Published in final edited form as:

Ecol Psychol. 2014 January 1; 26(1-2): 60–68. doi:10.1080/10407413.2014.874891.

Movement Forms: A Graph-Dynamic Perspective

Elliot Saltzman and

Department of Physical Therapy and Athletic Training, Boston University, Haskins Laboratories

Ken Holt

Department of Physical Therapy and Athletic Training, Boston University, Wyss Institute for Biologically Inspired Engineering

Abstract

The focus of this paper is on characterizing the physical movement forms (e.g., walk, crawl, roll, etc.) that can be used to actualize abstract, functionally-specified behavioral goals (e.g., locomotion). Emphasis is placed on how such forms are distinguished from one another, in part, by the set of topological patterns of physical contact between agent and environment (i.e., the set of *physical graphs* associated with each form) and the transitions among these patterns displayed over the course of performance (i.e., the form's *physical graph dynamics*). Crucial in this regard is the creation and dissolution of loops in these graphs, which can be related to the distinction between open and closed kinematic chains. Formal similarities are described within the theoretical framework of *task-dynamics* between physically-closed kinematic chains (physical loops) that are created during various movement forms and functionally-closed kinematic chains (functional loops) that are associated with task-space control of end-effectors; it is argued that both types of loop must be flexibly incorporated into the coordinative structures that govern skilled action. Final speculation is focused on the role of graphs and their dynamics, not only in processes of coordination and control for individual agents, but also in processes of inter-agent coordination and the coupling of agents with (non-sentient) environmental objects.

1. Introduction

The analysis of movement forms described in this paper had its origin in a discussion with 2nd year clinical doctorate physical therapy (DPT) students in a course called Scientific Basis of Human Movement (“SciBasis”) at Boston University. One of us (the first author) was told by the students that there was a conflict between the definition of “end effector” for bipedal locomotion (i.e., the body’s center of mass [COM]) that was just presented in class, and the definition they had learned in their previous Functional Anatomy course (i.e., the feet). The conflict was ultimately resolved by the realization that, in fact, end-effectors are defined at two levels of system description. The definition used in SciBasis resided at the abstract functional level of task description, where end-effector is the term used to capture the essence of a task, and refers to the agent-related property that is most directly involved in creating functionally appropriate, task-specific motion and force patterns for the activity at

hand. At this level, end-effectors could be specific parts of the agent's body (e.g., the hand in gripping, the hip in hip-checking), more abstract functions of body parameters (e.g., the body's COM, which is a function of joint angles and segmental lengths and masses), or functional extensions of the agent (e.g., tools, assistive devices [such as canes, crutches, walkers], sports implements or musical instruments). In this context, the task of locomotion is defined as the translation of the body's COM through an abstract navigation space (e.g., Elder, et al., 2007; Fajen & Warren, 2007); the particular *movement form* (e.g., walking, crawling, rolling) that is used to actualize the locomotor task is irrelevant at this level of description.

In contrast, the definition of end effector used for bipedal locomotion in Functional Anatomy resided at the concrete physical level of task description. In this case, end-effectors denote the specific parts of the body that are used to generate propulsive forces at the physical contact interfaces between agent and support surface in order to translate the body's center of mass through the environment; at this level, however, the end-effectors are essential to the definition of the particular movement form (e.g., feet for bipedal walking, hands and feet for crawling, extended body surface for rolling) that is used to actualize the locomotor task. In fact, one can use the spatial layout of physical contacts between end effectors and the environment to begin to formalize the concept of movement form in terms of the topological structure of the *physical graph* that characterizes the *kinematic chain* defined by the agent-environment system.

