Interior micro-CT with an offset detector

Kriti Sen Sharma^{a)}

Department of Electrical and Computer Engineering, Virginia Tech, Blacksburg, Virginia 24061

Hao Gong

VT-WFU School of Biomedical Engineering and Sciences, Virginia Tech, Blacksburg, Virginia 24061

Omid Ghasemalizadeh

Department of Mechanical Engineering, Virginia Tech, Blacksburg, Virginia 24061

Hengyong Yu

VT-WFU School of Biomedical Engineering and Sciences, Wake Forest University Health Sciences, Winston-Salem, North Carolina 27157

Ge Wang

Biomedical Imaging Center/Cluster CBIS/BME, Rensselaer Polytechnic Institute, Troy, New York 12180

Guohua Cao^{b)}

VT-WFU School of Biomedical Engineering and Sciences, Virginia Tech, Blacksburg, Virginia 24061

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Purpose: The size of field-of-view (FOV) of a microcomputed tomography (CT) system can be increased by offsetting the detector. The increased FOV is beneficial in many applications. All prior investigations, however, have been focused to the case in which the increased FOV after offset-detector acquisition can cover the transaxial extent of an object fully. Here, the authors studied a new problem where the FOV of a micro-CT system, although increased after offset-detector acquisition, still covers an interior region-of-interest (ROI) within the object.

Methods: An interior-ROI-oriented micro-CT scan with an offset detector poses a difficult reconstruction problem, which is caused by both detector offset and projection truncation. Using the projection completion techniques, the authors first extended three previous reconstruction methods from offset-detector micro-CT to offset-detector interior micro-CT. The authors then proposed a novel method which combines two of the extended methods using a frequency split technique. The authors tested the four methods with phantom simulations at 9.4%, 18.8%, 28.2%, and 37.6% detector offset. The authors also applied these methods to physical phantom datasets acquired at the same amounts of detector offset from a customized micro-CT system.

Results: When the detector offset was small, all reconstruction methods showed good image quality. At large detector offset, the three extended methods gave either visible shading artifacts or high deviation of pixel value, while the authors' proposed method demonstrated no visible artifacts and minimal deviation of pixel value in both the numerical simulations and physical experiments.

Conclusions: For an interior micro-CT with an offset detector, the three extended reconstruction methods can perform well at a small detector offset but show strong artifacts at a large detector offset. When the detector offset is large, the authors' proposed reconstruction method can outperform the three extended reconstruction methods by suppressing artifacts and maintaining pixel values. © 2014 American Association of Physicists in Medicine. [<http://dx.doi.org/10.1118/1.4876724>]

Key words: micro-CT, interior tomography, offset detector

1. INTRODUCTION

Microcomputed tomography (micro-CT) (Refs. 1 and 2) is a powerful tool for a variety of applications including cancer research,^{3,4} failure analysis,⁵ bone analysis,⁶ and evolutionary biology.⁷ Compared to clinical CT, micro-CT has much higher spatial resolution (1–100 μm) and much smaller field-of-view (FOV). When optimizing micro-CT scanner geometry, there is usually a trade-off between the spatial resolution and the FOV. A geometry with higher magnification leads to a higher spatial resolution but smaller FOV. When a certain spatial resolution is required but the object

is larger than the FOV, projection truncation unavoidably occurs.

The FOV of a micro-CT system can be doubled by offsetting the detector in a direction tangential to the acquisition trajectory.^{8,9} The “offset detector” (or “displaced detector”) acquisition leads to an asymmetric coverage of the object at different projection angles. Consequently, this asymmetry must be accounted for during reconstruction. Over the past years, many reconstruction techniques have been developed for micro-CT with offset-detector acquisition, and they can be categorized into three groups. The first group employs a weighting scheme together with the filtered back projection

FIG. 1. (a) Fan-beam geometry for interior tomography with offset detector array. The enlarged effective FOV (FOV1) and the original FOV (FOV2) are marked by dash and solid circles, respectively. (b) Mathematical notations for the offset detector geometry, where the detector array is scaled to go through isocenter of the scan.

(FBP) technique.^{8–12} The weighting scheme accounts for the fact that a central overlapping region is sampled over the entire 360°, while the remaining is sampled over 180° only. The weighting is carried out either before the ramp filtering step (preconvolution weighting), or after (postconvolution weighting). The second group employs first the local derivative filter (instead of the nonlocal ramp filter), and subsequently the back-projection filtration (BPF) or derivative back-projection (DBP).^{13,14} This group of methods also involves an inverse Hilbert filtering step.¹⁵ The BPF/DBP methods provide theoretically exact reconstruction schemes for offset detector CT/micro-CT,¹³ in contrast to the FBP method which typically involve some approximations.⁸ The third group is rather recent and involves iterative reconstruction techniques that employ a weighted projection updating scheme.^{16,17} The iterative techniques have been shown to reduce left-right shading artifacts,¹⁷ and allowed high-quality reconstruction when using a smaller number of projections.¹⁶

All previous studies on micro-CT with offset detector have been focused to the case where the FOV after offset-detector acquisition covered the entire transaxial extent of the object. In this paper, we studied a new case where the FOV after the detector offset still covers an interior ROI inside the object (i.e., interior micro-CT with offset detector). The fan-beam geometry of an interior micro-CT with offset detector is shown in Fig. 1(a). The detector is illustrated by the thick black line, and the center of system FOV is marked by the white crosshair. By offsetting the detector, and acquiring projections over 360°, the effective FOV [i.e., the enlarged FOV2 in Fig. 1(a)] is larger than the original FOV without detector offset [FOV1 in Fig. 1(a)]. Additionally, even after the offset-detector acquisition, the effective FOV is still not large enough to cover the full transaxial extent of the object. Therefore, reconstruction of projections acquired from such an interior micro-CT with an offset detector must deal with both detector offset and data truncation.

