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Statistical analysis of quiet stance sway in 2-D

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Abstract

Subjects exposed to a rotating environment that perturbs their postural sway show adaptive changes in their voluntary spatially directed postural motion to restore accurate movement paths but do not exhibit any obvious learning during passive stance. We have found, however, that a variable known to characterize the degree of stochasticity in quiet stance can also reveal subtle learning phenomena in passive stance. We extended Chow and Collins (Phys Rev E 52(1):909–912, 1995) one-dimensional pinned-polymer model (PPM) to two dimensions (2-D) and then evaluated the model's ability to make analytical predictions for 2-D quiet stance. To test the model, we tracked center of mass and centers of foot pressures, and compared and contrasted stance sway for the anterior–posterior versus medio-lateral directions before, during, and after exposure to rotation at 10 rpm. Sway of the body during rotation generated Coriolis forces that acted perpendicular to the direction of sway. We found significant adaptive changes for three characteristic features of the mean square displacement (MSD) function: the exponent of the power law defined at short time scales, the proportionality constant of the power law, and the saturation plateau value defined at longer time scales. The exponent of the power law of MSD at a short time scale lies within the bounds predicted by the 2-D PPM. The change in MSD during exposure to rotation also had a power-law exponent in the range predicted by the theoretical model. We discuss the Coriolis force paradigm for studying postural and movement control and the applicability of the PPM model in 2-D for studying postural adaptation.

Keywords

Adaptation; Coriolis force; Posture; Pinned polymer; Stabilogram; Stochastic

Introduction

During quiet upright stance, the body shows low-amplitude oscillatory behavior known as postural sway. Because of its latent “random-walk like” nature, passive postural sway is best characterized with variables used to measure stochastically driven phenomena. One such method is stabilogram diffusion analysis. The stabilogram diffusion function (SDF) is the mean square displacement (MSD) of the center of pressure (CoP) as a function of the time

interval between CoP comparisons (Gurfinkel 1973; Collins and De Luca 1993; Newell et al. 1997; Dijkstra 2000; Peterka 2000; Duarte and Zatsiorsky 2001; Riley and Turvey 2002). The SDF provides statistical measures that are robust to the non-stationary behavior of the CoP (Ferdjallah et al. 1999). Its parameters have been shown to be sensitive to vision (Collins and De Luca 1995; Riley et al. 1997a), touch (Riley et al. 1997b), gravito-inertial force magnitude (Collins et al. 1995b), age (Collins et al. 1995a), and disease state (Mitchell et al. 1995).

The SDF of a classical random walk shows a linear increase in value as a function of the time interval between comparisons and is characterized by the diffusion coefficient, D , which is equal to half the slope of the SDF. For a more general random walk process, the SDF has the form of a power function with the SDF value proportional to the time interval between comparisons raised to an exponent. Having identified multiple correlated random walk structures in the CoP signals of postural sway over different time scales, Collins and De Luca (1993) proposed an underlying nonlinear control mechanism that is active at different time scales. At short intervals ($\tau < 1$ s), they found that CoP excursions resembled simple drift, with its path being over-correlated compared to a classical random walk, suggesting an uncontrolled open-loop operation for the postural system. At intermediate time scales, the CoP excursions more closely resembled a diffusive, classical random walk, or Brownian motion, with past and future displacements uncorrelated. At longer time scales ($\tau \gg 1$ s), the CoP traces were anticorrelated, which Collins and DeLuca interpreted as reflecting active closed-loop feedback control that ensures the CoP remains localized within a bounded area. Passive sway thus could have a deterministic but involuntary component, a stochastic component, and a voluntary control component.

A subject standing on the axis of rotation in a rotating reference frame experiences a movement-contingent Coriolis force during body sway, $\vec{F}_{\text{Coriolis}} = -2m(\vec{\Omega} \times \vec{v})$, where m is the mass of the moving object, $\vec{\Omega}$ is the angular velocity of the rotating environment, and $\vec{v}$ is the linear velocity of the object relative to the rotating frame. The Coriolis force is orthogonal to the object's velocity direction and has the unique feature of not involving mechanical contact with the body unlike mechanical or robotic devices for delivering perturbations, which directly touch the body activating receptors that provide information about force magnitude and direction. It is under the indirect control of the subject in that the magnitude and direction of the Coriolis perturbation are contingent upon body sway velocity. If the eyes are closed, the sources from which information about the perturbation can be gained are (a) the vestibular receptors, (b) proprioceptive receptors including muscle spindles and golgi tendon organs, (c) sensory receptors in the soles of the feet, and (d) efference copy signals associated with voluntary sway movements (Lackner and DiZio 1994, 2002).

Typically, when voluntary goal-oriented movements are made during exposure to Coriolis force perturbations in a rotating room, adaptive accommodations occur (Soeda et al. 2003; Lackner and DiZio 1994, 2003; Kurtzer et al. 2005). Movements are initially deviated but over repeated trials become less deviated. However, for quiet stance sway in the rotating room—which is also perturbed by Coriolis forces—little evidence for adaptive learning has been demonstrated. It could be that: (1) no postural learning occurs in quiet stance, (2)

involuntary sway velocities are so small in magnitude that the orthogonal Coriolis perturbation is below sensory threshold, (3) quiet stance provides no well-defined goal-oriented task, for which a performance error can be estimated, and hence, there is no learning behavior in contrast to a goal-oriented voluntary sway task where sensory-motor systems can estimate the error, (4) learning behavior is not observable in the background stochasticity of the system, or (5) we are dealing with the wrong variables for evidence of adaptive changes. Our goal was to see whether a variable could be identified that would reflect learning behavior in Coriolis-perturbed quiet stance. Such a factor might help distinguish the stochastic, deterministic, voluntary, and involuntary components of postural sway.

We took as our starting point Chow and Collins (1995) stochastic continuum pinned-polymer model (PPM) of spontaneous postural sway. The PPM is an abstract mathematical mapping that has similar constraints and dynamics to those observed in CoP trajectories of spontaneous postural sway. It has the great advantage of making such sway amenable to quantitative modeling. Postural sway in a rotating room generates Coriolis perturbations transverse to sway directions and hence represents a 2-D test scenario. Consequently, to accommodate the Coriolis perturbation factor, it was necessary to extend Chow and Collins' 1-D PPM mapping to an equivalent 2-D PPM. This required adding some additional features to the PPM, which are described in the modeling section below.

Materials and methods

Subjects

Five healthy, adult subjects (age range: 20–65) participated after giving informed consent to a protocol approved by the Brandeis Committee for the Protection of Human Subjects. They were without vestibular or sensory-motor impairments that could have influenced their performance.

Apparatus, experimental task, and conditions

We measured the reaction forces at the feet and the center of mass (CoM) and CoP trajectories of subjects passively standing under both stationary and rotation conditions.

Subjects were tested in the Graybiel Laboratory slow rotation room (SRR), a circular windowless room 7.1 m in diameter. They stood on an AMTI Accusway force plate in the center of the SRR at the axis of rotation. The device measured the three components of the ground reaction forces and their moments exerted under each foot, sampled at 100 Hz. The subjects, stationed at the center of the SRR, did not experience any significant centrifugal force during rotation. However, they did experience a Coriolis force contingent upon the velocity and direction of their CoM motion that was estimated by monitoring the position of infrared emitting diodes (IREDs) attached to the shoulder, sternum, waist, hip, knee, and ankle using an Optotrak™ motion analysis system (Northern Digital, Inc.). The IRED positions were sampled at 100 Hz with respect to a calibrated reference point in 3-D space with a spatial accuracy better than 0.5 mm³.

