An Analytical Approach for Synthesizing Line Actuation Spaces of Parallel Flexure Mechanisms

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In this study, we present an analytical approach for synthesizing line actuation spaces of a parallel flexure mechanism (PFM) that can help designers to arrange linear actuators within the PFM in a correct and optimal way. On the basis of screw theory and upon an assumption of small deformations, an important synthesis criterion stated as "any actuation space of a flexure mechanism is always linearly independent of its constraint space" has been derived and disclosed for the first time. Guided by this criterion, a general synthesis process for the line actuation spaces of PFMs is introduced and demonstrated with several selective examples. The proposed synthesis criterion and process will enable designers to (i) systematically formulate line actuation spaces in the format of screw systems; (ii) likely yield a multiple solution to actuation spaces; and (iii) potentially determine an optimal result from those alternatives for actuator placement.

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1 Introduction

A PFM is defined as a flexure system [1] with a rigid moving platform directly connected to the base by at least two flexure elements or limbs. Compared with their serial counterparts, PFMs have potentially more favorable performance, such as higher stiffness, high natural frequencies, and higher precision; they have therefore been extensively used in applications such as ultra-precision motion stages [2], precision robotics [3], and micro-electro-mechanical systems fabrication [4].

Despite numerous designs that exist in the literature [2–6], type synthesis is regarded as an effective tool to find multiple solutions in the conceptual design phase of PFMs. Currently, systematic approaches for achieving a comprehensive type synthesis including the rigid-body replacement approach [7], building block synthesis [8], constraint-based design [9,10], freedom and constraint topology approach [11,12], and screw algebra method [13]. A large variety of known and unknown PFMs generating a specified motion pattern have been developed. However, little work has been devoted to actuator placement for these architectures.

In fact, placing actuators is considered an important aspect in the type synthesis of parallel mechanisms [14] including PFMs. Improper actuator placement in a PFM may cause additional errors such as parasitic errors [15] and even lead to actuation invalidity or redundancy. Hopkins and Culpepper [15] were perhaps the first to introduce actuation spaces to study actuation selection and layout of flexure systems. In Ref. [15], an actuation space of a flexure system is depicted as a geometric shape that represents the actuator locations and orientations, and actuation spaces vary with different flexure systems. As such, a total of 26 actuation spaces are enumerated, and an optimal actuator placement related to each space that can minimize the error motions of the moving platforms is also provided to match different freedom and constraint spaces. A twist-wrench stiffness matrix is introduced to both qualitatively and quantitatively analyze the error motions in the optimization process. However, designers have to input complex information covering all structural and size parameters of a PFM in order to calculate the twist-wrench stiffness matrix. It is worth mentioning that in the early phase of design, such an optimization process may cause the synthesis problem to become complicated.

Therefore, a simple analytical approach is introduced in this study for synthesizing line actuation spaces of PFMs with a specified motion pattern with an aim to helping designers form an initial concept of correctly and quickly placing actuators without the need of complex calculations. The mathematical tool used is screw theory [16]. An important synthesis criterion regarding actuation spaces, i.e., any actuation space of a flexure system is always linearly independent of its constraint space, is derived using screw theory. On the basis of this criterion and the following synthesis approach, suitable actuation spaces corresponding to representative PFMs can be obtained. For simplification purpose, those parallel flexure systems with orthogonal motions along the coordinate axes of the functional stage are selected as examples, resembling the type synthesis of orthogonal motions accomplished by Su and Tari [17,18]. The generation and application of the synthesis criterion for actuation spaces is the major contribution of this paper.

2 Actuation Space

In this section, several fundamental elements within a flexure system, including a deformation twist [19], a constraint wrench, and an actuation force, are all defined within the framework of screw theory.

