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Stages: Sub-Fourier Dynamic Shim Updating Using Nonlinear Magnetic Field Phase Preparation

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Abstract

Heterogeneity of the static magnetic field in magnetic resonance imaging may cause image artifacts and degradation in image quality. The field heterogeneity can be reduced by dynamically adjusting shim fields or dynamic shim updating (DSU), in which magnetic field homogeneity is optimized for each tomographic slice to improve image quality. A limitation of this approach is that a new magnetic field can be applied only once for each slice, otherwise image quality would improve somewhere to its detriment elsewhere in the slice. The motivation of this work is to overcome this limitation and develop a technique using nonlinear magnetic fields to dynamically shim the static magnetic field within a single Fourier-encoded volume or slice, called sub-Fourier dynamic shim updating. However, the nonlinear magnetic fields are not used as shim fields; instead, they impart a strong spatial dependence to the acquired MR signal by nonlinear phase preparation, which may be exploited to locally improve magnetic field homogeneity during acquisition. A theoretical description of the method is detailed, simulations and a proof-of-principle experiment are performed using a magnet coil with a known field geometry. The method is shown to remove artifacts associated with magnetic field homogeneity in balanced steady-state free precession (bSSFP) pulse sequences. We anticipate that this method will be useful to improve the quality of magnetic resonance images by removing deleterious artifacts associated with a heterogeneous static magnetic field.

Introduction

Homogeneity of the main magnetic field inside of a magnetic resonance imaging (MRI) device is critical to obtain images of sufficient quality without distortions or artifacts and spectroscopic data with high spectral resolution. Nevertheless, imperfections in the coil windings introduce magnetic field heterogeneity and objects containing a heterogeneous magnetic susceptibility distribution distort the main magnetic field B_0 . Furthermore, variations in the anatomy of air cavities and biological tissues between subjects, together

with variation introduced by subject movement, respiration and other physiological motion, can cause additional spatial and temporal deterioration of the magnetic field homogeneity. To correct for errors in the field homogeneity it is customary to employ room temperature (RT) shimming. In this method, the uncorrected magnetic field distribution is first measured and a compensating magnetic field is introduced by adjusting currents in the static magnetic field B_0 shim magnet coils prior to data acquisition (1-4).

Despite the use of shim magnets, complete magnetic field homogeneity within an extended volume is a problem. Shim magnets, which produce magnetic fields consisting of low-order spherical harmonic functions, are unable to fully compensate for the complex magnetic field heterogeneities in human MRI. The addition of higher-order, shim magnets has progressively smaller improvement in the homogeneity of the magnetic field and arrives with increasing cost and difficulty of implementation (5,6). Consequently, it is usually desirable to decompose the volume of interest into a spectroscopic volume of interest (7), a set of 2D slices or separate 3D slabs (6,8,9) or, conceptually, smaller subvolumes to reduce the magnetic field heterogeneity across the total volume (10). These techniques use an approach called dynamic shim updating (DSU), in which both linear and higher order magnetic fields are adjusted dynamically throughout the MRI scan (9,11,12).

A limitation of DSU is that the magnetic fields can only be modified between Fourier-encoded volumes; for example, it is possible to use one magnetic field to reduce field heterogeneity in one slice and then use a different magnetic field to reduce heterogeneity in a second slice. Optimal field homogeneity can be achieved independently for each slice to obtain better homogeneity and image quality overall. In general, decomposing a single Fourier-encoded volume into multiple Fourier-encoded volumes and performing DSU, such as using a 2D Fourier-encoded multislice acquisition instead of a 3D Fourier-encoded acquisition (6), or hypothetically, using a multidimensional Fourier-encoded acquisition to further decompose the volume (10), can significantly improve local field homogeneity. It is not usually desirable, however, to perform DSU during the acquisition of a single Fourier-encoded volume, such as a single slice, because the entire volume contributes to the acquired signal. In this case, if the local magnetic field homogeneity were improved over part of the slice, it would degrade field homogeneity and also image quality elsewhere in the slice.

Our motivation was to overcome this limitation and permit DSU within a single Fourier-encoded volume. The method that we use is nonlinear prephasing, also called phase scrambling. In this method, an additional nonlinear phase is imparted on the MRI signals to introduce spatial dependence to the acquired k-space data (13-17). Since the k-space signal has spatial dependence, it is possible to locally change the magnetic field without affecting image quality elsewhere in the Fourier-encoded volume. In several recent studies, the homogeneity of the magnetic field was corrected within a Fourier-encoded volume at the time of excitation in EPI (18-21). The phase attributed to the magnetic field heterogeneity or background phase was corrected using an additional compensating phase during the excitation pulse. Instead of correcting the field using the RF pulse phase, we use dynamic shim fields and nonlinear gradient prephasing to correct the field heterogeneity. We seek to find what effect these methods have on the acquired spatial resolution, implement an

approach for steady-state MRI in spoiled or balanced gradient echo imaging, and explore how quickly the dynamic shim fields can be updated without perturbing image quality elsewhere.