2. Movement forms, kinematic chains, physical graphs, and their contact topologies

A kinematic chain is composed of rigid bodies that are connected pairwise by joints, e.g. the forearm and upper arm are body segments that are connected by the elbow joint, and the upper arm is connected to the trunk by the shoulder joint. The simplest topology for a kinematic chain is that of an *open chain* for which the proximal end of one segment of the chain is anchored ("rooted") to the environment and all other segments are free to move (Levangie & Norkin, 2011; Zatsiorsky, 1998). Figure 1a shows a planar open chain defined by 3 segments and 3 joint angles; since each joint angle can change independently of the others, this chain has 3 degrees of freedom. A topologically distinct, more complex kinematic structure called a *closed chain* is created when the chain contains one or more closed loops (Figure 1B shows a planar closed chain with one loop). Such loops can be created in two ways. The first way is to create a *segment-only loop* in which, for example, a more distal segment in the chain is joined to a more proximal segment, e.g., when placing your hand on your hip, a loop is created starting at the hand, and proceeding up the arm and down the trunk back to the hand. The second type of loop is a *segment+environment loop* that is defined by starting at one segment-environment 'joint', then traversing the segmental chain to a 'joint' between another segment and the environment, and finally traversing the environment back to the original segment-environment 'joint', e.g., during bipedal stance, such a loop is created starting at the 'joint' between the floor and the left foot, going up the left leg, across the trunk, down the right leg to the ground-right foot 'joint', and finally back through the environment (floor) to the original ground-left foot 'joint'.¹

Such loops are created by adding physical *constraints* that create new ‘joints’ in the system. For example, the closed chain of Figure 1b is created from the open chain in Figure 1a by adding two positional constraints to the distal end of segment-3 that set its y-value to zero (i.e., brings it onto the x-axis) and its x-value to a given distance from the origin, without constraining the segment’s orientation. Note that the closed chain in Figure 1b can be considered to define either a segment-only or a segment+environment loop, depending on whether the base segment, S_0 , is considered to represent a body segment (e.g., the trunk) or an environmental object (e.g., the floor). Since the degrees-of-freedom of a constrained system (Figure 1b) are equal to those of the original (unconstrained) system (Figure 1a) minus the number of added constraints (e.g., Saltzman, 1979), there are $3 - 2 = 1$ degree of freedom in the newly created closed chain. The result is a classic 4-bar kinematic mechanism in which no joint angle can change independently of the others.

It has been proposed elsewhere (Vaughan, et al, 1982) that movement forms can be classified according to their topological features. Using terms introduced in the discussion above, such features of a movement form’s physical graph include: the presence of open chains, trees (branched chains), and closed chains; the number of segments in physical contact with the environment and with other segments; and the resulting number of independent segment-only and segment+environment loops. Further, individual movement forms display characteristic dynamics of their physical graph topology over the course of their performance, e.g., the bipedal stride cycle for walking is defined, starting with the left foot, by: a) left leg single-limb stance phase during which only the left foot is in contact with the floor, defining a tree topology ‘rooted’ in the left foot; followed by b) double support phase during which both feet are in contact with the floor, defining a single-loop topology; followed by c) right leg single-limb stance phase during which only the right foot is in contact with the floor, defining a tree topology ‘rooted’ in the right foot; followed by d) double support phase during which both feet are again in contact with the floor, defining a single-loop topology; followed by e) restarting the cycle at (a). Such physically observable *graph dynamics* are particularly evident in the more complex footfall patterns of quadruped gaits (Figure 2), during which 0, 1, 2, 3, or all 4 legs can be in contact with the ground at a given moment in time.

3. Physically vs. functionally closed chains, task-dynamics, and uncontrolled manifolds

The type of constraints described above resulted from physical connections (‘joints’) either between segments or between segments and objects/surfaces that are anchored in the environment. Such constraints are called *hard constraints*, and are created by environmental forces such as gravity or friction (e.g., forces that ‘fix’ the feet to the ground during stance or fix the hands to the ground during pushups) or by agent-generated forces (e.g., grip forces

¹In graph theoretic terms (e.g., Bulca, 1998; Harary, 1969), the open chain in Figure 1A is described as a *tree* in which only a single path exists between any two nodes (body segments and environmental objects) through the set of nodes and edges (joints) in the chain; due to the loop inherent in the closed chain of Figure 1b, however, a *cycle* is present and there are two paths available, one clockwise and one counterclockwise, between any two segments. Since we are dealing with systems in which nodes (body segments and environmental objects) are physically defined and joints (edges) are physically enforced, we refer to the graphs of such systems as *physical graphs*.