In this paper, we first extended three previous methods for offset-detector reconstruction from global micro-CT to interior micro-CT, by incorporating the projection completion

technique.¹⁸ We then describe a novel hybrid reconstruction method, which is based on the combination of two extended methods and the frequency split scheme.^{19,20} Specifically, the three extended methods are based on the preconvolution weighting method,^{8,9} the postconvolution weighting method,⁸ and the weighted simultaneous algebraic reconstruction technique (SART),^{16,17,21,22} respectively. The hybrid method is based on the combination of the extended methods from the postconvolution weighting and the weighted SART. Performance of these four methods (three extended methods and one hybrid method) were evaluated via numerical simulations and physical experiments.

The paper is organized as follows. In Sec. 2, we first provided details for the development of the four reconstruction methods, then described their evaluation methods using both numerical simulations and physical experiments. In Sec. 3, we compared the reconstruction results from the four reconstruction methods via both visual inspection of artifacts and quantitative analysis. Finally, we conclude the paper with Sec. 4.

2. METHOD

2.A. Reconstruction method

The mathematical notations are introduced in Fig. 1(b). Two coordinate systems are used in mathematical formulation. One is the fixed x-y coordinate system with its origin coinciding with the isocenter of the scan trajectory. The other rotates about the same origin and one of its axes overlays the detector array. For ease of mathematical formulation, the detector is scaled to pass through the isocenter of the scan trajectory. D is the distance from x-ray source to the origin. β is the angular position of the x-ray source with respect to (w.r.t.) the fixed y-axis. Respectively, Θ and Δ are the length of the shorter and longer segments of the detector w.r.t. the isocenter (θ and δ are the corresponding angular values). Thus, $(\Theta + \Delta)$ is the total length of the detector array. The percentage p of offset of the detector can be calculated as

$$p = \frac{\frac{\Theta + \Delta}{2} - \Theta}{\Theta + \Delta} \times 100\%. \quad (1)$$

The spatial and angular positions of a specific detector cell w.r.t. the isocenter in the rotating coordinate system are denoted as s and α , respectively. The four algorithms are explained in Secs. 2.A.1–2.A.4. As an example, Fig. 2 shows the sinograms w.r.t. the major steps of the four methods at 37.6% detector offset. All the sinograms in Fig. 2 correspond to the central slice of the micro-CT phantom. The corresponding global sinogram is shown as a reference in Fig. 2(a).

2.A.1. Method I: Preconvolution weighting

The first reconstruction method is an extension of the preconvolution weighting scheme described in Ref. 8, or equivalently the fan-beam weighting scheme.⁹ The detailed steps of

FIG. 2. Sinograms depicting various steps for interior reconstruction with an offset detector array. All sinograms have 360° angular coverage. (a) Global sinogram shown as reference. (b) Truncated sinogram with offset detector acquisition. (c) Sinogram after offset-detector weighting. (d) Projection completion by cosine extrapolation up to object boundary. (e) Sinogram reflection. (f) Splicing to remove discontinuity at edge. (g) Projection completion. (h) Ramp filtering. (i) Offset-detector weighting. (b') Same as subfigure (b), but without zeros at missing data-points. (j) Projection completion at external edge. (k) Result of one iteration of SART + TV algorithm. (l) Projection correction.

this algorithm are described in the following:

- Zero padding is applied to the measured sinogram. The missing data within the ranges $s \geq \Theta$ and $s \leq -\Delta$ due to detector-offset and interior acquisition is filled with zeros [Fig. 2(b)].
- The weighting function described in Ref. 9 is used to fix the data redundancy of the sinogram in Fig. 2(b). The sinogram after offset-detector weighting is shown in Fig. 2(c)

$$\omega(s, \beta) = \begin{cases} 1 & -\Delta \leq s \leq -\Theta \\ \frac{1}{2} \left(-\sin \left(\frac{\pi \arctan(\frac{s}{D})}{2 \arctan(\frac{\Theta}{D})} \right) + 1 \right) & -\Theta \leq s \leq \Theta \\ 0 & \Theta \leq s \leq \Delta \end{cases} \quad (2)$$

- In order to reduce artifacts caused by data truncation, projection completion was carried out by extrapolating

the truncated sinogram up to the object boundary by cosine-extrapolation¹⁸ [Fig. 2(d)].

- Traditional fan-beam FBP reconstruction was employed on the processed sinogram to reconstruct the image.

2.A.2. Method II: Postconvolution weighting

The second reconstruction method is an extension of the postconvolution weighting scheme of Ref. 8. The steps of this algorithm are as follows:

- The missing data due to offset-detector acquisition in the range $\Theta \leq s \leq \Delta$ is recovered by the technique of sinogram reflection,²³ i.e., by using [Fig. 2(e)]

$$R_{\beta+\pi+2\alpha}(-s) = R_{\beta}(s), \quad (3)$$

where $R_{\beta}(s)$ is the projection measured at angle β , s is the spatial position of a detector element, α is the angular position of a detector element as shown in Fig. 1(b).