DEFINITION 1. Deformation Twist and freedom Space

It is assumed that a deflection occurring in a flexure system is sufficiently small to be able to be represented by a deformation twist $\hat{T}$, which is written as

$$\hat{T}_i = (\gamma_i | \delta_i) = (\theta_{ix} \ \theta_{iy} \ \theta_{iz} | \delta_{ix} \ \delta_{iy} \ \delta_{iz}) \quad (1)$$

Correspondingly, an n -dimensional freedom space Π_T , reflecting a specified motion pattern, can be spanned by n independent deformation twists, i.e.,

$$\Pi_T = [\hat{T}_1^T \ \dots \ \hat{T}_i^T \ \dots \ \hat{T}_n^T]^T \quad i = 1, 2, \dots, n \quad (2)$$

DEFINITION 2. Line Constraint Wrench and Line Constraint Space

According to line screw theory [16], a line constraint wrench $\hat{W}$ is mathematically represented by a pure force $\hat{F}$, written as

$$\hat{W} = \hat{F} = (\mathbf{F} | \mathbf{M}) = (F_x \ F_y \ F_z | M_x \ M_y \ M_z) \quad (3)$$

where

$$F_x M_x + F_y M_y + F_z M_z = 0, \quad F_x^2 + F_y^2 + F_z^2 \neq 0$$

Therefore, an m -dimensional line constraint space is indeed spanned by m independent line constraint wrenches, i.e.,

$${}^l\Pi_W = [\hat{F}_1^T \ \dots \ \hat{F}_j^T \ \dots \ \hat{F}_m^T]^T \quad j = 1, 2, \dots, m \quad (4)$$

DEFINITION 3. Actuation Force and Line Actuation Space

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Generally, an actuator can also be represented by a screw. As a special case, a piezoelectric actuator (such as PZT) works by pushing or pulling along its axis; it can therefore be regarded as an actuation force, which is essentially a line actuation wrench, written as

$$\hat{A} = (\mathbf{F}_A | \mathbf{M}_A) = (F_{Ax} \ F_{Ay} \ F_{Az} | M_{Ax} \ M_{Ay} \ M_{Az}) \quad (5)$$

where

$$F_{Ax}M_{Ax} + F_{Ay}M_{Ay} + F_{Az}M_{Az} = 0, \quad F_{Ax}^2 + F_{Ay}^2 + F_{Az}^2 \neq 0$$

As such, an r -dimensional line actuation space is spanned by r independent actuation forces, i.e.,

$${}^l\Pi_A = [\hat{A}_1^T \cdots \hat{A}_k^T \cdots \hat{A}_r^T]^T \quad k = 1, 2, \dots, r \quad (6)$$

3 Synthesis Criterion

In any flexure system, $\hat{T}$, $\hat{W}$, and $\hat{A}$ can be linked to each other via the global compliance matrix of the flexure system.

In this study, we assume that the deformation caused by flexures is sufficiently small so that the principle of linear elastic theory is applicable. On this assumption, the relationship between $\hat{T}$ and $\hat{A}$ can be established according to beam linear deformation theory, i.e.,

$$\hat{T} = \Delta \hat{C} \hat{A} \quad (7)$$

where $\mathbf{C}$ denotes the global compliance matrix of the flexure system, which is a real symmetric matrix. $\Delta = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{I} & \mathbf{0} \end{bmatrix}_{6 \times 6}$ represents an operator interchanging the first and last three components of a screw.

According to classical screw theory [18], the reciprocal relationship between $\hat{W}$ and $\hat{T}$ always satisfies

$$\hat{W} \circ \hat{T} = 0 \quad (8)$$

where the symbol $\circ$ denotes the reciprocal product of these two screws.

According to Eqs. (7) and (8), we can derive the important relationship between actuation wrenches and constraint wrenches within a flexure system, “actuation wrenches are always linearly independent of constraint wrenches,” and the proof is implemented as follows.

Suppose that actuation wrenches are linearly dependent of constraint wrenches. In this case, any actuation wrench can be represented by linear combination of a set of basis constraint wrenches, i.e.,

$$\hat{A} = \sum_{i=1}^n k_i \hat{W}_i \quad (9)$$

where at least one of k_i is nonzero.

