Nonlinear Cascade Strategy for Longitudinal Control of Electric Vehicle

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The problem of controlling the longitudinal motion of front-wheels electric vehicle (EV) is considered making the focus on the case where a single dc motor is used for both front wheels. Chassis dynamics are modelled applying relevant fundamental laws taking into account the aerodynamic effects and the road slope variation. The longitudinal slip, resulting from tire deformation, is captured through Kiencke's model. Despite its highly nonlinear nature the complete model proves to be utilizable in longitudinal control design. The control objective is to achieve a satisfactory vehicle speed regulation in acceleration/deceleration stages, despite wind speed and other parameters uncertainty. An adaptive controller is developed using the backstepping design technique. The obtained adaptive controller is shown to meet its objectives in presence of the changing aerodynamics efforts and road slope. [DOI: 10.1115/1.4024782]

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1 Introduction

In recent years, modern automatic control tools have been resorted to deal with several vehicle control problems, e.g., trajectory tracking [1], power management optimization [2], direct wheel torque control [3]. On the other hand, power propulsion systems in EVs have been given a significant attention both in the automobile industry and in academic research centers. EVs can be classified into various categories according to their configurations, functions and power sources. Pure EVs do not use petroleum, while hybrid cars take advantages of energy management between gas and electricity [3]. Owing to pure EVs there are, on one hand, indirectly driven EVs powered by electric motors through transmission and differential gears box and, on the other hand, directly driven vehicles propelled by in-wheel or wheel motors [4]. DC motors are commonly used in EVs propulsion [5]. In the present study, we are considering pure EVs in their basic configuration, i.e., single propulsion dc motor is coupled to both front wheels through one differential gear box (Fig. 1).

Presently, the focus is made on vehicle longitudinal speed control, under the assumption of equally distributed torque and friction forces between both sides. A high performance speed control strategy, in all driving conditions, is crucial for both passenger's and vehicle's safety. It is also quite useful in vehicle platooning [6], smart energy management [7], and autonomous driving [8]. As a matter of fact, a good speed control entails the reduction of energy consumption improving the vehicle drive efficiency [9].

The controller must be designed bearing in mind comfort and security requirements. More specifically, one must seek two control objectives in both acceleration/deceleration driving modes:

- (i) Tight regulation of longitudinal vehicle speed in presence of varying speed reference signal.
- (ii) Keeping tire sliding within a prescribed interval.

Most previous longitudinal controllers were based on simple models neglecting important nonlinear aspects such as rolling

resistance, aerodynamic effects, road load [4]. Ignoring these significant nonlinear effects is likely to limit the achievable performance level. In particular, the load road must be accounted for using an appropriate tire model. In this respect, several models have been proposed including Guo's model [10], Pacejka's model [11], Dugoff's model [12], Gim's model [13] and Kiencke's model [14]. In Ref. [15], Kiencke's model has been retained as it proved to be a good compromise between accuracy and simplicity. In addition to tire modeling, the vehicle model development also includes the modeling of the mechanical chassis dynamics. Chassis dynamics in both acceleration/deceleration longitudinal driving modes are described using motion fundamental laws. The chassis model developed this way, in Ref. [15], is presently completed with the model of the electric drive subsystem.

The latter includes three main components: (i) a battery of accumulators serving as embedded electric power source; (ii) a buck type dc–dc power converter operating according to the PWM (pulse width modulation) principle; (iii) a dc drive motor connected to the gear box. The obtained complete vehicle model turns out to be composed of two main blocks (Fig. 2):

- (i) The electromechanical part acted on with the control signal u of the underlying dc–dc converter; this part generates a motor torque T_m which undergoes a transformation through the gear box yielding the torque T_v .
- (ii) The pure mechanical part acted on with the torque T_v and generating the vehicle speed V_v .

Fig. 1 Physical structure of electric vehicle

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Fig. 2 Physical structure of electric vehicle

This system cascade structure motives the use of a cascade control strategy including two control loops, designed in two main steps using the backstepping approach [16]. In the first step, the outer loop involving the speed controller is designed based on the mechanical part model. During this design stage, the wheel torque T_v is viewed as a virtual control and a corresponding stabilizing control law is established. As, the torque is not the actual control input (for block 2 in Fig. 2), the corresponding stabilizing function T_v^* serves as reference trajectory for the inner regulator. In the second design step, the inner loop involving the torque regulator is designed, based on the electromechanical part of the model; the aim is to enforce suitable matching of the torque reference trajectory T_v^* by the actual wheel torque T_v . The cascade controller is provided with an adaptation capability to cope with the uncertainty that prevails on the wind speed and several other model parameters. The whole nonlinear adaptive cascade controller is formally shown to guarantee the stability and regulation purposes it is designed for. Furthermore, it is observed through numerical simulations that the controller is quite robust against the uncertainties characterizing the environmental characteristics.

The paper is organized as follows: system modeling is dealt with in Sec. 2; the cascade nonlinear adaptive controller is designed and analyzed in Sec. 3; the controller performances are illustrated by simulation in Sec. 4. A conclusion and reference list end the paper.

2 Modeling of “Chassis Vehicle—DC Motor” Association in Longitudinal Motion

2.1 Modeling of Two-Wheel Vehicle Chassis Longitudinal Motion With One Driving Wheel. Except for aerodynamic forces, all external efforts pressing on a vehicle are generated at the wheel-road contact. Understanding and modeling forces and torques developed at wheel-road contact is essential for studying properly the vehicle dynamics. First, let us note that a vehicle motion is composed of two types of displacements: translations along the x, y, z axes and rotations around these same axes. Making use of vehicle symmetry, a projection on the longitudinal axis is performed to reduce the four-wheel model to a longitudinal type model.

Figure 3 illustrates the forces involved in a longitudinal vehicle model; all notations are described in Table 1. It turns out that the vehicle longitudinal dynamics are characterized by two state variables, i.e., the vehicle speed V_v and the front-wheel speed V_w . As the slip coefficient depends on the current driving mode (acceleration or deceleration), the vehicle is characterized [15] by two state-space representations where the following notations are used:

Fig. 3 The forces acting on vehicle

T_v is the torque applied to the wheel,
 $x_1 = V_w$ is the wheel speed,
 $x_2 = V_v$ is the vehicle speed.

2.1.1 State-Space Representation in Deceleration Mode $V_w \leq V_v$. The deceleration-mode state-space representation, as established in Ref. [15] has the following form:

$$\begin{aligned}\dot{x}_1 &= \alpha_1 T_v + f_1(x_1, x_2) \\ \dot{x}_2 &= f_2(x_1, x_2)\end{aligned}\quad (1a)$$

where the functions f_1, f_2 are defined as follow:

$$\begin{aligned}f_1(x_1, x_2) &= \left[\alpha_2 + \alpha_3 \frac{x_1}{x_2} + \alpha_4 \exp\left(\alpha \frac{x_1}{x_2}\right) \right]^{-1} \\ &\times \left[\alpha_5 + \alpha_6 \frac{x_1}{x_2} + \alpha_7 (x_2 - V_a)^2 + \alpha_8 \frac{x_1}{x_2} (x_2 - V_a)^2 \right. \\ &\left. + \alpha_9 \exp\left(\alpha \frac{x_1}{x_2}\right) + \alpha_{10} (x_2 - V_a)^2 \exp\left(\alpha \frac{x_1}{x_2}\right) \right]\end{aligned}\quad (1b)$$

$$\begin{aligned}f_2(x_1, x_2) &= \alpha_{11} + \alpha_{12} (x_2 - V_a)^2 + \left[\alpha_2 + \alpha_3 \frac{x_1}{x_2} + \alpha_4 \exp\left(\alpha \frac{x_1}{x_2}\right) \right]^{-1} \\ &\times \left[\alpha_{13} + \alpha_{14} \frac{x_1}{x_2} + \alpha_{15} (x_2 - V_a)^2 + \alpha_{16} \frac{x_1}{x_2} (x_2 - V_a)^2 \right. \\ &\left. + \alpha_{17} \exp\left(\alpha \frac{x_1}{x_2}\right) + \alpha_{18} (x_2 - V_a)^2 \exp\left(\alpha \frac{x_1}{x_2}\right) \right]\end{aligned}\quad (1c)$$

In Eq. (1a), the functions f_1, f_2 feature linear parameterizations leading to the following parameterized form of Eq. (1a)

$$\begin{aligned}\dot{x}_1 &= \alpha_1 T_v + \sum_{i=1}^{12} \theta'_i g'_i(x_1, x_2) \\ \dot{x}_2 &= \sum_{j=1}^{15} \beta'_j h'_j(x_1, x_2)\end{aligned}\quad (1d)$$

To alleviate the text, the expressions of the functions g'_i, h'_j , and those of the coefficients α_i, θ'_i , and β'_j are placed in the Appendix.

