Analysis and measurement of the modulation transfer function of harmonic shear wave induced phase encoding imaging

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Shear wave induced phase encoding (SWIPE) imaging generates ultrasound backscatter images of tissue-like elastic materials by using traveling shear waves to encode the lateral position of the scatters in the phase of the received echo. In contrast to conventional ultrasound B-scan imaging, SWIPE offers the potential advantages of image formation without beam focusing or steering from a single transducer element, lateral resolution independent of aperture size, and the potential to achieve relatively high lateral resolution with low frequency ultrasound. Here a Fourier series description of the phase modulated echo signal is developed, demonstrating that echo harmonics at multiples of the shear wave frequency reveal target k -space data at identical multiples of the shear wavenumber. Modulation transfer functions of SWIPE imaging systems are calculated for maximum shear wave acceleration and maximum shear constraints, and compared with a conventionally focused aperture. The relative signal-to-noise ratio of the SWIPE method versus a conventionally focused aperture is found through these calculations. Reconstructions of wire targets in a gelatin phantom using 1 and 3.5 MHz ultrasound and a cylindrical shear wave source are presented, generated from the fundamental and second harmonic of the shear wave modulation frequency, demonstrating weak dependence of lateral resolution with ultrasound frequency.

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I. INTRODUCTION

Conventional ultrasound backscatter imaging relies on geometric focusing of ultrasound echoes across an extended aperture.¹ In these systems the focal beam width, which determines the lateral resolution, is proportional to the ultrasound wavelength and inversely proportional to aperture size.² In particular, the maximum lateral spatial frequency k_x of the target that can be sensed, assuming a linear imaging model, is set by the ultrasound wavelength and the combination of the transmit and receive aperture widths.^{3,4} Thus the relatively low frequency required for, e.g., transcranial ultrasonography^{5,6} limits the achievable spatial resolution.

Soft tissue supports not only compressional waves, but shear waves as well.⁷ This property enables the wide variety of shear wave elasticity imaging methods.^{8–10} It is likely that there are additional useful combinations of multiple wave types for imaging.¹¹ We have recently presented a method in which lateral resolution of the target is obtained by using traveling shear waves to encode the lateral position of the scatters in the phase of the received echo.¹² In this method, called shear wave induced phase encoding (SWIPE), the range of lateral spatial frequencies detectable is not set by the aperture size, but rather by the range of wavenumbers of the applied shear wave.

Here we develop a Fourier series description of the phase modulated signal showing that echo harmonics at multiples of the shear wave frequency reveal target k -space data at identical multiples of the shear wavenumber. These harmonics develop from the non-linear nature of phase modulation, and exist with purely linear propagation of shear and

ultrasound waves. The relationship between the harmonic terms to the k -space data and the ability to obtain higher spatial frequency data about the target with lower frequency shear waves was not considered in Ref. 12.

These new expressions allow for the calculation of the lateral modulation transfer function (MTF) of a SWIPE imaging system using any given phase modulated harmonic. The amplitude of the harmonics is determined by the shear wave amplitude and ultrasound frequency. Practical considerations limit shear wave amplitude in tissue, therefore MTFs are calculated here for constraints of maximum shear wave acceleration and maximum induced shear. These calculations are used to predict the cutoff spatial frequency at the -10 and -20 dB levels as a function of the product of ultrasound frequency and the acceleration or shear limits. MTFs of the SWIPE system are compared with those of a conventionally focused transducer, allowing the relative signal-to-noise ratio (SNR) of the SWIPE system to be evaluated.

Finally, results of a phantom experiment are presented, wherein nylon monofilament targets in a gelatin block are imaged using SWIPE. Images are formed at 1 and 3.5 MHz using both fundamental and, for the first time, modulation harmonic signals. The images demonstrate the validity of the SWIPE imaging method with both fundamental and harmonic-modulated signals and the weak dependence of lateral resolution on ultrasound wavelength using this technique is demonstrated.

II. THEORY

A. Echo from shear-wave-deformed elastic solid

Here we describe the echo signal from an elastic solid deformed by a shear wave, following the model developed

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in Ref. 12 [Eqs. (1)–(7)]. The echo signal from scatterers in the xz plane can be described as¹³

$$r(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(x, z, t) g(x, z) dx dz, \quad (1)$$

where $g(x, z)$ is the scattering potential at (x, z) , and $h(x, z, t)$ is the echo signal in response to a point target at location (x, z) . In a conventional imaging system an effort is made to make h as compact as possible to maximize spatial resolution. Here in contrast h need not be particularly well localized with respect to x , as the lateral resolution is obtained through the shear wave phase encoding process, rather than aperture-domain focusing. Instead, in SWIPE imaging the lateral extent of h determines the illuminated area that is ultimately imaged, not the lateral resolution.

