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Correlation Imaging with Arbitrary Sampling Trajectories

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Abstract

The presented work aims to develop a generalized linear approach to image reconstruction with arbitrary sampling trajectories for high-speed MRI. This approach is based on a previously developed image reconstruction framework, "correlation imaging" (1). In the presented work, correlation imaging with arbitrary sampling trajectories is implemented in a multi-dimensional hybrid space that is formed from the physical sampling space and a virtually defined space. By introducing an undersampling trajectory with both uniformity and randomness in the hybrid space, correlation imaging may take advantage of multiple image reconstruction mechanisms including coil sensitivity encoding, data sparsity and information sharing. This hybrid-space implementation is demonstrated in multi-slice 2D imaging, multi-scan imaging, and radial dynamic imaging. Since more information is used in image reconstruction, it is found that hybrid-space correlation imaging outperforms several conventional techniques. The presented approach will benefit clinical MRI by enabling correlation imaging to be used to accelerate multi-scan clinical protocols that need different sampling trajectories in different scans.

Keywords

Correlation imaging; Correlation function; High-speed MRI

Introduction

High speed imaging has been an active area of research interest since the beginning of magnetic resonance imaging (MRI), mostly for the benefit of reduced scan time and improved diagnostic image quality (2–6). To date, a variety of high speed imaging methods have been developed. These methods can accelerate data acquisition by reducing the number of gradient encoding steps, i.e., by collecting undersampled data. They differ by the way how images are reconstructed. For example, parallel imaging (7–13) makes use of coil sensitivity encoding. Image-space methods like SENSE (8) can remove image aliasing associated with undersampling by calculating the inverse of a multi-channel coil sensitivity

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encoding matrix. Since coil sensitivity encoding introduces k-space data correlation, missing data can be estimated from collected data samples. This has led to a group of k-space reconstruction methods like GRAPPA (10). Another category of high-speed MRI methods, compressed sensing (14–16), takes advantage of MRI data sparsity. By simultaneously enforcing data sparsity in a transformed space (e.g. wavelet space) and data fidelity in the sampling space, images may be reconstructed iteratively. In dynamic imaging, image reconstruction may benefit from information sharing among time sequential images. A number of data sharing methods have been developed for improved temporal resolution. This category includes generalized series reconstruction (17–19), keyhole or data replacement strategy (20–22), constrained reconstruction (23,24), dynamic imaging by modeling (25,26), UNFOLD (27), and reduced field of view (FOV) imaging (28,29).

Another major difference of existing high speed MRI methods is in the sampling strategy. MRI data may be collected with Cartesian or non-Cartesian encoding. Uniform undersampling in Cartesian imaging generates periodic and concentrated aliasing that can be described by a group of linear equations. Owing to its simplicity and flexibility, this strategy has been widely used in parallel imaging and data sharing methods, including SMASH (9), SENSE (8), PILS (11), GRAPPA (10), k-t SENSE/BLAST (12), k-t GRAPPA (13), generalized series reconstruction (17–19), keyhole (20–22), and constrained reconstruction (23,24). In contrast, image reconstruction in non-Cartesian imaging is more difficult because k-space non-uniformity introduces randomness in image-space aliasing. This issue may be addressed by methods like HYPR (30), BOSCO (31), projection reconstruction using FOCUSS (32), radial FOCUSS (33), spiral GRAPPA (34), auto-calibrated parallel spiral imaging (35) and iterative reconstruction using total variation (36). Many extended parallel imaging methods have the ability to reconstruct both Cartesian and non-Cartesian data. These include Iterative SENSE (37), PARS (38), JSENSE (39,40), pseudo-Cartesian GRAPPA (41), GROG (42,43), and kSPA (44). Most compressed sensing methods use random undersampling and iterative image reconstruction algorithms (14–16,45). They are also suitable for both Cartesian and non-Cartesian imaging.

High-speed MRI may benefit from the integration of multiple image reconstruction mechanisms. In dynamic high-speed MRI, coil sensitivity encoding and information sharing mechanisms can be utilized in aggregate. Previous works demonstrated that this integration outperforms either parallel imaging or data sharing methods alone (3,12,13,30,46,47). Here correlation imaging developed in a previous study (1) is enhanced by integrating multiple image reconstruction mechanisms. The presented work is inspired by an observation that MRI reconstruction needs data correlation either in the sampling space or in a transformed space. For example, parallel imaging seeks neighboring k-space data correlation introduced by coil sensitivity encoding. Compressed sensing takes advantage of transformed-space data sparsity that manifests as data correlation in the original space. Data sharing methods make use of information shared by time-sequential images, implying the existence of temporal data correlation. Since correlation imaging converts image reconstruction to the estimate of correlation functions that quantify data correlation (1), it naturally provides an approach to integrating multiple image reconstruction mechanisms for improved high-speed MRI.

The presented work is the continuation of a large effort to accelerate multi-scan MRI protocols in clinical practice (1). This work makes two contributions: First, a new approach to image reconstruction with arbitrary sampling trajectories is developed. This allows for the use of a sampling randomization to improve image reconstruction (14–16,45). In addition, correlation imaging with arbitrary sampling trajectories provides a generalized high-speed MRI reconstruction framework for multi-scan MRI protocols that use different sampling trajectories in different scans. It should be noted that the proposed algorithm is linear. Compared with other iterative approaches, such as SPIRiT (48), a linear approach is more suitable for clinical applications owing to its flexibility and simplicity. Second, the theoretical framework of correlation imaging is extended from k- to hybrid-space. This provides a generalized model for making use of multiple physical mechanisms that may introduce data correlation in clinical MRI.