- b. The splicing technique⁸ was applied to reduce the discontinuities at the edge where the real data meets the recovered data [Fig. 2(f)].
- c. To reduce the artifacts caused by data truncation (interior acquisition), projection completion was carried out in the same way as method I (Sec. 2.A.1) [Fig. 2(g)].
- d. Next, the projections were filtered by the smoothed Ram-Lak filter.¹⁵ Hanning window was used to do the smoothing [Fig. 2(h)].
- e. After the ramp filtering step, the weighting function of Eq. (2) was applied to the filtered projections over the extended range $-\Delta' \leq s \leq \Delta'$. The overlap range $-\Theta \leq s \leq \Theta$ still remains the same [Fig. 2(i)].
- f. The filtered and weighted projections were back-projected to reconstruct the image.

2.A.3. Method III: Weighted iterative reconstruction (WIR)

An iterative reconstruction algorithm was formulated by combining compressed-sensing based reconstruction,^{21,22} iterative reconstruction for offset-detector CT,^{16,17} and some special modifications for interior reconstruction. The importance of each step is discussed in Sec. 4. The steps for this method are as follows:

- a. The missing data within the range $\Theta \leq s \leq \Delta$ was not filled with zeros [Fig. 2]. Projection completion was conducted as method I [Fig. 2(j)].
- b. Data consistency was enforced by a modified version of the SART algorithm of²⁴

$$f_j^{k+1} = f_j^k + \lambda \omega(s) \sum_{i \in P(\psi)} \frac{a_{ij}}{\sum_{i \in P(\psi)} a_{ij}} \left(\frac{(R_i - A_i f^k)}{\sum_{j=1}^N a_{ij}} \right), \quad (4)$$

where R_i is the i th detector element of the measured projections, f_j is the j th image pixel, k is iteration number, A_i is a row of the system matrix composed of weight elements a_{ij} , and λ is the relaxation parameter which is chosen to be 0.8. $A_i f^k$ yields the forward-projected result at the i th detector bin. The summation of a_{ij} over $i \in P(\psi)$ is over all rays intersecting j th pixel at fixed angle ψ . The main modification from Ref. 24 was that the redundancy weighting technique of Refs. 16 and 17 was integrated with weighting function $\omega(s)$ of Eq. (2).

- c. TV minimization by gradient descent was employed after the completion of current iteration of SART.²² The α parameter for TV minimization is the maximal step for the steepest descent. It was set as 0.2. The number of TV subiterations was fixed at 5. The resultant image after one iteration of SART+TV is shown in Fig. 2(k).
- d. After the completion of SART+TV, the reconstructed image was forward projected to get the forward projected sinogram $A_i f^k$. The extrapolated sinogram data $R_\beta(s)$ within $-\Delta' \leq s \leq -\Delta$ is replaced by the corresponding part of the forward projected sinogram

[Fig. 2(l)]. The splicing technique of Ref. 8 was employed to ensure smooth merging of measured and forward-projected data around $s = -\Delta$. This step is referred to as “projection correction” in the rest of the paper.

- e. Steps (b)–(d) were repeated till convergence criterion ($\|f^{k+1} - f^k\| < \epsilon$) was met (where ϵ was set at 0.00001), or maximum number of iterations (set at 20) was reached.

2.A.4. Method IV: Frequency-split displaced detector reconstruction (FSDDR)

As we shall show in the experimental results, the WIR (method III) provides more accurate reconstruction of the high frequency components, but suffers from certain low frequency artifacts. The postconvolution weighting method (method II) provides better low-frequency reconstruction, but does not reconstruct the high frequency components as well as WIR. To benefit from the advantages of these two reconstruction methods, we employed a frequency split scheme^{19,20} to devise a novel interior reconstruction algorithm for offset detector CT. The details of the algorithm are listed below:

- a. Denote the images reconstructed by method II and III as f^{post} and f^{WIR} , respectively.
- b. The low-frequency component of f^{post} is extracted by applying 2D image convolution using a Gaussian kernel $G(\sigma)$ with standard deviation σ . σ was fixed at 115.5 μm in the spatial domain. Denote the low-frequency component by f^{Lo}

$$f^{\text{Lo}} = f^{\text{post}} * G(\sigma). \quad (5)$$

- c. The high-frequency component of f^{WIR} is calculated by applying 2D image convolution and image subtraction. Denote the high-frequency component by f^{Hi}

$$f^{\text{Hi}} = f^{\text{WIR}} - f^{\text{WIR}} * G(\sigma). \quad (6)$$

- d. The final image is obtained by using frequency split merging method^{19,20}

$$f^{\text{FSWIR}} = f^{\text{Lo}} + f^{\text{Hi}}. \quad (7)$$

2.B. Evaluation method

2.B.1. Numerical phantom simulation

A modified Shepp-Logan phantom was created for this work. It was made by overlaying a small Shepp-logan phantom on a larger one [Fig. 1(a)]. This combination is referred as the “double Shepp-Logan phantom.” In the simulation, the ROIs were located within the inner Shepp-logan phantom. The source-to-isocenter distance (SID) and source-to-detector distance (SDD) were 106.28 and 459.45 mm, respectively. These parameters were selected to match the setting of the physical experiments described later. The simulated detector had 154 elements and each element had a width of 50 μm .

FIG. 3. Micro-CT system components. Inset on left marks the x-y-z axes, where the x-y plane is the central plane of the cone-beam, and z is the vertical direction.

Without detector offset, the original FOV had a diameter of 1.78 mm, smaller than the inner Shepp-Logan phantom. In the simulation, the detector was offsetted (relative to its center) by amount of 9.4%, 18.8%, 28.2%, and 37.6% of the detector length. For each offset detector case, 720 projections were acquired over 360° with an angular step of 0.5° . X-ray projections were calculated using a distance driven ray-sum model.²⁵ X-ray photon noise statistics are known to be well approximated by the Poisson noise model.^{15,26} Poisson noise was added to the projection data to simulate realistic acquisition conditions. The noise level was fixed at a moderately high projection signal-to-noise ratio (SNR) value of 75.²¹ In each case, the simulation was repeated five times to measure the variability of the reconstruction w.r.t. the noise.