Subjects stood shoeless on the force platform, with feet side-by-side (ankle to ankle distance ≈ 20 cm) and arms by their sides. They were asked to stand passively with eyes closed. Trials were 25 s in duration. Data were collected for 3 experimental conditions: (1) Pre-rotation (Pre), the subjects performed 4 baseline trials in the stationary SRR, (2) Per-rotation (Per), the SRR was accelerated at $1^\circ/\text{s}^2$ to 10 rpm, and after 2 min at constant velocity subjects performed 20 quiet stance trials, (3) Post-rotation (Post), 2 min after the room was stopped the subject performed 10 quiet stance trials. We label the first two trials in the Per-condition as Per-initial, and the last two as Per-final. Similarly, the first two trials in the Post-condition are labeled as Post-initial, and the last two as Post-final.

Data analysis

CoP components—From the forces and moments directly measured by the force plate, we determined the antero-posterior (AP) and medio-lateral (ML) horizontal components of the CoP trajectory:

$$\text{CoP}^{\text{AP}} = \frac{-M^{\text{ML}}}{F^{\text{VERT}}} + \text{OO}^{\text{AP}} \text{ and } \text{CoP}^{\text{ML}} = \frac{M^{\text{AP}}}{F^{\text{VERT}}} + \text{OO}^{\text{ML}}, \quad (1)$$

where $\mathbf{M} = [M^{\text{AP}} \ M^{\text{ML}} \ M^{\text{VERT}}]$ represents the moments about the origin of the platform, $\mathbf{F} = [F^{\text{AP}} \ F^{\text{ML}} \ F^{\text{VERT}}]$ represents the ground reaction forces (all three components of $\mathbf{M}$ and $\mathbf{F}$ are directly measured by the force plates), and OO represents coordinate transformation constants that transform the platform centered coordinate system to an ankle centered one. By convention, we use positive signs for sway in the forward and in the leftward directions.

Two point correlation function of quiet stance CoP—The SDF is the MSD of CoP as a function of the time interval between CoP comparisons. The two-point correlation function is given by $C(t_2 - t_1) = \langle [q(t_2) - q(t_1)]^2 \rangle$, $q \in [x, y]$, where $x(t)$ and $y(t)$ are the positions of the ML and AP components of the CoP, respectively, at time t . We calculated the $C(\tau)$ function for all the conditions (Pre, Per_{initial}, Per_{final}, Post_{initial}, and Post_{final}) and used logarithmic plots to extract the slopes. We tested the scaling of $C(\tau)$ as $\tau \rightarrow 0$ and compared it with the scalings estimated by the model described below.

Cross-spectral coherence—If $P_{\text{AP}}(f)$ and $P_{\text{ML}}(f)$ are the power spectral densities of AP and ML sway, and $P_{\text{AP ML}}(f)$ the cross-power spectral density, then the magnitude squared coherence between AP and ML sway, $C_{\text{AP ML}}(f)$ is estimated using:

$$C_{\text{AP ML}}(f) = \frac{|P_{\text{AP ML}}(f)|^2}{P_{\text{AP}}(f)P_{\text{ML}}(f)}$$

A periodic hamming window, dividing the AP and ML data into 8 equal overlapping (50 %) sections, was used to compute the spectral densities.

Modeling: pinned string model for quiet stance

The CoP trajectory during quiet stance resembles a confined random walk. Chow and Collins (1995) likened its dynamics to that of a point on a string under tension pinned to a

stable position by an elastic force, a 1-D pinned-polymer model. An orthogonal Coriolis force is generated by sway in our test situation involving rotation. Consequently, we needed a 2-D model with cross-coupling between the two directions in the dynamical equations to represent the resulting behavior. Upright quiet stance has an inverted pendulum-like instability that is countered by a control system, the dominant component of which can be modeled as stochastic forcing. Under the assumptions that the dynamics of the motion of the CoP can be effectively mimicked by the dynamics of a particular point along the string, consider a flexible string under tension T , and nearly aligned along the vertical $\hat{z}$ direction, embedded in an elastic sheet or placed in a visco-elastic bath. We are interested in modeling the dynamics of any point on the string in the 2-D plane orthogonal to $\hat{z}$, as a function of the height variable z and time t . If the string is nearly aligned with the z direction, then z can be assumed to be the variable running along its length (Fig. 1). Let the point P be at distance $z = z_0$ along the length of the string. We assume that the total mass m is uniformly distributed along the string, the mass density given by ρ . Further, assume that the boundary effects are negligible or that the string is long enough such that boundaries can be considered far away from the point P of our interest. A piece of the string of unit length around point P will be acted on by an elastic restoring force with constant K , a viscous drag force with coefficient μ , and a stochastic control force modeled by stochastic fluctuations $\eta(z, t)$ provided by the bath. We can determine the two-point correlation curve given by the mean square displacement $C(t_2 - t_1) = \langle [q(z_0, t_2) - q(z_0, t_1)]^2 \rangle$, where $q(t)$ is the position of one of the orthogonal components in the plane orthogonal to $\hat{z}$. Postural sway can be characterized as a bounded, correlated random walk (Collins and Deluca 1993; Newell et al. 1997; Riley and Turvey 2002), with the positive slope of $C(\tau)$ at $\tau < 1$ s indicating positively correlated activity and at longer time scales showing negatively correlated activity.

If x and y are small displacements of point P in the horizontal plane, orthogonal to the z direction, and $\vec{\Omega} = [0 \ 0 \ \Omega]$ is the rotational velocity of the entire system, $\vec{v}$ the relative velocity in the rotational frame of reference, then the Coriolis force per unit length around point P is given by:

$$\begin{aligned}
 \vec{F}_{\text{Coriolis}} &= -2\rho \left(\vec{\Omega} \right. \\
 &\quad \times \vec{v} = -2\rho\Omega(\hat{z} \\
 &\quad \times \vec{v}) \equiv -2\rho\Omega \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & 1 \\ \partial_t x & \partial_t y & \partial_t z \end{vmatrix} = -2\rho\Omega \{-\hat{x}\partial_t y - \hat{y}(-\partial_t x) + 0\hat{z}\} \\
 &= 2\rho\Omega(\hat{x}\partial_t y \\
 &\quad - \hat{y}\partial_t x) \\
 &= 2\rho\Omega[\partial_t y \\
 &\quad - \partial_t x 0] = 2\rho\Omega [\Theta_x \partial_t y \Theta_y \partial_t x 0],
 \end{aligned} \tag{2}$$

where $\Theta = [1 \ -1 \ 0]$

In the continuum limit, the equations of motion will be:

$$\rho \partial_t^2 y + \mu \partial_t y = T \partial_z^2 y - K y + \eta_y(z, t) + 2\Theta_y \rho \Omega \partial_t x \quad \rho \partial_t^2 x + \mu \partial_t x = T \partial_z^2 x - K x + \eta_x(z, t) + 2\Theta_x \rho \Omega \partial_t y \quad (3)$$

For quiet stance, these equations allow the possibility of asymmetric control along the AP versus ML direction. We divide by μ to reduce the equations to a more convenient form (cf. Chow and Collins 1995),

$$\beta \partial_t^2 y + \partial_t y = \nu \partial_z^2 y - \alpha y + \eta_y(z, t) + \Theta_y \xi \partial_t x \quad \beta \partial_t^2 x + \partial_t x = \nu \partial_z^2 x - \alpha x + \eta_x(z, t) + \Theta_x \xi \partial_t y \quad (4)$$

where $\beta = \rho/\mu$, $\nu = T/\mu$, $\alpha = K/\mu$, $\xi = 2\beta\Omega$, and η absorbs the upfront μ^{-1} .

We Fourier transform these equations in space and time using:

$$y(z, t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-2\pi i(uz + \nu t)} y_{u, \nu} du d\nu \text{ and } \eta_y(z, t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-2\pi i(uz + \nu t)} \eta_{u, \nu}^y du d\nu \quad (5)$$

and similar equations for $x(z, t)$, and $\eta_x(z, t)$.