that fix the hands to a chin-up bar or to a door handle). Such hard constraints create *physically-closed* chains. However, there is also a class of constraints called *soft constraints* that serve to functionally link or ‘join’ an end-effector to a task-relevant ‘attracting’ target location in the environment despite the absence of a physical connection between end-effector and target location (e.g., Turvey, 1990). Such constraints are due to the *task-dynamics* (e.g., Saltzman & Kelso, 1987; Saltzman, et al., 2006) underlying the control and coordination of skilled movements during actions such as holding a coffee cup in space, keeping the head horizontal when walking, or keeping the center of mass in a safe region during quiet standing. Such soft constraints create *functionally closed* chains that are no less real or effective in constraining an agent’s movement patterns than are the hard constraints associated with physically closed chains that are enforced by physical connections between agent and environment. Thus, the kinematic chain in Figure 1a can also be viewed as a 3-joint arm whose distal tip is an end-effector for reaching, and that can be functionally constrained to reach an arbitrarily located ‘attracting’ target in the x-y task space. Importantly, the creation and dissolution of either hard or soft constraints result in graph-dynamic topological changes that, respectively, create and dissolve loops in the associated kinematic chains spanning agent and environment.

Task-dynamics provides a means of modeling the coordinative structures (controllers) that harness an agent’s articulator components (joint angles) in order to shape coordinated, functionally-specific movement patterns for the end-effectors involved in given task contexts, e.g., for a reaching-while-standing task, the coordinative structure would control motion of the reaching hand toward its target, while simultaneously ensuring that the COM stays safely within the agent’s base of support. Central to the coordinative structure’s operation is the system Jacobian (e.g., Saltzman, et al., 2006; Saltzman & Munhall, 1989). The Jacobian is a matrix that contains a separate row for each task-space dimension of each actively controlled end-effector, with columns that correspond to the agent’s set of joint angles; each of its J_{ij} elements is defined as the partial derivative of the i^{th} task coordinate with respect to the j^{th} joint angle and is a function of the current posture (joint angles configuration). The Jacobian is used in two ways in the task-dynamic framework: a) to define the forward kinematic transform from joint angular velocities to task-space velocities; and b) for redundant systems (in which there are more joints than task-dimensions), to define the inverse kinematic transform from task-space accelerations to joint angular accelerations using the Jacobian pseudoinverse. Importantly, each row of the Jacobian participates in the creation and maintenance of a task-specific, functionally closed chain for its associated task-space dimension of end-effector motion, and exerts its control throughout the timespan of task performance.

Using such a control scheme in a redundant system allows simultaneous task-demands (e.g., reach a desired target without losing balance) to be met by creating joint-angular motion patterns along the *controlled manifold* (CM) in joint space that correspond to task-appropriate patterns of motion of the end effectors in task-space. Importantly, the joint-angular motion patterns are also endowed with a flexibility or resilience of response to unforeseen perturbations that allows a given set of end-effector task-space goals to be achieved using a variety of postures. This adaptive postural variation occurs within the

system's *uncontrolled manifold* (UCM; e.g., Scholz, et al., 2000) which, for a given position of the task-space end-effectors, is the set of postures corresponding to that position. The geometry of the UCM is a function of the Jacobian's nullspace which, for a given posture, defines those motion patterns in joint space that cause no motion of any of the end-effectors in task space; the CM is the orthogonal complement to the UCM. Thus, task-dynamic coordinative structures not only create motion patterns of the articulators (along the CM) that create task-specific patterns of motion of the end effectors in task space, but they also allow contextually responsive adjustments of articulatory postures (along the UCM) that 'do no harm' to the attainment of task-space goals.