According to Eqs. (8) and (9)

$$\hat{T} \circ \hat{A} = \sum_{i=1}^n k_i (\hat{T} \circ \hat{W}_i) = 0 \quad (10)$$

Obviously, Eq. (10) is false. For example, when an actuation force $\hat{A} = (f_x, 0, 0; 0, 0, 0)$ acts on a flexure system, the system may generate a translational twist $\hat{T} = (0, 0, 0; v_x, 0, 0)$. It is clear that $\hat{T} \circ \hat{A} = f_x v_x \neq 0$ in this case. Therefore, it can be concluded that actuation wrenches are always linearly independent of constraint wrenches.

To summarize the above formulations, the relationship between an actuation space Π_A , a freedom space Π_T , and a constraint space Π_W within a flexure system is illustrated in Fig. 1.

Fig. 1 Mapping of actuation space, freedom space, and constraint space

Most importantly, a **synthesis criterion** of actuation spaces has been disclosed, which states the following:

For a small-deformation flexure system, the allowable actuation forces are always linearly independent of its constraint wrenches.

4 A general Synthesis Process

Because linear actuators generating large pushing or pulling forces are more likely to be used for ultra-precision flexure systems, such as lead zirconium titanate, a ceramic perovskite material (PZT), we select line actuation spaces as case studies to describe a general synthesis process of actuation spaces.

Step 1: For a given motion pattern, its corresponding n -dimensional freedom space is formulated by a twist matrix, i.e.,

$$\Pi_T = [\hat{T}_1^T \ \hat{T}_2^T \ \cdots \ \hat{T}_n^T]^T \quad (11)$$

Step 2: A constraint space Π_W , reciprocal to the freedom space Π_T , is spanned by $(6-n)$ independent basis constraint wrenches, which is written as

$$\Pi_T \Delta \Pi_W^T = \mathbf{0} \quad (12)$$

$$\Pi_W = [\hat{W}_1^T \cdots \hat{W}_j^T \cdots \hat{W}_{6-n}^T]^T \quad j = 1, 2, \dots, 6-n \quad (13)$$

In general, there always exists a line constraint subspace ${}^l\Pi_W$ spanned by $(6-n)$ independent constraint forces if the existence conditions provided in reference [17] are satisfied. In this case, each constraint force can be further formulated as

$$\Pi_T \Delta {}^l\Pi_W^T = \mathbf{0} \quad (14)$$

$${}^l\Pi_W = [\hat{F}_1^T \cdots \hat{F}_j^T \cdots \hat{F}_{6-n}^T]^T \quad j = 1, 2, \dots, 6-n \quad (15)$$

$$f : \{\hat{F} = (\mathbf{F} | \mathbf{M}) | \mathbf{F} \cdot \mathbf{M} = 0, \ \mathbf{F} \neq 0, \ \hat{T}_i \circ \hat{F} = 0, \ i = 1, 2, \dots, n\} \quad (16)$$

Step 3: By calculating the null space of Π_W , actuation spaces can be obtained as

$$\Pi_A \Pi_W^T = \mathbf{0} \quad (17)$$

$$\hat{F}_j \cdot \hat{A}_k = 0, \quad j = 1, 2, \dots, 6-n; \quad k = 1, 2, \dots, r \quad (18)$$

According to linear algebra, the reciprocal product of two screws can be transformed into their dot product, and thus, we can obtain linearly independent vectors from the actuation space by swapping the rotational vector and the translational vector of these twists, i.e.,

$$\Pi_A = \Pi_T \Delta \quad (19)$$

Step 4: With the proposed **synthesis criterion** and from linear algebra, a general formula relating to actuation forces can be

Fig. 2 The flowchart of actuation space synthesis process

derived by the following equation. Correspondingly, line actuation spaces can be obtained if they exist.

$$\hat{A} = \sum_{k=1}^n \lambda_k \hat{A}_k + \sum_{j=1}^{6-n} \mu_j \hat{W}_j, \quad \lambda_k \neq 0 \quad (20)$$

Step 5: Choose a set of available and even optimal actuation forces from the result in Step 4 in terms of such practical considerations as locations and orientations of the actuator placement.