For the k th spatial and the ω th temporal Fourier mode, we obtain

$$(-\lambda_{k, \omega} - i\omega) y_{k, \omega} + i\omega \Theta_y \xi x_{k, \omega} = \eta_{k, \omega}^y \text{ and } (-\lambda_{k, \omega} - i\omega) x_{k, \omega} + i\omega \Theta_x \xi y_{k, \omega} = \eta_{k, \omega}^x \quad (6)$$

where

$$\lambda_{k, \omega} = \beta \omega^2 - \nu k^2 - \alpha \text{ and } \xi = 2\beta\Omega. \quad (7)$$

Solving for $y_{k, \omega}$, $x_{k, \omega}$:

$$y_{k, \omega} = \frac{(-\lambda_{k, \omega} - i\omega) \eta_{k, \omega}^y - i\omega \Theta_y \xi \eta_{k, \omega}^x}{(-\lambda_{k, \omega} - i\omega)^2 + \Theta_x \Theta_y \omega^2 \xi^2} \text{ and } x_{k, \omega} = \frac{(-\lambda_{k, \omega} - i\omega) \eta_{k, \omega}^x - i\omega \Theta_x \xi \eta_{k, \omega}^y}{(-\lambda_{k, \omega} - i\omega)^2 + \Theta_x \Theta_y \omega^2 \xi^2} \quad (8)$$

Henceforth, we analyze the y (AP) direction, but similar equations can be derived for the x (ML) direction.

We define:

$$\psi = 1 - \Theta_x \Theta_y \xi^2 \quad (9)$$

Applying Eq. (9) in Eq. (8) gives:

$$y_{k, \omega} = \frac{(-\lambda_{k, \omega} - i\omega) \eta_{k, \omega}^y - i\omega \Theta_y \xi \eta_{k, \omega}^x}{\lambda_{k, \omega}^2 - \omega^2 \psi + 2i\omega \lambda_{k, \omega}} \quad (10)$$

Following Chow and Collins (1995), we assume stochastic forcing that is uncorrelated along z , the length of the string, but has a temporal correlation given by $D(\tau)$. For simplicity, we assume a case where:

(1) the functions η^x and η^y that perturb sway in the ML and AP directions are independent, and have no cross-correlations in the two orthogonal directions, i.e., $\langle \eta^x \eta^y \rangle = 0$; and (2) the statistics based on η^x and η^y are symmetric about the two directions, i.e., they have the same stochastic properties with the same autocorrelation function, $\langle \eta^x \eta^x \rangle = \langle \eta^y \eta^y \rangle$. These assumptions can be summarized by the equation:

$$\langle \eta_i(z', t') \eta_j(z, t) \rangle = D(t' - t) \delta(z' - z) \delta_{ij}, \quad (11)$$

where i, j refers to the two directions. In the Fourier space, this correlation can be written as:

$$\langle \eta_{k,\omega}^i \eta_{k',\omega'}^j \rangle = N(\omega; p) \delta(\omega' + \omega) \delta(k' + k) \delta_{ij}, \quad (12)$$

where $N(\omega; p)$ is a function of the Fourier frequency ω and a set of other noise parameters represented by p . We will choose a more exact form of $N(\omega; p)$ later.

Using Eqs. 10 and 12, we can now determine the autocorrelation function to be:

$$\langle y_{k,\omega} y_{k',\omega'} \rangle = N(\omega; p) \delta(\omega' + \omega) \delta(k' + k) [f_{k,\omega} f_{k',\omega'} + g_{k,\omega} g_{k',\omega'}], \quad (13)$$

$$\text{where } f_{k,\omega} = \frac{-(\lambda_{k,\omega} + i\omega)}{\lambda_{k,\omega}^2 - \omega^2 \psi + 2i\omega \lambda_{k,\omega}} \text{ and } g_{k,\omega} = \frac{-i\omega \Theta_y \xi}{\lambda_{k,\omega}^2 - \omega^2 \psi + 2i\omega \lambda_{k,\omega}} \quad (14)$$

To find the single-point autocorrelation function of point P at $z = z_0$, i.e., $S(\omega) = \langle y_{z_0,\omega} y_{z_0,\omega} \rangle$, we first inverse Fourier transform $\langle y_{k,\omega} y_{k',\omega'} \rangle$ in ω' , k' , and k , and use the identity for delta functions, $\int_{-\infty}^{\infty} du f(u) \delta(u+a) = f(-a)$, while integrating over ω' and k' , to obtain:

$$\langle y(z, \omega) y(z', \omega') \rangle = N(\omega; p) \int \frac{dk}{2\pi} e^{ik(z'-z)} [f_{k,\omega} f_{-k,-\omega} + g_{k,\omega} g_{-k,-\omega}] \quad (15)$$

where the integral limits go from $-\infty$ to ∞ . From Eq. (7), $\lambda_{k,\omega} = \beta\omega^2 - \nu k^2 - \alpha$ is an even function in ω and k , i.e., $\lambda_{-k,-\omega} = \lambda_{k,\omega}$. Then from Eq. (14):

$$f_{-k,-\omega} = \frac{-(\lambda_{k,\omega} - i\omega)}{\lambda_{k,\omega}^2 - \omega^2 \psi - 2i\omega \lambda_{k,\omega}} = f_{k,\omega}^* \quad (16)$$

where f^* is the complex conjugate of f . Similarly, we can show $g_{-k,-\omega} = g_{k,\omega}^*$. Then:

$$f_{k,\omega} f_{-k,-\omega} = \frac{\lambda_{k,\omega}^2 + \omega^2}{(\lambda_{k,\omega}^2 - \omega^2 \psi)^2 + 4\omega^2 \lambda_{k,\omega}^2} \text{ and } g_{k,\omega} g_{-k,-\omega} = \frac{\omega^2 (\Theta_y \xi)^2}{(\lambda_{k,\omega}^2 - \omega^2 \psi)^2 - 4\omega^2 \lambda_{k,\omega}^2} \quad (17)$$

Using Eq. (17), the single-point temporal correlation (S) at a point $z = z' = z_o$, entered in Eq. (15), is then given by:

$$S_{\text{ROT}}(\omega) \equiv y(z_o, \omega)y(z_o, \omega) = N(\omega; p) \int \frac{dk}{2\pi} \frac{\lambda_{k,\omega}^2 + \omega^2(1 + \Theta_y^2 \xi^2)}{(\lambda_{k,\omega}^2 - \omega^2 \psi)^2 + 4\omega^2 \lambda_{k,\omega}^2} \quad (18)$$

Since we have $\Theta = [1 \ -1 \ 0]$, as defined below Eq. (2), this implies $\Theta_y^2 = 1$, $\Theta_x \Theta_y = -1$, and from our definition in Eq. (9), $\psi = 1 + \xi^2$. Then:

$$S_{\text{ROT}}(\omega) = N(\omega; p) \int \frac{dk}{2\pi} \frac{\lambda_{k,\omega}^2 + \omega^2 \psi}{(\lambda_{k,\omega}^2 - \omega^2 \psi)^2 + 4\omega^2 \lambda_{k,\omega}^2} \quad (19)$$

We can relate the two-point correlation function $C(\tau)$, given by the MSD, to the single-point temporal autocorrelation function $S(\tau)$ obtained by inverse Fourier transforming Eq. (19).

The two-point correlation function is given by:

$$C_{\text{ROT}}(\tau) = 2[S_{\text{ROT}}(0) - S_{\text{ROT}}(\tau)] \quad (20)$$

Special case In the absence of a Coriolis force, in a stationary room, $\Omega = 0$, which makes $\xi = 0$ and thus $\psi = 1$. The denominator in the integral in Eq. 19 above is reduced to the form

$(\lambda_{k,\omega}^2 + \omega^2)^2$. These give:

$$S_{\Omega=0}(\omega) = N(\omega; p) \int \frac{dk}{2\pi} \frac{1}{(\lambda_{k,\omega}^2 + \omega^2)} \quad (21)$$

Eq. (21) corresponds to the result obtained by Chow and Collins for one dimension, where $\Omega = 0$.