Table 1 Notations of longitudinal vehicle model

l_f	distance between CoG and the front wheel base	m
l_r	distance between CoG and the rear wheel base	m
l	distance between the bases of the two wheels	m
h	height of the gravity centre	m
F_{aex}	aerodynamic drag force	N
F_{aez}	aerodynamic carrying force	N
g	gravity acceleration	m s^{-2}
M_v	vehicle mass	kg
F_{tf}	front-wheel drive force	N
F_{tr}	rear-wheel drive force	N
F_{vf}	load on the front-wheel	N
F_{vr}	load on the rear-wheel	N
ϕ	road slope	rad

2.1.2 *State-Space Representation in Acceleration Mode* $V_v < V_w$. The acceleration-mode state-space representation, established in Ref. [15], is given the compact form

$$\begin{aligned}\dot{x}_1 &= \alpha'_1 T_v + f'_1(x_1, x_2) \\ \dot{x}_2 &= f'_2(x_1, x_2)\end{aligned}\quad (2a)$$

where the functions f_1, f_2 are defined as follows:

$$\begin{aligned}f'_1(x_1, x_2) &= \left[\alpha'_2 + \alpha'_3 \frac{x_2}{x_1} + \alpha'_4 \exp\left(\alpha' \frac{x_2}{x_1}\right) \right]^{-1} \\ &\times \left[\alpha'_5 + \alpha'_6 \frac{x_2}{x_1} + \alpha'_7 (x_2 - V_a)^2 + \alpha'_8 \frac{x_2}{x_1} (x_2 - V_a)^2 \right. \\ &\left. + \alpha'_9 \exp\left(\alpha' \frac{x_2}{x_1}\right) + \alpha'_{10} (x_2 - V_a)^2 \exp\left(\alpha' \frac{x_2}{x_1}\right) \right]\end{aligned}\quad (2b)$$

$$\begin{aligned}f'_2(x_1, x_2) &= \alpha'_{11} + \alpha'_{12} (x_2 - V_a)^2 + \left[\alpha'_2 + \alpha'_3 \frac{x_2}{x_1} + \alpha'_4 \exp\left(\alpha' \frac{x_2}{x_1}\right) \right]^{-1} \\ &\times \left[\alpha'_{13} + \alpha'_{14} \frac{x_2}{x_1} + \alpha'_{15} (x_2 - V_a)^2 + \alpha'_{16} \frac{x_2}{x_1} (x_2 - V_a)^2 \right. \\ &\left. + \alpha'_{17} \exp\left(\alpha' \frac{x_2}{x_1}\right) + \alpha'_{18} (x_2 - V_a)^2 \exp\left(\alpha' \frac{x_2}{x_1}\right) \right]\end{aligned}\quad (2c)$$

Similarly, the functions f'_1, f'_2 feature linear parameterizations leading to the following parameterized form of Eq. (2a):

$$\begin{aligned}\dot{x}_1 &= \alpha'_1 T_v + \sum_{i=1}^{12} \theta''_i g''_i(x_1, x_2) \\ \dot{x}_2 &= \sum_{j=1}^{15} \beta''_j h''_j(x_1, x_2)\end{aligned}\quad (2d)$$

where the involved functions and notations g''_i, h''_j coefficient α''_i , coefficients θ''_i , and coefficients β''_j are appended.

2.1.3 *Complete State-Space Representation of Two-Wheel Vehicle With One Driving Wheel*. Combining Eqs. (1a) and (2a), one gets a single global model representing the vehicle in all operation modes. This is given the following compact form:

$$\begin{aligned}\dot{x}_1 &= \alpha_1^*(x_1, x_2) T_v + G_1(x_1, x_2) \\ \dot{x}_2 &= G_2(x_1, x_2)\end{aligned}\quad (3a)$$

Combining Eqs. (1d) and (2d), one gets the following equations:

$$\begin{aligned}\dot{x}_1 &= \alpha_1^*(x_1, x_2) T_v + \sum_{i=1}^{12} \theta_i g_i(x_1, x_2) \\ \dot{x}_2 &= \sum_{j=1}^{15} \beta_j h_j(x_1, x_2)\end{aligned}\quad (3b)$$

with

$$\begin{aligned}G_1(x_1, x_2) &= \sigma(x_1, x_2) f_1(x_1, x_2) + (1 - \sigma(x_1, x_2)) f'_1(x_1, x_2) \\ &= \sum_{i=1}^{12} \theta_i g_i(x_1, x_2)\end{aligned}\quad (4a)$$

$$\begin{aligned}G_2(x_1, x_2) &= \sigma(x_1, x_2) f_2(x_1, x_2) + (1 - \sigma(x_1, x_2)) f'_2(x_1, x_2) \\ &= \sum_{j=1}^{15} \beta_j h_j(x_1, x_2)\end{aligned}\quad (4b)$$

$$\alpha_1^*(x_1, x_2) = \sigma(x_1, x_2) \alpha_1 + (1 - \sigma(x_1, x_2)) \alpha'_1 \quad (4c)$$

$$\sigma(x_1, x_2) = \frac{1 - \text{sgn}(x_1 - x_2)}{2} \quad (4d)$$

$$\begin{aligned}g_i(x_1, x_2) &= \sigma(x_1, x_2) g'_i(x_1, x_2) + (1 - \sigma(x_1, x_2)) g''_i(x_1, x_2) \\ &\text{where } i \in \{1, \dots, 12\}\end{aligned}\quad (4e)$$

$$\begin{aligned}h_j(x_1, x_2) &= \sigma(x_1, x_2) h'_j(x_1, x_2) + (1 - \sigma(x_1, x_2)) h''_j(x_1, x_2) \\ &\text{where } j \in \{1, \dots, 15\}\end{aligned}\quad (4f)$$

$$\theta_i = \sigma(x_1, x_2) \theta'_i + (1 - \sigma(x_1, x_2)) \theta''_i \quad (4g)$$

$$\beta_j = \sigma(x_1, x_2) \beta'_j + (1 - \sigma(x_1, x_2)) \beta''_j \quad (4h)$$

It is shown in Ref. [15] that the model the model (Eqs. 3(a) and 3(b)) is singularity-free within the following validity model:

$$D_v = \left\{ (x_1, x_2) \in \mathbf{R}^2 : 1 - \varepsilon_l < \frac{x_1}{x_2} < 1 + \varepsilon_h \right\} \quad (5)$$

2.2 *Modeling of DC Drive System*. The ‘power converter—dc motor’ association is now modeled based on the equivalent circuit representation of Fig. 4. The modeling is performed under the following common assumptions:

- (i) The dc motor, presently a permanent magnet type, is axial-flux and operates in the linear range of the B-H curve of the magnetic material.
- (ii) The flux flows straightly across the air-gaps between stator and rotor; the fringing flux is ignored.
- (iii) The power converter dynamics are much more rapid compared to the motor electromechanical dynamics. Therefore, the power converter transfer function simplifies to a constant gain G .
- (iv) The vehicle is driven by two front wheels coupled to the dc motor through a gear box without losses. So, the tractive effort of two rear tires is zero. The wheel mass is negligibly small compared with the vehicle mass and the tire, yawing, pitching and rolling dynamics are not considered.

From Fig. 4, it is seen that the ‘motor-vehicle’ set is driven by the PWM buck dc–dc power converter. The latter is a two-quadrant high-speed switching circuit involving diodes (D_1, D_2) and (T_1, T_2) conventional transistors or power metal oxide semiconductor field effect transistor or gate turn-off thyristor. The two-quadrant feature is necessary because the motor is expected to operate both in acceleration and deceleration modes. Indeed, the current absorbed by the dc motor changes sense implying sign change of the motor torque. Then, the dc motor behaviour will be described by two different equations separately established in each operation mode. In this respect, note that the physical power converter (and consequently the whole controlled system) is acted on by the binary control signal. So, the outputs $s_1(t)$ and $s_2(t)$ of the PWM block are taking values in the discrete set $\{0, 1\}$. According to the PWM principle, $s_1(t)$ and $s_2(t)$ are generated as follows:

Acceleration mode	$s_1(t) = \begin{cases} 1 & \text{if } t_k \leq t \leq t_k + v_1(t_k)T_s \\ 0 & \text{if } t_k + v_1(t_k)T_s \leq t \leq t_{k+1} \end{cases}$	$s_2(t) = 0$
Deceleration mode	$s_1(t) = 0$	$s_2(t) = \begin{cases} 1 & \text{if } t_k \leq t \leq t_k + v_2(t_k)T_s \\ 0 & \text{if } t_k + v_2(t_k)T_s \leq t \leq t_{k+1} \end{cases}$

