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On Components, Latent Variables, PLS and Simple Methods: Reactions to Rigdon's Rethinking of PLS

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Abstract

Rigdon (2012) suggests that partial least squares (PLS) can be improved by killing it, that is, by making it into a different methodology based on components. We provide some history on problems with component-type methods and develop some implications of Rigdon's suggestion. It seems more appropriate to maintain and improve PLS as far as possible, but also to freely utilize alternative models and methods when those are more relevant in certain data analytic situations. Huang's (2013) new consistent and efficient PLSe2 methodology is suggested as a candidate for an improved PLS.

An important feature of science is that theories, methodologies, conclusions etc. are regularly reevaluated, unworkable or untestable ideas rejected, and new ideas and approaches proposed and perhaps provisionally accepted. Professor Rigdon (2012) has proposed re-evaluating the field of partial least squares (PLS) methodology. He thoroughly and coherently reviews many inadequacies of factor analysis based structural equation models (SEM), i.e., latent variable models (LVM), and proposes to eliminate LVM as a viable research strategy. In view of the historical commonalities between PLS and LVM, a rejection of LVM would require also rejecting PLS. To avoid this logical difficulty, Rigdon proposes morphing PLS into a composite variable model (CVM) based on compounds of measured variables. However, this creates a difficulty for PLS since CVM has no need for PLS, e.g., Hwang, Ho, and Lee (2010, p. 228) state that "GSCA [their variant of CVM] is recommended as an alternative to partial least squares for general SEM purposes" (p. 228). While GSCA may not be the final word in CVM (e.g., Henseler, 2012), at a minimum, the relations of PLS to CVM and LVM require some further analysis.

We certainly agree with Rigdon that observed and composite variable methods have an important role in data analysis and statistics – indeed, CVM are probably far more prevalent overall than LVM¹ -- and that development of new methodological and statistical tools for their use is a valuable goal. However, in contrast to Rigdon's proposal to eliminate PLS as an LVM, we think it is more important to elevate PLS as a desirable LVM by improving its mathematical and statistical basis. Before we explain how this might be accomplished, we

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¹To illustrate, teachers nearly universally have used a composite score to grade student performance.

discuss some features of CVM and LVM not emphasized by Rigdon. A thorough review of LVM is given in Hoyle (2012). Recent reviews of the use of PLS in management and marketing research are given by Hair, Sarstedt, Pieper, and Ringle (2012), Hair, Sarstedt, Ringle, and Mena (2012), Ringle et al. (2012), Rönkkö and Evermann (2013) and Henseler et al. (2014).

Errors of Measurement

There is a lot of virtue in classical test theory and its implications (e.g., Bentler, 2009). It can serve as a foundation for looking at the real world, and in turn, provide a viewpoint toward CVM and LVM. It seems obvious to many that any observed score is liable to be fallible, i.e., to contain unwanted noise or random error (plus various potential sources of bias not discussed here). Invariably a particular score value may seem a bit arbitrary for its intended use -- a student's score on an exam, a nurse's measure of blood pressure, an economist's statement on food inflation, a rating of teacher quality from student performance, and, as we recently saw in the US election, a preference given in a political survey -- no doubt only represent approximations to their intended target's concept score. Of course the basic equation $X = T + E$ is an abstraction that may not hold precisely in any particular situation², but it provides the basis for a nice skepticism about any particular X , implies the potential usefulness of other X s to reflect the same T (multiple indicators), and of course leads to further questions such as whether perhaps $T = a_1F_1 + \dots + a_kF_k$, that is, whether an LVM based on latent factors might be appropriate.³

If one does not want to reify any X , one can also be suspicious of a CVM, since a basic building block of any CVM is a linear compound of X s. Any such linear combination will automatically inherit the E s when they exist. Rigdon's espousal of CVM ignores these errors. We suggest that it is important to remember them, i.e., to remind oneself that under classical test theory and certain assumptions, composites will have smaller error than the individual X s, even though in practice we may never have enough X s in the composite to make the errors completely disappear (Bentler, 1972, 2007; Li & Bentler, 2011). Furthermore, since a given composite may well be a predictor variable in another equation of a CVM, we are forced to face the old issue that errors in predictor variables will almost always lead to parameter estimates for their coefficients that are biased proportionally to the magnitude of this irrelevant error (e.g., Cochran, 1968; Fuller, 1987). A main point of LVM is to avoid such biases (e.g., Wansbeek & Meijer, 2000). It would not be bad for PLS also to avoid such biases.