The whole synthesis process is demonstrated with a flowchart in Fig. 2.

During the synthesis process of line actuation spaces, it is required to calculate the actuation forces in terms of the **synthesis criterion**. However, while calculating these actuation forces, the

resultant actuation vectors may have multiple solutions or alternatives. All these in turn provide the optimum possibility of choosing suitable actuation forces from the actuation spaces.

5 Selective Examples

As the focus of this paper is synthesis of line actuation spaces that seeks optimum arrangement of linear actuators for a given motion pattern, we are interested in only some simple cases with a parallel connection by one or more wire flexures directly. In particular, we study the synthesis of actuation spaces within PFMs with twist axes along the coordinate axes. Without loss of generality, we provide three cases, i.e., one 1-DOF and two 2-DOF motion patterns, following the same synthesis process.

5.1 The Orthogonal Freedom Space. A six-dimensional freedom space is represented by six linearly independent twists $\hat{T}_i$, and thus, we can select a set of special cases, i.e., all twist axes are parallel to the coordinate axes and pass through the origin, i.e.,

$$\begin{cases} \hat{R}_x = (1 \ 0 \ 0 \mid 0 \ 0 \ 0) \\ \hat{R}_y = (0 \ 1 \ 0 \mid 0 \ 0 \ 0) \\ \hat{R}_z = (0 \ 0 \ 1 \mid 0 \ 0 \ 0) \\ \hat{P}_x = (0 \ 0 \ 0 \mid 1 \ 0 \ 0) \\ \hat{P}_y = (0 \ 0 \ 0 \mid 0 \ 1 \ 0) \\ \hat{P}_z = (0 \ 0 \ 0 \mid 0 \ 0 \ 1) \end{cases} \quad (21)$$

In the whole context, $\hat{R}$ and $\hat{P}$ represent a pure rotation and translation, respectively.

5.2 1-DOF Motion Pattern: $\hat{R}_x$

- (1) In this motion pattern, there exists only one independent twist within its freedom space, which is represented by

$$\hat{R}_x = [1 \ 0 \ 0 \mid 0 \ 0 \ 0] \quad (22)$$

- (2) By substituting Eq. (22) into Eq. (14), we obtain

$$\begin{cases} \hat{R}_x \circ \hat{F} = 0 \Rightarrow M_x = 0 \\ \mathbf{F} \cdot \mathbf{M} = 0 \Rightarrow F_x M_x + F_y M_y + F_z M_z = 0 \\ \mathbf{F} \neq 0 \Rightarrow F_x^2 + F_y^2 + F_z^2 \neq 0 \end{cases} \quad (23)$$

- (3) A set of independent bases is therefore derived from the above equations, which form a five-dimensional constraint space illustrated in Fig. 3(a).

Fig. 3 A flexure mechanism with motion pattern $\hat{R}_x$. (a) a line constraint space; (b) three selective actuation forces.

$$\begin{cases} F_x \neq 0, F_y = 0, F_z = 0, M_y = 0, M_z = 0 \\ F_x \neq 0, F_y = 0, F_z = 0, M_y \neq 0, M_z = 0 \\ F_x = 0, F_y \neq 0, F_z = 0, M_y = 0, M_z = 0 \\ F_x = 0, F_y \neq 0, F_z = 0, M_y = 0, M_z \neq 0 \\ F_x = 0, F_y = 0, F_z \neq 0, M_y = 0, M_z = 0 \end{cases} \quad (24)$$

$$\begin{cases} \hat{F}_1 = (1 \ 0 \ 0 \mid 0 \ 0 \ 0) \\ \hat{F}_2 = (1 \ 0 \ 0 \mid 0 \ 1 \ 0) \\ \hat{F}_3 = (0 \ 1 \ 0 \mid 0 \ 0 \ 0) \\ \hat{F}_4 = (0 \ 1 \ 0 \mid 0 \ 0 \ 1) \\ \hat{F}_5 = (0 \ 0 \ 1 \mid 0 \ 0 \ 0) \end{cases} \quad (25)$$