Taking h to be locally shift invariant, as in the far-field of an unfocused transducer, implies that

$$\begin{aligned} h(x, z, t) &= h\left(x, t - \frac{2z}{c_l}\right) \\ &= h_e\left(x, t - \frac{2z}{c_l}\right) \exp\left(j\omega_l\left(t - \frac{2z}{c_l}\right)\right), \end{aligned} \quad (2)$$

where c_l is the ultrasound wave speed, ω_l is ultrasound center frequency, and h_e is the relatively slowly-varying pulse envelope. With this model, the result of a small displacement u_z of a point target in the $+z$ direction is a negligible change in the envelope component and a phase shift of the carrier component,

$$\begin{aligned} h\left(x, t - \frac{2(z + u_z)}{c_l}\right) &= h_e\left(x, t - \frac{2(z + u_z)}{c_l}\right) \\ &\quad \times \exp\left(j\omega_l\left(t - \frac{2z}{c_l}\right)\right) \\ &\quad \times \exp\left(j\omega_l \frac{2u_z}{c_l}\right) \\ &\approx h\left(x, t - \frac{2z}{c_l}\right) \exp(2jk_l u_z), \end{aligned} \quad (3)$$

where $k_l = \omega_l/c_l$ is the ultrasound wavenumber.

Scatterers in an elastic target subject to a z polarized, $+x$ going plane shear wave undergo a displacement in the $+z$ direction of

$$u_z = a \sin(\omega_s t - k_s x - \phi), \quad (4)$$

where a is the shear wave displacement amplitude, ω_s and k_s are the shear wave frequency and wavenumber, and ϕ is the phase of the shear wave source relative to the ultrasound pulse emission. The time t is well approximated by z/c_l for a scattering site with rest position z , because the maximum phase error due to target displacement, $\omega_s a/c_l$, is on the order of 10^{-4} .¹⁴ Substitution of this expression into Eq. (1) allows the echo signal to be written as

$$r(t, k_s, \phi) = \int_{-\infty}^{\infty} \hat{g}(x, t) \exp\left[-j\beta \sin\left(\omega_s \frac{z}{c_l} - k_s x - \phi\right)\right] dx, \quad (5)$$

where

$$\beta = 2ak_l \quad (6)$$

and

$$\hat{g}(x, t) = \int_{-\infty}^{\infty} h(x, t - 2z/c) g(x, z) dz. \quad (7)$$

Here $\hat{g}$ is the object function *convolved* in the z direction by h , and *shaded* in the x direction by h . The impulse response h is designed to be short in time and compact in z , as in a normal ultrasound imaging pulse, so that target range is estimated from time-of-flight.

B. Harmonics, Bessel/Fourier series

The earlier development¹² used a Taylor series approximation to simplify the complex exponential term in Eq. (5). However, an exact expression may be derived by expanding the exponential term in a Fourier series using the identity

$$e^{j\beta \sin \xi} = \sum_{n=-\infty}^{\infty} J_n(\beta) e^{jn\xi}, \quad (8)$$

where J_n is the n th order Bessel function of the first kind. This identity and rearranging the order of integration and summation yields

$$r(t, k_s, \phi) = \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j(\theta_z - n\phi)} G(t, nk_s), \quad (9)$$

where $\theta_z = \omega_s z/c = \omega_s t/2$ is a depth-dependent phase shift, due to the time required for the ultrasound pulse to propagate to depth z , and

$$G(t, nk_s) = \int_{-\infty}^{\infty} \hat{g}(x, t) e^{-jnk_s x} dx \quad (10)$$

is the spatial Fourier transform of the target evaluated at a spatial frequency nk_s at depth $z = ct/2$. The n th term of the sum [Eq. (9)] is the lateral spatial Fourier transform of $\hat{g}$ evaluated at a spatial frequency of nk_s scaled by $J_n(\beta) e^{j(\theta_z - n\phi)}$.

A discrete-time signal on ϕ can be formed by repeated interrogation of the target at integer multiples of a phase step, i.e., $\phi = m\phi_0$. Substitution of this identity into Eq. (9) yields the slow-time signal

$$r(t, k_s, m) = \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j(\theta_z - nm\phi_0)} G(t, nk_s). \quad (11)$$

This repeated interrogation of the target at a pulse repetition frequency (PRF) of $2\pi f_s/\phi_0$ is similar to the development of the “slow-time” signal in pulsed wave (PW) and Color Doppler,¹⁵ with the added requirement that the acquisition be phase locked to the vibration source. This signal may then be filtered along ϕ to yield a single Fourier coefficient $G(nk_s)$. Repeating the process for shear wavenumbers of

$k_1 \leq k_s \leq k_2$ allows determination G over a range of spatial frequencies from nk_1 to nk_2 .