Fig. 4 The controlled system including the buckconverter, the dc motor and the vehicle

where $T_s > 0$ is a fixed sampling period, the t_k 's are sampling instants and $v_1(\cdot)$ and $v_2(\cdot)$ are functions, called duty ratio for T_1 (acceleration mode) and T_2 (deceleration mode), respectively, that continuously varies in the interval $[0, 1]$. One can define a global duty ratio $v(t) = \sigma(x_1, x_2)v_1(t) + (1 - \sigma(x_1, x_2))v_2(t)$, where $\sigma(x_1, x_2)$ is defined in Eqs. (4d). The point is that the (instantaneous) model involving $s_1(t)$ and $s_2(t)$ cannot be based upon to design a continuous control law as it involves a binary control input, namely, $s_1(t)$ and $s_2(t)$. To overcome this difficulty, it is commonly resorted to the averaging process over the cutting intervals [17]. This process is shown to give rise to average versions (of the above model) involving as a control input the mean value of u which is nothing other than the duty ratio $v(\cdot)$. To keep simpler notations, no distinction is made in the sequel between the converter-motor system variables (those shown in Fig. 4) and their averaged versions. For instance, the function u will also refer to its averaged version $v(\cdot)$. This convention, which is justified by the fact that the sampling period T_s practically too small (less than 1 ms), entails no confusion as only averaged signals are involved in the rest of the paper. Based on the above observations, the converter-motor behaviour is now analytically described considering the different operation modes.

2.2.1 Longitudinal Acceleration Mode. In this operation mode, the dc motor develops a positive torque and so its behavior undergoes the following equation:

$$v = GEu = Ri + L \frac{di}{dt} + \frac{1}{kr_{\text{eff}}} KV_v \quad (6a)$$

$$T_v = 1/ki \quad (6b)$$

where E denotes the voltage source generated by the embedded battery of accumulators in the vehicle; i dc motor armature circuit current; R and L rotor winding resistance and inductance; e winding back electromotive force which we know is related to the motor shaft angular velocity Ω_m by the relation $e = K \Omega_m$ with K called electromotive force coefficient; T_v produced torque applied on the gear box; G is the buck dc-dc power converter gain; v converter output voltage; r_{eff} is the wheel effective radius and k is the gear box ratio. Since, the wheels roll without slipping, one has $V_v = r_{\text{eff}} \Omega_v$, with Ω_v the angular velocity of the wheel. That is, $V_v = kr_{\text{eff}} \Omega_m$.

Combining the two Eqs. (6a) and (6b), yields the following single equation, used in subsequent developments:

$$\dot{T}_v = \frac{K}{kL} GEu - \frac{R}{L} T_v - \frac{K^2}{k^2 L r_{\text{eff}}} V_v \quad (7)$$

2.2.2 Longitudinal Deceleration Mode. In this mode, the dc motor develops a negative torque and so its motion is described by the equations:

$$v = uGE = -Ri - L \frac{di}{dt} + \frac{1}{kr_{\text{eff}}} KV_v \quad (8a)$$

$$T_v = 1/ki \quad (8b)$$

Combining the two Eqs. (8a) and (8b) one gets

$$\dot{T}_v = -\frac{K}{kL} GEu - \frac{R}{L} T_v + \frac{K^2}{k^2 L r_{\text{eff}}} V_v \quad (9)$$

2.2.3 Complete DC Motor Model in Acceleration/Deceleration Mode. Combining the two representations (7) and (9), one gets a single complete model describing the dc motor in both acceleration/deceleration modes:

$$\dot{x}_3 = Hu + G_3(x_2) - \tau x_3 \quad (10)$$

where $x_2 = V_v$ and $x_3 = T_v$ with

$$H = \beta \frac{K}{kL} GE \quad (11a)$$

$$G_3(x_2) = -\beta \frac{K^2}{k^2 L r_{\text{eff}}} x_2 \quad (11b)$$

$$\beta = \text{sgn}(x_1 - x_2) = \begin{cases} +1 & \text{if } x_1 > x_2 \quad (\text{acceleration}) \\ -1 & \text{if } x_1 \leq x_2 \quad (\text{deceleration}) \end{cases} \quad (11c)$$

$$\tau = \frac{R}{L} \quad (11d)$$

2.3 Complete State-Space Representation of Two-Wheel Vehicle With One Driving DC Motor Coupled to the Wheel. Combining the chassis state-space representation (3b) and the dc motor Eq. (10), one gets the following single global model representing the whole 'vehicle chassis and dc motor' association, in all operation modes (Fig. 2):

$$\dot{x}_1 = \alpha_1^*(x_1, x_2)x_3 + \sum_{i=1}^{12} \theta_i g_i(x_1, x_2) \quad (12a)$$

$$\dot{x}_2 = \sum_{j=1}^{15} \beta_j h_j(x_1, x_2)$$

$$\dot{x}_3 = Hu + G_3(x_2) - \tau x_3$$

The complete model validity domain, taking account (5), is defined by:

$$D = D_v \times R = \left\{ (x_1, x_2, x_3) \in R^3 : 1 - \varepsilon_1 < \frac{x_1}{x_2} < 1 + \varepsilon_h \right\} \quad (12b)$$

3 Adaptive Controller Design for Electric Vehicle Longitudinal Control

An adaptive cascade controller (Fig. 5) will now be developed using the backstepping design approach [16]. Accordingly, an outer regulator is first designed to control the chassis part, considering its input $x_3 = T_v$ as a virtual control signal. Then, an inner regulator is designed for the dc motor to make its output x_3 match

Fig. 5 Cascade control strategy for the chassis-motor association

well the control trajectory x_3^* generated by the outer regulator. To design the adaptive control and parameter update laws design, let us introduce the following notations:

- unknown parameters, $\tau \in \mathbf{R}$, $\eta = [\theta_1 \dots \theta_{12} \beta_1 \dots \beta_{15}]^T \stackrel{\text{def}}{=} [\hat{\theta}^T \hat{\beta}^T]^T \in \mathbf{R}^{27}$
- parameter estimates, $\hat{\tau}, \hat{\eta} = [\hat{\theta}_1 \dots \hat{\theta}_{12} \hat{\beta}_1 \dots \hat{\beta}_{15}]^T = [\hat{\theta}^T \hat{\beta}^T]^T$
- parameter estimation errors: $\tilde{\tau} = \hat{\tau} - \tau$, $\tilde{\eta} = \hat{\eta} - \eta = [\tilde{\theta}_1 \dots \tilde{\theta}_{12} \tilde{\beta}_1 \dots \tilde{\beta}_{15}]^T$, $\tilde{\theta} = \hat{\theta} - \theta$, $\tilde{\beta} = \hat{\beta} - \beta$

3.1 Outer Loop Design. The outer regulator aims at controlling the vehicle chassis making its outputs (V_w, V_v) match well a given reference signal (V_w^*, V_v^*) . During this design stage, the signal T_v is viewed as a virtual control input. The controller design is based on the chassis motion model (3b) (or equally the first couple of equations in (12a)). The prime control objective is to enforce the chassis and wheel speeds to track their reference trajectories denoted x_1^* and x_2^* , respectively. In view of the considered control objective, it is convenient to introduce the following tracking errors:

$$\begin{aligned} z_1 &\stackrel{\text{def}}{=} x_1 - x_1^* \\ z_2 &\stackrel{\text{def}}{=} x_2 - x_2^* \end{aligned} \quad (13)$$

According to the backstepping design technique, the focus will now be made on asymptotically stabilizing the (z_1, z_2) -system. To this end, the dynamics of this system are first determined by deriving of the (z_1, z_2) -errors with respect to time. Using Eq. (12a), one gets

$$\begin{aligned} \dot{z}_1 &= \alpha_1^*(x_1, x_2)x_3 + \sum_{i=1}^{12} \theta_i g_i(x_1, x_2) - \dot{x}_1^* \\ \dot{z}_2 &= \sum_{j=1}^{15} \beta_j h_j(x_1, x_2) - \dot{x}_2^* \end{aligned} \quad (14)$$