2.B.2. Physical phantom experiments

Physical phantom experiments were carried out on a micro-CT system developed in-house. The system consists of a microfocus x-ray source (Oxford Instruments, Inc.), a motorized rotation stage (Velmex, Inc.), a flat panel x-ray detector (Hamamatsu Corp), and two motorized translational stages (Velmex, Inc.). Figure 3 shows the components of the micro-CT system. The x-ray tube and detector were stationary, and the sample was rotated during data acquisition. The detector offset was realized by translating the detector on the horizontal translational stage. A customized micro-CT phantom [Fig. 4(b)] was made by modifying a previous one [Fig. 4(a)] developed by our group.²⁷

FIG. 4. Micro-CT phantom fabrication. (a) The 8 mm phantom before modification, (b) the modified micro-CT phantom used in this study, (c) the reconstructed central slice of the phantom. The original FOV of the interior scan (i.e., without offset-detector) is marked by dotted circle.

TABLE I. Settings for scans acquired on the Xplorer system for the micro-CT phantom (RA = rotation axis).

Scan parameter	Interior scan	Global scan
Source setting	80 kV, 250 μ A	80 kV, 250 μ A
Source-isocenter distance (mm)	106.28	387.80
Source-detector distance (mm)	459.45	451.81
Number of detector elements	1012	1012
Detector frame rate (fps)	1	1
Detector element size (μ m)	50	50
FOV without detector offset (mm)	11.68	39
Start-angle (deg)	0	0
End-angle (deg)	359.5	359.5
Number of projections	720	720
Image pixel size (effective) (μ m)	11.55	42.90

The system geometry and acquisition parameters are listed in the “Interior scan” column of Table I. The detector was offsetted horizontally (i.e., in the x direction relative to its center) by the same amount as the numerical simulation. Moreover, a global micro-CT scan was acquired using the parameters in “Global scan” column of Table I. The global scan provided a reference for the interior scans. The central slice of the global scan was reconstructed by the conventional fan-beam FBP algorithm and is shown in Fig. 4(c).

2.B.3. Image quality analysis

Visual inspection and quantitative error analysis were performed to evaluate the image quality. Visual inspection was first conducted over the reconstruction results of methods I–IV to evaluate image quality under the combined influence of interior-ROI reconstruction and detector offset. To evaluate the influence of detector offset in an isolated fashion, visual inspection was then carried out over the difference images between the reconstruction results of methods I–IV and the corresponding interior reference images. As to numerical simulation, the interior reference images were acquired using the projections measured by a hypothetically extended detector with 0% offset, which is termed as “effective-FOV reconstruction with 0% offset” (EFRZO) for the convenience of description. Specifically, EFRZO used the truncated projection data covering the interior range $-\Delta \leq s \leq \Delta$. Since the detector was not displaced in data acquisition, EFRZO did not employ the offset-detector weighting. For methods I–III, EFRZO separately employed FBP (w.r.t. methods I and II) and SART+TV (w.r.t. method III) with projection completion. For method IV, EFRZO combined FBP and SART+TV using frequency split scheme. The method of using an extended detector was not possible in physical experiments as the detector had a fixed width ($\Delta + \theta$). Thus, for physical experiments, the interior reference images were acquired using the projections measured by the same detector with 0% offset, which is termed as “original-FOV reconstruction with 0% offset” (OFRZO) for the convenience of description. The OFRZO was carried out in a similar way to the EFRZO,

FIG. 5. The selection of ROIs for pixel value calibration: ROI1 - ‘water’ region, ROI2 - ‘air’ region; (a) the central region of ground truth in numerical simulation, (b) the corresponding reconstruction by preconvolution weighting at 0% detector offset, (c) the zoom-in central region of the global FBP reconstruction of physical phantom, (d) the corresponding reconstruction by preconvolution weighting at 0 detector offset. Display window of (a) and (b) [−1700, 2000]. Display window of (c) and (d) [−1200, 2000].

except that it used the truncated projection data covering the interior range $-\frac{\Delta+\theta}{2} \leq s \leq \frac{\Delta+\theta}{2}$.

In quantitative error analysis, ROIs were first extracted from the aforementioned difference images used in the visual inspection, to quantify the error induced by detector offset. On the other hand, ROIs were also extracted from the difference images between the reconstruction results of methods I–IV and the global reference images (i.e., FOV covering the complete objects), to quantify the error induced by the net effects of detector offset and interior reconstruction. In numerical simulation, the ground truth image of the double Shepp-logan phantom was chosen as the global reference image. The ground truth of our numerical phantom is the computer generated image which was used to create the simulated projection data. As to physical experiments, the global FBP reconstruction was used to provide the global reference image, since the ground truth image of the physical phantom is not available. The global FBP reconstruction employed standard FBP algorithm and used the nontruncated projection data. Image registration was performed before quantitative error analysis. Then, the extracted ROIs were used to calculate the relative mean square error (RMSE) as follows:

$$\text{RMSE} = \frac{\sum_i^N (I_i^{\text{recn}} - I_i^{\text{ref}})^2}{\sum_i^N (I_i^{\text{ref}})^2}, \quad (8)$$

where I_i^{recn} is the i th pixel in the reconstructed image, and I_i^{ref} is the i th pixel in the reference image (i.e., global reference images or interior reference images). In addition, pixel value calibration was performed before the comparison to bring the reconstructed images to the same scale. Specifically, two ROIs from two piecewise-constant regions were selected as ‘water’ region and ‘air’ region, respectively (Fig. 5). The averaged pixel values of these two ROIs were used as the references for pixel value calibration. The formula of calibration is shown below

$$I_i^{\text{cali}} = (I_i^{\text{org}} - I_w) * \frac{1000}{I_w - I_A}, \quad (9)$$