Note that development has assumed linear propagation of both the shear wave and the ultrasound pulse. The harmonics that develop in this model are the result of the phase modulation of the echo signal by the scatterer motion and *not* due to nonlinear propagation of either wave. Notice also that a shear wave of spatial frequency k_s yields not only the Fourier coefficient $G(k_s)$, but the coefficients of integer multiples of this spatial frequency, $G(nk_s)$, including negative values of n . As a result, Fourier coefficients may be obtained for multiples of the shear wave spatial frequency, subject to the consideration that $J_n(\beta)$ ultimately decreases with increasing n for any given β . Signal-to-noise considerations will therefore ultimately limit the maximum spatial frequency that can be recovered. In addition, negative spatial frequencies can be recovered, without requiring a backwards-propagating shear wave.

The effect of ultrasound wavelength on lateral resolution achieved with SWIPE is indirect and occurs through its scaling of the modulation parameter β . Larger values of β , due to shorter ultrasound wavelength or greater shear wave amplitude, can increase the amplitude of the Bessel coefficients $J_n(\beta)$, thereby increasing the SNR of the higher harmonic terms of Eq. (9). There is no fixed maximum spatial frequency coefficient in this image formation model; instead there is only a soft limit set by the decreasing SNR of higher spatial frequency terms.

In contrast, conventional receive focusing or beamforming has a well-defined maximum spatial frequency set by the aperture width and ultrasound wavelength. The sensitivity p to a point source of frequency $f_l = c_l/\lambda_l$ in the focal zone of a focused transducer is given by

$$p(x, y, z) = p_0 \int_{-\infty}^{\infty} A(x_0, y_0) \exp\left(j \frac{2\pi}{\lambda z} (xx_0 + yy_0)\right) dx_0 dy_0, \quad (12)$$

where p_0 is a scaling term that includes acoustic impedance, A is the transducer apodization function, and (x_0, y_0) is a point in the aperture plane. For the sake of comparison to the SWIPE model, consider the lateral component through the one-dimensional (1D) integral

$$p(x, z) = p_0 \int_{-\infty}^{\infty} A(x_0) \exp\left(j \frac{2\pi}{\lambda z} xx_0\right) dx_0. \quad (13)$$

Using the substitution $f_x = x_0/\lambda z$ the integral is rewritten as a Fourier transform of the apodization function A scaled by λz ,

$$p(x, z) = \lambda z p_0 \int_{-\infty}^{\infty} A(\lambda z f_x) \exp(j 2\pi x f_x) df_x. \quad (14)$$

The lateral component of the receiver spatial frequency response of a conventional focused imaging system is just the scaled receive apodization function $A(\lambda z f_x)$. The maximum spatial frequency is determined by the lateral extent of the aperture, the depth, and the ultrasound wavelength. For instance, a transducer of width L of uniform apodization is described by $A(x_0) = \Pi(x_0/L)$, where

$$\Pi(x) = \begin{cases} 1 & \text{if } |x| < 1/2 \\ 0 & \text{else.} \end{cases}$$

The spatial frequency response of the system, $\Pi(\lambda z f_x/L)$, is constant up to a maximum spatial frequency of $L/2\lambda z$ and zero beyond.⁴ The receive lateral spatial frequency response of SWIPE and a conventionally focused transducer are compared below.

C. Cylindrical shear wave source considerations

The development so far has considered a plane harmonic shear wave. In the case of a cylindrical shear wave source, as described in Sec. III, the complex shear wave amplitude can be modeled by¹⁶

$$u_z(x) = \frac{j\pi C}{2} H_0^{(1)}(k_s x) \approx \sqrt{\frac{-\pi C^2}{2k_s x}} e^{j(k_s x - \pi/4)}, \quad (15)$$

where C is a constant with units of meters and $H_0^{(1)}$ is the zeroth order Hankel function of the first kind. The approximation holds for large $k_s x$. In contrast to the plane wave case the shear wave amplitude decreases as $x^{-1/2}$, which results in lower values of β and dimmer images for targets distal to the shear wave source, even in the absence of any shear wave attenuation. In a similar vein, for a given x , the shear wave amplitude decreases as $\omega^{-1/2}$, attenuating higher spatial frequency components.

Furthermore, and perhaps more importantly, there is a spatial phase dependence in addition to the $k_s x$ term in the plane wave case. For large values of $k_s x$ this dependence is in the form of a constant additional $\pi/4$ phase lag. Thus, for targets sufficiently far from a cylindrical shear wave source the plane wave development may be used by assuming a $\pi/4$ phase lag between the shear wave source and the target in addition to the $k_s x$ phase lag. The phase error between the exact and approximate solutions decreases rapidly with $k_s x$, with less than 0.1 radian difference between the two at $k_s x = 1$. This value is achieved in the 5 kPa phantom used in the experiments at a frequency of 35 Hz with a target 1 cm from the source.