The two-point correlation is given by:

$$C_0(\tau) = 2[S_{\Omega=0}(0) - S_{\Omega=0}(\tau)] \quad (22)$$

For the case where $\Omega \neq 0$, Eq. (19) is the new generalized result, where the extra term $\psi = 1 + \xi^2$ is the result of the 2-D Coriolis coupling in the dynamical equations when the environment is rotating. In the absence of a Coriolis force, since $\xi = 0$ for $\Omega = 0$, $S_{\text{ROT}}(\omega) = S_{\Omega=0}(\omega)$. The form of Eq. (19) is contingent on assuming symmetry of stochastic control in the AP and ML directions. In the absence of symmetry, it will have a more complicated form.

Following Chow and Collins (1995), we assume noise to have short time correlations that arise from “neuromuscular effects, such as intrinsic muscle force fluctuations and/or feedback-loop delays”. We therefore assume the function $N(\omega; p)$ in the Fourier space is such that for short time scales, where ω is much larger than a certain threshold $\delta(\omega \gg \delta)$,

we have a power law $\propto \omega^{-2\gamma}$ with $0 < 2\gamma < 1$; and for long time scales, where ω is much smaller than the threshold ($\omega \ll \delta$), the forcing is uncorrelated. Thus, with two parameters of threshold and the power law exponent ($p = \{\delta, \gamma\}$), $N(\omega; p)$ can be approximated by the form:

$$N(\omega; \delta, \gamma) \propto \frac{\delta^{2\gamma}}{(\omega^2 + \delta^2)^\gamma} \quad (23)$$

Using Eqs. (19, 21), in the limit $\omega \rightarrow \infty$, $S_{\text{ROT}} \& S_{\Omega=0} \propto \omega^{-2(1+\gamma)}$ (see “Appendix” Eqs. 39, 65). Both terms give the same inverse power in ω . In the time domain, they yield (see “Appendix” Eq. 66):

$$S(\tau) \propto \tau^{1+2\gamma} \quad (24)$$

Thus in the $\tau \rightarrow 0$ limit, the scaling of the correlation function in the presence of a Coriolis force field is the same as that in its absence. If C_{rot} and C_o are the correlation functions defined for rotating and stationary reference frames, respectively, then the residual $C_{\text{rot}} - C_o$ should have the same scaling as the individual correlation functions. We note that our scaling law in the frequency domain, in the absence of a rotation induced forced field, matches exactly with Chow and Collins (1995); however, the corresponding power in the time domain is $1 + 2\gamma$ rather than $1 + \gamma$ as obtained by Chow and Collins using dimensional arguments. The “Appendix” provides further detailed justification and explanation of the mathematical underpinnings.

The Coriolis force has the same properties as the Lorentz force when a rotational angular velocity vector is represented as an effective magnetic field ($B_{\text{eff}} = \Omega$), and mass is interpreted as an effective charge ($q_{\text{eff}} = 2m$) on the polymer (see Fig. 1). For mass density ρ , the charge density will then be $q_{\text{eff}} = 2\rho$. So, to model quiet stance in a rotating room, the necessary additions to the PPM are to place a charge of magnitude equal to twice the mass on the polymer distributed along the string and to place the bath in a magnetic field equal in magnitude to the room’s rotational angular velocity and in a direction orthogonal to the plane of particle motion. This charged PPM in a magnetic field can model involuntary postural sway in the rotating room in the same way that Chow and Collins (1995) PPM maps spontaneous sway in a stationary environment. We thus have the analytic tools and the experimental environment to explore the nature of passive postural sway.

Results

Two-point correlation function

The stabilogram plot in Fig. 2a, where the top panel is anterior–posterior (AP) and the bottom panel is mediolateral (ML), shows the two-point correlation function across all conditions over lag times ranging up to 10 s. The pre-rotation profile exhibits an initial rise for small time scales, an enlargement of which is shown in Fig. 2b for $0 \leq \tau \leq 400$ ms. For longer time scales, the functions level off to plateaus for all except the initial per-condition (at $\tau > 1$ s in Fig. 2a). Figure 2c shows the same information as Fig. 2a, but plotted on a log–

log scale. The linearity observed at $\tau \ll 1$ s verifies that the initial rise of the two-point correlation shows power law dependence over the time lag parameter. The onset of rotation increases the MSDs. By the final per-rotation trial, the function decreases in the AP direction (top panels in Figs. 2 and 3) but is still not the same as during the baseline period. Post-rotation, a residual effect in the correlation function remains even after the last post-rotation trial, the $C(\tau)$ traces do not completely match those of the baseline pre-rotation condition. However, the post-initial and post-final traces almost overlap. In the ML direction (bottom panels in Figs. 2 and 3) by the final per-rotation trial, the correlation function decreases but is still not the same as during the baseline period. When rotation is stopped for short time scales ($\tau < 1$ s), a residual effect in the correlation function remains even after the last post-rotation trial. However, for long time scales ($\tau > 1$ s), the $C(\tau)$ trace for the initial post-rotation time not only does not match those of the baseline pre-rotation condition, but goes below it. The post-final trace almost overlaps with the post-initial trace for short time scales, but diverges over time scales >1 s. In fact, by the final post-rotation trial, long time scale correlation does not merely show a gradual return to baseline level, but leaves a positive residual. These results contrast with our earlier findings for voluntary sway where we observed a return to baseline pattern by the final per-rotation trial (Bakshi 2009).

We performed repeated measures MANOVA on three characterizing variables in the logarithmic two-point correlation function plots for the five conditions (pre-rotation, initial per-rotation, final per-rotation, initial post-rotation, and final post-rotation). The first variable was the slope of the linear portion of the log-log plot up to 100 ms (i.e., 10^{-2} s τ 10^{-1} s in Fig. 2c). This slope is the same as the exponent of the power law of the two-point correlation function at short time scales. Our second variable was the intercept of the linear best fit between 10^{-2} s τ 10^{-1} (i.e., intersection of the line with the $\tau = 10^0$ ordinate in Fig. 2c). This intercept equals the logarithm of the proportionality constant of the power law at short time scales. The third variable was the logarithmic value of the two-point correlation function at the approximate neck junction region between the linear and the plateau ends of the log-log plots. We chose this at $\tau = 10^0 = 1$ s. Within subjects, Pillai's trace statistics verified that the conditions differed significantly with respect to the dependent variables, AP: $V = 1.35$, $F(12,48) = 3.31$, $p < 0.01$; ML: $V = 1.4$, $F(12,48) = 3.53$, $p < 0.01$.

For the AP direction, separate univariate ANOVAs identified significant condition effects for each of the three dependent variables: exponent, $F(4,16) = 5.71$, $p < 0.01$; intercept, $F(4,16) = 7.80$, $p < 0.01$; and plateau, $F(4,16) = 21.42$, $p < 0.01$. Within subjects contrasts between various conditions revealed that for the intercept variable significant differences with respect to pre-condition were observed for: the initial per-condition, $F(1,4) = 16.18$, $p < 0.02$, and the final per-condition, $F(1,4) = 9.85$, $p < 0.04$. For the exponent (slope) variable, the pre-condition differed significantly with initial per, $F(1,4) = 13.12$, $p < 0.03$; final per, $F(1,4) = 8.33$, $p < 0.05$; and initial post-conditions, $F(1,4) = 9.39$, $p < 0.04$. For the plateau variable, the pre-condition differed significantly from all the other conditions, initial per ($p < 0.01$), final per ($p < 0.01$), initial post ($p < 0.04$), and final post-condition ($p = 0.01$), $F(1,4) = 35.14$, 122.37, 10.03, 21.62, respectively. Comparisons between per- and post-conditions with multiple-comparison Bonferroni corrections showed significant ($p = 0.01$) mean

differences only for the final per-condition exponent value ($M = 1.67$, $SE = 0.10$) with respect to the final post-condition ($M = 1.34$, $SE = 0.07$) exponent.