Clearly, x_3 stands as a virtual control in the error model (14). Now, one needs to find a Lyapunov function candidate that is positive definite in (z_1, z_2) , such that its derivative along (14) is linear in x_3 . The following choice meets these requirements:

$$V_1(z_1, z_2, \tilde{\eta}) = c_1 |z_1| + \frac{1}{2} c_2 z_2^2 + \frac{1}{2\gamma} \tilde{\eta}^T \tilde{\eta} \quad (15)$$

where c_1, c_2 , and γ are positive design parameters that will influence the control performances. Note that V_1 is a positive definite function of $(z_1, z_2, \tilde{\eta})$ on $\mathbf{R}^2$. Derivation of $V_1(z_1(t), z_2(t), \tilde{\eta})$ with respect to time, along the trajectory of Eq. (14), yields:

$$\begin{aligned} \dot{V}_1 &= \text{sgn}(z_1) \left(c_1 \alpha_1^*(x_1, x_2)x_3 + c_1 \left(\sum_{i=1}^{12} \theta_i g_i(x_1, x_2) - \dot{x}_1^* \right) \right. \\ &\quad \left. + c_2 \text{sgn}(z_1) z_2 \left(\sum_{j=1}^{15} \beta_j h_j(x_1, x_2) - \dot{x}_2^* \right) \right) + \frac{1}{2\gamma} \tilde{\eta}^T \dot{\tilde{\eta}} \end{aligned} \quad (16)$$

Substituting $\hat{\theta} - \tilde{\theta}$ and $\hat{\beta} - \tilde{\beta}$ to θ and β in Eq. (16) yields

$$\begin{aligned} \dot{V}_1 &= \text{sgn}(z_1) \left(c_1 \alpha_1^*(x_1, x_2)x_3 + c_1 \left(\sum_{i=1}^{12} \hat{\theta}_i g_i(x_1, x_2) - \dot{x}_1^* \right) \right. \\ &\quad \left. + c_2 \text{sgn}(z_1) z_2 \left(\sum_{j=1}^{15} \hat{\beta}_j h_j(x_1, x_2) - \dot{x}_2^* \right) \right) \\ &\quad - c_1 \text{sgn}(z_1) \left(\sum_{i=1}^{12} \tilde{\theta}_i g_i(x_1, x_2) \right) - c_2 z_2 \left(\sum_{j=1}^{15} \tilde{\beta}_j h_j(x_1, x_2) \right) \\ &\quad + \frac{1}{2\gamma} \tilde{\eta}^T \dot{\tilde{\eta}} \end{aligned} \quad (17)$$

Equation (16) suggests to let $x_3 = x_3^*$ with:

$$\begin{aligned} x_3^* &= -\frac{1}{c_1 \alpha_1^*(x_1, x_2)} \times \left(c_1 \left(\sum_{i=1}^{12} \hat{\theta}_i g_i(x_1, x_2) - \dot{x}_1^* \right) \right. \\ &\quad \left. + c_2 \text{sgn}(z_1) z_2 \left(\sum_{j=1}^{15} \hat{\beta}_j h_j(x_1, x_2) - \dot{x}_2^* \right) + c v \text{sgn}(z_1) \right) \end{aligned} \quad (18)$$

where $c > 0$ is a new design parameter. There is no singularity in Eq. (18) because $\alpha_1^*(x_1, x_2) \neq 0$. Indeed, the substitution of the right side of Eq. (18) to x_3 in Eq. (16) yields:

$$\begin{aligned} \dot{V}_1 &= -c v - c_1 \text{sgn}(z_1) \left(\sum_{i=1}^{12} \tilde{\theta}_i g_i(x_1, x_2) \right) \\ &\quad - c_2 z_2 \left(\sum_{j=1}^{15} \tilde{\beta}_j h_j(x_1, x_2) \right) + \frac{1}{2\gamma} \tilde{\eta}^T \dot{\tilde{\eta}} \end{aligned} \quad (19)$$

with v is defined as follows:

$$v(z_1, z_2) = c_1 |z_1| + \frac{1}{2} c_2 z_2^2 \quad (20)$$

In order to construct the parameter adaptive law, introduce the vector function ψ :

$$\psi(x_1, x_2) \stackrel{\text{def}}{=} [c_1 \text{sgn}(z_1) g_1 \dots c_1 \text{sgn}(z_1) g_{12} \quad c_2 z_2 h_1 \dots c_2 z_2 h_{15}]^T \quad (21)$$

Then, the right side of Eq. (19) can be given the more compact form:

$$\dot{V}_1 = -c v(z_1, z_2) + \frac{1}{\gamma} \tilde{\eta}^T (-\gamma \psi + \dot{\tilde{\eta}}) \quad (22)$$

This suggests the following adaptive law:

$$\dot{\tilde{\eta}} = \dot{\eta} = \gamma\psi \quad (23)$$

Substituting the right side of Eq. (23) to $\dot{\tilde{\eta}}$ in Eq. (22) yields

$$\dot{V}_1 = -cv(z_1, z_2) \quad (24)$$

This is interesting as it shows that, by the stabilization function (18), the Lyapunov function V_1 is made negative semi-definite. The point is that x_3 only stands as a virtual control input in the chassis model (3b). Therefore, one can not let $x_3 = x_3^*$. Nevertheless, the expression (18) of the stabilization function is retained and a new error is introduced between the virtual control input x_3 and x_3^*

$$z_3 \stackrel{\text{def}}{=} x_3 - x_3^* \quad (25)$$

Using Eq. (18), the error equations (14) are rewritten in terms of the new error z_3

$$\begin{aligned} \dot{z}_1 &= \alpha_1^*(x_1, x_2)z_3 - \frac{c_2}{c_1} \text{sgn}(z_1)z_2 \left(\sum_{j=1}^{15} \hat{\beta}_j h_j(x_1, x_2) - \dot{x}_2^* \right) \\ &\quad - \frac{c}{c_1} \text{sgn}(z_1) \\ \dot{z}_2 &= \sum_{j=1}^{15} \hat{\beta}_j h_j(x_1, x_2) - \dot{x}_2^* \end{aligned} \quad (26)$$

Also, the derivative $\dot{V}_1$ is expressed in function of z_3

$$\begin{aligned} \dot{V}_1 &= \text{sgn}(z_1) \left(c_1 \alpha_1^*(x_1, x_2)z_3 + c_1 \left(\sum_{i=1}^{12} \hat{\theta}_i g_i(x_1, x_2) - \dot{x}_1^* \right) \right. \\ &\quad \left. + \alpha_1^*(x_1, x_2)x_3^* \right) + c_2 \text{sgn}(z_1)z_2 \left(\sum_{j=1}^{15} \hat{\beta}_j h_j(x_1, x_2) - \dot{x}_2^* \right) \end{aligned} \quad (27)$$

which, in view of Eq. (18), simplifies to

$$\dot{V}_1 = -cv + c_1 \text{sgn}(z_1) \alpha_1^*(x_1, x_2)z_3 \quad (28)$$

3.2 Inner Regulator Design. The aim of the inner regulator is to contribute, together with the outer regulator, to make the error system (z_1, z_2, z_3) asymptotically stable. First, it follows from Eqs. (12a) and (25) that the error z_3 undergoes the following differential equation:

$$\dot{z}_3 = Hu + G_3(x_2) - \tau x_3 - \dot{x}_3^* \quad (29)$$

Note that the actual control u has emerged for the first time in Eq. (29). To find a suitable control law for u , introduce the following Lyapunov function:

$$V(z_1, z_2, z_3, \tilde{\eta}, \tilde{\tau}) = V_1(z_1, z_2, \tilde{\eta}) + \frac{1}{2} z_3^2 + \frac{1}{2\sigma} \tilde{\tau}^2 \quad (30)$$

where σ is a positive design parameters that will influence the control performances. Using Eqs. (28) and (29), the time-derivative of V turns out to be the following:

$$\begin{aligned} \dot{V} &= \dot{V}_1(z_1, z_2, \tilde{\eta}) + z_3 \dot{z}_3 + \frac{1}{\sigma} \tilde{\tau} \dot{\tilde{\tau}} \\ &= -cv + \text{sign}(z_1) c_1 \alpha_1^*(x_1, x_2)z_3 + z_3 (Hu + G_3(x_2) - \tau x_3 - \dot{x}_3^*) \\ &\quad + \frac{1}{\sigma} \tilde{\tau} \dot{\tilde{\tau}} \end{aligned} \quad (31)$$

$$\begin{aligned} \dot{V} &= -cv + z_3 (Hu + G_3(x_2) - \dot{x}_3^* + c_1 \alpha_1^*(x_1, x_2) \text{sgn}(z_1) - \hat{\tau} x_3) \\ &\quad + \tilde{\tau} x_3 z_3 + \frac{1}{\sigma} \tilde{\tau} \dot{\tilde{\tau}} \end{aligned} \quad (32)$$