We introduce here a technique to dynamically shim the magnetic field within a Fourier-encoded volume during steady-state MRI pulse sequences using pulsed nonlinear magnetic fields. The theoretical framework for the method is shown using the concept of local k-space and a representative nonlinear magnetic field. The effect on the reconstructed image resolution is shown. It is demonstrated that it is possible to eliminate or reduce these artifacts using dynamic shim updating in simulations and a proof-of-principle experimental study.

Theory

Local K-Space Interpretation of the MR Signal

The pulse sequence depicted in Fig. 1 is a balanced steady-state free-precession (bSSFP) sequence (22,23) with several distinguishing features. Besides the short TR and balanced magnetic field gradients, there are two additional magnetic field gradients employed for dynamic shim updating. These two gradient systems are represented by additional gradient pulses appearing in the pulse sequence diagram. The first gradient system G_{shim} consists of several B_0 shim magnet coils used to improve the homogeneity of the magnetic field. In conventional dynamic shim updating approaches, the shim magnets are only updated between consecutive spatially encoded volumes; that is, the shim is updated for each excited 3D slab or 2D slice or else the local magnetic field homogeneity would be improved only to its deterioration elsewhere. This limitation is overcome by the second gradient system G_{sem} , which consists of nonlinear spatial encoding magnetic fields (SEM) whose purpose is to impart on the magnetization a nonlinear phase prior to spatial encoding.

The reason for why the second gradient system overcomes this limitation is that it introduces considerable spatial dependence to the acquired MR signals. From another perspective, the nonlinear magnetic fields temporally reposition gradient echoes arising from different locations within the object. There are some requirements on the nonlinear magnetic field used for nonlinear phase preparation. It must be strong and/or long enough to separate gradient echoes with sufficient temporal separation within the bandwidth of the receiver. Mathematically, the z-component of the nonlinear magnetic field $B_z(\mathbf{x})$ imparts on the transverse magnetization a phase proportional to the field amplitude and duration τ ,

$$\phi(\mathbf{x}) = \gamma \int_{\tau_0}^{\tau - \tau_0} B_z(\mathbf{x}) dt. \quad [1]$$

To understand how the phase preparation affects the acquisition, it is helpful to expand the phase preparation as a series function

$$\phi(\mathbf{x}) \approx \phi(x_0) + \nabla \phi(x_0) (\mathbf{x} - x_0) + \cdots \quad [2]$$

Expanding around the origin and ignoring the higher order terms (24), we now introduce the k-space shift vector

$$\mathbf{k}_s(\mathbf{x}) := \nabla \phi(\mathbf{x}_0). \quad [3]$$

$\mathbf{k}_s(\mathbf{x})$ is called the k-space shift vector because it indicates the distance in the frequency domain that a k-space signal is moved on account of the nonlinear magnetic field. The subscript s (for shift) distinguishes it from the usual spatial encoding vectors $\mathbf{k}$; the latter are not dependent on position. Fourier encoding is performed by sampling vectors $\mathbf{k}$ in k-space using linear magnetic field gradients. The set $\mathbf{K}$ of all vectors $\mathbf{k}$ represents the k-space coverage. The set of k-space vectors

$$\mathbf{K}_l = \mathbf{k}_s + \mathbf{K} \quad [4]$$

is called *local k-space*, because of the position dependence of $\mathbf{k}_s$, where $\mathbf{K}_l$ consists of all vectors of the form $\mathbf{k}_l = \mathbf{k}_s + \mathbf{k}$. Depending on the profile of the nonlinear magnetic field, each position within the object may have a unique $\mathbf{K}_l$. For example, each position has associated with it a unique $\mathbf{K}_l$ for a quadrupolar magnetic field, but obviously does not for a linear magnetic field.

The local k-space framework is a useful mathematical and visualization tool to predict the temporal repositioning of gradient echoes arising from nonlinear phase preparation of MRI signals. Local k-space was previously introduced to design a useful encoding trajectory and its corresponding, spatially varying resolution for objects encoded using a PatLoc gradient system (25) and the determination of a particular shape used for multidimensional localization by nonlinear phase preparation (26), but parallels its meaning to analyze k-space shifts induced by local field heterogeneity (27).