Rigdon recommends the use of various types of components, including those associated with decades-old methods that seem to have died out. Other older component methods also come to mind, e.g., Bentler (1976) proposed a CVM related to a relaxed LVM and Bartholomew (1984, 1985) proposed to use particular components that are rationalized on the basis of their closeness to factors. However, it has been noted that in such cases it would be possible just to use the factors themselves (Bentler, 1985a). While we are impressed by the surprisingly

²Especially when the observed data is binary. In such a case, we may take X to be a continuous latent variable with a probit or logit relation to its observed counterpart Y .

³When this is done, the interpretation of E shifts to uniqueness, incorporating both specificity and random error.

flexibility and ability to do clever things with newer component methods such as Hwang, Ho, and Lee's (2010) methodology for interactions, nonetheless, all component methods are equivalent in not being able to avoid incorporating errors. Bentler and de Leeuw (2011) showed that any components, including principal components, are linear combinations of common factors and unique factors. The latter contain random error. Why would one be interested in a linear combination that contains noise when such noise can be avoided with LVM?⁴

Scores

A lot of hand-wringing has gone into the issue of factor score indeterminacy. As Rigdon noted, this is an old problem that arises not only in factor analysis but also in more general models. However, many years ago Bentler (1980) noted that factor indeterminacy does not hinder model comparison and evaluation: "...although an infinite set of LVs [latent variables] can be constructed under a given LV model to be consistent with given MVs [measured variables], the goodness-of-fit of the LV model to data (as indexed, for example by a χ^2 statistic) will be identical under all possible choices of such LVs. Consequently, the process of evaluating the fit of a model to data and comparing the relative fit of competing models is not affected by LV determinacy. Thus, in my view, theory testing with LV models remains a viable research strategy in spite of factor indeterminacy" (p. 442). As far as we know, this proposition has not been disproven.

There is no doubt that individual scores for specific observations are sometimes needed. In such an application it is impossible to avoid relying on observed variables and composites. While the ideal nature of such a composite will depend on the application, LVM may still be useful. For example, a composite may be an estimate of a latent factor and used, with appropriate corrections, in factor score regression (Skrondal & Laake, 2001; Hoshino & Bentler, 2013), a predictive score based methodology like PLS. Or, it may be desirable to obtain a composite with maximal reliability or minimal noise (e.g., Bentler, 1968; Raykov, 2004).

Simplicity

Rigdon recommends methods and procedures that are simple. For example, he supports the use of equal or correlation weights in regression rather than standard least-squares estimated beta weights. We thoroughly agree that researchers should have simple models and methods available, but not necessarily because they are simple. We would rationalize any virtue of parameter simplicity using standard statistical terms such as bias, efficiency, and mean squared error, as well as reduced variance and lack of statistical degradation with fewer parameters. In the SEM context, Bentler and Mooijart (1989) provide some details. In the regression context, Bentler and Woodward (1979) developed a methodology for regression on composites that allows statistical tests of whether simple weights carry all of the information in the prediction or whether differential weighting adds any significant increment. They showed that simple a priori weights can be improved upon: rather than

⁴Bentler and Molenaar (2012) discuss models that seem to contain no factors but nonetheless are LVM.

accept either unit or correlation weights a priori, they developed a methodology to find and use optimal binary (+1, -1) or trinary (+1, 0, -1) weights. Trinary weights simplify by automatically dropping unimportant predictors. This overlooked methodology will be available in EQS 7 (Bentler, in preparation).

