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A Large Signal Model for CMUT Arrays with Arbitrary Membrane Geometries Operating in Non-Collapsed Mode

Sarp Satir [Student Member, IEEE], Jaime Zahorian [Student Member, IEEE], and F. Levent Degertekin [Senior Member, IEEE]

Abstract

A large signal, transient model has been developed to predict the output characteristics of a CMUT array operated in the non-collapse mode. The model is based on separation of the nonlinear electrostatic voltage-to-force relation and the linear acoustic array response. For linear acoustic radiation and crosstalk effects, the boundary element method is used. The stiffness matrix in the vibroacoustics calculations is obtained using static finite element analysis of a single membrane which can have arbitrary geometry and boundary conditions. A lumped modeling approach is used to reduce the order of the system for modeling the transient nonlinear electrostatic actuation. To accurately capture the dynamics of the non-uniform electrostatic force distribution over the CMUT electrode during large deflections, the membrane electrode is divided into patches shaped to match higher order membrane modes, each introducing a variable to the system model. This reduced order nonlinear lumped model is solved in the time domain using Simulink. The model has two linear blocks to calculate the displacement profile of the electrode patches and the output pressure for a given force distribution over the array, respectively. The force to array displacement block uses the linear acoustic model, and the Rayleigh integral is evaluated to calculate the pressure at any field point. Using the model, the transient transmitted pressure can be simulated for different large signal drive signal configurations. The acoustic model is verified by comparison to harmonic FEA in vacuum and fluid for high and low aspect ratio membranes as well as mass-loaded membranes. The overall Simulink model is verified by comparison to transient 3D FEA and experimental results for different large drive signals; and an example for a phased array simulation is given.

I. Introduction

There have been significant developments in modeling of CMUT behavior since the initial small signal equivalent circuit models [1, 2]. In these early models, a single CMUT membrane was modeled as a lumped parallel plate actuator based on Mason's equivalent circuit for piezoelectric actuators [3]. The equivalent circuits are simple to solve using circuit analysis techniques and provide insight into the CMUT behavior. However, these models are based on linearization of the parallel plate actuator and neglect the higher order membrane modes that influence CMUT element behavior [4]. Moreover, these techniques do not describe the large signal dynamics of CMUTs, which is desirable for operation in transmit mode and behavior in an array configuration.

To address the aforementioned limitations, a number of approaches have been proposed and implemented based on equivalent circuit modeling [5-9]. These efforts significantly improve

upon the ability to model nonlinear behavior in two dimensional arrays. In general, these models focus on CMUTs and arrays with uniform cross-sectional geometries and circular membrane shapes. However, the design space for CMUT optimization is extensive, including the ability to manufacture arbitrary membrane shapes (elliptical, rectangular, trapezoidal, etc.) with varied cross-sections, using substrate embedded springs, and incorporating multiple electrodes into a single membrane [10-12]. As an alternative to equivalent circuit models, finite element analysis (FEA) is commonly used to perform modal, harmonic, and transient simulations of CMUTs in a 3D fluidic environment [13, 14]. While FEA modeling is capable of dealing with the complex CMUT geometries, it becomes computationally expensive as the array size increases, predominantly associated with the modeling of the fluidic environment. For optimization of individual CMUT membrane geometries and array configurations, this approach is not ideal due to the significant computational time necessary for each simulation.

An alternative approach for small signal modeling of CMUT arrays, formulated by Meynier et al., utilized finite difference approximations of Timoshenko's thin plate equations to model the CMUT membranes [15, 16]. The acoustic radiation modeling was accomplished using the boundary element method (BEM) which only meshes the vibrating surface area of the CMUT array and does not require 3D fluidic meshing [17]. Since this is a 2D surface mesh over the CMUT membranes, the computational load is significantly reduced as compared to 3D FEA. For small signal analysis of CMUT arrays, this model has been shown to be accurate, as in the case of thermal-mechanical noise modeling of CMUT arrays as presented in [18]. However, this method is limited to the small signal analyses around the bias point of CMUTs with thin membranes, and is not valid for nonlinear transmit modeling. In [19], this method was extended to transient analysis of the displacement of a single CMUT membrane.

In this paper we present a computationally efficient, transient model capable of large signal analysis of CMUT arrays with arbitrary configurations. The model is formulated such that every electrode in a CMUT array is a lumped system, resulting in a multi-input multi-output (MIMO) model where the number of variables to be solved is equal to the number of electrodes in the array. Later in this study, the division of electrodes into patches will be described for the necessary conditions in Section IV.A. The model is based on separation of the linear structural acoustics problem and the nonlinear electrostatic force calculation. A similar approach was taken in [20] where the nonlinear CMUT behavior was modeled as a nonlinear electrostatic transformer and a linear mass-spring-damper system for a single CMUT membrane, neglecting higher order membrane shapes. By exploiting this separability, the solution of the distributed complex linear acoustic behavior of an array can be incorporated into a reduced order lumped nonlinear model. As the linear vibroacoustics problem is numerically solved as a function of frequency using BEM, this solution can be transformed into a lumped model where the variables are average electrode displacements and total forces acting on the electrodes. This reduces the number of variables to solve from the total number of nodes in the nodal mesh for BEM to the number of the electrodes, thus reducing the computational load significantly. This lumped MIMO model is then used in the transient simulation of the array by calculating the total electrostatic forces acting on individual electrodes with the mean electrode displacements and the individual input drive

signals. It should be noted that the linear relationship between electrostatic forces and electrode displacements are only valid for a non-collapsed CMUT membrane. For example, over the collapsed region, the membrane will not be displaced any further by electrostatic forces, introducing nonlinearity. Therefore, collapsed mode operation cannot be modeled with the current approach.

Further model improvements expand upon the boundary element method presented in [15], which is limited to CMUT membranes that are rigidly fixed at the boundary nodes and utilize thin plate approximations. This study will present an alternative method to calculate the CMUT membrane stiffness using FEA. For any given CMUT geometry, a number of static simulations are performed, equal to the number of nodes on the CMUT surface to calculate the stiffness matrix [21]. This allows for modeling of different edge boundary conditions as well as varied cross-sectional geometries and embedded springs. This stiffness matrix is computed for a single membrane and is reused for each membrane in an array and as such the FEA computational load is minimized.

This work will start first with an overview of the model at the block diagram level. Then the linear acoustic model will be described along with examples of its verification using FEA. Implementation of the nonlinear electrostatic actuation aspects and derivation of the MIMO and MISO models for transient analysis in Simulink will then be described for array displacement and output pressure calculations. Several examples will be given to provide insight into the application of the model, and the results will be compared to FEA and experiments. Finally, a discussion of the use of the model for iterative optimization will be given.