This manuscript first presents the theory for k- and hybrid-space correlation imaging with arbitrary sampling trajectories and its practical implementation for Cartesian and non-Cartesian imaging. In the experimental work, correlation imaging with arbitrary sampling trajectories is validated in reference to SPIRiT (48). The ability of hybrid-space correlation imaging to utilize coil sensitivity encoding, data sparsity and information sharing is demonstrated in multi-slice 2D imaging, multi-scan imaging and radial cardiac imaging.

Theory

This section presents a generalized linear approach to image reconstruction of arbitrarily sampled data for k- and hybrid-space correlation imaging. Practically, it is more advantageous to implement correlation imaging in a hybrid space. For example, multi-slice 2D imaging is considered in the 3D hybrid space formed from a 2D k-space and a 1D image-space.

Notation in the following theoretical work

N : channel number in data acquisition.

$\mathbf{k}, \mathbf{k}', \mathbf{k}''$: 2D or 3D k-space position vectors.

$\mathbf{r}$: a multi-dimensional position vector in image space.

$\mathbf{k}$: a linear k-space position shift vector.

$*$: operation of conjugate.

$\{d_i(\mathbf{k}), i=1,2,\dots,N\}$: N-channel data with full Fourier encoding.

$t_s(\mathbf{k})$: a binary-valued (0 or 1) trajectory function for undersampling from fully sampled data.

$\{u_i^m(\mathbf{k}), i=1,2,\dots,N\}$: a N-channel linear predictor for image reconstruction of channel m .

$\{c_{ij}(\mathbf{k}), i=1,2,\dots,N; j=1,2,\dots,N\}$: auto- and cross-channel correlation functions.

$c_t(\mathbf{k})$: the correlation function of an undersampling trajectory $t_s(\mathbf{k})$.

$\{D_i(\mathbf{r}), i=1,2,\dots,N\}$: N-channel images formed from fully Fourier encoded data $\{d_i(\mathbf{k}), i=1,2,\dots,N\}$.

$T_s(\mathbf{r})$: the point spread function of an undersampling trajectory $t_s(\mathbf{k})$.

$\{U_i^m(\mathbf{r}), i=1,2,\dots,N\}$: Image-space representation of a N-channel linear predictor
 $\{u_i^m(\mathbf{k}), i=1,2,\dots,N\}$.

$\{C_{ij}(\mathbf{r}), i=1,2,\dots,N; j=1,2,\dots,N\}$: Image-space representation of auto- and cross-channel correlation functions $\{c_{ij}(\mathbf{k}), i=1,2,\dots,N; j=1,2,\dots,N\}$.

$C_t(\mathbf{r})$: Image-space representation of an undersampling-trajectory correlation function
 $c_t(\mathbf{k})$.

Error function and k-space solution

Correlation imaging (1) is formulated as a linear prediction problem. As shown in Figure 1, the linear predictor estimates imaging data in channel m , $m=1, 2, \dots, N$, from multi-channel undersampled data:

$$\tilde{d}_m(\mathbf{k}, \Delta\mathbf{k}) = \sum_{i=1}^N \sum_{\text{all } \mathbf{k}'} d_i(\mathbf{k} - \mathbf{k}') t_s(\mathbf{k} - \mathbf{k}' - \Delta\mathbf{k}) u_i^m(\mathbf{k}') \quad (1)$$

In this work, the error function for linear prediction is defined as:

$$\varepsilon = \sum_{\text{all } \Delta\mathbf{k}} \sum_{\text{all } \mathbf{k}} \left| d_m(\mathbf{k}) - \tilde{d}_m(\mathbf{k}, \Delta\mathbf{k}) \right|^2 = \sum_{\text{all } \Delta\mathbf{k}} \sum_{\text{all } \mathbf{k}} \left| d_m(\mathbf{k}) - \sum_{i=1}^N \sum_{\text{all } \mathbf{k}'} d_i(\mathbf{k} - \mathbf{k}') t_s(\mathbf{k} - \mathbf{k}' - \Delta\mathbf{k}) u_i^m(\mathbf{k}') \right|^2 \quad (2)$$

It should be noted that the previous work (1) does not use the error summation over sampling-space shift $\mathbf{k}$. Here this error summation is used to address image reconstruction with non-uniform sampling. As shown in Figure 2, a non-uniform sampling trajectory, if shifted by different $\mathbf{k}$ relative to the full k-space data, may introduce different sampling patterns in each region, resulting in a reconstruction error function dependent on the shift. This k-space variant problem cannot be resolved using a linear approach. Due to averaging effects, the error summation in Eq. 2 gives an error function independent on $\mathbf{k}$ and image reconstruction with non-uniform sampling becomes a sampling-space invariant problem. This provides a mathematical approach to seeking a linear solution for image reconstruction of arbitrarily sampled data.