For the ML direction, separate univariate ANOVAs identified significant condition effects for each of the three dependent variables: exponent, $F(4,16) = 3.86$, $p < 0.03$; intercept, $F(4,16) = 6.00$, $p < 0.01$; and plateau, $F(4,16) = 6.46$, $p < 0.01$. Within subjects contrasts revealed that for all three variables significant differences with respect to pre-condition were observed for the initial per-condition (exponent: $F(1,4) = 54.20$, $p < 0.01$, intercept: $F(1,4) = 39.75$, $p < 0.01$, and plateau: $F(1,4) = 40.04$, $p < 0.01$).

Scaling at $\tau \rightarrow 0$

We measured the autocorrelation functions in the $\tau \rightarrow 0$ limit for the quiet stance data and compared them to theoretical predictions of our stochastic model applied to quiet stance. The correlation functions are plotted in logarithmic coordinates to obtain the slopes. We calculated the slopes of the log of the correlation values $C(\tau)$ versus log of the latency window τ for timescales between 10 ms to 100 ms (i.e., 10^{-2} s $\leq \tau \leq 10^{-1}$ s (Fig. 2c). The autocorrelation functions calculated from the AP data give a power scaling of no less than $C(\tau) \propto \tau^{1+0.5}$ and no more than $C(\tau) \propto \tau^{1+0.7}$ (Fig. 2c) for $\tau \rightarrow 0$ limit. This spans at least two decades. The model predicted a scaling of $\propto \tau^{1+2\gamma}$, with $0 < 2\gamma < 1$. The rotating frame conditions give scaling exponents ($\sim 1 + 0.7$ for per-initial, $\gtrsim 1 + 0.6$ for per-final), which are a little greater than the stationary frame condition exponents ($\sim 1 + 0.5$ for pre-rotation, $\sim 1 + 0.6$ for the post-rotation conditions). They also exhibit different intercepts, which implies that the diffusion coefficient D is the variable that describes the changes that happen from the pre-rotation to the rotation to the post-rotation conditions. The autocorrelation functions calculated from the ML data give a power scaling of no less than $C(\tau) \propto \tau^{1+0.6}$ and no more than $C(\tau) \propto \tau^{1+0.9}$ for $\tau \rightarrow 0$ limit. This also spans at least two decades. In the $\tau \rightarrow 0$ limit, while the initial rotation condition exponent ($\gtrsim 1 + 0.8$) differs significantly from the baseline stationary frame condition, the final per-rotation and the post-conditions do not give significantly different scaling compared with the pre-rotation exponent ($\sim 1 + 0.7$).

We note that the rotating frame condition plots give greater or the same scaling as the stationary frame condition. However, in the $\tau \rightarrow 0$ limit, the range of the exponents, irrespective of AP or ML direction, lies within $1 + 0.5$ to $1 + 0.9$. Thus, together for both AP and ML directions, our experimental scaling results give $0.5 \leq 2\gamma \leq 0.9$. The ranges agree with the model prediction of a scaling $\propto \tau^{1+2\gamma}$, with $0 < 2\gamma < 1$.

We plotted the residual difference $C_{\text{rot}} - C_o$ (Fig. 3a, and log-log scale 3b) and estimated its scaling at $\tau \rightarrow 0$ to be $\propto \tau^{1+0.9}$ for both the Per-initial and Per-final rotation conditions. This scaling also spans at least 2 decades. For $\tau \rightarrow 0$, we consider the scaling for the residual to also be $\propto \tau^{1+2\gamma}$, with $2\gamma < 1$. We got a value of $2\gamma = 0.9$, which equals the upper bound of the range of 2γ obtained experimentally and differs slightly from the average value of $2\gamma = 0.7$ obtained for the stationary reference frame.

The scaling of $C(\tau)$ at $\tau \rightarrow 0$ thus agrees with the theoretical predictions across all the conditions. The differences of correlation in the rotating frame versus the stationary frame, i.e., the residuals, also follow a power law as is predicted by our modeling.

Velocity component histograms

The relative distributions of the AP and ML components of the velocities are shown for all conditions in Fig. 4a, and their widths ($2 \times$ standard deviation) are shown in Fig. 4b. All the distributions peak near null velocity. The initial per-rotation distributions are wider than those of the other conditions, with its average width about 70–80 % larger compared with baseline pre. The distribution widths become <40 % wider by the final per-rotation period, and post-rotation return to near baseline levels. We performed a two-way ANOVA, the factors being sway directions and rotation conditions. There was no effect of sway direction (AP versus ML); however, the condition factor was significant ($p < 0.001$). The initial and final rotation condition differed significantly from pre-rotation ($p = 0.016$ and 0.046 , respectively); however, post-rotation did not differ, and there was no significant interaction.

Cross-covariance

Figure 5a shows the cross-correlation of mean-removed AP and ML sway (cross-covariance) for lag times of 0–5 s, normalized such that the auto-covariances for each of the two sway directions at zero lag identically scale to 1. We calculated the 95 % confidence interval around the *per minus pre* covariance difference. The AP versus ML covariance difference was significantly different from zero only between $\tau = 0.59$ – 1.03 s. In this time lag region, during rotation, the AP versus ML sways show a negative peak cross-covariance of ~ -0.4 . Figure 5b, c show the cross-covariances of the AP velocity component with the ML velocity and the ML sway component, respectively. In stationary room conditions, these cross-covariances were not statistically different from zero for all time lags between 0 and 5 s. Room rotation at short time lags did not increase the cross-covariance magnitude any >0.2 .

Cross-spectral coherence

Figure 6a shows the coherence across conditions as a function of frequency at a resolution of 0.1 Hz. Its values indicate how well the AP sway component corresponds to ML sway at each frequency. The coherence decreases from a value of 0.5 to below 0.2 across the spectrum 0–50 Hz. Figure 6b shows cross-coherence during the baseline and during rotation, enlarged for the region where the coherence is highest, i.e., frequency range 0–5 Hz. The cross-spectral coherence across this lower frequency spectrum shows a difference between room rotation versus room stationary at around 1 Hz. The inset in Fig. 6b shows the difference of the coherence of the initial per-condition with respect to the baseline. We calculated the 95 % confidence interval around the *per minus pre* coherence difference. It was significantly different from zero between $f = 0.87$ – 1.02 Hz. The inverse of this, $1/f = 0.98$ – 1.15 s, corresponded to the region of significant covariance difference.

Discussion

The variables typically used to measure motor learning are those of classical non-stochastic mechanics. For example, voluntary sway during rotation shows deviations followed by clear adaptive accommodations that restore intended movement paths. After rotation ceases, clear mirror-image negative aftereffects occur, followed by a gradual return to baseline pre-rotation behavior (Bakshi 2009). If we interpret a shift of the per-final trace toward pre-rotation baseline as reflecting an adaptive adjustment of voluntary sway, then we must conclude that we fail to notice a corresponding adaptation for passive sway during quiet stance. However, unlike a voluntary sway task where the goal is to sway in a particular pattern, no such goal or constraint is imposed during passive standing which is inherently a 2-D phenomena requiring control of amplitude but not direction. If the goal is not to fall, the magnitudes of the CoM and CoP variations matter but their directions matter less so.

Quiet stance postural sway because of its latent stochastic nature requires variables typically used to characterize stochastic dynamical phenomena. Unlike for voluntary sway during rotation where variables like mean sway amplitude and sway direction characterize the nature of adaptive learning, for quiet stance, we find that stochasticity related measures like the distribution of velocity magnitude (Fig. 4), and the two-point correlation function (Fig. 2) exhibit adaptive changes. From the histograms in Fig. 4, it can be seen that the widths of the velocity magnitude distributions first increased with onset of room rotation, then reduced toward baseline with continuous exposure, and attained near baseline levels after rotation was stopped. From the stabilograms in Fig. 2, it is clear that the Coriolis cross-coupled perturbations generated by passive sway during rotation affected several parameters of the MSD function: the slope and intercept at short time scales, and the plateau for long time scales.