II. Overview of the Transient Model

The model presented in this paper can be divided into three main sections, as described by the three blocks in Fig.1, with voltage input and pressure output as indicated. In block A, the drive signal vector, $V(t)$, contains the applied voltages for each of the CMUT electrodes as a function of time. The voltage inputs of block A and the average electrode displacements are used to calculate the total electrostatic forces iteratively for each electrode in the array. This block accounts for sources of nonlinearity in large signal CMUT modeling, where stress stiffening effects are neglected as the vacuum gaps are much smaller than the lateral dimensions and the induced strain will be small even under full deflection [22]. The sources of the nonlinearity in CMUT operation are the voltage square and inverse gap square dependence of the electrostatic force acting on the electrodes as explored in [23], and the change in force distribution on the electrodes as the CMUT membrane deflects. To capture the nonlinearity due to the change of electrostatic force distribution on the CMUT electrode, the electrode may be divided into patches and modeled as separate electrodes, depending on the electrode size. This incorporates higher order modes of the membrane in electrostatic force calculation, which increases the model accuracy significantly [24]. The method of electrode separation into patches will be explained in detail in Section IV.A.

Block B describes the vibroacoustic behavior of the array as a linear MIMO system relating the total electrostatic forces acting on each electrode patch and their respective average

displacements. This block is derived from the linear acoustic analysis of the CMUT array using the boundary element method presented in [15]. Using the BEM, nodal displacements are calculated as a function of frequency for multiple cases where each electrode patch in the array is excited individually with unit pressure applied to the patch. As the mechanical dynamics of the CMUT array are linear, the superposition of the individual solutions can be used for the solution of arbitrary excitation configurations. To reduce the order of the distributed model, the nodal frequency response data is lumped into frequency domain relationships that are taken from total forces acting on electrode patches to the average displacement of each patch. The calculated relations are analogous to self and mutual mechanical impedances of electrode patches, where the variables would be patch velocities and total forces acting on them. A MIMO finite impulse response (FIR) filter is then described by using the calculated frequency response data to obtain a time domain solution. As the linear acoustic problem is solved via BEM, a MIMO FIR filter block is constructed which relates the total forces acting on each electrode patch and their mean displacements such that:

$$\mathbf{U}(t) = H \{ \mathbf{F}(t) \}, \quad (1)$$

where $H\{\}$ is the MIMO FIR filter operator, $\mathbf{F}(t)$ and $\mathbf{U}(t)$ are the input (electrostatic force) and output (mean electrode patch displacement) vectors, respectively.

This linear MIMO model block couples the dynamics of individual electrode patches through acoustic interaction, and therefore it models the linear mechanical behavior of the array, including acoustic crosstalk and higher order membrane shapes. Fluid coupling is considered as the only source of crosstalk where the mechanical coupling between CMUT membranes through the substrate is neglected.

Electrostatic force (block A) and membrane displacement (block B) calculations completely define the electromechanical behavior of the modeled CMUT array. The solution of the transient model is then used to calculate the time domain pressure signal at an arbitrary point of interest.

Block C can be considered a multi-input single-output (MISO) system where the block inputs are the total electrostatic forces acting on array electrode patches, and the output is the pressure at the desired point in the immersion fluid. After the electrostatic forces acting on array electrode patches are obtained with the transient simulation in Simulink in Block B, the time domain pressure signal at point $\vec{r}$ is computed by

$$p^{\vec{r}}(t) = \Im^{-1} \left\{ \left[\begin{array}{cccc} P_1^{\vec{r}}(\omega) & P_2^{\vec{r}}(\omega) & \cdots & P_M^{\vec{r}}(\omega) \end{array} \right] \Im \left\{ \left[\begin{array}{c} F_1(t) \\ F_2(t) \\ \vdots \\ F_M(t) \end{array} \right] \right\} \right\} \quad (2)$$

where $P_j^{\vec{r}}(\omega)$ is the frequency domain relationship relating the total electrostatic force acting on j^{th} patch and the pressure at point $\vec{r}$, and $\Im\{[\mathbf{F}(t)]\} = \Im\{[F_1(t); F_2(t); \dots; F_M(t)]\}$ is

the discrete Fourier transform of the force vector calculated with the Simulink model for the input voltage vector $\mathbf{V}(t)$.

III. Acoustic Response Calculation for Arbitrary CMUT Membrane and Array Geometry

The blocks in Fig. 1 that describe the vibroacoustic behavior are derived using the computationally efficient boundary element method previously introduced by Meynier et al. to simulate CMUT arrays in fluidic environments [15]. In this method, the vibrating array surface is meshed and the force balance is solved for each node, reducing the distributed CMUT array behavior to an N degree of freedom vibration problem where N is the total number of nodes in the array mesh. For simulation accuracy, N should be selected such that every node can be considered as an baffled acoustic point source with uniform radiation pattern for the frequency range of interest, and mesh convergence is satisfied for stiffness matrix, $\mathbf{K}$, calculation [25]. An example membrane discretized into a 5×5 grid is shown in Fig.2 with a $dx \times dy$ nodal area. The force balance equation for each node i can be expressed as:

$$p_{app,i} = m\ddot{u}_i + \mathbf{k}_i \mathbf{u} + \sum_{n=1}^N \frac{\rho_f S \ddot{u}_n(t - \frac{r_{n,i}}{c_f})}{2\pi r_{n,i}} \beta(r_{n,i}, u_n) \quad (3)$$

Here the external mechanical pressure applied on node i , $p_{app,i}$, is balanced by the membrane reaction forces and fluid loading where u_i is the displacement of i^{th} node and $\mathbf{u} = [u_1 u_2 \dots u_N]^T$ is the displacement vector of all nodes in the array mesh. The first term in right hand side of Eq.(3) is the inertia of the node i with local mass density m , which is membrane density $\times$ thickness. The second term is the stiffness which is in vector form as $\mathbf{k}_i = [k_{1,i} k_{2,i} \dots k_{N,i}]$. For cases where analytical solutions can be obtained, such as Timoshenko's thin plate equation, $\mathbf{k}_i$ can be calculated numerically using the Finite Difference (FD) approximation of these solutions for the meshed membrane [15, 16]. The last term in Eq.(3) is the fluid loading term which makes use of the Green's function of a baffled point source. The term is the sum of the pressure contributions acting on node i from the displacement of each node in the array mesh where ρ_f and c_f are the density and speed of sound in the immersion fluid, respectively, S is the nodal area, $dx \times dy$, $r_{n,i}$ is the distance between node i and node n , and $\beta(r_{n,i}, u_n)$ is the attenuation in the immersion fluid.