Representation of the Nonlinear Magnetic Field

A quadratic (bowl-shaped) nonlinear magnetic field is shown as an example to illuminate the theory of the previous section. This field is a 2D quadratic field and resembles the Z2 gradient field used commonly for B₀ shimming.

$$B_z(x, y) = x^2 + y^2. \quad [5]$$

In practice, it is possible to construct this field with some additional variation in the z -direction. Equation [5] isolates the two-dimensional dependence of the second order spherical harmonic field Y_2^0 . The k-space signal is represented by

$$S(k_x, k_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho(x, y) e^{i[k_x x + k_y y + \phi(x, y)]} dx dy. \quad [6]$$

The usual spatial encoding vectors k_x and k_y traverse a local k-space for each position within the object. From Equation [2], the local k-space is determined by the nonlinear magnetic field and its gradient moment q ,

$$\phi(x, y) = q(x^2 + y^2). \quad [7]$$

From Equation [3], the k-space shift vector field for this nonlinear magnetic field is the gradient of the phase preparation function

$$\mathbf{k}_s(x, y) = 2q(x\hat{x} + y\hat{y}). \quad [8]$$

The dependence of the acquired k-space signal on the gradient moment q has been previously described (13,14,16,26). Conceptually, these ideas are depicted in Fig. 2 and illustrate that the nonlinear magnetic field introduces a spatial dependence to the k-space signal. For the purposes of dynamic shimming, there is a quadrupolar gradient moment q at which the k-space signals from different positions are maximally separated, but which does not cause significant attenuation of the k-space signal. This situation is depicted in Fig. 2C in the third image from the left. This particular q is determined by shifting the k-space signals originating from the edge of the field of view to the maximally encoded spatial

frequency in each direction, by choosing $x = \frac{FOV_x}{2}$ and $y = \frac{FOV_y}{2}$ in Eq. [6].

$$\mathbf{k}_s\left(\frac{FOV_x}{2}, \frac{FOV_y}{2}\right) = 2q(FOV_x\hat{x} + FOV_y\hat{y}) = k_{x,max}\hat{x} + k_{y,max}\hat{y} \quad [9]$$

Reconstruction of the spin density from the acquired MR signals is performed by discrete Fourier transformation

$$\hat{\rho}(x, y) = \sum_{-k_{y,max}}^{k_{y,max} - \Delta k_y} \sum_{-k_{x,max}}^{k_{x,max} - \Delta k_x} S(k_x, k_y) e^{ik_x x + k_y y} \quad [10]$$

The resulting image approximates the true spin density, but with an additional phase modulation associated with the nonlinear magnetic field,

$$\hat{\rho}(\mathbf{x}) \approx \rho(\mathbf{x}) e^{iq(x^2 + y^2)}. \quad [11]$$

Steady-State Dynamic Shim Updating

bSSFP is a fast imaging sequence in which the gradient-induced signal phase is zero between RF pulses (zero gradient moment) (23). This results in high SNR efficiency and useful fluid-totissue contrast; however, heterogeneity of the magnetic field introduces artifacts in balanced steady-state MRI sequences. These artifacts deteriorate image quality by sharply modulating the observed spin density $\hat{\rho}(\mathbf{x})$. Steady-state artifacts appear whenever the product of the magnetic field heterogeneity $\omega_0(\mathbf{x})$ and the repetition time TR is an integral multiple of 2π . In particular, for a phase alternating, bSSFP sequence this criteria is

$$\Delta\omega_0 TR = \dots, -3\pi, -\pi, \pi, 3\pi, \dots \quad [12]$$

Qualitatively, the RF pulses drive the magnetization to an oscillating steady-state in which the MR signal is minimized.

Dynamic shim updating is utilized to homogenize the magnetic field such that all values of the frequency-response $m(\mathbf{x})$ lie within the range $[-\pi, \pi]$ and no artifacts occur. After incorporating contributions from the frequency response function $m(\mathbf{x})$, the magnetic field heterogeneity $\omega_0(\mathbf{x})$, and the time-dependent magnetic field produced by the dynamic shim magnet coils $\omega_k(\mathbf{x})$, the MR signal is

$$S(k_x, k_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho m e^{i[k_x x + k_y y - \Delta\omega_0 t + \Delta\omega_k t + qx^2 + qy^2]} dx dy. \quad [13]$$

The dynamic B_0 shim magnetic coils produce a magnetic field that is a linear combination of spherical harmonic magnetic fields

$$\Delta\omega_k(\mathbf{x}) = \gamma B_{l,m} = \gamma \Sigma_{l,m} Y_l^m(\theta, \varphi). \quad [14]$$

The dynamic shim magnet coils affect the acquired signal $S(k_x, k_y)$ in two ways. First, they modulate the frequency-response function $m(\mathbf{x})$. Second, they compensate for the additional phase $\omega_0(\mathbf{x})t$ accumulated during spatial encoding. This second case is particularly important if the magnetic field heterogeneity is large relative to the acquisition time in which case it will cause significant distortion of the reconstructed image such as in EPI.