FIG. 6. Reconstructed images at different detector offset by using four reconstruction methods: (I) Preconvolution weighting (top row), (II) postconvolution weighting (the second row), (III) weighted iterative reconstruction (the third row), and (IV) frequency-split displaced detector reconstruction (bottom row). Display window [−600, 150].

where I_w is the averaged pixel value of ‘water’ region, I_A is the averaged pixel value of ‘air’ region, I_i^{cali} is the pixel value of the i th pixel after calibration, and I_i^{org} is the i th pixel value before calibration.

3. RESULTS

3.A. Artifacts visual inspection

The reconstructed images of double Shepp-logan phantom and physical phantom are shown in Figs. 6 and 7, respectively. The effective FOV is correspondingly enlarged as the amount of detector offset is increased. The corresponding difference images of numerical and physical phantoms used in visual inspection (Sec. 2.B.3) are shown in Figs. 8 and 9, respectively. The region outside the FOV was cropped out with a circular image support.

For the reconstruction results of numerical phantom, all of the images appear to be good in Fig. 6, except the subfigure (e) (w.r.t. 37.6% detector offset of method I) where a bright circular artifact is observed in the center. As to the difference images of numerical phantom, methods I and II yield the high deviation at the edge of the objects [Figs. 8(b)–8(e) and 8(g)–8(j)]. Figures 8(d) and 8(e) (method I) also have circular artifacts in the center. In comparison, methods III and IV [Figs. 8(l)–8(o) and 8(q)–8(t)] only demonstrate noise-like pattern without visible artifacts or high deviation at the edge of objects. In the first column of Fig. 8, the image difference is zero due to 0% detector offset.

As to the reconstruction results of physical phantom, method I provides good image quality at lower detector offset [Figs. 7(a) and 7(b)], but it creates clear shading artifacts at higher detector offset [Figs. 7(c)–7(e)]. These artifacts are similar to those observed in previous studies on offset

FIG. 7. Reconstruction images of the central slice of the physical phantom at different detector offsets: (I) Preconvolution weighting (top row), (II) postconvolution weighting (the second row), (III) weighted iterative reconstruction (the third row), and (IV) frequency-split displaced detector reconstruction (bottom row). Display window $[-1200, -200]$.

detector reconstruction.^{8,17} Method III also demonstrates shading artifacts at higher detector offsets [Figs. 7(m)–7(o)]. Methods II and IV do not demonstrate severe artifacts in all cases of detector offset [Figs. 7(g)–7(j) and 7(q)–7(t)]. As to the difference images of physical phantom, methods I and III yield worsening shading artifacts as the detector offset is increased [Figs. 9(a)–9(e) and 9(k)–9(o)]. Methods I and II present higher deviation at the edge of objects [as marked by

FIG. 9. The difference images w.r.t. OFRZO: (I) Preconvolution weighting (top row), (II) postconvolution weighting (the second row), (III) weighted iterative reconstruction (the third row), and (IV) frequency-split displaced detector reconstruction (bottom row). Display window $[-300, 400]$. The arrows indicate the region which demonstrates visible higher deviation at the edge of objects in subfigures (b)–(d) and (f)–(h).

the arrows in Figs. 9(b)–9(e) and 9(g)–9(j)]. Compared with methods I and II, method III reduces the deviation at the edge of objects. Finally, method IV suppresses both the shading artifacts and the deviation at the edge of objects.

3.B. Quantitative error analysis

Figure 10 shows the RMSE curves of the reconstruction results of the numerical phantom. Figure 11 plots the RMSE of the reconstruction results of the physical phantom. The insets of Figs. 10 and 11 show the ROIs used for the calculation of RMSE (marked by the red square).

Figure 10(a) shows the RMSE plots w.r.t. the ground truth of numerical phantom. The RMSE curve of method I is higher than that of other methods at high detector offset. It varies from 1.08% to 3.26% as the detector offset is increased. This phenomenon resulted mainly from the worsening artifacts and deviation at the edge of the objects in the ROIs. At high detector offset, the RMSE of Method II is within the range of [1.10%, 1.23%], and it is higher than that of methods III and IV. The RMSE of method III decreases as the detector offset is increased. The RMSE of Method IV is within the range of [0.44%, 0.65%], which is also the smallest among the four methods in all cases of detector offset. Therefore, method IV provides the most accurate reconstruction results in the numerical simulation.

Figure 10(b) demonstrates the RMSE w.r.t. EFRZO of numerical phantom. It is clearly observed that the RMSE of all methods increases as the detector offset is increased, but the rate of increase varies among different methods. Method I

FIG. 8. The difference images w.r.t. EFRZO at different detector offset percentages by using all four reconstruction methods—(I) Preconvolution weighting (top row), (II) postconvolution weighting (the second row), (III) weighted iterative reconstruction (the third row), and (IV) frequency-split displaced detector reconstruction (bottom row). Display window $[-400, 150]$.