Eq.(3) defines a unique equation for each node in the array mesh and results in N coupled equations which can be represented in matrix form. Considering harmonic excitation with pressure amplitude p_{app} at frequency ω , the vibroacoustic response of the array is described in frequency domain as a system of linear equations:

$$\mathbf{u}(\omega) = \mathbf{G}(\omega)^{-1} \mathbf{p}_{app} \quad (4)$$

Here $\mathbf{p}_{app}$ is the vector comprised of external pressures acting on array nodes and $\mathbf{u}(\omega)$ is the nodal displacement vector. $\mathbf{G}(\omega)$ is the force balance matrix which is calculated from the stiffness, $\mathbf{K}$, mass, $\mathbf{M}$, and mutual impedance matrix, $\mathbf{Z}_r(\omega)$ matrices such that:

$$\mathbf{G}(\omega) = \mathbf{K} - \omega^2 \mathbf{M} + j\omega \mathbf{Z}_r(\omega). \quad (5)$$

The stiffness matrix in Eq. (5), $\mathbf{K}$, describes how the normal static force acting on a given element area influences the displacement of the entire membrane surface. For static loading of the membrane, assuming that the lateral dimensions are much larger than the thickness, the stiffness can be approximated by the thin plate equation:

$$P(x, y) = K * u(x, y) = \frac{\partial^2 M_x}{\partial x^2} + \frac{\partial^2 M_y}{\partial y^2} + \frac{\partial^2 M_{xy}}{\partial x \partial y} \quad (6)$$

where $P(x, y)$ is the distribution of pressure acting on the CMUT membrane [16]. This is given as the flexural plate operator where M_x , M_y , and M_{xy} are the bending moments on an element volume of the membrane:

$$\begin{aligned} M_x &= D \left(\frac{d^2 u}{dx^2} + \nu \frac{d^2 u}{dy^2} \right), \quad M_y = D \left(\frac{d^2 u}{dy^2} + \nu \frac{d^2 u}{dx^2} \right), \\ M_{xy} &= D(1 - \nu) \frac{d^2 u}{dx dy}, \quad D = \frac{Eh^3}{12(1 - \nu^2)}. \end{aligned} \quad (7)$$

where ν is Poisson's ratio, E is the modulus of elasticity, and h is the membrane thickness.

For static case ($\omega = 0$), Eq. (6) can be written in matrix form for a meshed membrane such that

$$\mathbf{p}_{app} = \mathbf{K} \mathbf{u} \quad (8)$$

where $\mathbf{p}_{app}$ and $\mathbf{u}$ are the pressure and displacement vectors of the surface nodes respectively and $\mathbf{K}$ is the stiffness matrix describing the reaction stresses in an elemental area of the plate [15]:

$$\mathbf{K} = \begin{bmatrix} k_{1,1} & k_{2,1} & \cdots & k_{N,1} \\ k_{1,2} & k_{2,2} & \cdots & k_{N,2} \\ \vdots & \vdots & \ddots & \vdots \\ k_{1,N} & k_{2,N} & \cdots & k_{N,N} \end{bmatrix} \quad (9)$$

This unique $\mathbf{K}$ expression for the force balance at a node is generated through finite difference (FD) approximations to estimate the derivatives used to describe the bending moments in Eq. (6). Displacement of each node is a linear function of the nodal displacement values from surrounding nodes and the distance between nodes, resulting in a system of N force balance equations. Therefore in matrix form, the membrane stiffness can be described as an $N \times N$ matrix. The fixed boundary condition sets nodal displacements at

the membrane edges to zero. The stiffness matrix generated using this approach is a sparse matrix containing information that relates one node and its immediate surrounding nodes.

The mass matrix, $\mathbf{M}$, is a diagonal $N \times N$ matrix consisting of the local surface density and thickness at each node. For an array with arbitrary membrane geometries the mass matrix is:

$$\mathbf{M} = \begin{bmatrix} \rho_1 h_1 & 0 & \cdots & 0 \\ 0 & \rho_2 h_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \rho_N h_N \end{bmatrix}, \quad (10)$$

where ρ_n and h_n are the local density and membrane thickness at node n , respectively.

The last matrix in Eq. (5) is the fluid coupling matrix which couples the nodal displacements through acoustic propagation. The fluid loading on node i due to the displacement of node n can be expressed as $j\omega Z_r^{i,n}(\omega)u_n$ where

$$Z_r^{i,n}(\omega) = \frac{j\rho_f \omega S}{2\pi} \frac{e^{-jkr_{i,n}}}{r_{i,n}} 10^{-\alpha(\omega)r_{i,n}/20}, \quad (11)$$

k is the wavenumber, $\alpha(\omega)$ is the attenuation coefficient as a function of frequency in dB/m, and $r_{i,n}$ is the distance between node i and node n . This formulation is based on the Green's function of a baffled point radiator in a semi-infinite fluid. It assumes that the mesh is fine enough that the normal velocity is uniform over the nodal area and the node can be considered as a point source in the frequency range of interest. Since Green's function accounts for the radiation boundary conditions of the acoustics problem, the fluid is not meshed in the BEM, which allows for the significant reduction in computational time as compared to FEA [15]. The formulated fluid coupling is included in the force balance matrix as

$$\mathbf{Z}_r(\omega) = \begin{bmatrix} Z_r^{1,1} & Z_r^{2,1} & \cdots & Z_r^{N,1} \\ Z_r^{1,2} & Z_r^{2,2} & \cdots & Z_r^{N,2} \\ \vdots & \vdots & \ddots & \vdots \\ Z_r^{1,N} & Z_r^{2,N} & \cdots & Z_r^{N,N} \end{bmatrix}. \quad (12)$$

For diagonal elements in Eq. (12), where $r_{i,i} = 0$, the node is modeled as an infinitesimally small circular piston with an effective radius, a_{eff} , such that:

$$Z_r^{i,i}(\omega) = \rho_f c_f \left[\frac{1}{2} (ka_{eff})^2 + j \frac{8}{3\pi} (ka_{eff}) \right]. \quad (13)$$

It should be noted that the fluid coupling matrix $\mathbf{Z}_r(\omega)$ is symmetric, such that $Z_r^{i,n} = Z_r^{n,i}$. As the force balance matrix $\mathbf{G}(\omega)$ is calculated, Eq. (4) can be solved for nodal displacements when the array is excited with the harmonic load distribution, $\mathbf{p}_{app}e^{j\omega t}$.

Verification of this linear acoustic model was performed experimentally using CMUT arrays with $35 \mu\text{m} \times 35 \mu\text{m}$, $2 \mu\text{m}$ thick membranes [18]. The small signal thermal-mechanical noise of the test array under applied DC bias was measured and was compared to the FD-BEM simulations, showing good agreement. In [18], CMUT array behavior was linearized around the operation point corresponding to the applied DC bias value, including a spring softening effect in the simulations.