D. SWIPE lateral spatial frequency response under constraints

As described above, SWIPE allows measurement of the Fourier transform coefficients G of the target at spatial frequency nk_s to be measured, weighted by $J_n(\beta)$, where n is the harmonic number. The spatial frequency response is therefore determined by the dependence of $J_n(\beta)$ on nk_s . Referring to Eqs. (10) and (11), a point target has a unity Fourier transform G , and the variation in the received signal as a function of k_s is due to $J_n(\beta)$. In an ideal SWIPE system β would be independent of k_s .

In practice, the value of β is dependent on ω_s and is limited by the maximum vibration amplitude that can be generated or tolerated in practice. For example, the electromagnetic shaker shear wave source used in the experiments described here is capable of generating a maximum

acceleration that is relatively independent of frequency. The limit on shear is set by what the target will tolerate without damage. While soft tissue can undergo relatively large strains compared to most engineering materials without damage, there is nonetheless a maximum strain that may be safely applied.

The spatial frequency response of SWIPE can be characterized for maximum allowable acceleration and strain. For the plane shear wave described by Eq. (4), the shear wave amplitude a can be given in terms of maximum acceleration A_M ,

$$a = \frac{A_M}{\omega_s^2} = \frac{A_M}{(c_s k_s)^2} \quad (16)$$

and maximum shear ϵ_M ,

$$a = \frac{\epsilon_M}{k_s}. \quad (17)$$

The spatial frequency response of the n th harmonic SWIPE system is $J_n[2k_l a(k_s)]$. Because $J_n(\beta)$ does not increase with increasing β beyond its first maximum, a is limited to a maximum amplitude $a_{\max}$ for those values of k_s that yield excessively large values of β . The spatial frequency responses of a SWIPE system with a perfectly unfocused aperture and a conventional focused F/3 aperture are plotted in Fig. 1 for a maximum acceleration constraint and in Fig. 2 for a maximum shear constraint.

The spatial frequency corresponding to a given level of the demodulated SWIPE signal can be calculated for the

acceleration and strain constraints in Eqs. (16) and (17). This allows the effect of varying ultrasound frequency, strain limits, and maximum acceleration on the spatial frequency response to be readily seen. The minimum value of β required for $J_n(\beta)$ to obtain a given decibel level is denoted $\beta_{n,\text{dB}}$. The shear wave amplitude a in terms of the constraints of Eqs. (16) and (17) is then applied to the equation $\beta_{n,\text{dB}} = 2k_l a$. This equation may be solved for $f_x = nk_s/2\pi$, the detected cycles per unit length, where the additional factor of n is due to the spatial frequency multiplication in Eq. (10). The resulting expressions for f_x are

$$f_x = \frac{n}{c_s} \sqrt{\frac{f_l A}{\pi c_l \beta_{n,\text{dB}}}} \quad (18)$$

for a maximum acceleration constraint A and

$$f_x = \frac{2n f_l \epsilon}{c_l \beta_{n,\text{dB}}} \quad (19)$$

for a maximum strain constraint ϵ . Plots of f_x at the -10 and -20 dB levels are shown in Fig. 3 for a range of products of ultrasound frequency and strain ($f_l \epsilon$) and ultrasound frequency and acceleration ($f_l A$). In contrast to a conventionally focused transducer, where spatial resolution increases linearly with frequency, the spatial frequency response varies as the square root of the ultrasound frequency under the maximum acceleration constraint.

FIG. 1. (Color online) Plots of SWIPE system MTFs for the case of maximum acceleration constraint. Acceleration is limited to a maximum of 100 m/s^2 (top row) or 250 m/s^2 (bottom row) for frequencies of 0.5 MHz (a) and (d), 1.0 MHz (b) and (e), and 2.5 MHz (c) and (f). The lateral spatial frequency responses associated the signal phase modulated at 1, 2, and 3 times the shear wave excitation frequency f_s are shown as solid, dashed, and dotted lines, respectively. Notice both the higher spatial cutoff frequency and faster decay rates with increasing harmonic order, with the result that the harmonic with the highest signal level varies with spatial frequency under this constraint. The spatial frequency response of a fixed F/3 aperture is plotted as a dashed-dotted line.

FIG. 2. (Color online) Plots of SWIPE system MTFs for the case of maximum strain constraint. Strain is limited to a maximum of 0.01 (top row) or 0.05 (bottom row) for frequencies of 0.5 MHz (a) and (d), 1.0 MHz (b) and (e), and 2.5 MHz (c) and (f). The lateral spatial frequency responses associated the signal phase modulated at 1, 2, and 3 times the shear wave excitation frequency f_s are shown, along with the response of a fixed $F/3$ aperture. In all cases the fundamental modulated component provides the highest signal level at a given spatial frequency under this maximum strain constraint.

III. METHODS

The goal of the experiments described here is to demonstrate image reconstruction using SWIPE using both fundamental and harmonic components of the modulated signal. A further aim is to demonstrate improved image quality by correcting for the frequency-dependent phase response of the

shaker and the phase shift associated with a cylindrical wave source.