- (4) According to Eqs. (19) and (22), an actuation space is obtained by swapping the rotational vector and translational vector of the twist $\hat{R}_x$, and a general expression of formulating actuation forces is further derived as

$$\hat{A}_1 = [0 \ 0 \ 0 \mid 1 \ 0 \ 0] \quad (26)$$

$$\hat{A} = [k_1 + k_2 \ k_3 + k_4 \ k_5 \mid \lambda_1 \ k_2 \ k_4] \quad \lambda_1 \neq 0 \quad (27)$$

- (5) Find those available actuation forces from Eq. (27).

$$\begin{cases} k_3 \neq 0, k_1 = k_2 = k_4 = k_5 = 0 \\ k_4 \neq 0, k_1 = k_2 = k_3 = k_5 = 0 \\ k_5 \neq 0, k_1 = k_2 = k_3 = k_4 = 0 \end{cases} \quad (28)$$

$${}^I\Pi_A = \begin{cases} \hat{A}_1 = (0 \ 1 \ 0 \mid 1 \ 0 \ 0), & \text{or} \\ \hat{A}_2 = (0 \ 1 \ 0 \mid 1 \ 0 \ 1), & \text{or} \\ \hat{A}_3 = (0 \ 0 \ 1 \mid 1 \ 0 \ 0) \end{cases} \quad (29)$$

- (6) Select one suitable actuation force from ${}^I\Pi_A$, which should satisfy the **synthesis criterion** as well as other factors such as desired actuator placement.

$$\begin{cases} \hat{A}_1 \cdot \hat{F}_1 = 0, \hat{A}_1 \cdot \hat{F}_2 = 0, \hat{A}_1 \cdot \hat{F}_3 = 0, \hat{A}_1 \cdot \hat{F}_4 = 0, \hat{A}_1 \cdot \hat{F}_5 = 0 \\ \hat{A}_2 \cdot \hat{F}_1 = 0, \hat{A}_2 \cdot \hat{F}_2 = 0, \hat{A}_2 \cdot \hat{F}_3 = 0, \hat{A}_2 \cdot \hat{F}_4 = 0, \hat{A}_2 \cdot \hat{F}_5 = 0 \\ \hat{A}_3 \cdot \hat{F}_1 = 0, \hat{A}_3 \cdot \hat{F}_2 = 0, \hat{A}_3 \cdot \hat{F}_3 = 0, \hat{A}_3 \cdot \hat{F}_4 = 0, \hat{A}_3 \cdot \hat{F}_5 = 0 \end{cases} \quad (30)$$

The resultant actuation forces are illustrated in Fig. 3(b). It can be deduced that $\hat{A}_3$ is definitely the most suitable actuation force among the three alternatives because it minimizes the interference between the constraints and actuators.

Note that all constraints discussed in this study are depicted as black lines with two points at both ends, and all actuation forces are depicted as black lines with an arrow at one end.

5.3 2-DOF Motion Pattern: $\hat{R}_x \hat{R}_y$

- (1) A twist matrix containing two twists $\hat{R}_x \hat{R}_y$ is formulated as

$$\Pi_T = \begin{bmatrix} \hat{R}_x \\ \hat{R}_y \end{bmatrix} = \begin{bmatrix} 1 \ 0 \ 0 \mid 0 \ 0 \ 0 \\ 0 \ 1 \ 0 \mid 0 \ 0 \ 0 \end{bmatrix} \quad (31)$$

- (2) By substituting Eq. (31) into Eq. (14), it can be obtained a set of independent basis wrenches illustrated in Fig. 4(a).