FIG. 10. Plots of RMSE in numerical simulation: (Inset) The ROI used for error calculation is marked on the ground truth (display window $[-1700, 2000]$). (a) RMSE w.r.t. ground truth. (b) RMSE w.r.t. EFRZO. The y-axis is plotted on a logarithmic scale for both subfigures.

still gives the highest RMSE in all cases of detector offsets, which is consistent with the observation from visual inspection. Method II generates much lower RMSE than method I, but the RMSE curve is still higher than those of methods III and IV. The RMSE curves of methods III and IV are very close (within the range of $[0.14\%, 0.19\%]$) in all cases of detector offset. In numerical simulation, methods III and IV demonstrate similar capability of suppressing the reconstruction error induced by detector offset.

Figure 11(a) shows the plots of RMSE w.r.t. the global FBP reconstruction of the physical phantom. It is observed that the RMSE of all methods increases as the detector off-

FIG. 11. Plots of RMSE in physical experiment: (Inset) The ROI used for error calculation is marked on the reconstruction of the 0% detector offset case (display window $[-1200, 2000]$). (a) RMSE w.r.t. global FBP reconstruction. (b) RMSE w.r.t. OFRZO. The y-axis is plotted on a logarithmic scale for both subfigures.

set is increased. At high detector offsets, methods I and III provide RMSE clearly higher than that of methods II and IV, which resulted mainly from the shading artifacts. The RMSE of method II varies from 5.94% to 12.56%. The RMSE of method IV is within the range of $[6.15\%, 10.05\%]$. Compared with method II, it can be observed that method IV further suppresses the error as the detector offset is increased. Therefore, method IV provides the most accurate reconstruction results with detector offset in physical phantom experiments.

Figure 11(b) plots the RMSE w.r.t. OFRZO of physical phantom. Again, the RMSE of all methods increase as the detector offset is increased. The RMSE of method I increases from 2.4% to 12.59%, and it is still the highest in all cases of detector offsets. The RMSE of method III rapidly surpasses that of methods II and IV as the detector offset is increased. The RMSE of Method IV is between 1.24% and 4.22%. Method IV provides lower RMSE than that of method II, which is similar to the case shown in Fig. 10(b). Again, method IV demonstrates the best capability of reducing the reconstruction error induced by detector offset in physical phantom experiments.

4. DISCUSSION AND CONCLUSIONS

We investigated image reconstruction methods for interior micro-CT with detector offset. We first extended three previous methods (methods I–III) and proposed a novel method IV (FSDDR) using frequency split scheme. We then extensively evaluated the four reconstruction methods using the computer simulated datasets and the realistic datasets from our inhouse micro-CT. The relative performance of the various methods was reported.

For interior micro-CT without detector offset, the reconstructed images can suffer from the cupping artifact at edges and the deviation of CT numbers because of projection truncation.^{22,28,29} Such distortion can be reduced by a multitude of interior reconstruction algorithms^{22,29–37}—the most common of which are the projection completion techniques based methods.^{18,38} For interior micro-CT with detector offset, there are two types of truncation. The first is due to offset detector acquisition, and the second is due to interior micro-CT. Consequently, the reconstruction algorithms need to be modified accordingly. As discussed in Sec. 1, there are at least three groups of methods for offset detector reconstruction. Among these methods, the DBP/BPF based exact reconstruction methods^{13,14} are not directly applicable for interior micro-CT with offset detector, because the inverse Hilbert filtering step requires that measured projection data be available over an interval larger than the object support.¹⁴ Therefore, we focused our efforts on the FBP-type and iterative reconstruction schemes.

In all the reconstruction methods (methods I–IV), projection completion was carried out by cosine-extrapolation up to objection boundary. Object boundary can be acquired from external physical measurements, a low-resolution global scan,²⁸ or optical measurements by a time-of-flight camera.³⁹ Approximately accurate knowledge of the object boundary has been shown to significantly reduce the truncation

FIG. 12. Central ROI by method III for the 9.4% detector offset case, (a) without using projection completion, and (b) with projection completion.

errors.^{34,39} As shown in Fig. 12, the reconstruction generates strong bias when projection completion is not applied even in purely iterative reconstruction technique (method III). The cosine-extrapolation was selected because of its ease of use and computational simplicity. Note that other methods^{22,27,28,32} can be used to achieve even higher accuracy in the projection completion.

In method I (the extended preconvolution weighting), Eq. (2) obeys the two requirements imposed by Parker:⁴⁰ (1) It creates a smooth edge at the truncation edge and (2) it also weights the projections so that doubly sampled or redundant data have a unit total weight.⁹ Yet, the major shading artifacts and high deviation of pixel value were observed on both numerical and physical data. Such artifacts are well known and have been reported previously for preconvolution weighting methods.^{8,17} It shows the limitation of the method I for the case with a highly offsetted detector. With high percentage of detector offset, the range of oversampled data $-\theta \leq s \leq \theta$ is narrow. After applying the smooth fan-beam weighting scheme, a sharp transition still remains at the boundary of the oversampled region. Such a sharp transition is then amplified by the ramp filter of FBP algorithm to generate the artifact in the center of the reconstructed image.¹⁵

Method II (the extended postconvolution weighting) does not suffer from the major shading artifacts which reside in the low frequency components. However, it still demonstrated some deviations at edges of objects away from the central region of ROI [marked by arrows in Figs. 9(g)–9(j)]. Due to nonlinearities in the projection process (e.g., beam-hardening, etc.), the recovered data may not be equal to the ground truth. The splicing technique of Ref. 8 was utilized to reduce discontinuity at the edge where estimated data meet the measured data. The major difference between methods II and I is that the sequence of application of offset-detector weighting function, filtering, and backprojection is different. This method is known as the postconvolution weighting function because offset-detector weighting is applied after ramp filtering. Reconstruction in this order has been known to reduce artifacts at high offset percentages, but still introduces some estimation errors in the reflection and splicing steps.⁸ Therefore, method II can provide low frequency components of high quality, but the corresponding high frequency components contain estimation error.

