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Spatially Variant Positron Range Modeling Derived from CT for PET Image Reconstruction

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Abstract

The influence of a finite positron annihilation distance represents a fundamental limit to the spatial resolution of PET scanners. It is appreciated that this effect is a minor concern in whole-body F18 imaging, but it does represent an issue when imaging with higher energy isotopes such as N13 or Rb82. This effect is especially relevant for imaging tasks along tissue gradients such as the cardiac/lung boundary and diaphragm/lung boundary. This work presents a method to determine the positron range effect from a CT scan and to model this effect as shift-variant, anisotropic kernels. The positron annihilation distance across boundaries of tissues is represented with a simple model, which can be quickly derived from CT scans and applied in the reconstruction of PET images. The positron range compensation map is applied in a modified OSEM algorithm to simulated and measured data.

I. Introduction

Positron range is generally accepted as a minor concern in the resolution of F18 imaging with whole-body PET systems [1]–[4]. For isotopes with higher positron energies, the influence of positron range can result in resolution degradation which is spatially variant dependent on the variations in absorber media. This effect is especially relevant for imaging tasks along tissue gradients such as the cardiac/lung boundary and diaphragm/lung boundary. This work adopts a simple model of the positron annihilation range for different density tissues and relates these variant ranges to the Hounsfield units in the CT image. The model allows for the CT image from a PET/CT exam to guide the generation of a shift-variant, anisotropic positron range compensation map. The positron range map is applied in a modified OSEM algorithm to simulated and measured data.

II. Positron Range in Uniform Media

Positron range degrades the spatial resolution in PET imaging. The majority of work modeling positron range for imaging tasks assumes that the medium is homogenous with the density of water [4]. In homogenous tissue, the influence of positron range can be modeled as a isotropic 3D kernel. We computed the maximum positron range from a formula fit to empirical beta decay data by Katz and Penfold [5]:

$$R_{max} \left[\text{mg/cm}^2 \right] = \begin{cases} 412E^{1.265-0.0954\ln E} & 0.01 \leq E \leq 2.5 \text{ MeV} \\ 530E - 106 & 2.5 \text{ MeV} < E < 20 \text{ MeV} \end{cases} \quad (1)$$

where the energy of the positron, E , is in MeV and the maximum range, R_{max} is independent of material. The evaluation of (1) over the positron energy spectra for common isotopes lead to 1D annihilation probability density functions presented in figure 1. The mean of these range distributions are in keeping with published data as presented in table I.

The Katz and Penfold empirical equation is based on the theory that the ability to stop beta particles depends primarily on the density of electrons in the media (density of the media in mg/cm^3). Using (1) and positron energy spectra, we computed the average positron annihilation distance as a function of absorber density as shown in figure 2.

To test the validity of the model, we compared the model with simulations using GEANT4 in the GATE simulation package [9]. GEANT4 uses a multiple scattering model based on Lewis's theory [10], and we performed condensed simulations of the transport of positrons through matter. Figure 3 presents the GEANT4 and the proposed model 1D annihilation distance in terms of the fraction of the number of positrons annihilated versus distance for different media. Comparison of curves shows agreement between the simulations and the Katz & Penfold empirical decay.

The previous discussion has focused on the 1D annihilation probability: the probability that a positron will annihilate at a distance, d , from the site of decay. To form the positron range blurring kernels, we determine the 3D annihilation probability: the probability that a positron will annihilate at a location $(x, y, Z)T$ when it originated at the site $(x, y, z)o$. The 3D annihilation density is formed by normalizing the 1D annihilation density by the surface area of the sphere at each location. Figure 4 compares the 3D annihilation probability density of the GEANT4 simulations and the proposed use of Katz & Penfold's eq(1).

Derenzo [3] proposed that these 3D annihilation probabilities can be fitted by the sum of two exponentials,

$$P(x) = Ce^{-k_1x} + (1 - C)e^{-k_2x} \quad (2)$$

In an effort to model the positron annihilation density for different absorbers, we fit this dual exponential function to the annihilation probability density derived from the Katz & Penfold empirical equation for a range of materials.

We define the absorbing material with the CT image acquired during a PET/CT exam. Considering that CT imaging is non-quantitative, there is no direct link between CT number (Hounsfield units) and tissue density (mg/cm^3). We assume that the bilinear/trilinear scaling methods which convert the CT image to linear attenuation coefficients at 511keV ($\mu @ 511\text{keV}$) for conventional PET attenuation correction are accurate (This assumption generally holds for physiologic tissues) [11]. The mass attenuation coefficients ($\mu/\text{density}$) at 511keV of tissues of interest are all close to $0.096 \text{ cm}^2/\text{g}$. Dividing the $\mu @ 511\text{keV}$ map

(l/cm) by 0.096 cm²/g provides an approximate map of the density of the tissues. Figure 5 provides the values of the parameters (k_1 , k_2 , C) that fit the annihilation probability densities as a function of an absorber's linear attenuation coefficient at 511keV. This figure provides a lookup table of the anticipated annihilation density for a given PET attenuation value.

III. Positron Range in Non-uniform Media

In non-homogenous tissue, the positron range could be modeled as anisotropic kernels which ideally would be derived from Monte Carlo simulations for each patient's tissue distribution. The Monte Carlo approach is generally too computationally intensive for clinical PET imaging. A primary challenge of quickly determining the range effect in non-homogenous media is how to model the range across boundaries.

Bai *et al.* have explored modeling positron range in non-homogenous materials for a small animal system [6]. They compensated for variant positron ranges by either A) anisotropically truncating an isotropic point probability density function dependent on tissue type, or B) performing successive convolution operations of tissue dependent range kernels to determine range models across tissue boundaries.