$$\begin{cases} \hat{R}_x \circ \hat{F} = 0, \hat{R}_y \circ \hat{F} = 0 \Rightarrow M_x = M_y = 0 \\ \mathbf{F} \cdot \mathbf{M} = 0 \Rightarrow F_x M_x + F_y M_y + F_z M_z = 0 \\ \mathbf{F} \neq 0 \Rightarrow F_x^2 + F_y^2 + F_z^2 \neq 0 \end{cases} \quad (32)$$

$$\begin{cases} \hat{F}_1 = (1 \ 0 \ 0 \mid 0 \ 0 \ 0) \\ \hat{F}_2 = (1 \ 0 \ 0 \mid 0 \ 0 \ 1) \\ \hat{F}_3 = (0 \ 1 \ 0 \mid 0 \ 0 \ 0) \\ \hat{F}_4 = (0 \ 0 \ 1 \mid 0 \ 0 \ 0) \end{cases} \quad (33)$$

- (3) By substituting Eq. (31) into Eq. (19), an actuation space consisting of two actuation couples can be obtained

$$\Pi_A = \begin{cases} \hat{A}_1 = (0 \ 0 \ 0 \mid 1 \ 0 \ 0) \\ \hat{A}_2 = (0 \ 0 \ 0 \mid 0 \ 1 \ 0) \end{cases} \quad (34)$$

- (4) According to Eq. (20), a set of actuation forces can be further derived, which may span a suitable line actuation space, as illustrated in Fig. 4(b).

$${}^I\Pi_A = \begin{cases} \hat{A}_1 = (010 \mid 100) \\ \hat{A}_2 = (100 \mid 010) \end{cases} \quad (35)$$

Fig. 4 A flexure mechanism with motion pattern $\hat{R}_x \hat{R}_y$. (a) a line constraint space; (b) a suitable line actuation space.

Fig. 5 A flexure mechanism with motion pattern $\hat{P}_x \hat{P}_y$ (a) a line constraint subspace; (b) a suitable line actuation space

5.4 2-DOF Motion Pattern: $\hat{P}_x \hat{P}_y$

(1) A twist matrix containing two twists $\hat{P}_x \hat{P}_y$ is written as

$$\Pi_T = \begin{bmatrix} \hat{P}_x \\ \hat{P}_y \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & | & 1 & 0 & 0 \\ 0 & 0 & 0 & | & 0 & 1 & 0 \end{bmatrix} \quad (36)$$

(2) By substituting Eq. (36) into Eq. (14), three independent constraint forces can be obtained from the above equations, as illustrated in Fig. 5(a).

$$\begin{cases} \hat{F}_1 = (0 & 0 & 1 & | & 0 & 0 & 0) \\ \hat{F}_2 = (0 & 0 & 1 & | & 1 & 0 & 0) \\ \hat{F}_3 = (0 & 0 & 1 & | & 0 & 1 & 0) \end{cases} \quad (37)$$

(3) According to the **synthesis criterion**, two actuation forces are obtained by swapping the rotational vectors and translational vectors of the deformation twists, as illustrated in Fig. 5(b).

$${}^t\Pi_A = \begin{cases} \hat{A}_1 = (1 & 0 & 0 & | & 0 & 0 & 0) \\ \hat{A}_2 = (0 & 1 & 0 & | & 0 & 0 & 0) \end{cases} \quad (38)$$

6 Conclusions and Future Work

This study provides a mathematical formulation to analytically synthesize line actuation spaces of PFMs within the framework of screw theory. With the global compliance matrix of the flexure system, the mappings between its freedom space, constraint space, and actuation space are formulated and linked to each other. On the basis of the mappings, an important synthesis criterion, i.e., **“constraint wrenches are always linearly independent of actuation forces with the assumption of the small deformations,”** is derived as an effective guidance for a synthesis process of actuation spaces generating a specified motion pattern. This work not only provides designers with multiple alternatives to arrange the actuator placement for a given flexure system but it also potentially helps them to find an optimal way to place linear actuators.

Future work will also take rotary actuators into consideration to make the actuation space more abundant.

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