By letting the partial derivative of the error function with respect to the conjugate of each element of the linear predictor be equal to zero, a group of linear equations can be generated and used to find the least-square solution to linear prediction. The equation with respect to the conjugate of an element $u_j(\mathbf{k}'')$

$$\begin{aligned}
& \sum_{i=1}^N \sum_{\text{all } \mathbf{k}'} \left[\sum_{\text{all } \mathbf{k}} d_i(\mathbf{k} - \mathbf{k}') d_j^*(\mathbf{k} - \mathbf{k}'') \sum_{\text{all } \Delta \mathbf{k}} t_s(\mathbf{k} - \mathbf{k}' - \Delta \mathbf{k}) t_s^*(\mathbf{k} - \mathbf{k}'' - \Delta \mathbf{k}) \right] u_i(\mathbf{k}') \\
&= \sum_{\text{all } \mathbf{k}} d_m(\mathbf{k}) d_j^*(\mathbf{k} \\
&\quad - \mathbf{k}'') \sum_{\text{all } \Delta \mathbf{k}} t_s^*(\mathbf{k} - \mathbf{k}'' - \Delta \mathbf{k})
\end{aligned} \tag{3}$$

Here by definition, the correlation function between two channels i and j is:

$$c_{ij}(\mathbf{k}) = \sum_{\text{all } \mathbf{k}'} d_j(\mathbf{k}' + \mathbf{k}) d_j^*(\mathbf{k}') \tag{4}$$

and the correlation function of an undersampling trajectory is:

$$c_t(\mathbf{k}) = \sum_{\text{all } \mathbf{k}'} t_s(\mathbf{k}' + \mathbf{k}) t_s^*(\mathbf{k}') \tag{5}$$

And because $t_s(k)$ is binary valued, the following equation can be derived:

$$\sum_{\text{all } \Delta \mathbf{k}} t_s^*(\mathbf{k} - \mathbf{k}'' + \Delta \mathbf{k}) = \sum_{\text{all } \Delta \mathbf{k}} t_s(\mathbf{k} - \mathbf{k}'' + \Delta \mathbf{k}) t_s^*(\mathbf{k} - \mathbf{k}'' + \Delta \mathbf{k}) = c_t(0) \tag{6}$$

By substituting Eqs. 4–6 into Eq. 3, a group of linear equations are generated:

$$\sum_{i=1}^N \sum_{\text{all } \mathbf{k}'} c_{ij}(\mathbf{k} - \mathbf{k}') c_t(\mathbf{k} - \mathbf{k}') u_i(\mathbf{k}') = c_t(0) c_{mj}(\mathbf{k}) \tag{7}$$

where the indexes j and $\mathbf{k}$ count all the elements of a N -channel linear predictor. With a predetermined undersampling trajectory $t_s(\mathbf{k})$, the linear predictor can be resolved from Eq. 7 if auto- and cross-channel correlation functions $c_{ij}(\mathbf{k})$'s are available. It should be noted that $t_s(\mathbf{k})$ is an arbitrary binary-valued function and Eq. 7 gives a solution to image reconstruction of arbitrarily sampled data.

Linear predictor in a multi-dimensional hybrid space

Multiple image reconstruction mechanisms can be integrated into correlation imaging by quantifying data correlation with correlation functions. A practical challenge to this integration is that different mechanisms introduce data correlation in different domains. For example, coil sensitivity encoding for parallel imaging introduces k -space data correlation. Data sparsity often introduces image-space data correlation. Information sharing for data sharing methods introduces temporal data correlation. To utilize different-domain data correlation, correlation imaging can be implemented in a multi-dimensional hybrid space.

Figure 3 gives a linear prediction model for correlation imaging in a hybrid space $(\mathbf{k}, \mathbf{I})$, where $\mathbf{k}$ represents a 2D or 3D k -space position vector and $\mathbf{I}$ is a multi-dimensional vector that indexes images with data correlation in an arbitrary domain. In this model, image reconstruction utilizes coil sensitivity encoding in the $\mathbf{k}$ dimension and other reconstruction mechanisms in the $\mathbf{I}$ dimension. For example, if $\mathbf{I}$ indexes a series of 2D image slices, image

reconstruction may utilize across-slice image smoothness introduced by MRI data sparsity. If $\mathbf{I}$ indexes a series of time sequential images, image reconstruction may utilize temporal data correlation. Since the models in Figures 1 and 3 differ only by the space where data are undersampled and reconstructed, one can derive the equations for hybrid-space reconstruction by replacing $\mathbf{k}$ in Eqs. 1–7 with $(\mathbf{k}, \mathbf{I})$. The linear prediction is then given by:

$$\tilde{d}_m(\mathbf{k}, \mathbf{I}, \Delta\mathbf{k}, \Delta\mathbf{I}) = \sum_{i=1}^N \sum_{\text{all } \mathbf{k}', \mathbf{I}'} d_i(\mathbf{k}-\mathbf{k}', \mathbf{I}-\mathbf{I}') t_s(\mathbf{k}-\mathbf{k}'-\Delta\mathbf{k}, \mathbf{I}-\mathbf{I}'-\Delta\mathbf{I}) u_i^m(\mathbf{k}', \mathbf{I}') \quad (8)$$

The hybrid-space linear predictor can be resolved using the following linear equations:

$$\sum_{i=1}^N \sum_{\text{all } \mathbf{k}', \mathbf{I}'} c_{ij}(\mathbf{k}-\mathbf{k}', \mathbf{I}-\mathbf{I}') c_t(\mathbf{k}-\mathbf{k}', \mathbf{I}-\mathbf{I}') u_i^m(\mathbf{k}', \mathbf{I}') = c_t(0, 0) c_{mj}(\mathbf{k}, \mathbf{I}) \quad (9)$$

where

$$c_{ij}(\mathbf{k}, \mathbf{I}) = \sum_{\text{all } \mathbf{k}', \mathbf{I}'} d_i(\mathbf{k}'+\mathbf{k}, \mathbf{I}'+\mathbf{I}) d_j^*(\mathbf{k}', \mathbf{I}') \quad c_t(\mathbf{k}, \mathbf{I}) = \sum_{\text{all } \mathbf{k}', \mathbf{I}'} t_s(\mathbf{k}'+\mathbf{k}, \mathbf{I}'+\mathbf{I}) t_s^*(\mathbf{k}', \mathbf{I}') \quad (10)$$