The thin plate equations and boundary conditions used to generate the $\mathbf{K}$ matrix above inherently limit the numerical modeling capabilities of the BEM approach. For CMUTs with thin membranes, the FD approximation of the analytical thin plate equations is a powerful and computationally efficient tool that results in an accurate description of membrane stiffness. However thin plate equations are not suitable to model high frequency ($> 40 \text{ MHz}$) CMUT imaging arrays comprised of $18 \mu\text{m}$ wide and $3 \mu\text{m}$ thick membranes [26]. Additionally, the interaction with neighboring membranes and attachment to substrate is not accurately reflected in the fixed edge boundary condition imposed by the thin plate equations. Fabricated CMUT membranes also incorporate a buried metal electrode with thickness typically on the order of $0.1\text{-}0.3 \mu\text{m}$ as compared to the bulk membrane material thickness of $2\text{-}5 \mu\text{m}$. This is difficult to include in the FD approximation of the thin plate equation as well as any non-uniform membrane geometry such as mass loaded or stiffened membranes [10-12]. Some CMUTs use embedded silicon rods to provide stiffness independent of the vibrating thin membrane, thus requiring a significantly different model [10]. Fig.3 shows a simple square membrane with fixed edges as compared to a CMUT with mass loading, electrode material, and alternate fixed boundary to illustrate some of the variety in membrane shapes.

A. Expansion of the BEM approach to arbitrary membrane geometry

To address the main limitations described above, static FEA can be used to generate an equivalent stiffness matrix, $\mathbf{K}$. For this purpose, FEA is implemented using the same BEM nodal locations over the $dx \times dy$ areas associated with the BEM meshing density. For each finite area centered on each node of the 2D membrane surface, a uniform pressure of 1 Pa is applied, and the resulting displacement for each nodal location is calculated with FEA. Fig.4 shows a sample FEA mesh with boxed region for pressure application. Each FEA simulation produces displacement information over the entire membrane where $u^{i,n}$ is the displacement of node n when unit pressure is applied to node i . With all N FEA simulations, the displacement matrix is fully populated, and the stiffness matrix can be calculated as:

$$\mathbf{K} = \begin{bmatrix} u^{1,1} & u^{2,1} & \dots & u^{N,1} \\ u^{1,2} & u^{2,2} & \dots & u^{N,2} \\ \vdots & \vdots & \ddots & \vdots \\ u^{1,N} & u^{2,N} & \dots & u^{N,N} \end{bmatrix}^{-1} \quad (14)$$

Through this formulation, the $\mathbf{K}$ matrix is completely populated with information as to how each node affects all other nodes on the surface. It should be noted that with this approach, the thin plate approximations are removed, realistic edge boundary conditions can be applied, and complex geometries can be evaluated for a CMUT as shown in Fig.3. Also, for a given geometry and mesh density, the FEA need only be performed once as it can be reused for an array of the same membranes. Membrane symmetry may also be exploited to reduce computation time. The resulting $\mathbf{K}$ matrix is directly combined with the BEM formulation for fluid loading to implement an efficient hybrid computation of linear CMUT dynamics.

To validate the stiffness matrix calculations and indicate the limitations of the thin plate approximation, harmonic simulations in vacuum were performed in COMSOL and compared to both the numerical method with the stiffness matrix based on thin plate approximations and the hybrid FEA-numerical method. The bulk membrane material chosen for the simulation, silicon nitride as deposited by plasma enhanced chemical vapor deposition (PECVD), has been used previously for CMUT fabrication with properties listed in Table I, [4, 11, 18, 26].

The FEA mesh convergence was verified for all simulations. The square CMUT membrane thickness was fixed at 2 μm with electrode coverage dimensions set to 75% of the edge dimensions which were adjusted from 40 μm to 10 μm , aspect ratios 20 to 5. These dimensions were chosen to be consistent with CMUT membranes previous fabricated for ultrasonic imaging applications [2, 10-12, 18, 26, 27]. Table II shows that the hybrid method and FEA simulations are in good agreement for all aspect ratios, deviating by less than 1.5% for 2 μm nodal meshing in the BEM domain. The element sizes for simulations were chosen based on convergence criteria of less than 5%. With the low aspect ratio of 5, the thin plate approximation deviates by 25% from FEA while the hybrid method deviates by less than 2%, as seen in Table II.

To further validate the hybrid method, a single mass loaded membrane with a buried electrode was simulated in water with the material properties listed in Table I and dimensions listed in Table III. The frequency response using the hybrid method was compared to FEA utilizing symmetry in a quarter sphere with an outer matched layer which models an infinite fluidic space. For the hybrid numerical method, the stiffness matrix was calculated using 1 $\mu\text{m} \times 1 \mu\text{m}$ elements. For each node, the mass matrix was calculated based on the average thickness and known density of the materials in the 1 $\mu\text{m} \times 1 \mu\text{m}$ regions. Fig.6(top), shows the FEA results with 1 MHz resolution from 20-70 MHz, along with the hybrid method, using 1 MHz resolution from 5-80 MHz, showing excellent agreement in center frequency and bandwidth, with 0.2 MHz difference in center frequency. It can also be seen that the FEA above 45 MHz shows oscillations which is a known issue with insufficient fluid meshing density at higher frequencies.

To expand upon the single membrane comparisons, the same membrane geometry was simulated using quarter symmetry, indicated by dashed lines in Fig.5 to simulate a 2×2 array of membranes with 24 μm pitch. It can be seen in Fig.6 (bottom), that the center frequency and bandwidth are in agreement with maximum 1.1 dB difference in the region where FEA

is accurate. The effect of multiple membranes in close proximity is shown by the increased center frequency, 32 MHz to 37 MHz, and corresponding -3 dB fractional bandwidth reduction from ~37% to ~21 %.

A key benefit to the hybrid analysis method is the possibility for significant reduction in computation time. For comparison purposes, a computer with an Intel i5 processor running at 2.67 GHz and 4 GB of RAM was utilized for single and multiple membrane configurations. The full FEA simulation took approximately ~10 hours for the single membrane case and approximately ~16 hours for the array of membranes. To compute the stiffness matrix, Comsol with MATLAB was used in an automated loop, requiring approximately 30 minutes. Once the stiffness matrix was computed, the frequency analysis required less than a minute to run for the single membrane and less than 2 additional minutes for the array.

IV. Transient Simulation of CMUTs for Large Input Signals

With the linear vibroacoustics solution obtained for membrane arrays with arbitrary geometry, the next step is the transient simulation of the forced response of the array driven by the large drive signals. To incorporate nonlinear electrostatic signals in the analysis, the Simulink model shown in Fig.7 was developed. The model takes the electrical drive signals of each transmit electrode patch as the input vector $V(t)$, and outputs the electrostatic forces acting on each patch as vector $F(t)$ as a function of time. It should be noted that every signal in the model is multiplexed and carries separate time domain signals for each electrode patch. The force vector $F(t)$ is then post-processed for calculation of time domain pressure at an arbitrary point in the immersion field. In the model, the MIMO FIR filter block represents the linear vibroacoustic response of the CMUT array suitable for time domain solution, which is block B in Fig.1. The nonlinear electrostatic force calculation block is implemented via a look-up table providing the relationship between the mean gap under the CMUT electrode patch and the normalized total electrostatic force acting on it. The details of the block diagram are explained in the next subsections.