The method used to correct the magnetic field heterogeneity is summarized in Fig. 3. In this method, k-space is traversed in a Cartesian fashion and the shim magnet coils are adjusted for each phase-encoded line. The currents delivered to the shim magnet coils are determined by minimizing the magnetic field heterogeneity within a subvolume V_{ky} surrounding the position y for which the echo is formed at spatial frequency k_y . Signal data was simulated using Eq. [13] and

$$\phi = -\Delta\omega_0 t + \Delta\omega_k t = 0. \quad [15]$$

Eq. [15] is valid for small t , which holds for short TE bSSFP acquisitions. A quadratic magnetic field profile was used for simulation

$$\Delta\omega_0(x, y) = Ay^2. \quad [16]$$

The magnetic field was optimized using the vendor supplied B_0 shim coils and workflow. Pulse sequence parameters and relaxation times were chosen as described in the section Methods: **1D Simulations**. Magnitude spoiled gradient echo images from the magnetic field map were used for the spin density. We used the steady-state solutions to the Bloch equations to calculate the frequency response m (see Appendix A). Only linear magnetic field gradients were used for magnetic field correction.

Dynamic B_0 shimming is realized by ensuring that the field is relatively homogeneous when the k-space data corresponding to a specific location in the object are scanned. In general, for dynamic shim updating in multiple dimensions, the necessary and sufficient condition of echo formation is that the zero frequency $\mathbf{K}_l = 0$ is encoded during the acquisition. This is the case whenever

$$\mathbf{k}_s = -\mathbf{k}. \quad [17]$$

Since the magnetic field is homogenized for each position within the object, it is possible in principle to homogenize the entire magnetic field within the excited volume.

There are some limitations to this approach. The adjustment of the shim magnet coils does reduce the image quality somewhat. The reduction in the image quality is associated with the effect of a time-varying frequency-response function on the point-spread function. Furthermore, there is some loss of signal-to-noise ratio and additional truncation artifacts that are introduced by the nonlinear magnetic field, especially at positions near the edge of the field-of-view (26). Hardware limits on the gradient strength, slew rate and eddy currents may further reduce the effectiveness of this approach.

Resolution

Adjustment of the B0 shim coils throughout the MRI scan causes the frequency-response function m to be time dependent. This will have some effect on the reconstructed image resolution. This principle can be examined by analyzing the point spread function associated with a particular set of shim coil adjustments. To simplify the discussion, we assume that the signal is encoded in the steady-state. Otherwise, following a change in the resonance frequency between two consecutive phase encoding steps (i.e. RF pulses),

$$\Delta\omega_{k_y}(y) \rightarrow \Delta\omega_{k_y+\Delta k}(y) \quad [18]$$

there will be some number of RF pulses for the magnetization to reach a new steady-state. If the adjustment of the magnetic field is small then the perturbation of the steady-state magnetization is also small. If, however, the adjustment of the magnetic field is large, there may be a large number of RF pulses where the magnetization response fluctuates greatly before returning to a new steady-state. In general, the transient signal of a bSSFP sequence will decay to a new steady-state, but this decay rate may be long compared to the imaging time and reducing transients is an important problem (28). To avoid this situation in practice, it is possible to apply a sufficient number of RF pulses between consecutive phase-encoding steps k_y during which the dynamic shim adjustment is fixed.

For each position y_0 , a point spread function $\hat{\rho}(y)$ can be calculated by approximating the spin density as a delta function $\rho(y) = \delta(y - y_0)$ and employing Eq. [6].

$$\hat{\rho}(y) = \sum_{-k_{y,max}}^{k_{y,max}-\Delta k_y} S(k_y) e^{ik_y y} dk_y \quad [19]$$

$$= e^{i[k_y y_0 + q y_0^2]} \sum_{-k_{y,max}}^{k_{y,max}-\Delta k_y} m(k_y) e^{ik_y y} dk_y \quad [20]$$

$$\hat{\rho}(y) = e^{i[k_y y_0 + q y_0^2]} \mathcal{F}^{-1}[m(k_y)] \quad [21]$$

Aside from the additional position-dependent phase, the key feature of Eq. [21] is that the steady-state artifacts, which were before associated with image space, are now in the frequency domain. Consequently these artifacts result in a loss of resolution, which depends on the location y_0 .

Naturally, by waiting for the magnetization to reach a steady-state between subsequent phase encoding steps the scan time may be significantly prolonged. However, by ensuring that there are smooth adjustments of the dynamic magnetic field, it is possible to avoid significant perturbations of the magnetization response.