Methods

Correlation imaging can accelerate MRI using multiple image reconstruction mechanisms. The presented theoretical work (Eqs. 1–10) provides a linear approach to improving this combinational reconstruction by optimizing undersampling. This section gives the practical implementation of correlation imaging and introduces a sampling randomization strategy for improved image reconstruction. Also presented are the experimental methods for demonstration.

Practical implementation: Data acquisition and image reconstruction

To resolve a linear predictor from Eq. 9, Eq. 10 may be used to estimate correlation functions in calibration. Computation efficiency can be improved if this calibration is performed in Fourier transformed space. The Fourier transform of Eq. 10 gives:

$$C_{ij}(\mathbf{r}, \mathbf{f}) = D_i(\mathbf{r}, \mathbf{f}) D_j^*(\mathbf{r}, \mathbf{f}) \quad (11)$$

and

$$C_t(\mathbf{r}, \mathbf{f}) = T_s(\mathbf{r}, \mathbf{f}) T_s^*(\mathbf{r}, \mathbf{f}) \quad (12)$$

where $\mathbf{f}$ is the Fourier conjugate vector to $\mathbf{I}$. These equations are used to estimate correlation functions.

Figure 4(a) shows how correlation imaging is implemented in Cartesian imaging. Here it is essential to have a set of calibration data. Practically, calibration may use either a set of low-resolution images collected before the real scan or auto-calibration signals (ACS) collected

simultaneously with the real scan. In calibration, Eq. 11 is used to calculate auto- and cross-channel correlation functions. Eq. 12 is used to calculate the correlation function of an undersampling trajectory. The calculated correlation functions are used to form Eq. 9 for resolving the linear predictor, which is in turn used to filter N-channel undersampled data for image reconstruction.

Figure 4(b) shows the implementation of correlation imaging for non-Cartesian imaging. Here the calibration uses the collected center k-space data. A non-uniform Fourier transform is first applied to non-Cartesian data using a gridding-based algorithm with iterative sampling density compensation described by previous studies (49,50). After transforming non-Cartesian data into Cartesian space, image reconstruction follows the same procedure as that in Cartesian imaging.

Practical implementation: Undersampling randomization in a hybrid-space

Undersampling is the cause of image aliasing and affects how aliasing is suppressed using different reconstruction mechanisms. Aliasing suppression with coil sensitivity encoding tends to be most effective with uniform undersampling while random undersampling may improve image reconstruction with data sparsity and information sharing (8,14,16). In hybrid-space correlation imaging, a hybrid undersampling strategy (Figure 5a) is introduced for multiple image reconstruction mechanisms. In the $\mathbf{k}$ dimension, data are undersampled uniformly with a $\mathbf{I}$ -dependent linear shift $\mathbf{k}(\mathbf{I})$. By utilizing a shift that randomly varies in the $\mathbf{I}$ dimension, undersampling gives uniformity in the $\mathbf{k}$ dimension and randomness in the $\mathbf{I}$ dimension. This generates a sampling trajectory compatible with both $\mathbf{k}$ -dimension coil sensitivity encoding and $\mathbf{I}$ -dimension data sparsity/information sharing in image reconstruction. From the calculated point spread function (Figure 5b), it is found that sampling randomization in the $\mathbf{I}$ dimension also suppresses the periodic aliasing associated with uniform undersampling in the $\mathbf{k}$ dimension.

MRI experiments

All the human subjects were scanned with the approval of the Institutional Review Board at the Cincinnati Children's Hospital Medical Center.

A multi-scan head imaging experiment was conducted using an 8-channel coil array and a 3.0 Tesla clinical scanner. Two sets of multi-slice 2D data were acquired sequentially using a T1-weighted TSE FLAIR sequence (TR/TI/TE 11000/2800/125 ms, TSE factor 30, FOV 240×240 mm, matrix 240×240, 30 axial slices with 3 mm thickness and 1 mm gap, phase encoding left-right) and a T2-weighted TSE sequence (TR/TE 3000/100 ms, TSE factor 12, flip angle 90°, FOV 240×240 mm, matrix 240×240, 30 axial slices with 3 mm thickness and 1 mm gap, phase encoding left-right).

A breath hold 2D cine cardiac imaging experiment was conducted using a 4-channel cardiac coil and a 1.5 Tesla clinical scanner. A set of time sequential images was collected with a 2D TrueFISP radial sequence (TR/TE 36.64/2.29 ms, flip angle 65°, FOV 300 mm, slice thickness 5 mm, 256 projections with 512 points/projection, 17 cardiac phases).

Algorithm implementation and performance characterization

Offline undersampling was used to simulate imaging acceleration. Offline image reconstruction experiments were conducted for feasibility demonstration. In the presented Cartesian reconstruction experiments, calibration uses 24 ACS lines for 1D undersampling trajectories. The reconstructed center k-space data were replaced by ACS for improved signal to noise ratio (SNR) at the final step of reconstruction. In the presented non-Cartesian imaging experiments, self-calibration was performed with 10% of the center k-space data. All the algorithms were implemented in MATLAB (MathWorks Inc., Natick, MA).