Equation (32) suggests that the control signal u should be selected so that

$$Hu + G_3(x_2) - \dot{x}_3^* + c_1 \alpha_1^*(x_1, x_2) \text{sgn}(z_1) - \hat{\tau} x_3 = -\frac{c_3}{2} z_3 \quad (33)$$

where $c_3 > 0$ is a new design parameter. Doing so, one gets the following control law:

$$u = -\frac{1}{H} \left(G_3(x_2) - \dot{x}_3^* + c_1 \alpha_1^*(x_1, x_2) \text{sgn}(z_1) - \hat{\tau} x_3 + \frac{c_3}{2} z_3 \right) \quad (34)$$

where H is expressed by Eq. (11a). Then, substituting the right side of Eq. (34) to u in Eq. (32) yields

$$\dot{V} = -cv - \frac{c_3}{2} z_3^2 + \tilde{\tau} \left(\frac{1}{\sigma} \dot{\tilde{\tau}} + x_3 z_3 \right) \quad (35)$$

This suggests the following parameter adaptive law:

$$\dot{\tilde{\tau}} = \dot{\tau} = -\sigma x_3 z_3 \quad (36)$$

Indeed, due to Eq. (36), Eq. (35) boils down to

$$\begin{aligned} \dot{V}(z_1, z_2, z_3, \tilde{\eta}, \tilde{\tau}) &= -cv - \frac{c_3}{2} z_3^2 \\ &= -c \left(c_1 |z_1| - \frac{c_2}{2} z_2^2 \right) - \frac{c_3}{2} z_3^2 \quad (\text{using Eq. (20)}) \end{aligned} \quad (37)$$

which shows that V is negative semidefinite. The cascade Controller thus developed includes the inner regulator defined by the control law (34) and the outer regulator defined by the stabilizing law (18). These laws involve the time-derivatives of the reference signals x_1^* and x_2^* , up to the second order. This requirement can always be complied with letting those reference signals be first order filtered versions of the desired wheel and vehicle speeds, denoted $x_1^d = V_w^d$ and $x_2^d = V_v^d$, respectively. Specifically, one has

$$\begin{aligned} x_1^* &= \frac{1}{1 + T_r s} x_1^d \\ x_2^* &= \frac{1}{1 + T_r s} x_2^d \end{aligned} \quad (38)$$

where the time constant T_r , freely chosen by the user, stands as a new design parameter. Then, it follows from Eq. (38) that the time-derivatives of x_1^* and x_2^* can be obtained from the measured signals x_1^d and x_2^d as follows:

$$\begin{aligned} \dot{x}_1^* &= \frac{s}{1 + T_r s} x_1^d \quad \text{and} \quad \ddot{x}_1^* = \frac{s^2}{1 + T_r s} x_1^d \\ \dot{x}_2^* &= \frac{s}{1 + T_r s} x_2^d \quad \text{and} \quad \ddot{x}_2^* = \frac{s^2}{1 + T_r s} x_2^d \end{aligned} \quad (39)$$

Clearly, the above time-derivatives are measurable as all involved filters are proper. Note that, the vehicle desired speed x_2^d is directly chosen by the user, generally as a step-like input reference. Then, the wheel speed x_1^d is deduced from x_2^d according to the rule

$$x_1^d = (1 + \lambda^*) x_2^d \quad (40a)$$

which in turn implies, using Eq. (38), that

$$x_1^* = (1 + \lambda^*)x_2^* \quad (40b)$$

where λ^* is a constant (representing the sliding) that is uniquely obtained from Eq. (3a) letting there $x_2 = x_2^d$, $x_1 = (1 + \lambda^*)x_2^d$ and setting $\dot{x}_2 = 0$. Doing so, one gets $G_2((1 + \lambda^*)x_2^d, x_2^d) = 0$, due to Eq. (3a). Then, Eq. (3c) yields

$$\sigma((1 + \lambda^*)x_2^d, x_2^d)f_2((1 + \lambda^*)x_2^d, x_2^d) + (1 - \sigma((1 + \lambda^*)x_2^d, x_2^d))f_2'((1 + \lambda^*)x_2^d, x_2^d) = 0 \quad (41a)$$

This is an algebraic equation that must be solved to get an adequate value of λ^* for any fixed x_2^d . In view of Eq. (5), the search domain is limited to the following:

$$1 - \varepsilon_l < \lambda^* < 1 + \varepsilon_h \quad (41b)$$

The control system thus established is illustrated by Fig. 5 and its performances are formally described in Theorem 3.1.

THEOREM 3.1. Consider the closed loop control system, sketched by Fig. 5, consisting of the following:

- the vehicle chassis and motor, both represented by the state-space model (12a),
- the cascade nonlinear adaptive controller, including the outer regulator (18) and the inner regulator (34), where c_1, c_2, c_3 are any positive design parameters and the reference signals are generated by Eqs. (38) and (40a).

Then, there exists a real number exists a $\mu^* > 0$ such that if $0 < \mu < \mu^*$ and $c_1|z_1(0)| + \frac{1}{2}c_2z_2^2(0) < \mu$ then

- All the closed-loop system signals remain bounded.
- The errors $(z_1(t), z_2(t), z_3(t))$ vanish asymptotically $\square$

Proof. Substituting the right sides of Eq. (34) to u and x_3^* in Eq. (29) yields, together with Eqs. (26), (23), and (36), the following closed-loop system representation in the (z_1, z_2, z_3) -coordinates:

$$\begin{aligned} \dot{z}_1 &= \alpha_1^*(x_1, x_2)z_3 - \frac{c_2}{c_1} \text{sgn}(z_1)z_2 \left(\sum_{j=1}^{15} \hat{\beta}_j h_j(x_1, x_2) - \dot{x}_2^* \right) \\ &\quad - \frac{c}{c_1} v \text{sgn}(z_1) \end{aligned} \quad (42a)$$

$$\dot{z}_2 = \sum_{j=1}^{15} \hat{\beta}_j h_j(x_1, x_2) - \dot{x}_2^* \quad (42b)$$

$$\dot{z}_3 = c_1 \alpha_1^*(x_1, x_2) \text{sgn}(z_1) + \frac{c_3}{2} z_3 \quad (42c)$$

$$\dot{\tilde{\eta}} = \gamma \psi \quad (42d)$$

$$\dot{\tilde{\tau}} = -\sigma x_3 z_3 \quad (42e)$$

Note that these equations hold provided that the control model (12a) is representative of the controlled system and this actually is the case as long as $(x_1, x_2, x_3) \in D$ (see Eq. 12b)). Under this condition, the positive definite Lyapunov function V , defined by Eq. (30), has a semi-negative definite time-derivative, given by Eq. (37), implying that the state vector $(z_1, z_2, z_3, \tilde{\eta}, \tilde{\tau})$ is bounded. To show that (x_1, x_2, x_3) actually stay in the domain D and that (z_1, z_2, z_3) converges to zero we make use of Lasalle's invariance principle. Accordingly, let $\mu > 0$ be any real number and consider the following sets:

$$\Omega_\mu = \left\{ [z_1, z_2, z_3, \tilde{\eta}^T, \tilde{\tau}]^T \in \mathbf{R}^{3I}, V(z_1, z_2, z_3, \tilde{\eta}, \tilde{\tau}) < \mu \right\} \quad (43a)$$

$$Z_\mu = \left\{ [z_1, z_2, z_3, \tilde{\eta}^T, \tilde{\tau}]^T \in \Omega_\mu, \dot{V}(z_1, z_2, z_3, \tilde{\eta}, \tilde{\tau}) = 0 \right\} \quad (43b)$$

M_μ is the largest subset of Z_μ that is invariant for the system (Eqs. 42(a)–42(e)) with

$$V(z_1, z_2, z_3, \tilde{\eta}, \tilde{\tau}) = c_1|z_1| + \frac{1}{2}c_2z_2^2 + \frac{1}{2\gamma}\tilde{\eta}^T\tilde{\eta} + \frac{1}{2}z_3^2 + \frac{1}{2\sigma}\tilde{\tau}^2 \quad (44a)$$

(using Eqs. (15) and (30))

Fig. 6 Cascade control strategy for the chassis-motor association

Table 2 Simulated operation conditions

		0 s–16 s	16 s–32 s	32 s–48 s	48 s–64 s	64 s–80 s	80 s–96 s	96 s–112 s	112 s–128 s
I	V_a^d (km/h)	80	80	80	80	80	80	80	80
	Noise variance	0	0	0	5%	0	0	0	0
II	V_a (km/h)	0	0	0	0	10	10.5	0	0
	Road slope (deg)	0	0	0	0	0	0	5	5.25
	Road state	AD	AW	AD	AD	AD	AD	AD	AD
III	V_a (km/h)	0	0	0	0	10	10	0	0
	Road slope (deg)	0	0	0	0	0	0	5	5
	Road state	AD	AD	AD	AD	AD	AD	AD	AD

Note: I: include the external inputs, II: contain the vehicle parameters, and III: describe the controller parameters.