Methods

Dynamic Shim Coil Adjustment Algorithm

A sliding window algorithm was devised to calculate the shim coil adjustment between consecutive phase encoded lines. The sliding window algorithm consisted of an optimization to determine the frequency of the RF pulse and receiver (carrier frequency) and the currents delivered to the B_0 shim coils used for dynamic shimming (linear magnetic field gradients). The result of this optimization is a coefficient matrix β_k . These coefficients are related to the dynamic shim currents by the coil resistance and determined using a linear least-squares approach. The coefficient matrix β_k is calculated as follows: first, an $N \times 3$ matrix X containing the positions of each voxel $\mathbf{r}$, e.g. in 2-dimension $\mathbf{r} = [x, y]$, where $\mathbf{r} \in V_k$, and V_k is the volume to be shimmed, is constructed, so that

$$X = \begin{pmatrix} 1 & x_1 & y_1 \\ \vdots & \vdots & \vdots \\ 1 & x_n & y_n \end{pmatrix}. \quad [22]$$

Then, the uncorrected magnetic field profile is formed by a $N \times 1$ matrix y and the chosen frequencies $\omega(\mathbf{r})$,

$$y = \begin{pmatrix} \Delta\omega_1 \\ \vdots \\ \Delta\omega_N \end{pmatrix}. \quad [23]$$

Finally, the least squares solution β'_k is found by

$$\beta'_k = \min_{\beta_k} \|y - X\beta_k\|_2^2 \quad [24]$$

subject to two constraints

$$\gamma \frac{d\beta'_{k_y}}{dk_y} < \alpha_1 \left(x = \frac{L}{2} \right), \quad [25]$$

$$\gamma \left| \beta'_{ky} \right| < \alpha_2. \quad [26]$$

Here, $\alpha_{1,2}$ are constraints on the total magnetic field amplitude and time rate of change of the magnetic field amplitude. These values are defined at the location where the magnetic field change is the largest, which for an MRI scan obtained at isocenter is the edge of the field-of-view. Typical values for $\alpha_{1,2}$ were 300 Hz and 20 Hz. The current magnetic field adjustment was used to constrain the subsequent magnetic field adjustment in an iterative fashion.

MRI

Experiments were performed on a 3 T MRI device (TIM Trio Model, Siemens Healthcare, Erlangen Germany) equipped with an insert comprised of two custom quadrupolar magnetic fields (a PatLoc system) (29-31). The equipped linear (imaging) magnetic field gradients had a nominal 40 mT/m gradient amplitude and resolution of less than 10 μ T/m. Each PatLoc magnet produced a magnetic field of 2.5 mT/m²/A with a maximum rise time 400 A/s and maximum current of 80 A. The PatLoc coils each produced a uniform (z -independent) quadrupolar magnetic field over 20 cm. Eddy-current compensation was performed in the PatLoc coils by preemphasis adjustment following estimation of the induced eddy-currents (32). No cross-term eddy-current correction was performed for the PatLoc system. A birdcage RF transmit and 8-channel receive coils were positioned inside the PatLoc coil for signal transmission and reception. A cylindrical container filled with saline, 0.1 mM MnCl₂ and plexiglass cylinders to provide features were used for imaging experiments. A linear magnetic field heterogeneity was produced by offsetting the linear shim coil in the phase-encoding direction.

To validate the approach, we constructed a distortion magnetic field, corrected it using sub-Fourier dynamic shimming, and compared this result with a globally optimal shim field up to second-order. A distortion magnet coil was constructed using copper magnet wire on a wood former. The magnetic field produced by the coil distorted the magnetic field in the phantom with quadratic variation in the anterior/posterior dimension (physical y -axis of the magnet). This field was corrected using sub-Fourier dynamic shim correction in the phase-encoding dimension with only zero and first order dynamic shim coils (linear coils) whose amplitude was found by Eq. [24]. This experiment was limited to 2D because of the PatLoc coil geometry.

Calibration of the Nonlinear Magnetic Fields and Magnetic Field Mapping

An imaging pulse sequence was developed to map the relevant magnetic fields. A 3D multiecho, multiple contrast sequence was used with 8 echoes in succession. The first four echoes were acquired without the nonlinear magnetic fields. The latter four echoes were acquired with small nonlinear gradient pulses delivered in the interecho period to impart an additional phase. B_0 maps were computed from the first four echoes and phase images were phase unwrapped along the temporal dimension. A field map was computed by L^2 norm minimization (4). The accumulated static field (background) phase was subtracted from the

latter four echoes to isolate the phase accumulation induced by the pulsed nonlinear field during the interecho period

$$\phi_q = \phi - X\beta', \quad [27]$$

The field profiles of the two nonlinear fields were then solved by L^2 norm minimization of the corrected phase images.

Pulse Sequence

A custom, phase alternated, bSSFP sequence (Fig. 1) was implemented with the following imaging parameters: matrix 256×256 , FOV = $210 \text{ mm} \times 210 \text{ mm}$, TE/TR = 5/10 ms, flip angle = 70 degrees. The duration of the pulsed nonlinear field was 600 μs . 2D Tukey window filtering ($\alpha = 0.05$) of the acquired k-space data was applied to suppress Gibb's artifacts during reconstruction. The TR was moderately longer than a clinical bSSFP pulse sequence to protect the custom-built nonlinear fields from damage.