Such a control scheme may be inadequate, however, if there are also physically closed kinematic chains involved. According to Bernstein (1967), the highest levels of skilled performance are attained when actively supplied forces serve to *complement* those supplied passively (e.g., by gravity, ground reaction forces, friction of environmental surfaces), in order to create the total set of forces appropriate for the task. In this regard, physically closed chains can be viewed as sources of passive constraint that should also be incorporated into the control schemes for highly skilled movements. This can be done by adding separate rows to the Jacobian for each independent physically closed chain that exists during all or part of a given task's performance (see also Berenson, et al. [2011] for a related proposal). For example, unimanual reaching during bipedal stance is a task whose topology does not change over its entire timespan. In this case, the Jacobian's composition will remain constant throughout the task, containing rows that are related both to the functionally closed chains associated with active control of the hand and COM, as well as to the physically closed chain (segment+environment loop) associated with bipedal stance. However, for tasks such as bipedal walking (see section 2), where the physical graph changes over the course of the stride cycle (alternating between trees and single-loops), the row composition of the Jacobian needs to change over time to reflect these physical graph dynamics. Doing so will ensure that the control geometry of the coordinative structure contains a CM and UCM that veridically reflect the entire set of task-specific active constraints as well as physically enforced passive constraints on joint angle kinematics during performance of the task.

4. Future directions

The work described in this paper represents a first step in applying the concepts of graph theory to characterizing movement forms according to the topological properties of their physical graphs, and examining the implication of these properties for skilled control and coordination. As was highlighted in the discussion of bipedal and quadrupedal gait patterns, a more complete analysis of movement forms must also take into account the graph dynamics that govern transitions over time between the topologically distinct phases of the forms. Recent empirical work on the spatiotemporal symmetry properties of interlimb coordination observed in a wide variety of gaits, and corresponding theoretical models of the neural central pattern generators (modeled as systems of coupled nonlinear oscillators) hypothesized to underlie these patterns, (e.g., Pinto & Golubitsky, 2006; Turvey, et al., 2012), offer a promising means of formalizing the graph dynamics of movement forms in general. We are encouraged by these results, and speculate further that the current

framework can be generalized beyond graphs that are defined solely by the topology of mechanical contacts ('joints') between an agent's body segments (e.g., hands on hip) or between agent and environment (e.g. monopedal or bipedal stance). Specifically, we propose that the framework be generalized to graphs that include *informational* links (edges) characterizing the *perceptual* coupling of agents to objects and surfaces in the environment that are either spatially separated from the agents (and are perceived via vision, audition, or olfaction) or in mechanical contact with agents (and are perceived via haptics or gustation) (Saltzman & Holt, 2009; Saltzman, 2012). Within such a framework, agents and (non-sentient) environmental objects and surfaces would define the graph's nodes, and links (edges) between nodes would be defined either physically or informationally; the adaptive behavior of agents would be characterized by the creation and dissolution of such links so that, at any moment in time, the behavior of the system spanning agents and environment would be shaped by the currently active *behavioral graph* (see also Warren, 2006) which would structure the flow of information and forces according to its topology.

Acknowledgments

This article is dedicated with deep affection and respect to the memory of Herbert Pick, Jr., whose fascination with the role of nested coordinate system geometries in the control and coordination of skilled action led, somehow, to the writing of this article.

FUNDING

The authors gratefully acknowledge the support of NIH grant 2-P01-HD-001994.

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Figure 1.

(a) Open kinematic chain; (b) Closed kinematic chain. Both panels show the planar case with a 'base' set of (x, y) axes; S_i denotes the i^{th} (line) segment; ϕ_j denotes the j^{th} (hinge) joint. The x-axis can be considered to define an additional 'base' segment, S_0 .

Figure 2.

Two quadruped gait footfall patterns. Ordinate labels denote foot (Left/Right and Hind/Front); abscissa labels indicate % gait cycle; dark bars show intervals of foot-ground contact. (adapted from http://commons.wikimedia.org/wiki/File:Gait_graphs.jpg; used with attribution to “HCA at en.wikipedia” under Creative Commons Attribution 3.0 Unported license [CC BY 3.0].