Method III (the extended version of weighted SART) yielded good image quality in numerical simulation. How-

ever, it demonstrated visible shading artifacts on physical data, which have never been reported before. It is also observed that method III well suppressed the deviation of pixel value at the edge of objects on both numerical and physical data. Therefore, method III can provide high frequency components of better quality. Moreover, it is necessary to explain the importance of some steps of method III, since it was utilized in both methods III and IV. Previous studies^{16,17} showed the advantages of using a smooth weighting function to modify the projection updates [i.e., weighted SART, Step (b) in Sec. 2.A.3]. TV minimization [Step (c) in Sec. 2.A.3] was incorporated as it allows improvement of the noise characteristics of iterative reconstruction.²¹ TV minimization had also been shown to allow few-view reconstruction for offset detector CT.¹⁶ The iteration update of Eq. (4) minimizes the objective $R - Af^2$. However, the extrapolated portion of the sinogram $R_\beta(s)$ in $-\Delta' \leq s \leq -\Delta$ should not be considered during the minimization process. The projection correction step [Step (d) in Sec. 2.A.3] ensured that only the measured portion (i.e., in $-\Delta \leq s \leq \Theta$) of the sinogram contributed to the minimization process. Moreover, the backprojection-reprojection process in projection correction improved the quality of sinogram completion. The forward projected sinogram data obey the sinogram consistency conditions,³⁴ but the sinogram data after basic projection completion by cosine-extrapolation¹⁸ do not satisfy the same conditions.

Method IV employed the frequency split scheme, and harnessed the advantages of different reconstruction algorithms in different frequency components. Our major findings suggested that the proposed method IV (FSDDR) provided the best reconstruction results by largely reducing the cupping/shading artifacts and the deviation of pixel value. The reduction of shading artifacts results from the low frequency component from method II (postconvolution weighting).⁸ Accurate high-frequency information was achieved from method III (weighted iterative reconstruction). The standard deviation σ of the Gaussian kernel [Eq. (5)] was empirically fixed at $115.5 \mu\text{m}$ which is equal to the total length of ten image pixels in interior scan (pixel size is shown in Table I). The corresponding full width at half maximum (FWHM) of the Gaussian kernel is $272.0 \mu\text{m}$ in the spatial domain.

In conclusion, our work showed that interior micro-CT with offset detector can be used to achieve an enlarged effective FOV in micro-CT. The four reconstruction methods presented here can suppress the artifacts, albeit to a various degree. Users can select from our array of methods depending on the accuracy needs of their study. The proposed method IV (FSDDR) is proved to be the most accurate of all algorithms studied in this paper. It can provide the most reliable results for all detector offset cases. This method will allow a micro-CT to scan a greatly enlarged region within a large object even with a small detector.