From the perspective of the two-point correlation function after rotation ends, the post-initial and post-final traces almost overlap and are significantly different from the baseline pre-rotation traces. For very short time scales, we analyzed the slope and the intercept using a log-log scale to determine the exponent and the proportionality constant of the power law relationship. The per-rotation initial and final conditions showed significant changes compared with pre-rotation for both the intercept and the exponent. For the post-rotation conditions, only the initial post-condition's exponent was significantly different from that of the pre-condition. For longer time scales, we calculated the plateau of the correlation and found that for all per- and post-conditions the plateau values were significantly different from those of the pre-condition.

To succeed in our goal to identify variables that might reveal the presence of learning behavior during quiet stance in a rotating reference frame, we extended the Chow and Collins (1995) stochastic model for quiet stance from a 1-D PPM to a 2-D PPM. In our 2-D extension of the model, we assumed “symmetry” or isotropy across the AP and ML directions for stochastic noise, i.e., the noise in both directions have the same autocorrelation function scaling. The observed relationship between the AP and ML sway reflects the coupling due to Coriolis forces across the axes. We find that when the room is rotating, a significant cross-covariation that was hitherto absent in no-rotation condition shows up for

time lags around ~1 s. Outside the found region of significant difference, the cross-covariance in rotating and stationary room condition are either absent or the same (Fig. 5). This increase in cross-covariance at relatively short time scale is consistent with the presence of a cross-axis biomechanical coupling due to Coriolis forces over and above the stochastic forcing. The cross-spectral coherences are relatively low in all conditions, but some coherence is present at lower frequencies (<5 Hz), which then declines to near negligible level for higher frequencies. Within the lower frequency range, we find that maximum relative cross-coherence emerges at about 1 Hz during room rotation when compared to the baseline stationary room condition. The increase in coherence from no-rotation to rotation-condition, and the time scale order of magnitude when it occurs ($1 \text{ Hz}^{-1} \sim 1 \text{ s}$), seen in Fig. 6b inset, is within the bounds where the peaks are observed in the cross-covariance (Fig. 5a). We interpret the baseline coherence spectra as representing the signature relationship of the inherent nature of the stochastic forcing in the two directions, while the additional coherence at 1 Hz originates from the emergence of Coriolis cross-coupling between the AP and ML directions during room rotation. In future work, we will model the frequency-dependent interrelationship between the two directions of sway across conditions and investigate how it affects the short- and long-term parameters of the SDF.

Because our theoretical model focuses on the nature of the short time scale limit of the two-point correlation function, our assumption is based upon the similar features of AP versus ML motion for time scales smaller than 200 ms (as seen in $\tau < 2 \times 10^{-1} \text{ s}$ in top and bottom panels of Fig. 2c). The model predicts a scaling $\propto \tau^{1+2\gamma}$, with $1 < 1 + 2\gamma < 2$. From the data, we estimated that at this short time scale, the scaling exponents, irrespective of AP or ML direction of sway, all lie in the range $1 + 0.5 \leq 1 + 2\gamma \leq 1 + 0.9$. This window gives us the maximum bounds on the accuracy of our assumption of symmetric scaling. In a case where the noise in the orthogonal directions is different, then from our data, we can impose constraints on the degree of “asymmetry”. If the noise across the two orthogonal directions differs only in their power law (scaling exponents) at small time scales, then the absolute difference of the exponents should not be more than 0.9–0.5, i.e., $|\text{exponent}_{\text{AP}} - \text{exponent}_{\text{ML}}| \leq 0.4$. If voluntary sway is mixed with involuntary and stochastic components of postural sway, this symmetry potentially could be further broken. Thus, in future work, it behooves us to extend our 2-D model with asymmetric noise roles for the two different body planes, and we will also make a quantitative comparison of learning in the stochastic versus the non-stochastic components of postural sway.

In conclusion, the extended PPM has provided a tool for revealing adaptation of 2-D stability to Coriolis force perturbations that was not evident using measures typically employed to characterize voluntary postural sway. It also provides opportunities for future investigation of the nature and timescale of the adaptation. In the PPM, the role in balance control of stiffness and of active anticipatory control by the CNS (Winter et al. 1996, 1998, 2001; Morasso and Schieppati 1999) can be mapped to the elastic restoring force with constant K , viscous drag with coefficient μ , tension T , and the parameter D of the stochastic control force fluctuations $\eta(z, t)$. This approach gives us a way to probe parametrically the source of the adaptive learning that we observed on presentation and removal of the Coriolis

force environment, and the nature of control for the involuntary versus voluntary and the stochastic versus nonstochastic components of postural sway.

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Appendix

Equations for the single-point temporal correlations for no-rotation and rotation conditions are: (1) No-rotation,

$$S_{\Omega=0}(\omega) = N(\omega; p) \int \frac{dk}{2\pi} \frac{1}{(\lambda_{k,\omega}^2 + \omega^2)} \quad (25)$$

(2) Rotation,

$$S_{\text{ROT}}(\omega) = N(\omega; p) \int \frac{dk}{2\pi} \frac{\lambda_{k,\omega}^2 + \omega^2(1 + \xi^2)}{(\lambda_{k,\omega}^2 - \omega^2(1 + \xi^2))^2 + 4\omega^2\lambda_{k,\omega}^2} \quad (26)$$

We evaluate the integrals and their power law nature over ω , under the limit $\omega \rightarrow \infty$.

From Eqs. (7) and (23) we have:

$$\lambda_{k,\omega} = \beta\omega^2 - \nu k^2 - \alpha \text{ and } N(\omega; p) \propto \frac{1}{(\omega^2 + \delta^2)^\gamma} \quad (27)$$

Case 1: room stationary

$$S_{\Omega=0}(\omega) \propto \frac{1}{(\omega^2 + \delta^2)^\gamma} \int_{-\infty}^{\infty} dk \frac{1}{(\beta\omega^2 - \nu k^2 - \alpha)^2 + \omega^2} \quad (28)$$

We can perform the integration using the method of contour integration in complex analysis.

We rescale the integration variable k to absorb ν , and define $a = \beta\omega^2 - \alpha$, and equivalently render the relation as

$$\propto \frac{1}{(\omega^2 + \delta^2)^\gamma} \oint_C dz \frac{1}{(a - z^2)^2 + \omega^2} \quad (29)$$

where the integration is along a contour C in the upper half of the complex plane that includes the horizontal axis and a semicircular arc of infinitely large radius (as the radius of arc tends to infinity, it makes a null contribution to the value of the integral).

We now define:

$$r_0 = (a^2 + \omega^2)^{1/2} \text{ and } \theta_0 = \tan^{-1} \frac{|\omega|}{|a|}, 0 \leq \theta_0 \leq \frac{\pi}{2}, \quad (30)$$

The denominator of the function within the integral in Eq. (26) is of order 4, and thus, it has

four singularities/poles given by: $z_{01} = +(a+i\omega)^{\frac{1}{2}} = r_0^{\frac{1}{2}} e^{i\frac{\theta_0}{2}}$ in quadrant 1,

$z_{02} = -(a-i\omega)^{\frac{1}{2}} = -r_0^{\frac{1}{2}} e^{-i\frac{\theta_0}{2}} = r_0^{\frac{1}{2}} e^{i(\pi-\frac{\theta_0}{2})}$ in quadrant 2,

$z_{03} = -(a+i\omega)^{\frac{1}{2}} = -r_0^{\frac{1}{2}} e^{i\frac{\theta_0}{2}} = r_0^{\frac{1}{2}} e^{i(\pi+\frac{\theta_0}{2})}$ in quadrant 3, and $z_{04} = +(a-i\omega)^{\frac{1}{2}} = r_0^{\frac{1}{2}} e^{-i\frac{\theta_0}{2}}$ in

quadrant 4. The above gives $z_{01} - z_{02} = r_0^{\frac{1}{2}} (e^{i\frac{\theta_0}{2}} + e^{-i\frac{\theta_0}{2}}) = 2r_0^{\frac{1}{2}} \cos \frac{\theta_0}{2}$ and similarly,

$$z_{01} - z_{03} = 2r_0^{\frac{1}{2}} e^{i\frac{\theta_0}{2}}; z_{01} - z_{04} = 2ir_0^{\frac{1}{2}} \sin \frac{\theta_0}{2} = z_{02} - z_{03}; z_{02} - z_{04} = -2r_0^{\frac{1}{2}} e^{-i\frac{\theta_0}{2}} \quad (31)$$