Table 3 Numerical characteristics of the simulated vehicle

AD: Asphalt dry, CW: Cobblestone wet	AD	CW
C_1 Primary tire parameter	1.1	0.5
C_2 Primary tire parameter	25	30
C_3 Primary tire parameter	0.5	0.2
M_v Chassis mass	560 kg	
J Wheel inertia	500 kg m ²	
r_{eff} Effective wheel radius	0.28 m	
μ_{rr} Rolling resistance coefficient	0.025	
χ Related height of the center of gravity	0.2	
Ψ Related position of center of gravity	0.43	
k_v Load correction factor	0.55	
ρ Density of air	1.202 kg/m ³	
C_x Aerodynamic drag coefficient	0.5	
C_z Aerodynamic lift coefficient	0.259	
S Frontal area vehicle	0.8 m ²	
Numerical characteristics of dc motor		
G Gain of the Buck dc–dc converter	100/E	
R Resistance	0.5 Ω	
L Inductance	0.01 H	
K The back voltage constant	1.23 (60/2 π)	
k Ratio of the gear box	1/5	

$$\dot{V}(z_1, z_2, z_3, \tilde{\eta}, \tilde{\tau}) = -cc_1|z_1| - \frac{cc_2}{2}z_2^2 - \frac{c_3}{2}z_3^2 \quad (\text{using Eq. (37)}) \quad (44b)$$

From Eqs. (43a) and (44a), one immediately gets

$$[z_1, z_2, z_3, \tilde{\eta}^T, \tilde{\tau}^T]^T \in \Omega_\mu \Rightarrow c_1|z_1| + \frac{1}{2}c_2z_2^2 < \mu \quad (45)$$

On the other hand, it is readily seen from Eqs. (19), (38), and (40b) that, for any positive (c_1, c_2) , there exists a $\mu^* > 0$ such that if $0 < \mu < \mu^*$ one has

$$c_1|z_1| + \frac{1}{2}c_2z_2^2 < \mu \Rightarrow (x_1, x_2, x_3) \in D$$

which, together with Eq. (45), implies that one has, if $0 < \mu < \mu^*$ then

$$[z_1, z_2, z_3, \tilde{\eta}^T, \tilde{\tau}^T]^T \in \Omega_\mu \Rightarrow (x_1, x_2, x_3) \in D \quad (46)$$

Now, by Lasalle's invariance principle, one has that, if $[z_1(0), z_2(0), z_3(0), \tilde{\eta}(0)^T, \tilde{\tau}(0)^T]^T \in \Omega_\mu$ then, $[z_1(t), z_2(t), z_3(t), \tilde{\eta}(t)^T, \tilde{\tau}(t)^T]^T$ stays in Ω_μ (and so in D by Eq. (46)) and converges to M_μ (as $t \rightarrow \infty$). Then, it follows using Eqs. (45) and (46) that, it follows that, if $0 < \mu < \mu^*$ and $[z_1(0), z_2(0), z_3(0), \tilde{\eta}(0)^T, \tilde{\tau}(0)^T]^T \in \Omega_\mu$ then, $(z_1(t), z_2(t), z_3(t))$ converges to zero. This completes the proof of Theorem 3.1. ■

4 Simulation

The performances of the cascade nonlinear controller (18) and (34) are now illustrated through numerical simulations, performed with Matlab-Simulink, according to the simulation setting of Fig. 6. The environmental parameters are described by Table 2. Note that the environmental parameter values used in the controller are deliberately let to be different from those used in the simulated vehicle, in order to check the controller robustness to modeling errors.

The vehicle characteristics, as well as those of the electric dc motor are shown in Table 3. The controller design parameters $(c, c_1, c_2, c_3, \gamma, \sigma, T_r)$ must be given suitable numerical values before online implementation of the control algorithm. Suitable values can simply be selected by the usual 'try-an-error' search method. Doing so, the following values have been retained:

Fig. 7 Controller tracking performances

Fig. 8 Controller robustness performances

- The model reference time constant in Eq. (38) is set to $T_r = 1$ s.
- The design parameters are $c = 10$, $c_1 = 10$, $c_2 = 0.2$, $c_3 = 1$, $\gamma = 10$, and $\sigma = 10$.

4.1 Adaptive Backstepping Controller Tracking Capability.

The controller tracking capability of the proposed cascade nonlinear controller is presently illustrated in the absence of modeling errors. The resulting tracking performances are illustrated by Figs. 7(a)–7(d) which shows the following progression in vehicle operation mode: Acceleration mode [0 s–16 s] and deceleration mode [16 s–32 s].

The speed reference signals x_1^* and x_2^* are, respectively, filtered versions of step like desired signals x_1^d and x_2^d (38). It is seen in Figs. 8(a) and 8(b) that both vehicle and wheel speeds match well their reference signals. Indeed, the steady-state tracking errors perfectly vanish.

Figure 7(c) shows that the torque motor $T_m = kT_v = kx_3$ in turn matches well its reference trajectory kx_3^* generated by the outer regulator (see Fig. 6). Figure 7(d) shows that the control action u , generated by the inner regulator, rapidly settles down after each speed reference change. In the light of the Fig. 7(e), one sees that the security is guaranteed due to a good behavior of the contact tire/road, i.e., the skating of the wheel on the road is avoided. This is guaranteed because the sliding is kept in the safety zone of λ . The safety zone, established in Ref. [15], is defined by

$$\lambda \in \begin{cases} [\lambda_{\min}, 0] & \text{if } x_1 \leq x_2 \\]0, \lambda_{\max}] & \text{if } x_1 > x_2 \end{cases} \quad \text{with } \lambda_{\min} = -8.74\% \quad \text{and } \lambda_{\max} = 10\% \quad (47)$$

Finally, Fig. 7(f) illustrates parameter adaptation by plotting the evolution of the parameter estimate $\hat{\tau}$, one see that the parameter estimate has to adjust to $\tau = 50$.

4.2 Adaptive Backstepping Controller Robustness Checking.

In this subsection, we will check to what extent the controller is able to maintain a suitable performance level when the

Fig. 9 Simulated experimental setting with PID's regulators

measurements (defining the operation conditions) are subject to external noise. First, the true vehicle operation protocol is described by Table 2, where “I,” “II,” and “III” in the first column refer to the external inputs, the vehicle parameters and the controller parameters, respectively. The changes concern the vehicle speed reference x_2^d and the environmental conditions: wind speed V_a , road slope ϕ and the state of the road characterized by (C_1, C_2, C_3) . Table 2 shows the following progression in vehicle operation mode:

- at time 16 s: the road state changes, passing from Asphalt dry $(C_1, C_2, C_3) = (1.1, 25, 0.5)$ to Cobblestone wet $(C_1, C_2, C_3) = (0.5, 30, 0.2)$.

- at time 32 s: the road state changes again going back to Asphalt dry $(C_1, C_2, C_3) = (1.1, 25, 0.5)$
- at time 64 s: an opposite front wind of 10 km/h comes into action.
- at time 80 s: the front wind speed changes slightly and becomes 10.5 km/h.
- at time 96 s: the road slope passes from 0 deg to 5 deg (upward road).
- at time 112 s: the road slope slightly changes and becomes 5.25 deg.

To illustrate the controller robustness, the environmental parameters are given different values in the controller design. The

(a)

(b)

Fig. 10 (a) Wheel speed responses. Solid: reference. Dashed: speed response for adaptive controller. Dotted: speed response for PID cascade controller (b) Sliding responses. Solid: sliding response for adaptive controller. Dashed: sliding response for PID cascade controller

Fig. 11 Wheel speed responses. Solid: reference. Dashed: speed response with $c = 10$. Dotted: speed response with $c = 10^{-2}$

differences in the environmental parameters, between the vehicle and the controller, are deliberately produced to evaluate the controller robustness to modelling errors. For instance:

- From time 16 s to 32 s: a change actually happens in the road state but asphalt dry parameters continue to be used in the controller.
- From time 80 s to 96 s: there is a change in the front wind speed (which becomes 10.5 km/h) but the controller still uses the ancient value (10 km/h).
- From time 112 s to 128 s: the road slope slightly changes and becomes 5.25 deg but the controller still assumes the value 5 deg.

Besides, from time 48 s to 64 s, the measurements of the vehicle and wheel speeds are affected by zero mean white noises whose instantaneous amplitudes represent 5% of the unnoisy values.

In the light of Fig. 8, it is seen that the controller robustness against the mentioned uncertainties is quite satisfactory. Indeed, the settling time (of the output responses) is less than 1.75 s and no overshoot in the acceleration mode. The tracking error perfectly vanishes when the operation conditions are perfectly known to the controller. It does not vanish in presence of parameter uncertainties or output noise. But, its asymptotic value is too small compared to the reference value. Indeed, the zooms in Figs. 8(a) and 8(b) show that the tracking performance deterioration is really insignificant since the steady state error is less than 0.6 km/h when the vehicle is going at 80 km/h, i.e., the tracking error represents 0.75% of the vehicle speed. This is a quite acceptable performance given the significant model uncertainty.