Parameters are parts of models, and model simplicity in LVM and CVM is, of course, dear to our heart. Rigdon did not propose a specific (simple or complex) CVM structure for use. The modeling complexity of LISREL (Jöreskog & Sörbom, 1996) with its many Greek parameter matrices led us to the far simpler Bentler-Weeks (1980) model and the EQS program (Bentler, 1985b) for non-matrix, and then, graphical modeling setups. It is odd to see SEM continue to be presented today in far more complex ways than is necessary (see e.g., Bentler 2010). Of course, too much simplicity can be harmful. Some program practices make it hard or impossible for a researcher to determine the exact model and statistical definitions of the estimators and tests given by the program output.

Model Testing

In his approach to CVM, Rigdon does not emphasize the need for model testing, perhaps because of the typical absence of a formal model testing methodology in CVM. For example, Hwang, Ho, and Lee (2010, p. 228) provide no χ^2 test of model fit for their recent approach to interaction modeling,⁵ while in contrast an LVM approach to interaction can provide such a test (Mooijaart & Bentler, 2010). We do recognize that model testing is but one endpoint of a modeling research program, since, in addition to prediction as emphasized in PLS, model exploration and model building also can be valuable enterprises (e.g., Jöreskog, 1993). But in our view it is a critical endpoint, in principle allowing a clean evaluation of given models and a comparison of alternative models (Bentler & Satorra, 2010; Satorra & Bentler, 2010; Yuan & Bentler, 2007).

Rather than hopelessly accept Rigdon's conclusion that "...PLS path modeling lacks an inferential test statistic like factor-based SEM's χ^2 statistic" (p. 9), we recommend instead to improve PLS so that it can provide such a test. In Huang's (2013) dissertation study, we build on Dijkstra's (2011) consistent PLS estimator to derive two consistent and efficient PLS estimators (PLSe1 and PLSe2) that asymptotically have minimum sampling variability.⁶ Our methodology also yields standard errors for parameter estimates and allows evaluation of models via χ^2 goodness of fit tests. The dissertation work is recently completed and may still need further improvements. The results show that PLSe2 is as good as standard SEM in these regards. We briefly describe PLSe2 and the results of our simulation studies in the following section.

PLSe2: An Efficient Estimator with Tests for Partial Least Squares

PLS lacks a classical parametric inferential framework (Vinzi, Trinchera & Amato, 2010). One has to resort to empirical confidence intervals and hypothesis testing procedures based

⁵Hwang et al. consider "a latent interaction as a product of interacting latent variables whose individual scores are uniquely determined as weighted composites of observed variables" (p. 229). This would not define latent variables or an LVM according to Bentler's (1982) definition (Bollen & Hoyle, 2012; Treiblmaier, Bentler, & Mair, 2011).

⁶We thank Professor Dijkstra for his contributions to this work.

on resampling methods such as jackknife and bootstrap (Chin, 1998). It also suffers from undesirable statistical properties of the estimates, e.g. coefficients are known to be biased (Cassel, Hackl, & Westlund, 1999, 2000) and only consistent at large (Dijkstra, 1983). Recently, Dijkstra (2011) proposed a new estimator PLSc (a PLS-consistent estimator to distinguish it from the traditional PLS estimators) which involves corrections to the PLS estimates to account for measurement error. He showed that his new PLSc estimator is very effective in reducing the bias in parameter estimates. That is, Dijkstra's (2011) consistent PLS estimator PLSc yields a consistent estimator $\hat{\Sigma}_c$ of the population correlation matrix Σ . If desired, this can be rescaled into a covariance matrix.

Based on Dijkstra's PLSc estimates, we investigated two efficient PLS estimation methodologies (PLSe1 and PLSe2). Here we present PLSe2 that utilizes Browne's (1974) generalized least squares (GLS) covariance structure estimation methodology to obtain an efficient estimator and the associated parameter and model evaluation methodology.