Factorization of the denominator inside the integral in Eq. (29) gives:

$$\propto \frac{1}{(\omega^2 + \delta^2)^\gamma} \oint_c dz \frac{1}{(z - z_{01})(z - z_{02})(z - z_{03})(z - z_{04})} \quad (32)$$

Since our contour is in the upper half of the complex plane, two poles z_1 and z_2 lie within the contour, and from Cauchy's integral theorem, the total contour integral can be reduced to the sum of contour integrals on contours C_1 and C_2 around the two poles. Thus:

$$\propto \frac{1}{(\omega^2 + \delta^2)^\gamma} \left\{ \oint_{C_1} dz \frac{1}{(z - z_{01})(z - z_{03})(z - z_{04})} + \oint_{C_2} dz \frac{1}{(z - z_{01})(z - z_{02})(z - z_{04})} \right\} \quad (33)$$

In Eq. (30), we apply Cauchy's integral formula $\oint_C dz \frac{f(z)}{(z - z_o)} = 2\pi i f(z_o)$ and ignore all upfront constants (because we are interested only in the proportionality), to give:

$$\propto \frac{i}{(\omega^2 + \delta^2)^\gamma} \left\{ \frac{1}{(z_{01} - z_{02})(z_{01} - z_{03})(z_{01} - z_{04})} + \frac{1}{(z_{02} - z_{01})(z_{02} - z_{03})(z_{02} - z_{04})} \right\} \quad (34)$$

Using Eq. (31) we then obtain:

$$\begin{aligned} &\propto \frac{1}{(\omega^2 + \delta^2)^\gamma} \frac{1}{r_0^{3/2}} \left\{ \frac{1}{(\cos \frac{\theta_0}{2})(e^{i\frac{\theta_0}{2}})(i \sin \frac{\theta_0}{2})} + \frac{1}{(-\cos \frac{\theta_0}{2})(i \sin \frac{\theta_0}{2})(-e^{-i\frac{\theta_0}{2}})} \right\} \\ &\propto \frac{1}{(\omega^2 + \delta^2)^\gamma} \frac{r_0^{-3/2}}{\sin \frac{\theta_0}{2} \cos \frac{\theta_0}{2}} \left\{ \frac{1}{e^{i\frac{\theta_0}{2}}} + \frac{1}{e^{-i\frac{\theta_0}{2}}} \right\} \propto \frac{1}{(\omega^2 + \delta^2)^\gamma} \frac{r_0^{-3/2}}{\sin \frac{\theta_0}{2}} \end{aligned} \quad (35)$$

We now apply the $\omega \rightarrow \infty$ limit

$$\lim_{\omega \rightarrow \infty} r_0 = \lim_{\omega \rightarrow \infty} \left[(\beta \omega^2 - \alpha)^2 + \omega^2 \right]^{1/2} = \lim_{\omega \rightarrow \infty} \omega^2 \left[\left(\beta - \frac{\alpha}{\omega^2} \right)^2 + \frac{1}{\omega^2} \right]^{1/2} \propto \omega^2 \quad (36)$$

$$\lim_{\omega \rightarrow \infty} \theta_0 = \lim_{\omega \rightarrow \infty} \tan^{-1} \frac{\omega}{\beta \omega^2 - \alpha} = \lim_{\omega \rightarrow \infty} \tan^{-1} \frac{1}{\omega \left(\beta - \frac{\alpha}{\omega^2} \right)} \propto \frac{1}{\omega} \quad (37)$$

$$\lim_{\omega \rightarrow \infty} \sin \theta_0 = \lim_{\omega \rightarrow \infty} \sin \frac{1}{\omega} \propto \frac{1}{\omega} \quad (38)$$

The limits in Eqs. (36, 38) when applied to Eq. (35) generate:

$$\lim_{\omega \rightarrow \infty} S_{\Omega=0}(\omega) \propto \lim_{\omega \rightarrow \infty} \frac{1}{(\omega^2 + \delta^2)^\gamma} \frac{r_0^{-\frac{3}{2}}}{\sin \frac{\theta_0}{2}} \propto \frac{1}{\omega^{2\gamma}} \frac{(\omega^2)^{-3/2}}{\frac{1}{\omega}} \propto \omega^{-2-2\gamma} \quad (39)$$

Case 2: room rotation

$$\begin{aligned} S_{\text{ROT}}(\omega) &= N(\omega; p) \int \frac{dk}{2\pi} \frac{\lambda_{k,\omega}^2 + \omega^2(1 + \xi^2)}{(\lambda_{k,\omega}^2 - \omega^2(1 + \xi^2))^2 + 4\omega^2 \lambda_{k,\omega}^2} \quad (40) \\ &= N(\omega; p) \int \frac{dk}{2\pi} \frac{\lambda_{k,\omega}^2 + \omega^2(1 + \xi^2)}{\lambda_{k,\omega}^4 + \omega^4(1 + \xi^4 + 2\xi^2) - 2\omega^2 \lambda_{k,\omega}^2(1 + \xi^2) + 4\omega^2 \lambda_{k,\omega}^2} \\ &= N(\omega; p) \int \frac{dk}{2\pi} \frac{1}{\lambda_{k,\omega}^4 + \omega^4 + 2\omega^2 \lambda_{k,\omega}^2 - 2\omega^2 \xi^2 (\lambda_{k,\omega}^2 - \omega^2) + \omega^4 \xi^4} = N(\omega; p) \int \frac{dk}{2\pi} \frac{\lambda_{k,\omega}^2 + \omega^2(1 + \xi^2)}{(\lambda_{k,\omega}^2 + \omega^2)^2 - 2\omega^2 \xi^2 (\lambda_{k,\omega}^2 + \omega^2 - 2\omega^2) + \omega^4} \quad (41) \\ &= N(\omega; p) \int \frac{dk}{2\pi} \frac{\lambda_{k,\omega}^2 + \omega^2(1 + \xi^2)}{(\lambda_{k,\omega}^2 + \omega^2)^2 - 2\omega^2 \xi^2 (\lambda_{k,\omega}^2 + \omega^2) + 4\omega^4 \xi^2 + \omega^4 \xi^4} \end{aligned}$$

The denominator in Eq. (41) is a quadratic polynomial, $y^2 - 2\omega^2 \xi^2 y + 4\omega^4 \xi^2 + \omega^4 \xi^4$, with

$$y = (\lambda_{k,\omega}^2 + \omega^2). \text{ It has the complex roots } \frac{2\omega^2 \xi^2 \mp \sqrt{4\omega^4 \xi^4 - 4(4\omega^4 \xi^2 + \omega^4 \xi^4)}}{2} = \omega^2 \xi^2 \mp 2i\omega^2 \xi$$

Thus, factorizing the denominator of $S_{\text{ROT}}(\omega)$ gives,

$$\begin{aligned} &= N(\omega; p) \int \frac{dk}{2\pi} \frac{\lambda_{k,\omega}^2 + \omega^2(1 + \xi^2)}{(\lambda_{k,\omega}^2 + \omega^2 - \omega^2 \xi^2 + 2i\omega^2 \xi) (\lambda_{k,\omega}^2 + \omega^2 - \omega^2 \xi^2 - 2i\omega^2 \xi)} = N(\omega; p) \\ &\quad \int \frac{dk}{2\pi} \frac{\lambda_{k,\omega}^2 + \omega^2(1 + \xi^2)}{(\lambda_{k,\omega}^2 + \omega^2(1 + i\xi)^2) (\lambda_{k,\omega}^2 + \omega^2(1 - i\xi)^2)} \quad (42) \end{aligned}$$