A. Equipment

The physical relationship of the phantom, shear wave source, and ultrasound transducer is illustrated schematically

FIG. 3. (Color online) Peak spatial frequencies at the -10 or -20 dB level under maximum acceleration (a) or maximum strain (b) constraints, as a function of the product of ultrasound frequency and maximum acceleration (a) or maximum strain (b). In the strain-constrained case the fundamental modulation component always provides the highest signal level for a given spatial frequency. In the acceleration constrained case, the third harmonic results correspond to the greatest spatial frequency at a level of -10 dB with respect to the unmodulated echo. However, at the -20 dB level, the fundamental corresponds to the greatest spatial frequency. This behavior corresponds to the combination of higher spatial cutoff frequencies and faster signal attenuation versus spatial frequency with increasing harmonic order (visible in Fig. 1) in the acceleration-constrained case.

in Fig. 4. The phantom was a 12 cm cube cast from 200 Bloom pig skin gelatin with a shear modulus of approximately 5 kPa. Embedded within the phantom were three parallel nylon monofilaments 0.13 mm diameter. The monofilaments were parallel to the top surface of the cube, and lay along a diagonal, with 2.5 mm spacing in depth and lateral position. A 5 mm diameter aluminum rod was centered on the top surface of the block and embedded to a depth of 7 cm. The monofilament group was centered 1 cm lateral to this rod.

The free end of the rod was coupled to an electromagnetic shaker (Model 4810, Bruel & Kjaer North America, Norcross, GA) driven by an amplifier (Model 2718, Bruel & Kjaer North America, Norcross, GA). The shaker drive signal was supplied by one of two channels of an arbitrary function generator (AFG 3022B, Tektronix Inc., Beaverton, OR). The second channel was used to trigger an ultrasound pulser-receiver (JSR DPR 35E, Imaginant Inc., Pittsford, NY) and digitizing oscilloscope (Waverunner 44MXi, Teledyne LeCroy, Chestnut Ridge, NY). The use of the dual-channel function generator ensured synchrony between the vibration source and the ultrasound echo acquisition.

Unfocused, single element pistons were used in pulse-echo mode for ultrasound echo acquisition. The 1 MHz transducer (Panametrics Transducers from Olympus NDT Inc., Waltham, MA) had a diameter of 13 mm, while the 3.5 MHz (Panametrics C384SU) had a diameter of 6 mm. These diameters were selected to ensure that the wire targets were in the far-field of each transducer. Echoes were digitized at a rate of 500 MS/s; a low-pass digital filter was applied to the over-sampled echo data to increase the effective dynamic range and narrow the passband of the oscilloscope.

B. Data collection

Echo data were collected for shear wave frequencies between 40 and 1500 Hz in 20 Hz steps. At vibration frequencies below approximately 100 Hz standing waves are significant, due to the low attenuation of gelatin and phantom size. The presence of standing waves causes an undesired shift in the modulation phase that corrupts image reconstruction. In an effort to avoid standing wave effects, pulsed shear waves were employed. Shear wave tone bursts

of duration not greater than the round-trip propagation time from the rod to the boundary (55 ms) were applied to the phantom. Ultrasound echo acquisition was synchronized with the tone bursts. To avoid transient effects associated with the start of the pulsed shear wave and the natural response of the shaker, no echoes are collected in the 24 ms after the start of the shear wave tone burst.

Ultrasound echoes were collected at an effective sample rate 8 times the shear wave frequency. This allowed straightforward filtering for the fundamental and second harmonic components of the phase modulated echo. At low shear wave frequencies, the PRF was simply 8 times the shake drive frequency. To avoid ultrasound reverberations from the tank boundaries at higher shear wave frequencies, the PRF was lowered by adding integer multiples of the shear wave period to the pulse repetition period. The PRF was set to be $8f_s/(8n+1)$, where n was the smallest integer large enough to keep PRF below a threshold: 400 Hz in the 1 MHz case, 1 kHz in the 3.5 MHz case. The greater attenuation of the 3.5 MHz ultrasound by the phantom enabled the use of the higher PRF. The attenuation of ultrasound in plain gelatin is much lower than that of tissue; a phantom with tissue-average attenuation of 0.5 dB/cm/MHz would allow use of higher PRFs.

The shaker drive voltage was varied with frequency to correspond to peak displacement amplitude and peak acceleration constraints. As noted above, there is a maximum useful displacement amplitude for a given modulation component, set by the maxima of $J_n(\beta)$. Displacement constraints corresponding to the maxima of $J_1(\beta)$ of 225 microns for 1 MHz and 64 microns for the 3.5 MHz case were selected. There was no constraint in these experiments on acceleration levels other than what could be generated by the shaker, 220 m/s.