A. Reduced Order Lumped Model Approximation for Electrostatic Force Calculation

Since in most CMUTs the ratio of lateral dimensions to the gap is large, the total electrostatic force acting on the membrane under the m^{th} electrode in the array can be approximated as

$$F_m^{es}(t) = \frac{-\epsilon A_m}{2} \left(\frac{V_m(t)}{g_m(t)} \right)^2, \quad (15)$$

which models the transducer as a parallel-plate capacitor [2]. Here ϵ is the permittivity of free space, A_m is the electrode area, $g_m(t) = g_0 + u_m(t)$ is the mean instantaneous gap under the electrode, g_0 is the initial gap, and u_m is the mean membrane displacement of the m^{th} array element with an input voltage V_m . Although this model is effective in capturing the nonlinear behavior of CMUTs as shown in [23], the parallel-plate equation which assumes uniform charge distribution within the CMUT electrode does not hold for large

displacements. As the membrane deflects, the charge and force distribution on the membrane also changes due to the local gap variation over the CMUT electrode. Therefore, the parallel plate approximation fails and the relationship between the mean gap and the total electrostatic force becomes a more complex function for a deflected membrane which needs to be dealt with separately.

A straightforward approach consistent with the BEM analysis would be to model each BEM nodal area under the electrode as a separate parallel plate device that is mechanically coupled to the rest of the structure. But this would result in a large system of equations. To reduce the size of the problem while still maintaining accuracy, the membrane electrode is modeled with multiple electrode patches with coupled dynamics accounting for the variable electrostatic force distribution on the membrane. The patch shapes and locations are chosen by separating the electrode into multiple displacement regions based on the modal analysis of the membrane. For example, consider a square CMUT membrane, as shown in Fig. 9, with a central electrode, where only the symmetric modes of the membrane are excited with electrostatic excitation. Although the first mode of the membrane is dominant in the motion, higher order symmetric modes are also excited. Especially when the membrane goes under large displacements while the electrostatic input voltage is applied, the force distribution on the electrode changes and the electrostatic forces are more localized at the center of the electrode due to gap dependence of the force. This localized force can excite the second symmetrical mode of the membrane (5th mode in Fig. 9) which has a resonance frequency of $\sim 3.6\times$ of the first mode, covering the useful frequency range for most CMUT applications. Therefore, one can expect that a small number of electrode patches shaped to match the in-phase out-of-phase structure of these first two dominant modes should capture the dynamics of the non-uniform electrostatic force distribution over the CMUT electrode.

For this particular geometry, which can represent a large group of CMUT devices with circular and hexagonal membranes, it has been observed that 2 electrode patches placed on the out of phase regions of the second symmetric mode result in an accurate prediction of the nonlinear behavior with full gap swing. The patch shape selection is automated and can be done efficiently. The mode shapes, and therefore the electrode patch shapes, of a single membrane are calculated as the eigenvectors of the matrix $\mathbf{M}^{-1}\mathbf{K}$, where the mass matrix, $\mathbf{M}$, and the stiffness matrix, $\mathbf{K}$, are already calculated for BEM formulation [28]. The mode shapes and the resulting 2 electrode patches for the 40 μm square membrane with full electrode coverage is shown in Fig.8. For increased accuracy, the CMUT electrode can be modeled with an increased number of patches considering higher order modes. However the tradeoff is a larger system with an increased number of inputs and outputs. For membranes with partial electrode coverage that does not extend into both out of phase regions of the second symmetric mode of the membrane in Fig.8 (bottom), a single electrode patch is sufficient to capture the large signal behavior accurately without the need of separation of the electrode into multiple patches.

The relationship between the average gap under an electrode patch and the total force acting on it is defined using the stiffness matrix $\mathbf{K}$, as discussed previously:

$$\mathbf{u} = \mathbf{K}^{-1} \boldsymbol{\delta}^{patch} \quad (16)$$

Here, $\mathbf{u}$ is the displacement vector of the membrane when the electrode patch is loaded with 1 Pa static pressure. The sifting vector, $\boldsymbol{\delta}^{patch}$, specifies the nodes of the active electrode patch such that $\boldsymbol{\delta}^{patch} = [\delta_1; \delta_2; \dots; \delta_i; \dots; \delta_N]$ and

$$\delta_i = \begin{cases} 1 & \text{if node } i \in \text{active electrode patch} \\ 0 & \text{else} \end{cases} \quad (17)$$

In order to calculate the relationship between the mean displacement and total electrostatic force on an electrode patch, the solution of Eq. (16) is used for the whole range of membrane displacement. The assumption made here is that the deflection profile within an electrode patch does not change in dynamic operation so the total electrostatic force acting on the electrode patch can be defined by sweeping the normalized deflection profile

calculated in Eq.(16) throughout the whole gap. In other words, $\frac{\mathbf{u}}{u_{max}} g_0 d$ gives any possible non-collapse displacement vector, where u_{max} is the maximum displacement value of the solution of Eq. (16), and d is the percentage of deflection such that $0 \leq d \leq 1$. For $d = 0$, the membrane is not deflected and for $d = 1$ the membrane touches the substrate.

For any given membrane displacement profile, the corresponding mean gap and electrostatic force is calculated as:

$$g_{mean}(d) = g_0 + \frac{1}{N_{patch}} \sum \frac{\mathbf{u}}{u_{max}} g_0 d$$

$$\frac{F^{es}(d)}{V^2} = \sum \frac{-\epsilon S}{2(g_{eff,0} + \frac{\mathbf{u}}{u_{max}} g_0 d)^2} \quad (18)$$

where summation is done over the nodes in the electrode patch of interest, N_{patch} is the number of nodes in the patch, S is the area associated with a single node, and $g_{eff,0}$ is the effective initial gap with an insulation layer with thickness $h_{isolation}$ and relative permittivity $\epsilon_{isolation}$:

$$g_{eff,0} = g_0 + \frac{h_{isolation}}{\epsilon_{isolation}} \quad (19)$$

Calculating Eq. (18) using the membrane displacement shape calculated in Eq. (16) results in the desired relationship between the lumped system variables: total electrostatic force acting on the electrode patch and its mean displacement. Note that in this approach, each nodal area is considered as a separate parallel plate contributing to the total force depending on the gap under that node, thus accounting for the non-uniform membrane deflection in the electrostatic force calculation. This is in contrast with the case where the whole electrode is modeled as a single parallel plate where the gap and corresponding charge distribution is assumed to be uniform. To emphasize this difference, the force vs. average displacement relationship is calculated for both cases for a CMUT which has a $2 \mu\text{m}$ thickness, $40 \mu\text{m} \times$