Assuming that a sample of size N is drawn from a multivariate normal population and $\mathbf{S}$ is the sample covariance matrix, Browne (1974, Propositions 1-3) proved that when a consistent estimator $\mathbf{W}_2$ of the population inverse covariance matrix Σ^{-1} is used and the normal theory GLS fit function:

$$F_{GLS}(\theta) = \frac{1}{2} \text{tr}\{[\mathbf{S} - \Sigma(\theta)] \mathbf{W}_2\}^2 \quad (1)$$

is minimized with respect to θ to yield the minimizing value $\hat{F}_{GLS}$ under the assumption of multivariate normality of variables, the test statistic $(N-1)\hat{F}_{GLS}$ is asymptotically distributed as a $\chi^2_{m^*-q}$ variate. Here q represents the number of free parameters in the model, $m^* = m(m+1)/2$ where m denotes the number of manifest variables (so m^* denotes the number of nonredundant elements of $\mathbf{S}$) and N is the sample size. This statistic can be used to test the validity of the hypothesized model. Browne (1974) showed that the estimator resulting from the minimization of $F_{GLS}(\theta)$ is consistent, asymptotically normally distributed, and asymptotically efficient in the Loewner sense. He showed that estimates of the variances of the estimator can be obtained from the diagonal of $(\dot{\sigma}' \mathbf{W}_N^{-1} \dot{\sigma})^{-1}$ evaluated at $\hat{\theta}$, where $\dot{\sigma}$ is the Jacobian matrix, $\mathbf{W}_N = .5 \mathbf{D}_m' (\mathbf{W}_2 \otimes \mathbf{W}_2) \mathbf{D}_m$, and $\mathbf{D}_m$ is the duplication matrix.

In practical implementations of Browne's theory, $\mathbf{W}_2 = \mathbf{S}^{-1}$ (for GLS) and $\mathbf{W}_2 = \hat{\Sigma}_{ML}^{-1}$ (for normal theory GLS) are used, where the latter is based on an iteratively updated estimator $\hat{\Sigma}$ under the model that leads to the normal theory maximum likelihood (ML) estimator (Lee & Jennrich, 1979). Our PLSe2 methodology makes use of Browne's results by taking

$\mathbf{W}_2 = \hat{\Sigma}_c^{-1}$, the consistent PLS-based weight matrix. This provides a new member for the class of normal theory GLS estimators. To implement PLSe2, one could use the existing framework of any SEM package that implements normal theory GLS and simply replace the default weight with the new weight obtained from Dijkstra's PLSc procedure.

When the assumption of multivariate normality is inappropriate, the test statistics and standard errors can be corrected using the variety of robust methods available in the field such as the Satorra-Bentler (1994) scaled and adjusted test statistics, and the appropriate sandwich estimator of asymptotic variances (e.g., Bentler & Dijkstra, 1985, eq. 1.4.5).

We demonstrate the performance of PLSe2 by comparing it with ML estimation as widely used in traditional SEM through a Monte Carlo simulation study. The data generating model resembles the one from Maruyama and McGarvey (1980). We took their sample correlation matrix and estimated a model as shown below. The parameter estimates are used to generate multivariate normally distributed data at sample sizes of 200, 500 and 1000.

The measurement model is: $\mathbf{y} = \mathbf{\Lambda}\boldsymbol{\eta} + \boldsymbol{\epsilon}$, specifically

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \\ y_9 \\ y_{10} \\ y_{11} \\ y_{12} \\ y_{13} \end{pmatrix} = \begin{pmatrix} \lambda_{1,1} & & & & & & & & & & & & \\ \lambda_{2,1} & & & & & & & & & & & & \\ \lambda_{3,1} & & & & & & & & & & & & \\ & \lambda_{4,2} & & & & & & & & & & & \\ & \lambda_{5,2} & & & & & & & & & & & \\ & & \lambda_{6,3} & & & & & & & & & & \\ & & \lambda_{7,3} & & & & & & & & & & \\ & & \lambda_{8,3} & & & & & & & & & & \\ & & & \lambda_{9,4} & & & & & & & & & \\ & & & \lambda_{10,4} & & & & & & & & & \\ & & & & \lambda_{11,5} & & & & & & & & \\ & & & & \lambda_{12,5} & & & & & & & & \\ & & & & \lambda_{13,5} & & & & & & & & \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \\ \eta_5 \end{pmatrix} + \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \epsilon_4 \\ \epsilon_5 \\ \epsilon_6 \\ \epsilon_7 \\ \epsilon_8 \\ \epsilon_9 \\ \epsilon_{10} \\ \epsilon_{11} \\ \epsilon_{12} \\ \epsilon_{13} \end{pmatrix},$$