To better appreciate this performance, a comparison is made between the proposed cascade nonlinear adaptive controller and a cascade controller involving proportional integral differential (PID's) regulators (Fig. 9). The operation conditions are those corresponding to the interval [0–16 s] in Table 2. Furthermore, one uses the easy road: Cobblestone wet (C_1, C_2, C_3) = (0.5, 30, 0.2). The behaviour of both cascade controllers (vehicle speed and sliding) is described by Figs. 10(a) and 10(b). In the light of the Fig. 10(b), one sees that the security is guaranteed by cascade nonlinear adaptive controller, due to the sliding is kept in the prescribed interval (47). Clearly, the performance of the cascade nonlinear controller is much better than the PID based cascade controller.

The parameters (the proportional action, the integral action, the derivate action) of the PID speed regulator and the PID torque regulator are, respectively: ($3 \cdot 10^3$, 10, $10 \cdot 10^{-3}$) and (10^{-3} , 1, 10).

4.3 Controller Performance Sensitivity to Design Parameters.

It would be useful to provide a systematic rule for selecting optimal values of the design parameters. Unfortunately, such a rule is generally not available in nonlinear control systems. Then, numerical simulation turns out to be the only alternative. The simulations show that the control performances are actually influenced by the values given to the design parameters. This is illustrated by Fig. 11 which shows the sensitivity of the vehicle speed to the design parameter c , while all others are given the constant values of Secs. 4.1 and 4.2. It is clearly seen in Fig. 11 that the value $c = 10$, retained in Sec. 4.1, leads to the best performances. As a matter of fact, the complexity of such simulation-based trial-and-error rule increases with the number of design parameters.

5 Conclusion

The problem of longitudinal control is addressed for EVs propelled by dc motors through a differential gear box. A cascade nonlinear control strategy is developed using the adaptive backstepping design technique, based on the recently developed model described by (12a). The latter accounts for aerodynamic effects, road slope variation, road state and dc motor characteristics. It is formally shown that the proposed cascade nonlinear controller performs well both in acceleration and deceleration modes, despite uncertainties on the environmental characteristics. This control allows that the state of the system with bounded parameters converge asymptotically to zero. For the system given by (12a) having 28 unknown parameters; then, the adaptive backstepping controller has to adjust 28 unknown parameters.

In order to verify the behavior of the controller based on Theorem 3.1, a set of simulations were presented and was found that the simulation results are in complete agreement with the theoretically expected results presented in Secs. 2 and 3. Indicate that the overall adaptive system performs quite well and the use of adaptive gains in the adaptive laws can be introduced to modify the transient behavior and the convergence to zero.

Appendix

DEFINITION OF THE MODEL PARAMETERS

Vehicle parameters in deceleration

$\alpha:$	$-C_2$
$\alpha_1:$	$\frac{r_{\text{eff}}}{J}$
$\alpha_2:$	$1 + \chi k_v (C_1 + C_3)$
$\alpha_3:$	$-k_v \chi C_3$
$\alpha_4:$	$-k_v \chi C_1 \exp(C_2)$
$\alpha_5:$	$-(1 - \Psi) M_v g \cos \varphi (\mu_{\pi} + k_v (C_1 + C_3)) \frac{r_{\text{eff}}^2}{J}$
$\alpha_6:$	$(1 - \Psi) M_v g \cos \varphi k_v C_3 \frac{r_{\text{eff}}^2}{J}$
$\alpha_7:$	$\frac{1}{2} (1 - \Psi) \rho S C_z (\mu_{\pi} + k_v (C_1 + C_3)) \frac{r_{\text{eff}}^2}{J}$
$\alpha_8:$	$-\frac{1}{2} (1 - \Psi) \rho S C_z k_v C_3 \frac{r_{\text{eff}}^2}{J}$
$\alpha_9:$	$(1 - \Psi) M_v g \cos \varphi k_v C_1 \exp(C_2) \frac{r_{\text{eff}}^2}{J}$
$\alpha_{10}:$	$-\frac{1}{2} (1 - \Psi) \rho S C_z k_v C_1 \exp(C_2) \frac{r_{\text{eff}}^2}{J}$
$\alpha_{11}:$	$-g \sin(\varphi)$
$\alpha_{12}:$	$-\frac{1}{2 M_v} \rho S C_x$
$\alpha_{13}:$	$(1 - \Psi) g \cos \varphi k_v (C_1 + C_3)$
$\alpha_{14}:$	$-(1 - \Psi) g \cos \varphi k_v C_3$
$\alpha_{15}:$	$-\frac{1}{2 M_v} (1 - \Psi) \rho S C_z k_v (C_1 + C_3)$
$\alpha_{16}:$	$\frac{1}{2 M_v} (1 - \Psi) \rho S C_z k_v C_3$
$\alpha_{17}:$	$-(1 - \Psi) g \cos \varphi k_v C_1 \exp(C_2)$
$\alpha_{18}:$	$\frac{1}{2 M_v} (1 - \Psi) \rho S C_z k_v C_1 \exp(C_2)$

Vehicle parameters in acceleration

$\alpha':$	C_2
$\alpha'_1:$	$\frac{r_{\text{eff}}}{J}$
$\alpha'_2:$	$1 + \chi k_v (C_1 - C_3)$
$\alpha'_3:$	$k_v \chi C_3$
$\alpha'_4:$	$-k_v \chi C_1 \exp(-C_2)$
$\alpha'_5:$	$-(1 - \Psi) M_v g \cos \varphi (\mu_{\pi} + k_v (C_1 - C_3)) \frac{r_{\text{eff}}^2}{J}$
$\alpha'_6:$	$-(1 - \Psi) M_v g \cos \varphi k_v C_3 \frac{r_{\text{eff}}^2}{J}$
$\alpha'_7:$	$\frac{1}{2} (1 - \Psi) \rho S C_z (\mu_{\pi} + k_v (C_1 - C_3)) \frac{r_{\text{eff}}^2}{J}$
$\alpha'_8:$	$\frac{1}{2} (1 - \Psi) \rho S C_z k_v C_3 \frac{r_{\text{eff}}^2}{J}$
$\alpha'_9:$	$(1 - \Psi) M_v g \cos \varphi k_v C_1 \exp(-C_2) \frac{r_{\text{eff}}^2}{J}$
$\alpha'_{10}:$	$-\frac{1}{2} (1 - \Psi) \rho S C_z k_v C_1 \exp(-C_2) \frac{r_{\text{eff}}^2}{J}$
$\alpha'_{11}:$	$-g \sin(\varphi)$
$\alpha'_{12}:$	$-\frac{1}{2 M_v} \rho S C_x$
$\alpha'_{13}:$	$(1 - \Psi) g \cos \varphi k_v (C_1 - C_3)$
$\alpha'_{14}:$	$(1 - \Psi) g \cos \varphi k_v C_3$
$\alpha'_{15}:$	$-\frac{1}{2 M_v} (1 - \Psi) \rho S C_z k_v (C_1 - C_3)$
$\alpha'_{16}:$	$-\frac{1}{2 M_v} (1 - \Psi) \rho S C_z k_v C_3$
$\alpha'_{17}:$	$-(1 - \Psi) g \cos \varphi k_v C_1 \exp(-C_2)$
$\alpha'_{18}:$	$\frac{1}{2 M_v} (1 - \Psi) \rho S C_z k_v C_1 \exp(-C_2)$

Definition of the coefficients $\theta'_i, \theta''_i, g'_i, g''_i, \beta'_j, \beta''_j, h'_j, h''_j; i \in \{1, \dots, 12\} j \in \{1, \dots, 15\}$