1D Simulations

1D simulations were performed to empirically measure the performance of the proposed method to reduce balanced SSFP banding artifacts (Matlab, Natick, MA, USA). Linear, quadratic and cubic field heterogeneity 1D profiles were simulated,

$$\Delta\omega_0(y) = Ay; \quad Ay^2; \quad Ay^3. \quad [28]$$

For field heterogeneity of order y^N , shim coils of order $y^{0 \dots N-1}$ were utilized for dynamic shim. The amplitude A was chosen to create a 1 kHz frequency range across the field-of-view. Pulse sequence parameters were chosen to be the same as those used in the section **Pulse Sequence**. Relaxation times $T_2 = 100 \text{ ms}$ and $T_1 = 1000 \text{ ms}$ were chosen to be similar to gray matter tissue at 1.5 T. The spin density profile was chosen to be a 1D Tukey window ($\alpha = 0.35$) to minimize Gibbs' artifacts resulting from Fourier transformation. Steady-state solutions to the Bloch equations were used to calculate the frequency response m^{28} . Signal data was simulated using the 1D analogue of Eq. [13].

Results

1D Simulations

The results of the 1D simulations are shown in Fig. 4. The spin density profile, a flat surface with tapered edges, used for the simulation is shown on the left. The reconstructed object obtained using the balanced SSFP sequence with several types of field heterogeneity and dynamic shim correction is shown on the right. The reconstructed object obtained without dynamic shim correction shows characteristic banding artifacts, which distort the flat spin density profile. Adjustment of the carrier frequency (zero order shim) could completely correct banding artifacts resulting from a linear magnetic field range $\omega_0 = [-500 \text{ } 500] \text{ Hz}$. This correction was robust, but incomplete for quadratic and cubic magnetic field profiles. Further improvement of the reconstructed spin density profile was possible with first and second order B_0 shim coils.

MRI Experiments

Fig. 5 shows the results of an experiment to correct for a known magnetic field heterogeneity using DSU and nonlinear phase preparation in a phantom. Steady-state artifacts are a result of heterogeneity in the magnetic field during RF pulse excitation, but a consequence is that changes in the magnetic field between RF pulses have no effect on the steady-state artifacts during that same period. Hence, dynamic shim updates were only performed between phase-encoding steps. To show that artifacts in the phase-encoding direction could be corrected, we designed a distortion coil with heterogeneity predominantly in the phase-encoding direction. The z-component of the magnetic field generated by the distortion coil is shown in Fig. 5A. The magnetic field varied from approximately 0 Hz at the vertex of the field to 120 Hz at the edge of the field-of-view 12 cm away. The shape of the field was nearly quadratic, so that the field gradient rapidly increased in the same direction from the vertex.

Using Eqs. [24-26], we determined the frequency shift (Hz) and linear field gradient (mT/m), which would correct for the local field heterogeneity in Fig. 5A. The trajectory of these two parameters for each phase encoding step is shown in Fig. 5B. We compared the results of Bloch simulations (Eq. [A10]) to the experimental results in Fig. 5C. We found excellent agreement between the images obtained in the uncorrected (Fig. 5C, left column) and corrected (Fig. 5C, right column) simulations and experiments. One noticeable difference between the simulation and experiment was a reduction in resolution and shading towards the bottom periphery of the phantom. We attribute these differences to the transient response of the magnetization during the bSSFP sequence and the size of the transients, which are determined by relaxation times and pulse sequence parameters such as steady-state catalyzing RF pulses and TR. In these experiments, steady-state catalyzing pulses were used prior to phase-encoding, but not between adjacent phase-encoding lines. We also compared the performance of the method with a globally optimal shim obtained using the linear and quadratic magnetic fields equipped on the scanner. We found that it was not possible to fully correct for the distortion field using the global shim as shown by the residual steady-state artifacts appearing at the bottom of the phantom (Fig. 5D), but these artifacts were eliminated using the DSU and nonlinear prephasing approach. The magnitude k-space data for the two images in Fig. 5D is shown in Fig. 5E, demonstrating that nonlinear prephasing introduced significant spatial dependence to the acquired signal. The magnitude k-space data in Fig. 5E very nearly resembles the object.

Discussion

The goal of this work was to demonstrate the feasibility of a method to correct MRI artifacts within a Fourier-encoded volume using nonlinear prephasing and dynamic adjustment of the B_0 shim coils. Two high performance quadrupolar magnet coils were used to produce a nonlinear phase, which introduced a strong spatial dependence to the detected MRI k-space signal. This enabled dynamic shim adjustment within a Fourier-encoded volume. This approach overcomes a limitation of conventional dynamic shim updating, which requires that magnetic field adjustment only be adjusted between consecutive spatial encodings. For example, this method allows for magnetic field adjustments within a single Fourier-encoded

volume such as a 2D slice or 3D slab. For MRI systems that use high performance magnet coils of at least second order, these approaches can be readily adapted. Otherwise, it is possible to produce a similar effect using tailored RF pulses (13,20).