$$\text{where } \mathbf{\Lambda} = \begin{pmatrix} .609 & & & & & & & & & & & & \\ .686 & & & & & & & & & & & & \\ .611 & & & & & & & & & & & & \\ & .481 & & & & & & & & & & & \\ & .364 & & & & & & & & & & & \\ & & .658 & & & & & & & & & & \\ & & .844 & & & & & & & & & & \\ & & .167 & & & & & & & & & & \\ & & & .631 & & & & & & & & & \\ & & & .507 & & & & & & & & & \\ & & & & .522 & & & & & & & & \\ & & & & .793 & & & & & & & & \\ & & & & .400 & & & & & & & & \end{pmatrix}.$$

The covariance matrix of $\boldsymbol{\epsilon}$ is a diagonal matrix $\boldsymbol{\Theta}$ whose diagonal elements are .51, .37, .50, .35, .63, .57, .29, .97, .60, .74, .73, .37 and .84.

The structural model is $\boldsymbol{\eta} = \mathbf{B}\boldsymbol{\eta} + \boldsymbol{\zeta}$, and

$$\begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \\ \eta_5 \end{pmatrix} = \begin{pmatrix} 0 & \beta_{1,2} & 0 & 0 & \beta_{1,5} \\ \beta_{2,1} & 0 & \beta_{2,3} & \beta_{2,4} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \\ \eta_5 \end{pmatrix} + \begin{pmatrix} \zeta_1 \\ \zeta_2 \\ \zeta_3 \\ \zeta_4 \\ \zeta_5 \end{pmatrix},$$

This model is non-recursive. The covariance matrix of ζ is a symmetric matrix ψ with 1s on the diagonal:

$$\text{where } B = \begin{pmatrix} 0 & .212 & 0 & 0 & .251 \\ .295 & 0 & .266 & 1.027 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

where $\psi_{3,4} = .379$.

The means of PLSe2 estimates across 300 Monte Carlo replications are compared with those from traditional ML estimation. We also compare Root Mean Square Error (RMSE), which can provide information of both the deviation of each parameter estimate from the true value and the variability of such distances:

$$RMSE = \left[\frac{1}{M} \sum_{i=1}^M (\hat{\theta}_i - \theta)^2 \right]^{\frac{1}{2}}, \text{ where } M \text{ is the number of replications and } \theta \text{ refers to a generic parameter with } \hat{\theta}_i \text{ the estimate from Monte Carlo replication } i.$$

Table 1 compares the parameter estimates of factor loadings, regression coefficients, and the residual factor correlation using ML and PLSe2. We can see that at each sample size, ML and PLSe2 produce almost identical parameter estimates and RMSE. Overall for both methods, bias reduces as sample size increases. At N=200, both methods recovers parameters quite well with PLSe2 showing slightly higher RMSE than ML. Such difference goes away at N=1000 when the two estimators become virtually identical and unbiased.

It is worth noting that the estimates of the regression coefficients are generally more biased than the estimates of the loadings or the factor correlation. This happens to both ML and PLSe2 and it is probably due to the complexity of the non-recursive model we used. For example, at N=200, RMSE of the loading estimates are generally less than 0.1 whereas RMSE of the regression coefficients estimates are all larger than 0.1. We also observe a more pronounced bias in $\beta_{4,5}$ at N=200 where both ML and PLSe2 overestimate this parameter by 14%. However, when sample size goes to 1000, such differences are diminished.

ML and PLSe2 both produce standard error estimates and minimum fit function chi-square statistics. Although standard error estimates are not tabulated, they are almost identical and are well calibrated, which means that the mean of the estimated standard errors are very close to the standard deviation of the empirical sampling distribution across 300 replications.