To validate correlation imaging with arbitrary sampling trajectories, SPIRiT iterative reconstruction was used as a reference method owing to its generality and robustness. The reference reconstruction was performed using the SPIRiT MATLAB software developed by Lustig (48). Following the experimental validation, hybrid-space correlation imaging was investigated in multi-slice 2D imaging, multi-scan imaging and radial cardiac imaging.

Images reconstructed without undersampling were used as image references. Reconstruction performance was characterized using an error image calculated by subtracting a reconstructed image from the reference image. Quantitative characterization was given by an RSS error defined as the square Root of the Sum of Squares (RSS) of an error image. In the presented results, this error was normalized with respect to the reference image and was given as a percentage number representing the ratio of the RSS of an error image to that of the reference image.

Results

In this section, image reconstruction with arbitrary sampling trajectories is investigated in reference to SPIRiT. To demonstrate its ability to enhance correlation imaging with multiple image reconstruction mechanisms, hybrid-space implementation with sampling randomization is investigated in multi-slice 2D imaging, multi-scan imaging and radial Cardiac imaging.

Experimental validation in reference to SPIRiT

Correlation imaging with arbitrary sampling trajectories were validated in reference to SPIRiT. The image reconstruction experiments were performed with two sets of 2D MRI data that have been used in the original SPIRiT literature (48): One was the 8-channel 2D Cartesian imaging data from a human brain scan and the other was the 8-channel 2D spiral imaging data from a phantom scan. These data were obtained from the SPIRiT software. CG-SPIRiT algorithm was used. The center 24×24 samples of Cartesian data and the center 10% samples of spiral data were used for calibration in the experiments. Correlation functions were calculated from these fully sampled low-resolution calibration data.

In Cartesian imaging (Figure 6), reconstruction experiments were first performed with the 2D uniform and random undersampling trajectories (reduction factor $R=4$) provided by the SPIRiT software (48). From the error images (Figure 6d, 1st and 2nd rows), it was found that correlation imaging gives more artifacts while SPIRiT generates higher noise. The calculated RSS errors of correlation imaging and SPIRiT are comparable. Since MRI data

are always fully sampled data in the frequency-encoding direction, reconstruction experiments were then performed with 1D undersampling practically used in 2D MRI. By comparing reconstruction error images with $R=4$ (Figure 6d, 3rd and 4th rows), it was again found correlation imaging gives more artifacts while SPIRiT generates higher noise. Quantitatively by calculating RSS errors with reduction factors from 2 to 6 (Figures 6b and 6c), it was found that correlation imaging performs slightly better with uniform sampling while SPIRiT slightly better with random sampling.

In spiral imaging (Figure 7), reconstruction experiments were performed with two different reductions factors ($R=3$ and 4). It was found that SPIRiT gives less noise while correlation imaging gives fewer artifacts in the error images. Quantitatively from the calculated RSS errors, it was found that two methods give comparable performance.

Correlation imaging using across-slice data sparsity in multi-slice 2D imaging

In multi-slice 2D imaging, correlation imaging can benefit from both k- and image-space data correlation. Here multi-slice 2D imaging is considered in a 3D hybrid space (k_x, k_y, z) , where k_x is the k-space frequency encoding direction, k_y is the k-space phase encoding direction and z is the image-space slice position index, (k_x, k_y) forms the **k** dimension and z forms the **I** dimension (Figure 3). By sampling randomization in the **I** dimension (Figure 5), multi-slice 2D imaging can be accelerated using 3D data correlation introduced by both coil sensitivity encoding and across-slice data sparsity.

Multi-slice 2D imaging relies primarily on 2D image reconstruction methods. Here SPIRiT is used as a reconstruction reference because it has been demonstrated to be superior to most linear 2D methods. Since it relies on coil sensitivity encoding alone, however, it is physically limited by multi-channel coil arrays. Typically, the maximal acceleration factor for parallel imaging alone with an 8-channel head coil is 4 in 2D imaging. This limitation may be overcome by utilizing data sparsity in addition to coil sensitivity encoding in hybrid-space correlation imaging.

A reconstruction experiment was conducted using multi-slice 2D data from the T2-weighted TSE scan. Figure 8 shows reconstruction results for the center slice with reduction factors ranged from 5 to 10. As expected, SPIRiT (Figure 8b) generates strong aliasing artifacts due to the limitation posed by the 8-channel coil array. It should be noted that the total reconstruction errors of these SPIRiT images are technically acceptable. However, because the aliasing artifacts manifest in such a concentrated pattern that they can easily cause misjudgments in disease diagnosis and treatment, the image reconstruction is not clinically acceptable. In this case, imaging acceleration is limited by artifacts. Figure 8(c) shows how 2D k-space data vary across slices (a center phase encoding line $k_y=0$ is used as an example). It can be seen that the data are spread out along the slice direction in hybrid space while they are concentrated after 1D slice-direction Fourier transform. This indicates multi-slice 2D imaging has Fourier-space data sparsity and thus image-space data correlation. By utilizing both across-slice data sparsity and coil sensitivity encoding in the 3D hybrid space, correlation imaging with sampling randomization gives considerably fewer aliasing artifacts than SPIRiT (Figure 8d), thereby providing an approach to performing multi-slice 2D imaging at a higher speed than parallel imaging.