The amplitude and phase of shaker vibration were measured using two methods. An accelerometer (Model U352CG8, PCB Piezotronics Inc., Depew, NY) coupled to the aluminum rod measured the amplitude and phase of the rod acceleration with respect to the shaker drive signal. In addition, ultrasound echoes were collected from the embedded tip of the rod, and the displacement amplitude and phase with respect to the drive signal measured from the time shift of the echoes. These two methods yielded consistent results.

C. Echo processing

A total of 512 echoes were collected at each shear wave frequency. Echoes were arranged in a matrix with each column containing one echo; this is similar to the arrangement of Doppler data with row index representing “fast time” and column index representing “slow time.”¹⁵ Referring to Eq. (11), the coefficient $G(t, nk_s)$ —the Fourier transform of the image slice at a depth $z = ct/2$ evaluated at a spatial frequency of nk_s —is modulated on the slow-time signal $J_n(\beta)e^{i(\theta_z - nm\phi_0)}$, where m is the slow-time index. Recovery of the $G(t, nk_s)$ is thus a matter of filtering the slow time signal for frequency $n\phi_0$. The Hilbert transform of each column was calculated to generate the complex radio frequency signal implied by Eq. (3). The Fourier transform of each row

FIG. 4. Schematic illustration of the data acquisition used in the SWIPE imaging experiments. The shaker drives a 5 mm diameter aluminum rod to act as a harmonic cylindrical wave source. The stationary, unfocused ultrasound transducer collects echoes from the wire targets embedded in the gelatin block, which move in the $\pm z$ direction with the shear wave propagation in the block.

was calculated to perform the filtering operation. Because the effective slow-time sampling rate was always 8 times the shear wave frequency ($\phi_0 = 2\pi f_s / \text{PRF} = \pi/4$), the phase modulation harmonics appeared in the same frequency bin for every shear wave frequency.

The sampled lateral k -space for the fundamental and second harmonic components were assembled into matrices with depth/fast-time rows and spatial frequency columns. Negative frequency components of k -space were provided by the negative frequency components of the Fourier transform of each row; this can be seen by substitution of $-n$ for n in Eq. (11). Correction for the phase shift between the shaker drive signal and shaker rod motion was performed by multiplication of each spatial frequency component by $e^{-j\phi(f_s)}$, where $\phi(f_s)$ is the phase difference. This phase shift was measured by the accelerometer and by ultrasonic motion tracking of the shaker rod, with similar results by each method. To eliminate phase shifts due to gross motion of the target, the phase of each modulated component was calculated with reference to the unmodulated (i.e., slow-time zero-frequency) component of the echo.

As described earlier, the spatial dependence of the phase of a cylindrical wave is not simply kx as in the plane wave case. A correction for this effect was calculated and applied as for the shaker phase dependence above. The correction at a given frequency was determined by evaluating the phase difference between Eq. (15) at the mean target range (10 mm) and the shaker rod radius (2.2 mm), and comparing this with the plane wave phase shift, i.e.,

$$\phi_{\text{cyl}} = \angle \left\{ H_0^{(1)}(k_s x_{\text{target}}) \cdot \overline{H_0^{(1)}(k_s x_{\text{rod}})} \cdot \exp[-jk_s(x_{\text{target}} - x_{\text{rod}})] \right\}, \quad (20)$$

where the overline indicates the complex conjugate.

K -space data were not directly collected at spatial frequencies corresponding to shear wave frequencies of -20 , 0 , and 20 Hz. Filling the “missing” data with zeroes leads to undesirable sidelobes in the reconstructed image. To avoid

such artifacts, k -space values were interpolated from the surrounding -40 and 40 Hz data. The interpolated values were given by

$$|M| \phi_{-40} \phi_{\Delta}^{f_s - 40/80},$$

where $|M|$ is the magnitude linearly interpolated between $|G(t, -2\pi \cdot 40/c_s)|$ and $|G(t, 2\pi \cdot 40/c_s)|$, $\phi_{-40} = G(t, -2\pi \cdot 40/c_s)/|G(t, -2\pi \cdot 40/c_s)|$, and $\phi_{\Delta} = G(t, 2\pi \cdot 40/c_s) \times \overline{G(t, -2\pi \cdot 40/c_s)} / |G(t, 2\pi \cdot 40/c_s)G(t, -2\pi \cdot 40/c_s)|$. This method smoothly interpolates both the phase and magnitude.

Images were reconstructed from the compensated k -space data by performing 1D inverse Fourier transforms to each row of data. The abrupt truncation of k -space at the maximum shear wavenumber results in a sinc lateral point-spread function (PSF), similar to that of a unapodized ultrasound transducer. A Hamming window applied prior to the inverse Fourier transform reduced the sidelobe levels at the expense of the mainlobe width.