The advantage of using ML or normal theory GLS estimation is that they both allow for formal model fit test. Since the model we estimate is correctly specified, the expected value for the chi-square statistic is equal to the model's degree of freedom ($df = m^* - q = 59$) and the expected variance of this test statistic is $2df = 118$. At N=200, the ML test statistic has a mean of 60.54 and variance of 115.27, which are very close to the expected values. The

PLS2 test statistic has a mean of 58.44 and a slightly elevated variance of 142.02. As sample size goes to 1000, both ML and PLS2 produce test statistics with empirical distribution closely following the theoretical expectation (mean of 58.98 and variance of 128.29 for ML; mean of 58.39 and variance of 127.05 for PLS2). With a correctly specified model, the expected model rejection rate at $\alpha = .05$ should be controlled at around 5%. Given 300 replications, the 95% confidence interval for the observed percentage of rejected model is between 2.5% and 7.5%. For both ML and PLS2, the observed model rejection rates at all sample sizes are between 4% and 6%, well within the 95% confidence interval.

Additional Complications

New proposals and developments are often followed by unanticipated consequences, and it should be noted that Rigdon's rejection of LVM has further logical consequences that would restrict data analysis unnecessarily. If one rejects latent variables in a basic SEM context such as PLS, to be consistent one also should reject latent variables in other contexts. Illustrative simple extensions of SEM that would have to be rejected include, for example, multiple group models for study of measurement invariance (e.g., Millsap, 2011), growth curve models in which mean structures are added to covariance structures (e.g., McArdle, 2012), mixture models (e.g., Yuan & Bentler, 2010), growth mixture models (e.g., Shiyko, Ram, & Grimm, 2012), and multilevel models for within-level and between-level effects (e.g., Bentler et al., 2011). In addition, models that do not appear to be factor-based SEM contain random variables that are actually latent variables and hence would have to be rejected. These include classical statistical models such as random coefficient models, mixed models with fixed effects and random effects, and hierarchical linear models. See e.g., An, Yang, and Bentler (2013) for a modern example, de Leeuw and Meijer (2008) for a review, and Bentler and Liang (2008) on a unified approach to mixed models and multilevel models. As noted by Muthén (2003), “..the idea of latent variables captures a wide variety of statistical concepts, including random effects, missing data, sources of variation in hierarchical data, finite mixtures, latent classes, and clusters” (p. 81). In addition to specific model types such as latent class models, a rejection of LVM would include ignoring potentially useful combinations of model types such as multilevel latent class analysis (Henry & Muthén, 2010).

Furthermore, we note that Rigdon is especially interested in developing a CVM with a currently unknown “complete and consistent approach to measurement which is factor-free” (p. 33). We are actually sympathetic to such a goal, since we have ongoing research on formative measurement with binary responses that extends Guttman scaling methodology (Bentler, 2011). However, Rigdon proposes to give up LVM measurement, which also would require rejecting item response theory (IRT), today's foundational methodology for educational and psychological measurement (e.g., Embretson & Reise, 2000; van der Linden & Hambleton, 2010). IRT is, of course, applicable to scale development and measurement in a wide variety of other fields. Whether in classical unidimensional or multidimensional formats, and various response functions, these methods describe the probabilities of endorsement of binary or ordinal items or response categories in terms of parameters as well as latent variable trait scores. This is an interesting and active field, especially in the multidimensional context (e.g., An & Bentler, 2011, 2012; Cai, 2010; Cai, Yang, & Hansen,

2011; Reckase, 2009; J. Wu & Bentler, 2012, 2013). Rather than reject IRT because it is an LVM, we feel that IRT, like the various methods previously mentioned, should be utilized when it can serve a useful purpose. EQSIRT was recently completed as a companion to EQS to facilitate IRT analyses (E. Wu & Bentler, 2013).

Conclusions

A variety of fields, including the management sciences, have to evaluate theories in the context of ill-measured constructs, error-contaminated variables, extraneous influences, and other challenges to accurate scientific inferences. We do not agree with Rigdon that the best approach to dealing with these challenges is to avoid latent variable models. We urge a realistic recognition of their virtues and limitations. A major historical problem with PLS estimators has been their inconsistency. We propose to use Dijkstra's (2011) correction to PLS so that the new PLSc estimator is consistent, and then propose an efficient estimator PLSe2 that is based on PLSc so that we can obtain standard error estimates and evaluate models via χ^2 goodness of fit tests. The results of our simulations show that PLSe2 is as good as standard ML estimation in SEM, thus providing one avenue for the resurrection of PLS as a fully justified statistical methodology that will be available in EQS 7 (Bentler, in preparation).