40 μm SiN_x membrane, full electrode coverage, and 100 nm gap thickness with no isolation. The CMUT electrode is divided into 2 patches, as discussed earlier, to include the effect of change in displacement profile and force distribution within the electrode as the membrane deflects. Fig.9 presents the calculated normalized electrostatic force as a function of mean gap under the actuation electrode patches. The calculation is done for both electrode patches separately. The proposed method is also compared to the parallel plate approximation where the normalized electrostatic force is calculated as a function of the mean gap via Eq. (15). In the figure, it can be seen that for large displacements, the total electrostatic forces acting on the electrode patches deviate from the parallel-plate approximation. Note that the center of the membrane has traveled full 100 nm when the average gap under the center electrode patch (patch 1) is about 32 nm. As expected, the deviation is larger for patch 1 which has a larger curvature and more charge non-uniformity as compared to the side electrode patch (patch 2) as the membrane displacement gets larger. More linear behavior of patch 2 in Fig. 10 suggests that the nonlinearity due to non-uniform charge distribution can be reduced by exciting the membrane with side electrode actuation as also presented in [27].

This relationship is included in the Simulink model as a look-up table which takes the average gap of an active electrode patch as its input with the normalized total electrostatic force as the output. The calculation is interpolated linearly by the lookup table in the Simulink model for gap values different than the calculated values. Then the total electrostatic force acting on the electrode patch is calculated with the input signal squared, $V^2(t)$.

B. Transient Vibroacoustic Response Calculation for the Reduced Order System

As every active electrode patch is modeled separately in our reduced order model, the dynamic problem in Eq. (4) is solved for multiple cases where a single electrode patch is excited in each case. For M total active electrode patches in the model, Eq. (4) becomes

$$[\mathbf{u}^1 \quad \mathbf{u}^2 \quad \dots \quad \mathbf{u}^m \quad \dots \quad \mathbf{u}^M] = \mathbf{G}(\omega)^{-1} [\delta^1 \quad \delta^2 \quad \dots \quad \delta^m \quad \dots \quad \delta^M]. \quad (20)$$

In eq. (20), $\mathbf{u}^m$ denotes the displacement vector of the whole meshed array when electrode m is with 1 Pa pressure at frequency of ω . It should be noted that the BEM system is an N degree of freedom coupled system where N is the total number of nodes in the BEM mesh. To reduce the system to an M degree of freedom system where M is the total number of electrode patches, the patches are modeled as lumped elements which take the total electrostatic force acting on it as its input and output the mean displacement. Considering that in an array, mesh number of nodes is much larger than the number of electrodes patches, ($N \gg M$), this approach greatly reduces the number of equations to solve simultaneously for the dynamic problem.

To model the array as an M input and M output system, the relationship between the total electrostatic force acting on m^{th} electrode patch to the mean displacement of n^{th} patch is calculated as a function of frequency as

$$\begin{aligned}\bar{u}_n(\omega) &= \frac{1}{N_{patch,n}} \sum \mathbf{u}^m \delta^n, \\ H_{m,n}(\omega) &= \frac{\bar{u}_n(\omega)}{S_m},\end{aligned}\quad (21)$$

using the solution of Eq. (20). Here the summation is done over the nodes in the n^{th} electrode patch and S_m is the area of m^{th} patch. In Eq. (21), $\bar{u}_n(\omega)$ is the mean displacement of the patch n and S_m is the total force acting on the patch m , which is equal its area since uniform 1 Pa pressure is applied to the patch in Eq. (20). The inverse Discrete Fourier Transform of $H_{m,n}(\omega)$ yields in the impulse response, $h_{m,n}(t)$, the displacement of patch n when m^{th} patch is excited with an impulse. Here the Hermitian symmetry of $H_{m,n}(\omega)$ is exploited for DFT calculation by substituting $H_{m,n}(-\omega) = H_{m,n}^*(\omega)$ since $h_{m,n}(t)$ is expected to be a real function. The impulse response relating the force acting on m^{th} patch and the displacement of n^{th} patch is incorporated in Simulink model as a discrete FIR filter, where the filter coefficients are the impulse response samples such that [29],:

$$\hat{u}_n[v] = \sum_{\tau=1}^T B_{\tau}^{m,n} \hat{F}_m[v - \tau] = \sum_{\tau=1}^T \hat{h}_{m,n}[\tau] \hat{F}_m[v - \tau] \quad (22)$$

Here u_n , F_m and $h_{m,n}$ are the discrete time domain displacement of n^{th} patch, total force acting on m^{th} patch and the impulse response relating u_n and F_m . T is the total number of samples and τ is the time step. The time step τ is determined by the maximum frequency the BEM problem in Eq. (20) is solved for and T is determined by the frequency resolution. For example if Eq. (20) is solved for frequencies up to 100 MHz with 50 kHz resolution, time resolution is 5 ns and T is 4000, which corresponds to total simulation time of 20 μ s. A similar approach was pursued to model the radiation impedance of a circular baffled piston for transient Simulink simulation of a single piston in [23]. In [23], the analytical frequency domain representation of the radiation impedance of the circular piston, which relates its velocity and the fluid load acting on itself due to its own motion, was incorporated in the model as an arbitrary response FIR filter block with the frequency response equal to the radiation impedance of the piston.

For an example case, the MIMO Simulink filter block for a single membrane with 2 inputs and 2 outputs for 2 electrode patches is presented in Fig. 10. The block takes 2 discrete time force signals as inputs, F_1 and F_2 , and outputs the multiplexed vector of respective displacements. The FIR filter coefficients are:

$$\begin{aligned}B_1 &= \begin{bmatrix} \hat{h}_{1,1} \\ \hat{h}_{1,2} \end{bmatrix} = \begin{bmatrix} \hat{h}_{1,1}[1] & \hat{h}_{1,1}[2] & \cdots & \hat{h}_{1,1}[T] \\ \hat{h}_{1,2}[1] & \hat{h}_{1,2}[2] & \cdots & \hat{h}_{1,2}[T] \end{bmatrix}, \\ B_2 &= \begin{bmatrix} \hat{h}_{2,1} \\ \hat{h}_{2,2} \end{bmatrix} = \begin{bmatrix} \hat{h}_{2,1}[1] & \hat{h}_{2,1}[2] & \cdots & \hat{h}_{2,1}[T] \\ \hat{h}_{2,2}[1] & \hat{h}_{2,2}[2] & \cdots & \hat{h}_{2,2}[T] \end{bmatrix},\end{aligned}\quad (23)$$

where the number of rows in each coefficient matrix is equal to the number of patches, M , which is 2 in this case. Each filter block takes the respective total force as its input and outputs the multiplexed displacements. The contributions from each block are then added

together giving the linear multiplexed MIMO displacement output vector. The memory block is needed to break the algebraic loop introduced by the FIR filters and the gap feedback in the Simulink model (Fig.7). The details of the algebraic loop phenomenon in the Simulink model is out of the scope of this paper and more information can be found in Simulink Documentation [30]. The extension of the approach to an arbitrary number of electrode patches is straightforward, requiring one FIR filter block per patch, modeling the array as an M input M output system with the total electrostatic forces acting on array electrode patches as the inputs and the mean displacements as the outputs. If the total electrostatic forces acting on each transmitting electrode patch are known, the mean displacements can be solved using the MIMO FIR block. Together with the electrostatic force calculation, this block completes the Simulink model which takes the time domain drive signals for each electrode patch as input and outputs the total electrostatic forces for pressure calculation.