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- ^{a)}Current address: GE Healthcare, Waukesha, WI 53186.
- ^{b)}Author to whom correspondence should be addressed. Electronic mail: ghcao@vt.edu
- ¹E. L. Ritman, "Micro-computed tomography-current status and developments," *Annu. Rev. Biomed. Eng.* **6**, 185–208 (2004).
 - ²S. R. Stock, *Microcomputed Tomography: Methodology and Applications* (CRC, Boca Raton, FL, 2010).
 - ³K. Machin and S. Webb, "Cone-beam X-ray microtomography of small specimens," *Phys. Med. Biol.* **39**(10), 1639–1657 (1994).
 - ⁴M. J. Paulus *et al.*, "High resolution X-ray computed tomography: An emerging tool for small animal cancer research," *Neoplasia* **2**(1–2), 62–70 (2000).
 - ⁵M. Feser *et al.*, "Sub-micron resolution CT for failure analysis and process development," *Meas. Sci. Technol.* **19**(9), 094001 (2008).
 - ⁶T. Yamashita, Y. Nabeshima, and M. Noda, "High-resolution micro-computed tomography analyses of the abnormal trabecular bone structures in klothe gene mutant mice," *J. Endocrinol.* **164**(2), 239–245 (2000).
 - ⁷J. D. Schiffbauer *et al.*, "The origin of intracellular structures in Ediacaran metazoan embryos," *Geology* **40**(3), 223–226 (2012).
 - ⁸P. S. Cho, A. D. Rudd, and R. H. Johnson, "Cone-beam CT from width-truncated projections," *Comput. Med. Imaging Graph.* **20**(1), 49–57 (1996).
 - ⁹G. Wang, "X-ray micro-CT with a displaced detector array," *Med. Phys.* **29**, 1634–1636 (2002).
 - ¹⁰J. Gregor, S. S. Gleason, and M. J. Paulus, "Conebeam x-ray computed tomography with an offset detector array," in *Proceedings of the International Conference on Image Processing, ICIP 2003, Barcelona, Catalonia, Spain, 2003* (IEEE, New York, 2003).
 - ¹¹V. Liu, N. R. Lariviere, and G. Wang, "X-ray micro-CT with a displaced detector array: Application to helical cone-beam reconstruction," *Med. Phys.* **30**, 2758–2761 (2003).
 - ¹²L. F. Yu *et al.*, "Application of asymmetric cone-beam CT in radiotherapy," in *Nuclear Science Symposium Conference Record*, edited by J. A. Seibert (IEEE, New York, 2004), Vols. 1–7, pp. 3249–3252.
 - ¹³S. Leng *et al.*, "Exact fan-beam image reconstruction algorithm for truncated projection data acquired from an asymmetric half-size detector," *Phys. Med. Biol.* **50**(8), 1805–1820 (2005).
 - ¹⁴D. Schäfer, M. Grass, and P. van de Haar, "FBP and BPF reconstruction methods for circular X-ray tomography with off-center detector," *Med. Phys.* **38**, S85–S94 (2011).
 - ¹⁵A. C. Kak and M. Slaney, *Principles of Computerized Tomographic Imaging* (SIAM, Philadelphia, PA, 1988), Vol. 33.
 - ¹⁶J. Bian *et al.*, "Optimization-based image reconstruction from sparse-view data in offset-detector CBCT," *Phys. Med. Biol.* **58**(2), 205–230 (2013).
 - ¹⁷E. Hansis *et al.*, "Iterative reconstruction for circular cone-beam CT with an offset flat-panel detector," *Nuclear Science Symposium Conference Record (NSS/MIC)* (IEEE, New York, 2010).
 - ¹⁸G. Tisson, "Reconstruction from transversely truncated cone beam projections in micro-tomography," Ph.D. thesis, Universiteit Antwerpen, Belgium, 2006.
 - ¹⁹E. Meyer *et al.*, "Frequency split metal artifact reduction (FSMAR) in computed tomography," *Med. Phys.* **39**, 1094–1116 (2012).
 - ²⁰G. Shechter *et al.*, "The frequency split method for helical cone-beam reconstruction," *Med. Phys.* **31**, 2230–2236 (2004).
 - ²¹K. Sen Sharma *et al.*, "Experimental studies on few-view reconstruction for high-resolution micro-CT," *J. X-ray Sci. Technol.* **21**(1), 25–42 (2013).
 - ²²H. Yu and G. Wang, "Compressed sensing based interior tomography," *Phys. Med. Biol.* **54**(9), 2791–2805 (2009).
 - ²³M. Nassi *et al.*, "Application of a reflection technique for improved temporal resolution with dynamic and ECG-gated computed tomography," *Proc. SPIE* **0206**, 103–108 (1979).
 - ²⁴G. Wang and M. Jiang, "Ordered-subset simultaneous algebraic reconstruction techniques (OS-SART)," *J. X-ray Sci. Technol.* **12**(3), 169–177 (2004).
 - ²⁵B. De Man and S. Basu, "Distance-driven projection and backprojection in three dimensions," *Phys. Med. Biol.* **49**(11), 2463–2475 (2004).
 - ²⁶J. Hsieh, *Computed Tomography: Principles, Design, Artifacts, and Recent Advances* (SPIE, Bellingham, WA, 2009).
 - ²⁷K. S. Sharma *et al.*, "Scout-view assisted interior micro-CT," *Phys. Med. Biol.* **58**(12), 4297–4314 (2013).
 - ²⁸M. Costin *et al.*, "A 2D multiresolution image reconstruction method in X-ray computed tomography," *J. X-ray Sci. Technol.* **19**(2), 229–247 (2011).
 - ²⁹K. Taguchi *et al.*, "Interior region-of-interest reconstruction using a small, nearly piecewise constant subregion," *Med. Phys.* **38**(3), 1307–1312 (2011).
 - ³⁰H. Kudo *et al.*, "Tiny *a priori* knowledge solves the interior problem in computed tomography," *Phys. Med. Biol.* **53**(9), 2207–2231 (2008).
 - ³¹O. Nalcioğlu, Z. Cho, and R. Lou, "Limited field of view reconstruction in computerized tomography," *IEEE Trans. Nucl. Sci.* **26**(1), 546–551 (1979).
 - ³²F. Noo, R. Clackdoyle, and J. D. Pack, "A two-step Hilbert transform method for 2D image reconstruction," *Phys. Med. Biol.* **49**(17), 3903–3923 (2004).
 - ³³D. Pak-Kong Lun and T. Hsung, "Region-of-interest tomography using multiresolution interpolation," in *Proceedings of the International Conference on Acoustics, Speech, and Signal Processing, ICASSP-97, Munich, Bavaria, Germany, 1997* (IEEE, New York, 1997).
 - ³⁴J. Xu, K. Taguchi, and B. M. Tsui, "Statistical projection completion in x-ray CT using consistency conditions," *IEEE Trans. Med. Imaging* **29**(8), 1528–1540 (2010).
 - ³⁵S. Tang and X. Tang, "Radial differential interior tomography and its image reconstruction with differentiated backprojection and projection onto convex sets," *Med. Phys.* **40**(9), 091914 (13pp.) (2013).
 - ³⁶Q. Xu *et al.*, "Statistical interior tomography," *IEEE Trans. Med. Imaging* **30**(5), 1116–1128 (2011).
 - ³⁷J. Yang *et al.*, "Theoretical study on high order interior tomography," *J. X-ray Sci. Technol.* **20**(4), 423–436 (2012).
 - ³⁸G. V. Gompel, "Towards accurate image reconstruction from truncated x-ray CT projections," Ph.D. thesis, University of Antwerp, 2009.
 - ³⁹M. Ismail *et al.*, "3D-guided CT reconstruction using time-of-flight camera," *Proc. SPIE* **7964**, 796429-1-796429-11 (2011).
 - ⁴⁰D. L. Parker, "Optimal short scan convolution reconstruction for fan beam CT," *Med. Phys.* **9**, 254–257 (1982).