IV. RESULTS

The measured displacement and acceleration of the shaker rod and wire target as a function of frequency are plotted in Fig. 5. Displacement was estimated from tracking of ultrasound echoes from the wire target and rod tip; scaling by the displacement amplitude by ω^2 yields the acceleration. The regions of constant displacement amplitude and constant acceleration are clearly visible in the plots of the rod response. In the case of the wire it can be seen that the acceleration decreases with increasing frequency in the “constant” acceleration band; this is due in part to the frequency-dependence of cylindrical wave propagation [Eq. (15)].

MTFs were calculated from the above displacement measurements, as well as measured directly from the modulated echo signal; the results are plotted in Fig. 6 for each of the displacement and acceleration constraints applied. The displacement-based MTF was found by the applying Eq. (6) to the measured displacement amplitude a at each frequency

FIG. 5. (Color online) Displacement and acceleration of shaker rod (a) and central wire target (b) measured in the gelatin phantom. Displacement was measured by cross-correlation tracking of echoes from the target; acceleration was calculated by scaling displacement measurements by ω (Ref. 2).

FIG. 6. (Color online) MTFs of the SWIPE system measured from the central wire target. The modulated signal level is plotted with a solid line, while the signal level expected for the measured displacement amplitudes (Fig. 5) is plotted as a dashed line. In (a) is the response with the 3.5 MHz transducer and the 65 micron, 220 m/s² constraint, (b) is the 1 MHz response under the 225 micron, 220 m/s² constraint, and (c) is the 1 MHz response under the 65 micron constraint.

to and calculating $J_n(\beta)$ for the fundamental ($n=1$) and second harmonic ($n=2$) modulation components. The directly measured magnitude of the modulated echo components were taken from the echo spectrum at 1 or 2 times the modulation frequency and scaled with respect to an unmodulated carrier. In both cases the estimates were performed in the vicinity of the peak echo amplitude. Good agreement is seen between the predicted and measured MTFs.

Under identical constraints, the signal level in the 3.5 MHz case is higher for any given spatial frequency compared to the 1 MHz case, due to the smaller ultrasound wavelength and correspondingly higher β . Inspection of Figs. 6(a) and 6(b) reveals comparable spatial frequency responses for the fundamental component, despite the factor of 3.5 difference in ultrasound frequency, achieved by increasing the peak displacement amplitude by the same factor for the 1 MHz case compared to the 3.5 MHz case. The two 1 MHz cases [Figs. 6(a) and 6(b)] demonstrate the same response in the acceleration constrained portions of the response; the lower displacement amplitude constraint results in a lower signal level for the displacement-constrained portion.

Images of wire targets generated from the fundamental (f_s) and harmonic ($2f_s$) phase modulation components using 1 and 3.5 MHz transducers are presented in Fig. 7. In contrast to the typical behavior of a conventionally beamformed image, the width of the wire target images are approximately equal for both the 1 and 3.5 MHz images. The height (axial length) of the wire responses is greater in the 1 MHz image, due to the lower frequency and bandwidth of the transducer. This is to be expected, as depth resolution is due solely to time-of-flight estimation, as in conventional imaging. The “tails” of the 1 MHz wire images should not be confused for sidelobes. This tail is clearly seen beneath the most proximal of the wire images (at ~ 34 mm), which additionally does not show a strong sidelobe to its right. This tail is associated with this wire, and is not a sidelobe of the center wire. The long tails in the 1 MHz image are due to the much lower bandwidth of this transducer, 0.44 MHz (40%) at -6 dB, compared to 2.1 MHz (61%) for the 3.5 MHz transducer.

Sidelobes in the point response are visible in the 40 dB dynamic range images. These sidelobe levels are in excess of the ideal response of the Hamming window, as illustrated

FIG. 7. Reconstructions of nylon monofilament targets embedded in gelatin phantom. Echo data were acquired at 1 MHz (left column) and 3.5 MHz (right column). Images are generated from phase modulation component at the shear wave frequency (f_s) and displayed with a dynamic range of 40 dB (top row). Images reconstructed from the double frequency phase modulation component ($2f_s$) are shown in the bottom row.

in Fig. 8. Examination of the k -space data indicates that the elevated sidelobe levels are due principally to deviations from ideal linear phase rather than from variations in amplitude. Figure 9 plots the residual phase error of the k -space data for the 3.5 MHz acquisition through the peak amplitude region of the echo.

V. DISCUSSION

MTFs and cutoff spatial frequencies were calculated for maximum strain and maximum acceleration constraints. The maximum acceleration case is notable in that lateral spatial resolution improves as the square root of ultrasound frequency [see Eq. (18)] rather than in direct proportion as in conventional imaging [Eq. (14)]. This result suggests that SWIPE may have an advantage when the use of low frequency ultrasound is required, e.g., to penetrate highly attenuating targets, as the rate of loss of resolution with decreasing ultrasound frequency is slower in the SWIPE case. The acceleration constrained MTF calculations (Figs. 1 and 3) suggest that use of the harmonic phase modulated components can provide enhanced resolution compared to

the fundamental modulated component. In the strain constrained case, the fundamental term is always preferred.