C. Pressure Calculation at an Arbitrary Point in the Immersion Fluid

Once the forces on the electrode patches are known, the pressure at point $\vec{r}$ when the m^{th} electrode patch is excited can be calculated as a function of frequency using the baffled point radiator model [31]:

$$p_m^{\vec{r}}(\omega) = \sum \frac{-\rho_f \omega^2 S}{2\pi} \frac{e^{-jk\mathbf{r}_{\vec{r}}}}{\mathbf{r}_{\vec{r}}} 10^{\frac{-\alpha \mathbf{r}_{\vec{r}}}{20}} \mathbf{u}^m, \quad (24)$$

where the summation is done over the whole meshed array with the nodal displacements calculated in Eq. (20), and $\mathbf{u}^m$ is the displacement of the nodes when only the m^{th} electrode patch is excited. Here $\mathbf{r}_{\vec{r}} = [r_1; r_2; \dots; r_i; \dots; r_N]$ is the distance vector composed of the distances between the point where pressure is calculated and the BEM nodes. Note that with this approach, the pressure is not calculated from average electrode patch displacements as pistons, rather all the high spatial resolution nodal displacement information from BEM is retained while performing the pressure calculation. Similar to the derivation of Eq. (21), a frequency domain relationship relating the total electrostatic force acting on m^{th} patch and the pressure at point $\vec{r}$ is calculated as:

$$P_m^{\vec{r}}(\omega) = \frac{p_m^{\vec{r}}(\omega)}{S_m}. \quad (25)$$

Using superposition principle, the system can be modeled as a MISO system and the pressure contributions from each excited electrode patch can be added up to find the total pressure at the point of interest such that

$$p^{\vec{r}}(\omega) = [P_1^{\vec{r}}(\omega) \quad P_2^{\vec{r}}(\omega) \quad \dots \quad P_M^{\vec{r}}(\omega)] \Im \left\{ \begin{bmatrix} F_1(t) \\ F_2(t) \\ \vdots \\ F_M(t) \end{bmatrix} \right\} \quad (26)$$

Since the electrostatic forces acting on array electrodes were obtained with the transient simulation in Simulink, the time domain pressure signal at point $\vec{r}$ is computed by inverse Fourier transform of Eq. (26).

V. Model Verification

A. Model Comparison with FEA

For comparison purposes, the CMUT element shown in Fig.8 is modeled in a 2×2 array configuration for large signal transient analysis and each full electrode is divided into 2 patches as shown in Fig. 9. The membrane pitch was set to $50 \mu\text{m}$. The simulation is set up such that all four array elements are excited with the same drive signal and the pressure at the array surface in the middle of the array is calculated both with Comsol and our model. In the first case, a 20 ns long 70 V pulse is applied to the array without DC bias which results in full gap membrane swing. The time domain pressure signals and their spectrums for both FEA and the Simulink model are presented in Fig.11. The two are in excellent agreement, with 1 dB max difference up to 40 MHz for this 10 MHz CMUT, which shows the capability of the model with large signal actuation.

In the second case, the same device is driven by a 50 V, 5 MHz, and 1 cycle tone burst with no DC bias, similar to suggested sub-harmonic drive mechanism for harmonic imaging with CMUTs [23]. The applied signal results in full-gap membrane swing to drive the CMUT array and also the electrostatic actuation signal is still active when the full gap is achieved, representing the worst case scenario in terms of nonlinearities. Fig.12 shows the time domain pressure signals and their spectra for the Simulink model and FEA. To emphasize the significance of multiple patches, the FEA result is also compared to the model without separating the transmit electrode into 2 patches. In Fig.12, it can be seen that the nonlinear behavior of the CMUT array is captured more accurately when the transmit electrode is divided into 2 patches, which incorporates the effect of the change in the displacement profile and charge distribution within the electrode when the CMUT membrane goes through large displacements, as discussed in Section IV.A.

B. Experimental Validation

A 16 membrane CMUT element was fabricated, shown in Fig.13 (left), for experimental verification of the model. The device is composed of $2.2 \mu\text{m}$ thick, $35 \mu\text{m}$ square SiNx membranes with $25 \mu\text{m}$ electrodes. The gap thickness is 50 nm at rest, the SiNx electrode isolation layer is 200 nm thick and the array pitch is $45 \mu\text{m}$. The low-temperature process presented in [32] has been used for transducer fabrication. The array was tested in vegetable oil and the transmitted pressure is measured with a broadband hydrophone which has a bandwidth of 80 MHz (Onda HGL-0085). The experimental setup is shown in Fig.13 (right). The attenuation in vegetable oil is measured as a function of frequency, shown in Fig.14, , and a curve is fit to the measured data assuming that the attenuation coefficient is in the form of Af^B , where f is frequency, and A and B are constants.

For model validation, the pressure 2.2 mm away from the array is measured for the same drive signal which is measured in the experimental setup and used in the simulations. The collapse voltage of the CMUT in test was measured and simulated to be 40 V. The

resonance frequency of the device in air for 25 V DC bias is simulated as 17 MHz, which is verified with a network analyzer measurement. Every membrane was modeled with an 11×11 mesh grid in the BEM formulation, and each electrode is divided into 2 patches. The time domain pressure signals for no DC bias and 45 V 30 ns pulse and its frequency spectrum is presented in Fig.15. The simulation and experiment match within 2 dB up to 40 MHz, which is 20 MHz above the upper 3 dB cut-off frequency of the transducer. The results match fairly well considering the 10% uncertainty in fabrication in terms of membrane and gap thickness.