Correlation imaging using across-scan data correlation in multi-scan imaging

In multi-scan imaging (1), correlation imaging may benefit from image similarities across scans. Figures 8 (a) and (e) are two images collected from two different scans (T2-weighted TSE and T1-weighted TSE FLAIR). Although the image contrast is different, the geometry of tissue boundaries is similar, which introduces across-scan data correlation. By including multi-slice images from both the T1-weighted TSE FLAIR and the T2-weighted TSE scans in the **I** dimension, across-slice and across-scan data correlation can be integrated into 4D hybrid-space correlation imaging for the T2-weighted TSE scan. Compared with 3D correlation imaging (Figure 8d), 4D correlation imaging (Figure 8f) gives less artifacts, permitting multi-slice 2D imaging to be run at an even higher speed in multi-scan MRI.

Correlation imaging using temporal data correlation in dynamic imaging with a radial trajectory

In radial cardiac imaging, correlation imaging can benefit from k-t space data correlation. Here image reconstruction is implemented in a 3D radial k-t space (k_r, k_θ, t_i), where k_r represents the k-space radial direction, k_θ represents the k-space angular direction, t_i is the index of time frames, (k_r, k_θ) forms the **k** dimension and t_i forms the **I** dimension (Figure 3). To randomize undersampling along the temporal dimension, a strategy similar to Figure 5 is used with an angular shift $\theta(t_i)$ varying randomly with time (Figure 9a). This undersampling strategy can improve image reconstruction using 3D k-t space data correlation introduced by both coil sensitivity encoding and temporal information sharing.

A GRAPPA-like reconstruction method was implemented using pseudo-Cartesian GRAPPA (41) in conjunction with k-t GRAPPA (13). A k-t FOCUSS reconstruction method was implemented using the original MATLAB software developed by Jung et al. (33). These methods were used as reconstruction references. The GRAPPA-like method can effectively utilize coil sensitivity encoding. Since k-t GRAPPA uses uniform sampling in the temporal direction, however, its image reconstruction with temporal information sharing is suboptimal. In contrast, k-t FOCUSS is a compressed sensing technique that relies only on temporal information sharing. It is expected that 3D k-t space correlation imaging will outperform these two methods because image reconstruction is optimized with an undersampling strategy compatible with both coil sensitivity encoding and temporal information sharing.

An image reconstruction experiment was conducted using 16 of 256 radial lines at each time frame. This acceleration factor is high considering the data are collected using a 4-channel coil. From the reconstruction results (Figure 9b), it was found that the GRAPPA-like method suffers from spiking artifacts and high noise. Although k-t FOCUSS gives high image quality at the first time frame, the temporal resolution is low. In contrast, hybrid-space correlation imaging gives both better image quality at the first time frame and higher temporal resolution in the dynamic image trajectory. Here the hybrid-space undersampling trajectory with k-space angular uniformity and temporal randomization plays a key role in correlation imaging. This trajectory effectively reduces the aliasing (Figure 5b) and improves image reconstruction using k-space and temporal data correlation in aggregate.

Discussion

Relation to parallel imaging, compressed sensing and data sharing methods

Correlation imaging is proposed for clinical MRI where multiple images, such as multi-slice 2D images, multi-scan images or multi-timeframe images are available. Hybrid-space correlation imaging provides a generalized framework for image reconstruction using data correlation arising from multi-image acquisition. The underlying data correlation is physically associated with those image reconstruction mechanisms in parallel imaging, compressed sensing and data sharing methods. However, the mathematical approach is different and the presented work is not intended to replace any of these conventional methods aimed at accelerating single-image acquisition.

In single-slice 2D or single-volume 3D Cartesian imaging, correlation imaging relies primarily on k-space data correlation arising from coil sensitivity encoding. The presented work is closely related to linear GRAPPA- or SENSE-based methods because image reconstruction is formulated as the same k-space convolution problem and uses the same physical information. In this case, it is not expected that a performance gain over conventional parallel imaging methods can be achieved. The benefit is expected to be flexibility and simplicity as correlation imaging provides a generalized linear framework for k- or image-space reconstruction of arbitrarily sampled data.

In clinical MRI, it is expected that the presented work will significantly benefit many MRI data collection methods, such as multi-slice 2D imaging, multi-scan imaging and dynamic imaging. By performing hybrid-space image reconstruction that can take advantage of data correlation underlying multi-image data acquisition, correlation imaging provides a gain over several parallel imaging, compressed sensing and data sharing methods. This gain arises from two physical factors: First, hybrid-space data correlation is associated with multiple image reconstruction mechanisms including coil sensitivity encoding, data sparsity and temporal information sharing. More physical information is used in image reconstruction than those conventional methods. Second, sampling randomization in the **I** dimension of a hybrid space effectively reduces aliasing generated from undersampling in the **k** dimension. This physically facilitates image reconstruction and improves the integration of multiple data correlation mechanisms.

Algorithm validation with respect to SPIRiT

It should be pointed out that the experiments presented in Figures 6 and 7 aim to validate correlation imaging algorithm using a well-established approach, SPIRiT. These experiments are not designed for a complete evaluation because the different image reconstruction mechanisms underlying the two methods generate many practical issues that make the comparison difficult. For example, correlation functions can be calibrated using either coil sensitivity profiles, i.e., coil sensitivity alone, or images, i.e., both coil sensitivity and image content. However, SPIRiT always uses images (ACS) to calibrate k-space convolution kernels. For the purpose of validation, the presented work uses the same calibration data as that in SPIRiT although it was found that correlation imaging calibration is more robust using coil sensitivity profiles than that using images in single-slice 2D

imaging. As a result, the performance of correlation imaging shows certain dependence on imaging data.