As described above, the reconstructed images presented are formed after estimating and equalizing the spatial

FIG. 8. (Color online) Realized and ideal PSFs for the 3.5 MHz SWIPE system. The solid line corresponds to the measured response to the central wire target, based on the fundamental signal. The dashed line is the response for ideal, Hamming-windowed data, i.e., linear phase and constant amplitude with spatial frequency.

FIG. 9. (Color online) Residual phase error for the central wire target at 3.5 MHz, obtained by subtracting a linear least-squares fit to the estimated phase. The phase noise does not appear to be completely random. Rather low-level variations are superimposed on a large-scale, single cycle oscillation, as well as more rapid oscillatory component.

frequency response over the entire range of shear wave frequencies interrogated. Thus the PSF is the Fourier transform not of the measured MTFs, but of the equalized and windowed k -space data. To the extent that it is possible to gather sufficient signal at higher spatial frequencies, e.g., by averaging multiple echoes at a given shear wave frequency, it is possible to obtain higher resolution. In contrast B-mode averaging improves SNR does not increase resolution for a given ultrasound frequency. Thus, while B-mode resolution is well defined in terms of ultrasound frequency and f -number, SWIPE resolution and SNR are inextricably linked, even for a fixed ultrasound frequency. Stationary targets that allow for significant time for echo averaging, therefore, seem particularly well suited to this method.

A challenge for this method is the generation of a shear wave field where the phase change with frequency is a linear function of lateral position x , as in the case of a plane wave, where $\psi = kx$. Initial work with surface sources¹² showed relatively poor results, as the phase dependence is not linear on frequency, due to the interaction of surface and shear waves, each with its own propagation velocity and frequency-dependent amplitude. The effects of source geometry were investigated; the results indicated that a shaded, extended internal source (as might be generated with focused acoustic radiation force) can provide near-ideal phase linearity.¹⁷ The rod geometry used in these experiments represents a non-ideal approximation to such a source, as the vibration amplitude does not vary along the length of the rod. While the use of cylindrical shear wave results in an additional, non-linear dependence of phase on lateral position, this effect is negligible for shear wave frequencies and source distances involved here.

Deviations from ideal point spread responses observed in these experiments appear to be due to noise in the phase of the modulated signal. The large-scale structure of the phase error signal in Fig. 9 suggests that there is a non-random component of this error. Standing wave effects due to the low attenuation of the gelatin target are one likely source. While an effort was made to minimize the effect of the transient response of the shaker in the pulsed acquisition system, it is possible that this is not entirely suppressed at low frequencies,

and acts as an additional source of phase error. The interaction of surface waves with the shear wave, and the limited applicability of the line source in an infinite media model at low frequencies are further possible sources of aberration.

The case of a fixed, unfocused transducer was treated here. In this case, all of the lateral resolution is achieved through the SWIPE method. In standard B-mode imaging, beamforming is used on transmit and receive, and the PSF is the product of transmit and receive contributions. Transmit focusing could likewise be applied to SWIPE imaging, with the same improvement in lateral resolution and sidelobe levels. Because the characteristics of the beam would be similar in B-mode and SWIPE if transmit beamforming were applied, this work focused on receive-side characterization.

VI. CONCLUSION

The results presented here suggest that the SWIPE method is capable of producing backscatter images of reasonable resolution with relatively low diagnostic ultrasound frequencies. Stationary targets that allow echo averaging enable improved SNR and the recovery of higher spatial frequency components with a concomitant improvement in resolution. In comparison to conventional focusing, this method appears to be most useful at low ultrasound frequencies and in cases of limited aperture size.

These properties suggest that transcranial imaging is a possible application of this method. In the adult human skull ultrasound in the range of 0.5–1 MHz is relatively weakly attenuated, with rapidly increasing loss at higher frequency.⁵ In addition to the difficulty posed by high attenuation, the ultrasound wavefront is also significantly distorted by the non-uniform thickness and much greater sound speed of the skull versus soft tissue. Correction of this wavefront distortion is a challenging problem and the subject of much research.^{18–20} SWIPE does not rely on coherent summation of echoes over an extended aperture for lateral resolution, and can be performed with point-like sources and detectors,¹² avoiding phase aberration concerns altogether. SWIPE performed with a finite aperture subject to phase aberrations would result in deviations from the non-aberrated beam pattern and target illumination. However, small translations of scatterers toward or away from the transducer would still generate the phase shifts required for SWIPE imaging, and lateral resolution would be unaffected. The detection of ultrasound echo phase shifts (as needed to implement SWIPE) is routinely performed in the form of transcranial Doppler sonography,⁶ and shear wave generation in the brain has been demonstrated for Magnetic Resonance Elastography.²¹ This possible application is an interesting topic for further study.

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