of Grid Connected Fuel Cell Generators," *IEEE Trans. Power Del.*, **17**(2), pp. 467–473.
- [11] Andujar, J., Segura, F., Duran, E., and Renteria, L. A., 2011, "Optimal Interface Based on Electronic in Distributed Generation Systems for Fuel Cell," *Renewal Energy*, **36**(11), pp. 2759–2770.
- [12] Farooque, M., and Maru, H., 2001, "Fuel Cells—The Clean and Efficient Power Generators," *Proc. IEEE*, **89**(12), pp. 1819–1829.
- [13] Mazumdar, S. K., 2004, "Solid-Oxide-Fuel-Cell Performance and Durability: Resolution of the Effects of Power-Conditioning Systems and Application Loads," *IEEE Trans. Power Electron.*, **19**(5), pp. 1263–1278.
- [14] Krumdieck, S., Page, S., and Round, S., 2004, "Solid Oxide Fuel Cell Architecture and System Design for Secure Power on an Unstable Grid," *J. Power Sources*, **125**(1), pp. 189–198.
- [15] Li, Y. H., Choi, S. S., and Rajakaruna, S., 2005, "An Analysis of the Control and Operation of a Solid Oxide Fuel Cell Power Plant in an Isolated System," *IEEE Trans. Energy Convers.*, **20**(2), pp. 381–387.