In general, SPIRiT is a non-linear iterative reconstruction technique that relies on self-consistency constraint arising from k-space data correlation. This technique uses the physical information provided primarily by multi-channel coil sensitivity encoding. In single-slice 2D and single-volume 3D imaging, SPIRiT outperforms many conventional linear parallel imaging techniques because it takes the most advantage of k-space data correlation by algorithm iteration. Although the performance difference between k-space correlation imaging and SPIRiT was found to be minor in the presented work (Figures 6 and 7), it is believed that SPIRiT would perform better in a larger scale experiments because correlation imaging trades image quality in k-space areas with high sampling density for compensating that in those areas with low sampling density. However, it is expected that hybrid-space correlation imaging will provide a gain over SPIRiT in multi-slice 2D, multi-scan and dynamic imaging where multiple images are available, because this approach may physically benefit from I-dimension data correlation mechanisms that may not introduce a strong self-consistency constraint.

Sampling density compensation

In the presented work, image reconstruction is formulated as a k-space invariant problem by minimizing an error function that sums reconstruction errors associated with all the different k-space shifts ($\mathbf{k}$ in Eq. 2) of a sampling trajectory. Since reconstruction errors increase as sampling density decreases, image quality in k-space areas with high sampling density may be traded for compensating that in those areas with low sampling density, resulting in a suboptimal solution to image reconstruction. To improve image reconstruction with a sampling trajectory that varies considerably in sampling density, correlation imaging needs sampling density compensation. In non-Cartesian imaging, this is automatically implemented using a non-uniform Fourier transform (49,50). In multi-dimensional (2D) random Cartesian undersampling, sampling density variation may be minimized using techniques like Poisson-disc sampling (51). In 1D random Cartesian undersampling that has limited freedom of randomization, however, the experimental results show that correlation imaging could be inferior to those iterative methods that treat image reconstruction as a k-space variant problem.

To improve image reconstruction with 1D random undersampling (e.g. Figure 6), every phase encoding line was resampled along the frequency encoding direction with an undersampling rate designed to equalize the neighboring sampling density variation along the phase encoding direction. By resampling with different k-space shifts for multiple times, the original dataset was divided into multiple datasets without information loss. These datasets were considered as those generated from the same image with different undersampling trajectories. By aligning them in the I dimension of a hybrid-space correlation imaging model, images may be reconstructed from data with lower sampling density variation. Figure 10(a) shows an example of sampling density compensation for 1D random undersampling: A 1D trajectory is equivalently divided into five 2D random trajectories with improved sampling uniformity. As demonstrated in Figure 10(b), image

aliasing cannot be effectively suppressed using correlation imaging without sampling density compensation for a 1D undersampling trajectory ($R=5$). By regrouping the data into five datasets with different 2D undersampling trajectories (Figure 10a), correlation imaging gives image quality comparable to that of SPIRiT. Since this data regrouping increases the number of images to be reconstructed, the gain in image quality is achieved at a cost of computation. It should be noted that this approach is generally suitable for correlation imaging with arbitrary sampling trajectories because MRI data are always fully sampled in the frequency encoding direction.

Resolved and unresolved problems

Multi-scan MRI acceleration is challenging because data are collected with different data acquisition strategies in different scans. The presented work addresses the sampling trajectory difference across scans. However, this work will still face other challenges found in the previous work, e.g. phase distortion in EPI sequence, image FOV inconsistency.

Conclusion

Correlation imaging provides a generalized linear approach to accelerating MRI using multiple image reconstruction mechanisms and arbitrary sampling trajectories. It can be optimized by seeking an undersampling trajectory compatible with different image reconstruction mechanisms. Hybrid-space correlation imaging with sampling randomization offers the ability to utilize coil sensitivity encoding, data sparsity and information sharing mechanisms in aggregate. This improves image reconstruction in Cartesian and non-Cartesian imaging, thereby providing a higher imaging speed.

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Figure 1.

Linear prediction model for correlation imaging in k-space: Image reconstruction is modeled as a linear estimate of real data from multi-channel undersampled data. Here, N is the channel number and $\{u_i(\mathbf{k}), i=1,2,\dots,N\}$ is a linear predictor. This model seeks the least square solution to a linear predictor for reconstructing single-channel data $d_m(\mathbf{k})$ from undersampled N -channel data $\{d_i(\mathbf{k}) \cdot t_s(\mathbf{k}), i=1,2,\dots,N\}$, where $t_s(\mathbf{k})$ is an arbitrary undersampling trajectory. The reconstructed data are generated by filtering the undersampled data with the linear predictor and then summing the filtered data over channels.

Figure 2.

An example of 1D non-uniform undersampling trajectory with different k-space shift Δk . Sampling non-uniformities introduce reconstruction error distribution that varies regionally in k-space (k-space variant). By error summation over all Δk 's relative to the full k-space, regional non-uniformities are removed due to averaging effects and reconstruction error distribution becomes k-space invariant. This allows for the use of a linear approach to finding a solution to image reconstruction. The solid lines represent sampled phase encoding lines and the dashed lines represent missing lines.

Figure 3.