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Table 1

Comparison of the mean of parameter estimates and RMSE across 300 replications using ML and PLSe2.

true value	N=200						N=500						N=1000					
	ML			PLSe2			ML			PLSe2			ML			PLSe2		
	mean	RMSE	mean	mean	RMSE	mean	mean	RMSE	mean	mean	RMSE	mean	mean	RMSE	mean	mean	RMSE	mean
$\lambda_{1,1} = .609$	.606	.070	.598	.074	.612	.043	.609	.044	.611	.029	.610	.029	.610	.029	.610	.029	.029	.029
$\lambda_{2,1} = .686$	.675	.073	.667	.076	.690	.044	.687	.045	.686	.030	.685	.030	.685	.030	.685	.030	.030	.030
$\lambda_{3,1} = .611$	.607	.065	.600	.069	.614	.041	.611	.041	.613	.031	.612	.031	.612	.031	.612	.031	.031	.031
$\lambda_{4,2} = .481$	.494	.102	.490	.103	.464	.077	.461	.078	.474	.053	.472	.053	.472	.053	.472	.053	.053	.053
$\lambda_{5,2} = .364$	.366	.076	.360	.077	.351	.055	.348	.056	.359	.036	.357	.036	.357	.036	.357	.036	.036	.036
$\lambda_{6,3} = .658$	.667	.095	.655	.099	.664	.061	.659	.062	.658	.044	.656	.044	.656	.044	.656	.044	.044	.044
$\lambda_{7,3} = .844$	.827	.094	.815	.099	.842	.067	.836	.068	.842	.048	.839	.048	.839	.048	.839	.048	.048	.048
$\lambda_{8,3} = .167$	.180	.073	.173	.075	.170	.052	.168	.051	.166	.037	.164	.037	.164	.037	.164	.037	.037	.037
$\lambda_{9,4} = .631$	.638	.103	.624	.107	.627	.057	.621	.058	.632	.046	.629	.046	.629	.046	.629	.046	.046	.046
$\lambda_{10,4} = .507$	.520	.095	.508	.096	.505	.054	.499	.056	.502	.041	.499	.041	.499	.041	.499	.041	.041	.041
$\lambda_{11,5} = .522$	.536	.093	.522	.092	.526	.063	.522	.063	.522	.047	.520	.047	.520	.047	.520	.047	.047	.047
$\lambda_{12,5} = .793$	.789	.111	.776	.118	.790	.074	.783	.076	.799	.060	.796	.060	.796	.060	.796	.060	.060	.060
$\lambda_{13,5} = .400$	.411	.086	.398	.084	.400	.056	.395	.057	.402	.039	.399	.039	.399	.039	.399	.039	.039	.039
$\beta_{1,2} = .212$	.227	.117	.225	.123	.203	.077	.201	.078	.208	.054	.206	.054	.206	.054	.206	.054	.054	.054
$\beta_{1,5} = .251$	.286	.102	.287	.109	.255	.069	.252	.072	.250	.047	.248	.047	.248	.047	.248	.047	.047	.047
$\beta_{2,1} = .295$	.284	.215	.285	.215	.324	.170	.327	.172	.309	.107	.309	.107	.309	.107	.309	.107	.107	.107
$\beta_{2,3} = .266$	.264	.208	.245	.219	.268	.121	.261	.122	.276	.089	.274	.089	.274	.089	.274	.089	.089	.089
$\beta_{2,4} = 1.027$	1.044	.342	1.032	.354	1.123	.297	1.127	.302	1.063	.182	1.063	.182	1.063	.182	1.063	.182	.182	.182
$\psi_{4,3} = .379$	.386	.110	.386	.116	.381	.067	.381	.068	.379	.047	.378	.047	.378	.047	.378	.047	.047	.047