C. Phased Array Operation Simulation

With the ability to apply arbitrary signals to multiple individual CMUT membranes, the model can be used for the investigation of phased arrays. As an example, a 16 element CMUT array shown in Fig. 16 is simulated when each of the array elements are excited with different input signals. The array size and symmetry constraints were set by the maximum size of the 3D transient FEA problem that can be handled by our computer using Comsol. The CMUT array comprised of 2 μm thick 40 μm square membranes with 100 nm gap and no electrode isolation. The pressure 100 μm away from the center of the array surface at $\vec{r}_f = [0, 0, 100 \mu\text{m}]$ is calculated both with the model and Comsol when the array is excited to focus at $\vec{r}_f$ and without focus. Appropriate time delays presented in Table IV are introduced to the drive signals for each membrane to focus the array, as determined by the distances between the center of membranes and $\vec{r}_f$:

$$t_{d,i} = \frac{|\vec{r}_f - \vec{r}_i| - |\vec{r}_f - \vec{r}_1|}{c_{fl}}, \quad (26)$$

$$V_i(t) = V(t - t_{d,i}).$$

Here, $\vec{r}_i$ indicates the coordinates of the center of i^{th} membrane, $V(t)$ is drive signal applied to the elements, $V_i(t)$ is the voltage signal acting on the i^{th} element in the array. The pressure signals at $\vec{r}_f$ are also presented in Fig.16 when a 65 V and 40 ns pulse is applied to the array elements with and without the focusing delay. It can be seen that the pressure at the desired point is increased when the focusing delay is applied, as expected. The results compare well to Comsol simulations and demonstrate the capability of the model for phased array simulations.

VI. Discussion and Conclusion

A key benefit of the modeling and simulation approach presented is its generality in terms of individual CMUT geometry, CMUT placement in an array, and large signal actuation capability of phased CMUT arrays. With this approach, it is possible to investigate the interaction of arrays of CMUTs with arbitrary geometry and actuation for expedited iterative design optimization.

As the model is based on a 2D surface mesh, increasing the array size has been shown to be computationally less expensive than using 3D transient FEA. As the Simulink model blocks are pre-calculated for a given array geometry, the transient SIMULINK model takes less

than a second to run. This is a significant advantage over FEA, as the model can be simulated for different drive signals for individual array elements iteratively in a significantly reduced computation time. The model is also capable of modeling CMUT behavior with external circuit elements since time domain instantaneous capacitance can be calculated as presented in [23], where a single CMUT element was modeled with series passive electronic elements. As a further development, a SPICE netlist of the driving circuitry can be incorporated in the SIMULINK model as a block, so the whole system can be simulated simultaneously.

Fig.17 describes the general process flow of the model along with main computational methods and blocks. For an initial set of parameters, and based on the individual membrane geometry, either the finite difference method or the hybrid-FEA method can be used to generate the stiffness matrix depending on the geometry. Single or multiple membrane geometries, each with a separate $\mathbf{K}$ matrix, can be arranged into an array with subsequent analysis using the BEM. With the MIMO and MISO blocks derived from these calculations, SIMULINK is used with voltage signals to compute the output pressure. As the model is generated in discrete steps, it can be seen that optimization or modification can be more or less computationally expensive depending upon the parameter modified such as mass loading, array spacing, gap thickness, and applied signal as described by the feedback block in Fig.17. For example, since the gap thickness has minimal influence on the frequency response, that parameter can be iterated quickly without modifying the MIMO and MISO blocks or changing the array geometry, and does not require recalculation of the $\mathbf{K}$ matrix. Since iterative optimization is an important feature of ultrasound transducer design, this model is generally well-suited for rapid and accurate design of CMUT arrays for a variety of applications and quantitative studies of fundamental transducer characteristics, such as in energy conversion efficiency and linearity.

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Fig. 1.
Block diagram describing the transient model with arbitrary voltage input and pressure output.

Fig. 2.

CMUT membrane divided into 5×5 matrix of nodes. $N=25$, with corresponding areas $dx \times dy$.

Fig. 3. (left) Simple square membrane with fixed edge. (right) Membrane with mass loading, metal electrode, and variable fixed boundary.

Fig. 4.

The $dx \times dy$ area for pressure loading shown on a simple meshed membrane in Comsol.

Fig. 5.
A mass loaded CMUT array with aluminum electrode. Symmetry boundary conditions are depicted by dashed lines for the FEA analysis.

Fig. 6.
 (top) Comparison of hybrid method to FEA using Comsol for a single mass loaded CMUT,
 (bottom) 2×2 array with $24 \mu\text{m}$ pitch.

Fig. 7.

SIMULINK block diagram with the input vector of voltage signals applied to array electrode patches. The output vector $F(t)$ is the time domain electrostatic forces acting on the electrode patches, which is post-processed for pressure calculation.

Fig. 8.

(top) First 6 modes of modeled membrane, calculated as the eigenvectors of the matrix $\mathbf{M}^{-1}\mathbf{K}$. The membrane is SiN_x , $2 \mu\text{m}$ thick, dimensions of $40 \mu\text{m} \times 40 \mu\text{m}$, and has full electrode coverage. (bottom) The actuation electrode separated to 2 patches using the out of phase displacement regions of 5th membrane mode.

Fig. 9. Relationships obtained for the mean gap and total electrostatic force for both electrode patches of the modeled membrane. The relationship is also compared to the parallel-plate approximation.

Fig. 10.

The MIMO FIR Block in the SIMULINK model relating total forces acting on electrode patches and mean patch displacements for an example case of a single membrane with 2 patches.

Fig. 11.

Simulated pressure at the array surface with FEA and the model from this study, with corresponding spectrum for a high amplitude, short pulse resulting in full gap swing

Fig. 12.

(top) Simulated time domain pressure signals at the array surface for a 1 cycle tone-burst with no DC bias resulting in full-gap swing, (bottom) Spectra of the pressure signals evaluated with FEA and our model with a single electrode patch and 2 electrode patches modeling the transmit electrode.

Fig. 13.
(left) 16-membrane CMUT array used for experiments and model validation. (right)
Experimental setup with a broadband hydrophone

Fig. 14. Measured attenuation coefficient of vegetable oil as a function of frequency and curve fit used in simulations

Fig. 15. Simulated and experimental pressure when the transducer is excited with no DC bias and 30 ns 45 V pulse

Fig 16.

(left) Simulated phased array geometry. (right) Pressure at 100 μm with and without focus when array is excited with a short pulse

Fig. 17.

Block diagram describing the modeling process flow and design feedback parameters that can be modified at each stage of the modeling.

Table I
Material Properties Used in Simulations

Property	PECVD SiN _x	Al
Density, kg/m ³	2040	2700
Poisson's ratio	0.22	0.35
Young's modulus, GPa	110	70

Table II
Simulation Results Comparing FEM, Hybrid Method and Finite Difference Method

Aspect Ratio	Center Frequency (MHz)		
	FEM	Hybrid	Finite Difference
20	15	15	15
15	27	27	27
10	57	58	59
5	190	193	237

Table III
Simulated Membrane Dimensions and Materials

Geometry	L × W × H, μm	Material
Base Membrane	$20 \times 20 \times 1$	SiN
Top Mass	$15 \times 15 \times 2$	SiN
Buried Electrode	$14 \times 14 \times 1$	Al

Table IV
Required Delays for Focus at [0,0,100 μm]

Patch	Delay
1a, 1b	~26 ns
2a, 2b, 3a, 3b	~12 ns
4a, 4b	0 ns