$\bullet \theta'_1 = \alpha_5$	$\bullet \theta''_1 = \alpha'_5$	$\bullet \beta'_1 = \alpha_{11} + \alpha_{12} V_a^2$	$\bullet \beta''_1 = \alpha'_{11} + \alpha'_{12} V_a^2$
$\bullet \theta'_2 = \alpha_6$	$\bullet \theta''_2 = \alpha'_6$	$\bullet \beta'_2 = \alpha_{12}$	$\bullet \beta''_2 = \alpha'_{12}$
$\bullet \theta'_3 = \alpha_7$	$\bullet \theta''_3 = \alpha'_7$	$\bullet \beta'_3 = \alpha_{12} V_a$	$\bullet \beta''_3 = \alpha'_{12} V_a$
$\bullet \theta'_4 = \alpha_7 V_a^2$	$\bullet \theta''_4 = \alpha'_7 V_a^2$	$\bullet \beta'_4 = \alpha_{13}$	$\bullet \beta''_4 = \alpha'_{13}$
$\bullet \theta'_5 = \alpha_7 V_a$	$\bullet \theta''_5 = \alpha'_7 V_a$	$\bullet \beta'_5 = \alpha_{14}$	$\bullet \beta''_5 = \alpha'_{14}$
$\bullet \theta'_6 = \alpha_8$	$\bullet \theta''_6 = \alpha'_8$	$\bullet \beta'_6 = \alpha_{15}$	$\bullet \beta''_6 = \alpha'_{15}$
$\bullet \theta'_7 = \alpha_8 V_a^2$	$\bullet \theta''_7 = \alpha'_8 V_a^2$	$\bullet \beta'_7 = \alpha_{15} V_a^2$	$\bullet \beta''_7 = \alpha'_{15} V_a^2$
$\bullet \theta'_8 = \alpha_8 V_a$	$\bullet \theta''_8 = \alpha'_8 V_a$	$\bullet \beta'_8 = \alpha_{15} V_a$	$\bullet \beta''_8 = \alpha'_{15} V_a$
$\bullet \theta'_9 = \alpha_9$	$\bullet \theta''_9 = \alpha'_9$	$\bullet \beta'_9 = \alpha_{16}$	$\bullet \beta''_9 = \alpha'_{16}$
$\bullet \theta'_{10} = \alpha_{10}$	$\bullet \theta''_{10} = \alpha'_{10}$	$\bullet \beta'_{10} = \alpha_{16} V_a^2$	$\bullet \beta''_{10} = \alpha'_{16} V_a^2$
$\bullet \theta'_{11} = \alpha_{10} V_a^2$	$\bullet \theta''_{11} = \alpha'_{10} V_a^2$	$\bullet \beta'_{11} = \alpha_{16} V_a$	$\bullet \beta''_{11} = \alpha'_{16} V_a$
$\bullet \theta'_{12} = \alpha_{10} V_a$	$\bullet \theta''_{12} = \alpha'_{10} V_a$	$\bullet \beta'_{12} = \alpha_{17}$	$\bullet \beta''_{12} = \alpha'_{17}$
		$\bullet \beta'_{13} = \alpha_{18}$	$\bullet \beta''_{13} = \alpha'_{18}$
		$\bullet \beta'_{14} = \alpha_{18} V_a^2$	$\bullet \beta''_{14} = \alpha'_{18} V_a^2$
		$\bullet \beta'_{15} = \alpha_{18} V_a$	$\bullet \beta''_{15} = \alpha'_{18} V_a$

$$\bullet g'_1(x_1, x_2) = d(x_1, x_2) = \left[\alpha_2 + \alpha_3 \frac{x_2}{x_1} + \alpha_4 \exp\left(\alpha \frac{x_2}{x_1}\right) \right]^{-1}$$

$$\bullet g'_2(x_1, x_2) = \frac{x_1}{x_2} d(x_1, x_2)$$

$$\bullet g'_3(x_1, x_2) = x_2^2 d(x_1, x_2)$$

$$\bullet g'_4(x_1, x_2) = d(x_1, x_2)$$

$$\bullet g'_5(x_1, x_2) = -2d(x_1, x_2)$$

$$\bullet g'_6(x_1, x_2) = x_1 x_2 d(x_1, x_2)$$

$$\bullet g'_7(x_1, x_2) = d(x_1, x_2) \frac{x_1}{x_2}$$

$$\bullet g'_8(x_1, x_2) = -2x_1 d(x_1, x_2)$$

$$\bullet g''_1(x_1, x_2) = d'(x_1, x_2) = \left[\alpha'_2 + \alpha'_3 \frac{x_2}{x_1} + \alpha'_4 \exp\left(\alpha' \frac{x_2}{x_1}\right) \right]^{-1}$$

$$\bullet g''_2(x_1, x_2) = \frac{x_2}{x_1} d'(x_1, x_2)$$

$$\bullet g''_3(x_1, x_2) = x_2^2 d'(x_1, x_2)$$

$$\bullet g''_4(x_1, x_2) = d'(x_1, x_2)$$

$$\bullet g''_5(x_1, x_2) = -2d'(x_1, x_2)$$

$$\bullet g''_6(x_1, x_2) = x_1 x_2 d'(x_1, x_2)$$

$$\bullet g''_7(x_1, x_2) = d'(x_1, x_2) \frac{x_2}{x_1}$$

$$\bullet g''_8(x_1, x_2) = -2x_1 d'(x_1, x_2)$$

<ul style="list-style-type: none"> • $g'_9(x_1, x_2) = d(x_1, x_2) \exp\left(\alpha \frac{x_1}{x_2}\right)$ • $g'_{10}(x_1, x_2) = x_2^2 d(x_1, x_2) \exp\left(\alpha \frac{x_1}{x_2}\right)$ • $g'_{11}(x_1, x_2) = d(x_1, x_2) \exp\left(\alpha \frac{x_1}{x_2}\right)$ • $g'_{12}(x_1, x_2) = -2d(x_1, x_2) \exp\left(\alpha \frac{x_1}{x_2}\right)$ • $h'_1(x_1, x_2) = 1$ • $h'_2(x_1, x_2) = x_2^2$ • $h'_3(x_1, x_2) = -2x_2$ • $h'_4(x_1, x_2) = d(x_1, x_2)$ • $h'_5(x_1, x_2) = d(x_1, x_2) \frac{x_1}{x_2}$ • $h'_6(x_1, x_2) = x_2^2 d(x_1, x_2)$ • $h'_7(x_1, x_2) = d(x_1, x_2)$ • $h'_8(x_1, x_2) = -2x_2 d(x_1, x_2)$ • $h'_9(x_1, x_2) = x_1 x_2 d(x_1, x_2)$ • $h'_{10}(x_1, x_2) = d(x_1, x_2) \frac{x_1}{x_2}$ • $h'_{11}(x_1, x_2) = -2x_1 d(x_1, x_2)$ • $h'_{12}(x_1, x_2) = d(x_1, x_2) \exp\left(\alpha \frac{x_1}{x_2}\right)$ • $h'_{13}(x_1, x_2) = x_2^2 d(x_1, x_2) \exp\left(\alpha \frac{x_1}{x_2}\right)$ • $h'_{14}(x_1, x_2) = d(x_1, x_2) \exp\left(\alpha \frac{x_1}{x_2}\right)$ • $h'_{15}(x_1, x_2) = -2d(x_1, x_2) \exp\left(\alpha \frac{x_1}{x_2}\right)$ 	<ul style="list-style-type: none"> • $g''_9(x_1, x_2) = d'(x_1, x_2) \exp\left(\alpha' \frac{x_2}{x_1}\right)$ • $g''_{10}(x_1, x_2) = x_2^2 d'(x_1, x_2) \exp\left(\alpha' \frac{x_2}{x_1}\right)$ • $g''_{11}(x_1, x_2) = d'(x_1, x_2) \exp\left(\alpha' \frac{x_2}{x_1}\right)$ • $g''_{12}(x_1, x_2) = -2d'(x_1, x_2) \exp\left(\alpha' \frac{x_2}{x_1}\right)$ • $h''_1(x_1, x_2) = 1$ • $h''_2(x_1, x_2) = x_2^2$ • $h''_3(x_1, x_2) = -2x_2$ • $h''_4(x_1, x_2) = d'(x_1, x_2)$ • $h''_5(x_1, x_2) = d'(x_1, x_2) \frac{x_2}{x_1}$ • $h''_6(x_1, x_2) = x_2^2 d'(x_1, x_2)$ • $h''_7(x_1, x_2) = d'(x_1, x_2)$ • $h''_8(x_1, x_2) = -2x_2 d'(x_1, x_2)$ • $h''_9(x_1, x_2) = x_1 x_2 d'(x_1, x_2)$ • $h''_{10}(x_1, x_2) = d'(x_1, x_2) \frac{x_2}{x_1}$ • $h''_{11}(x_1, x_2) = -2x_1 d'(x_1, x_2)$ • $h''_{12}(x_1, x_2) = d'(x_1, x_2) \exp\left(\alpha' \frac{x_2}{x_1}\right)$ • $h''_{13}(x_1, x_2) = x_2^2 d'(x_1, x_2) \exp\left(\alpha' \frac{x_2}{x_1}\right)$ • $h''_{14}(x_1, x_2) = d'(x_1, x_2) \exp\left(\alpha' \frac{x_2}{x_1}\right)$ • $h''_{15}(x_1, x_2) = -2d'(x_1, x_2) \exp\left(\alpha' \frac{x_2}{x_1}\right)$
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