In this work we adopt a crude, but fast, model to adjust for variations across boundaries. We perform a average of the fitted parameters (from figure 5) of annihilation densities for the originating voxel and the target voxel. This average defines a new dual-exponential function to describe the probability that an positron annihilated in a target voxel. Specifically, to determine the positron range blurring kernel for originating voxel, x_o , we define its contributions to a target voxel x_t as

$$\hat{P}_{ot} = \bar{C}_{ot} e^{-\bar{k}_{1_{ot}} d_{ot}} + \left(1 - \bar{C}_{ot}\right) e^{-\bar{k}_{2_{ot}} d_{ot}} \quad (3)$$

where d_{ot} is the distance from voxel x_o to voxel x_t , $\bar{C}_{ot}$ is the average of the C parameters from voxels x_o and x_t , and $\bar{k}_{[1,2]_{ot}}$ is the average of the k parameters from voxels x_o and x_t . The 3D positron blurring kernel is evaluated for each originating voxel over all potential target voxels. All of the kernels are normalize to have density equal to 1 leading to $\tilde{P}$ that provides an estimate of the positron range blurring for each pixel. Figure 6 presents the blurring kernels $\tilde{P}_{o*}$ for a decay of Rb82 in water next to a bone boundary and a lung boundary.

IV. Methods

A. Application to PET/CT Reconstruction

For a patient specific attenuation map, the positron blurring kernel can be calculated according to eq (3). This kernel is an obvious approximation based on the probability of positron annihilation in any given voxel being drawn from the average probability density functions of the source and target voxel. This approximation allows for a fast computation of the blurring kernel from an arbitrary 511keV attenuation map. We store the entire blurring map in a sparse matrix and apply it in a modified OSEM reconstruction such that the mean

of the Poisson distributed PET measurements, y , are given by $\bar{y} = P_{geo} \tilde{P} x$, where P_{geo} is the geometric projection matrix, $\tilde{P}$ is the proposed blurring kernels, and x is the image.

B. GATE Simulations

We applied the proposed positron range compensation to the reconstruction of data from GATE simulations. A phantom presented in figure 7 with lung, bone, and water regions was simulated to generate Rb82, 2D PET data for a cylindrical, clinical PET system (70cm transaxial FOV, 15cm axial FOV).

C. Patient Data

We applied the proposed method to reconstruct patient data from a clinical Rb82 cardiac PET/CT exam. We converted the patient CT image to the PET attenuation map for 511keV photons. The map guided the generation of unique blurring kernels, with a radius of 2cm, for each voxel location.

V. Results

The reconstructed images of the GATE simulated data appear in figure 8. The use of the positron range compensation yielded sharper boundary definition with little to no tails at the edge of the water lung boundary. On the negative side, in the absence of post-reconstruction smoothing, the positron range compensation does lead to hyper-resolved boundaries.

The images from the patient data are presented in figure 9. The reconstruction parameters (number of iterations and postrecon smoothing) were set to have visually similar resolution (as shown in profile). With “matched” resolution, the positron range method appears to have reduced noise.

VI. Conclusion

We presented a new method to link the CT image with an anticipated positron range effect. The method allows for the rapid estimation of a range blurring compensation map which can be applied in a modified OSEM algorithm. We presented initial results of the positron range compensation with simulated and measured Rb82 cardiac PET/CT data which suggest improved resolution/noise properties with the proposed approach. A thorough image quality evaluation of the proposed method is needed to quantify the benefit for cardiac PET imaging.

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Fig. 1. Positron 1D annihilation density in water based on positron spectra and eq (1).

Fig. 2.

Average positron range in media of different density based on eq (1).

Fig. 3. Fraction of positrons annihilated within fixed distances derived from GEANT4 simulations and proposed empirical model using Katz & Penfold.

Fig. 4.

Comparison of 3D annihilation density functions of the GEANT4 simulations and proposed empirical model for Rb82.

Fig. 5. Positron annihilation fitted parameters as a function of material as defined by its linear attenuation coefficient of 511keV photons.

Fig. 6.

Kernel used for decay of Rb82 in a pixel in water (+) next to the boundary of lung (left half of plot), rib bone (upper right quadrant), and water (lower right quadrant). The FWTM contour is plotted as the 2nd contour from center (green) and the FW at 200th maximum is the most extreme contour plotted.

Fig. 7.

Phantom used in GATE simulations with rectangular air, lung, bone and water regions. Uniform activity was simulated within each dotted region in the central water region to mimic activity in the myocardium next to water, lung, and bone. The right activity region contained twice the activity concentration as the left region. Colorbar presents density values (g/cm^3).

Fig. 8.

Images reconstructed from GATE simulations of two hot sources in water next to lungs and bone. Image A is reconstructed with conventional OSEM; Image B is reconstructed with proposed positron range kernel. Horizontal profiles (left) and vertical profiles through lung-water-lung regions (right) highlight hyper-resolved boundaries with positron range model (red).

Fig. 9.

Transaxial slice through patient images of Rb82 with reconstruction by 2D OSEM (left) and 2D OSEM with proposed positron range kernel (right). Profile through right and left ventricle appears to have matched boundary definition with reduced noise in reconstruction with positron range (yellow).

TABLE 1

Comparison of average positron range estimates (mm) in water

Isotope	Proposed	Bai [6]	Champion [7]	Partridge [8]
F18	0.61	0.51	0.66	0.6
N13	1.56	1.31	1.73	1.5
Rb82	6.57	-	7.49	5.9