There are some limits to this approach. It was shown theoretically that the resolution of the reconstructed image depends on the time-dependence of the dynamic shim. Therefore any potential improvement in magnetic field homogeneity may be compromised by a reduction in resolution. This effect is concisely described by Eq. [21], which relates the image resolution to the steady-state frequency-response function via the Fourier transform. Certainly the temporal separation of k-space signals arising from different locations enables these locations to be shimmed independently. However, these signals overlap considerably, therefore shimming of one position within the object should not be performed without consideration of the effect on higher spatial frequencies elsewhere in the object.

In steady-state sequences, the reduction in resolution originating from the time-dependence of the frequency-response function m is a significant concern. It was necessary to perform constrained least-squares minimization of the magnetic field over the subvolume to maintain a tolerable point spread function. Two constraints were the time rate of change of the dynamic shim magnetic field and its maximum amplitude. Constraining the maximum shim amplitude is used to maintain values within their hardware limits. Constraining the time rate of change of the dynamic shim magnetic field was necessary to ensure that the frequency response function m was smoothly varying so as not to perturb the steady-state. In addition, the minimum gradient raster time limits the rate at which the dynamic shim field can be adjusted in the frequency encoding direction, but is not a limitation in the phase-encoding direction since the gradient raster time is usually much less than the TR (e.g. bSSFP) or interecho spacing (e.g. EPI).

There are some other limitations to this approach. Artifacts appear at positions whose local k-space coverage has been truncated during acquisition. Mathematically, this situation occurs for any $\mathbf{k}_s(\mathbf{x}) \notin -\mathbf{K}$, i.e. the gradient echo arising from a particular position in the object is not sampled. For those positions whose echoes lie near the boundary of the k-space coverage $\mathbf{k}_s = -\mathbf{K}$, significant Gibb's ringing may occur. To ameliorate this artifact, 2D Tukey windowing of the acquired k-space data was performed albeit with some small reduction in resolution.

Eddy currents have a significant effect on the temporal characteristics of the magnetic field homogeneity. In particular, dynamic shim updating can have a deleterious effect on the magnetic field homogeneity if eddy currents are not fully eliminated (6). Furthermore, the resulting magnetic field homogeneity can be much worse than in the absence of dynamic shim adjustment. In this study, only eddy-currents of the same order were compensated by preemphasis of the second order magnet coils. Therefore, it is expected that the magnet coils produce residual low- and high-order cross-term eddy currents. Nevertheless, the coils were rapidly switched for a period of less than 4 ms thereby producing eddy-currents of opposite polarity and similar magnitude, substantially reducing its effect on the magnetic field homogeneity.

Fig. 5 demonstrates that a significant reduction in steady-state artifacts can be obtained under controlled conditions with a known distortion magnetic field. Further work must be done to improve this method for *in vivo* applications where the magnetic field heterogeneity may be much more complex and vary in three-dimensions. These techniques may be very useful to reduce artifacts originating from magnetic field heterogeneity in the prefrontal cortex in echo planar imaging applications. This method may also have value for cardiac applications, which routinely using bSSFP methods in complex heterogeneous fields at the interface between the myocardium and lung cavity. It should be mentioned that there are several existing approaches to mitigate artifacts in bSSFP, such as multiphase bSSFP acquisitions (33) or manipulation of the frequency response function with an alternating TR approach (34), among others, and it may be useful to combine this approach with others to maximally suppress artifacts.

In conclusion, we have shown theoretically and in experiments that it is possible to correct artifacts associated with a heterogeneous magnetic field within a single Fourier-encoded volume using a combination of nonlinear phase preparation and dynamic shim adjustment. This technique has the potential to improve image quality for a variety of medical imaging applications.

Appendix A

We derive here, as a reference, the bSSFP steady-state signal frequency response function and signal equation. Assuming instantaneous nutation of the magnetization by the pulse, which can be assumed if the applied RF field amplitude is much larger than the resonance frequency of the spin at the n th RF pulse

$$\omega_1 \gg \Delta\omega_n(\mathbf{x}) \quad [\text{A1}]$$

The steady-state of a balanced SSFP sequence can be described by spatially varying relaxation time constants

$$E_1 = e^{-\frac{T}{T_1}} \quad [\text{A2}]$$

$$E_2 = e^{-\frac{T}{T_2}} \quad [\text{A3}]$$

flip angle θ and phase accumulation per TR

$$\phi_n(\mathbf{x}) = \Delta\omega_n(\mathbf{x}) TR. \quad [\text{A4}]$$

Under these conditions, the steady-state magnetization following the n th RF pulse is

$$d_n(\mathbf{x}) = (1 - E_1 \cos \theta) (1 - E_2 \cos \phi) - E_2 (E_1 - \cos \theta) (E_2 - \cos \phi), \quad [\text{A5}]$$

$$m_{x,n}(\mathbf{x}) = \frac{(1 - E_1) E_2 \sin \theta \sin \phi}{d_n} \quad [\text{A6}]$$

$$m_{y,n}(\mathbf{x}) = \frac{(1 - E_1) \sin \theta (1 - E_2 \cos \phi) \cos \theta}{d_n} \quad [A7]$$

$$m_{z,n}(\mathbf{x}) = \frac{E_2 (1 - E_1) (E_2 - \cos \phi) + (1 - E_2 \cos \phi) \cos \theta}{d_n} \quad [A8]$$