Linear prediction model for correlation imaging in a multi-dimensional hybrid space: N , channel number; $\{d_i(\mathbf{k}, \mathbf{I}), i=1,2,\dots,N\}$, N -channel fully sampled data; $t_s(\mathbf{k}, \mathbf{I})$, undersampling trajectory; $\{u_i(\mathbf{k}, \mathbf{I}), i=1,2,\dots,N\}$, linear predictor; $\mathbf{k}$, 2D or 3D k -space position vector; $\mathbf{I}$, multi-dimensional vector that indexes image series with similarities. This model has the same structure as that in Figure 1 except that data are undersampled and reconstructed in a hybrid space.

Figure 4.

Implementation of correlation imaging for Cartesian (a) and non-Cartesian (b) imaging. Cartesian imaging requires additional calibration data while non-Cartesian imaging is self-calibrated. In non-Cartesian imaging, a non-uniform Fourier transform is first used to transfer non-Cartesian data into Cartesian space. In Cartesian space, two implementation schemes follow the same procedures.

Figure 5.

(a) A hybrid-space undersampling trajectory ($R=5$) that provides uniformity in the $\mathbf{k}$ dimension and randomness in the $\mathbf{I}$ dimension. (b) The point spread function calculated from the undersampling trajectory in (a) by Fourier transform. Above the 2D image is the 1D intensity plot across the center white dot (real image) along the $\mathbf{k}$ direction in Fourier space. It can be seen that $\mathbf{k}$ -dimension uniformity generates a periodic aliasing pattern. In reference to the real image intensity, the aliasing intensity is low due to the introduction of $\mathbf{I}$ -dimension randomness.

Figure 6. Reconstruction of 2D Cartesian brain imaging data using correlation imaging and SPIRiT. The data are provided by the SPIRiT software. (a) Reference image from fully sampled data. (b) RSS errors from reconstruction using 1D uniform sampling with different reduction factors ($R=2\sim6$). (c) RSS errors from reconstruction using 1D random sampling with different reduction factors ($R=2\sim6$). (d) Examples of reconstruction using 2D uniform (first row), 2D random (second row), 1D uniform (third row) and 1D random (fourth row) undersampling trajectories with a reduction factor $R=4$. From left to right in each row are

the undersampling trajectory, image space aliasing associated with the undersampling trajectory, SPIRiT reconstruction, and correlation imaging reconstruction. The 2D undersampling trajectories are provided by the SPIRiT software for demonstration. Practically, 1D undersampling is used in 2D MRI data acquisition. The percentage numbers in the error images are calculated RSS errors. In the undersampling trajectory figures, white color=1 and black color=0.

Figure 7.

Examples of reconstruction from spiral imaging data using correlation imaging and SPIRiT.

(a) Reference image from fully sampled data. (b) Image aliasing associated with undersampling ($R=3$ and 4): Images are generated from a non-uniform Fourier transform with zero padding. (c) SPIRiT and correlation imaging reconstruction results ($R=3$ and 4). The percentage numbers in the error images are calculated RSS errors.

Figure 8.

Hybrid-space correlation imaging in multi-slice 2D imaging and multi-scan imaging: The data were collected from two sequential multi-slice 2D scans with T1 TSE FLAIR and T2 TSE sequences. The T2 TSE data (30 slices) were undersampled for reconstruction experiments. The results for the center slice with reduction factors from 5 to 10 are presented. (a) Reference image from fully sampled data. (b) 2D SPIRiT reconstruction. (c) Multi-slice 2D data are sampled in a 3D hybrid space (k_x, k_y, z), where k_x is the k-space frequency encoding direction, k_y is the k-space phase encoding direction and z is the image-space slice direction. The 2D figures show the sampled data at $k_y=0$, i.e., the center k-space phase encoding lines from 30 slices. Data are spread out along the slice direction in the original space (top) while they show sparsity after 1D slice-direction Fourier transform (bottom). The axis f_z is Fourier-space slice direction. (d) 3D correlation imaging using across-slice data correlation introduced by Fourier-space slice direction data sparsity. (e) A T1 FLAIR image at the same position as that of (a). (f) 4D correlation imaging using across-

slice and across-scan data correlation. The percentage numbers in the error images are calculated RSS errors.

Figure 9.

Hybrid-space correlation imaging in radial cardiac imaging. (a) Undersampling strategy: 16 of 256 radial lines are used at every time frame. The trajectory is randomized temporarily using an angular shift $\theta(t_i)$ that randomly varies with time. (b) Reference and reconstructed images. The top row gives the reference and reconstructed 2D images at the first time frame. The bottom row gives the reference and reconstructed temporal trajectories along the white dotted lines shown in the top-row images. The aliasing images (2nd column) are generated by non-uniform Fourier transform. In reconstruction, correlation imaging uses the undersampling trajectory in (a). In radial GRAPPA, the angular shift of 16 radial lines increases linearly with time in a fashion similar to k-t GRAPPA. In k-t FOCUSS, sampling randomization is provided by the original k-t FOCUS MATLAB software.

Figure 10.

(a) Sampling density compensation for a 1D random undersampling trajectory (white color=1 and black color=0 in the figure): By resampling every phase encoding line in the frequency encoding direction, the original undersampled trajectory is divided into 5 new 2D undersampling trajectories with better sampling density uniformity. (b) Reconstruction results with and without sampling density compensation: SPIRiT iterative reconstruction is used as reference. The data and the 1D random undersampling trajectory ($R=5$) are the same as those in Figure 6. The percentage numbers in the error images are calculated RSS errors.