The SSFP response is the transverse component

$$m = m_{x,n} + i m_{y,n}. \quad [A9]$$

The MR signal is found by combining Eq. [13] and Eq. [A9], for small t ,

$$S(k_x, k_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho m e^{i[k_x x + k_y y + q x^2 + q y^2]} dx dy. \quad [A10]$$

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Figure 1.

Three kernels of a phase alternating bSSFP pulse sequence utilizing both dynamic shim updating and nonlinear phase preparation are shown. In addition to the usual balanced frequency encoding (*fe*) and phase encoding (*pe*) magnetic fields, there are balanced spatial encoding magnetic fields (*sem*) to produce a nonlinear phase and dynamic shim magnet coils (*shim*) to homogenize the magnetic field. Spatial encoding occurs each TR and constraints are placed on how rapidly the shim magnetic field may change between subsequent excitations to prevent significant perturbation of the steady-state. Notice that the dynamic B_0 shim coils can be discretely modulated each phase encoding step or continuously throughout frequency encoding. The detected signal shown during the period of frequency encoding shows quadratic phase modulation as a result of the nonlinear phase preparation.

Figure 2.

Effect of nonlinear magnetic field prephasing on the measured k-space signals (i.e. Z2 at isocenter, $2z^2 - x^2 - y^2$, where $z = 0$). (A) The relationship of the nonlinear magnetic field to the object. The nonlinear field (G_{SEM}) is applied prior to Cartesian k-space sampling as suggested in Fig. 1. (B) A depiction of the k-space shift vector field associated with the quadrupolar field. The length of the k-space shift vector is proportional to the nonlinear phase gradient. See, for example, Eqs. [3] and [8] in the text. (C) Increasing the nonlinear magnetic field moments q increases the relative k-space shift. We obtained these results by discretizing Eqs. [6] and [7] and computing the k-space signals from an oversampled image space grid. A small moment is insufficient to temporally separate gradient echoes arising from different positions. A large moment results in substantial outer volume suppression of the reconstructed image. The ideal gradient moment is one that maximally separates gradient

echoes, but does not cause significant Gibb's ringing $q = \frac{k_{max}}{L}$, where L is the size of the field-of-view.

Figure 3.

2D simulations demonstrating artifacts caused by a quadratic distortion field and correction by nonlinear prephasing and dynamic shim updating. (A) The spin density is represented in Eq. [A10] by a dense grid. (B) The distortion field (Eq. [28]) and spin density were used to compute the k-space signal (Eq. [A10]) and reconstructed image. The distortion field introduced bSSFP frequency response artifacts characterized by solid bands. (C) The calculated zero and first order shim coefficients used for the dynamic shim, computed from Eq. [24]. (D) Two phase encoded k-space lines (white dotted lines) and the corrected field applied to each of these lines. The vertex of the quadratic field is centered around the scanned k-space line. (E) The corrected k-space signal data. Notice that the artifacts visible in the k-space data in (B) and (D) are removed. (F) The final corrected image is free of visible artifacts.

Figure 4.

Results of a simulation to reconstruct a Tukey-window shaped spin density to demonstrate the improvement of the 1D frequency response profile m with the proposed method. The 1D frequency response profile was simulated for linear, quadratic and cubic magnetic fields heterogeneities without any correction and with DSU using zero, first and second order magnetic field correction. Shim order correction was up to N-1 the magnetic field heterogeneity order, since the same order would perfectly correct the frequency response profile.

Figure 5.

Correction for a distortion magnetic field using nonlinear phase preparation and dynamic shim updating. (A) The distortion field computed from the MR-derived magnetic field map. (B) Zero and first-order dynamic shim adjustments used to correct the field in (A). The shim is adjusted for each phase-encoded line. (C) Simulated (top row) and experimental images (bottom row) with and without dynamic shim updating. The bSSFP artifacts are completely corrected in the experimental data, and significantly reduced in the experimental data. Differences between the data are because of the dynamic frequency response function, which is not modeled by Eq. [A10]. (D) Comparison of a second order, globally optimal shim to first order, dynamic shim adjustment. Artifacts appear at the periphery of the phantom using a global shim, but are removed using the dynamic shim approach. (E) K-space data obtained using the globally optimal shim (log (S)) and dynamic shim (S).