Applying Eq. (27) we have:

$$\propto \frac{1}{(\omega^2 + \delta^2)^\gamma} \int \frac{dk}{2\pi} \frac{(\beta \omega^2 - \nu k^2 - \alpha)^2 + \omega^2(1 + \xi^2)}{[(\beta \omega^2 - \nu k^2 - \alpha)^2 + \omega^2(1 + i\xi)^2] [(\beta \omega^2 - \nu k^2 - \alpha)^2 + \omega^2(1 - i\xi)^2]} \quad (43)$$

We can evaluate the integration using the method of contour integration in complex analysis. We rescale the integration variable k to absorb ν , and define $a = \beta\omega^2 - \alpha$, and equivalently render the relation as

$$\propto \frac{1}{(\omega^2 + \delta^2)^\gamma} \oint_C dz \frac{(a - z^2)^2 + \omega^2(1 + \xi^2)}{\left[(a - z^2)^2 + \omega^2(1 + i\xi)^2\right] \left[(a - z^2)^2 + \omega^2(1 - i\xi)^2\right]} \quad (44)$$

where the integration is along a contour C in the upper half of the complex plane that includes the horizontal axis and a semicircular arc of infinitely large radius. As the radius of the arc tends to infinity, it makes a null contribution to the value of the integral.

The denominator of the function within the integral in Eq. (44) is of order 8, and thus it has eight singularities/poles. To obtain these poles, we solve for z in the equations:

$$(a - z^2)^2 + \omega^2(1 + i\xi)^2 = 0 \text{ and } \quad (45)$$

$$(a - z^2)^2 + \omega^2(1 - i\xi)^2 = 0 \quad (46)$$

If we define:

$$r = ((a - \omega\xi)^2 + \omega^2)^{1/2}, \theta = \tan^{-1} \frac{|\omega|}{|a - \omega\xi|}, 0 \leq \theta \leq \frac{\pi}{2}, \quad (47)$$

$$R = ((a + \omega\xi)^2 + \omega^2)^{1/2}, \Theta = \tan^{-1} \frac{|\omega|}{|a + \omega\xi|}, 0 \leq \Theta \leq \frac{\pi}{2}. \quad (48)$$

then roots of Eq. (45) are given by:

$$\begin{aligned} z_1 &= (a - \omega\xi + i\omega)^{\frac{1}{2}} = r^{\frac{1}{2}} e^{i\frac{\theta}{2}} \text{ in quadrant 1,} \\ Z_2 &= -(a + \omega\xi - i\omega)^{\frac{1}{2}} = -R^{\frac{1}{2}} e^{-i\frac{\Theta}{2}} = R^{\frac{1}{2}} e^{i(\pi - \frac{\Theta}{2})} \text{ in quadrant 2,} \\ z_3 &= -(a - \omega\xi + i\omega)^{\frac{1}{2}} = -r^{\frac{1}{2}} e^{i\frac{\theta}{2}} = r^{\frac{1}{2}} e^{i(\pi + \frac{\theta}{2})} \text{ in quadrant 3, and} \\ Z_4 &= (a + \omega\xi - i\omega)^{\frac{1}{2}} = R^{\frac{1}{2}} e^{-i\frac{\Theta}{2}} \text{ in quadrant 4.} \end{aligned} \quad (49)$$

Similarly the roots of Eq. (46) are:

$$\begin{aligned} Z_1 &= (a + \omega\xi + i\omega)^{\frac{1}{2}} = R^{\frac{1}{2}} e^{i\frac{\Theta}{2}} \text{ in quadrant 1,} \\ z_2 &= -(a - \omega\xi - i\omega)^{\frac{1}{2}} = -r^{\frac{1}{2}} e^{-i\frac{\theta}{2}} = r^{\frac{1}{2}} e^{i(\pi - \frac{\theta}{2})} \text{ in quadrant 2,} \\ Z_3 &= -(a + \omega\xi + i\omega)^{\frac{1}{2}} = -R^{\frac{1}{2}} e^{i\frac{\Theta}{2}} = R^{\frac{1}{2}} e^{i(\pi + \frac{\Theta}{2})} \text{ in quadrant 3, and} \\ z_4 &= (a - \omega\xi - i\omega)^{\frac{1}{2}} = r^{\frac{1}{2}} e^{-i\frac{\theta}{2}} \text{ in quadrant 4.} \end{aligned} \quad (50)$$

Thus for the k th quadrant, we obtain two poles z_k and Z_k , i.e., two poles per quadrant making a total of 8 poles.

Also factorizing the numerator $(a - z^2)^2 + \omega^2 (1 + \xi^2) = 0$ will give the following roots:

$$\begin{aligned} u_1 &= + \left(a + i\omega(1 + \xi^2)^{1/2} \right)^{\frac{1}{2}} = \varrho^{\frac{1}{2}} e^{i\frac{\varphi}{2}}, \\ u_2 &= - \left(a - i\omega(1 + \xi^2)^{1/2} \right)^{\frac{1}{2}} = - \varrho^{\frac{1}{2}} e^{-i\frac{\varphi}{2}}, \\ u_3 &= - \left(a + i\omega(1 + \xi^2)^{1/2} \right)^{\frac{1}{2}} = - \varrho^{\frac{1}{2}} e^{i\frac{\varphi}{2}}, \\ u_4 &= + \left(a - i\omega(1 + \xi^2)^{1/2} \right)^{\frac{1}{2}} = \varrho^{\frac{1}{2}} e^{-i\frac{\varphi}{2}}, \end{aligned} \quad (51)$$

where

$$\begin{aligned} \varrho &= (a^2 + \omega^2 (1 + \xi^2))^{1/2} \quad \text{and} \\ \varphi &= \tan^{-1} \frac{|\omega(1 + \xi^2)^{1/2}|}{|a|}, \quad 0 \leq \varphi \leq \frac{\pi}{2}, \end{aligned} \quad (52)$$

Then the factorization of the denominator and numerator inside the integral in Eq. (44) gives:

$$\propto \frac{1}{(\omega^2 + \delta^2)^\gamma} \oint_C dz \frac{(z - u_1)(z - u_2)(z - u_3)(z - u_4)}{(z - z_1)(z - z_2)(z - z_3)(z - z_4)(z - Z_1)(z - Z_2)(z - Z_3)(z - Z_4)} \quad (53)$$

Since our contour is in the upper half of the complex plane, four poles z_1, z_2, Z_1 and Z_2 lie within the contour, and from Cauchy's integral theorem the total contour integral can be reduced to the sum of contour integrals on contours c_1, c_2, C_1 and C_2 around the four poles.

Thus:

$$\begin{aligned} &\propto \frac{1}{(\omega^2 + \delta^2)^\gamma} \left\{ \oint_{c_1} dz \frac{(z - u_1)(z - u_2)(z - u_3)(z - u_4)}{(z - z_1)(z - z_2)(z - z_3)(z - z_4)(z - Z_1)(z - Z_2)(z - Z_3)(z - Z_4)} + \oint_{c_2} dz \frac{(z - u_1)(z - u_2)(z - u_3)(z - u_4)}{(z - z_1)(z - z_3)(z - z_4)(z - Z_1)(z - Z_2)(z - Z_3)(z - Z_4)} \right. \\ &\quad \left. + \oint_{c_3} dz \frac{(z - u_1)(z - u_2)(z - u_3)(z - u_4)}{(z - z_1)(z - z_2)(z - z_3)(z - z_4)(z - Z_1)(z - Z_2)(z - Z_3)(z - Z_4)} + \oint_{c_4} dz \frac{(z - u_1)(z - u_2)(z - u_3)(z - u_4)}{(z - z_1)(z - z_2)(z - z_3)(z - z_4)(z - Z_1)(z - Z_2)(z - Z_3)(z - Z_4)} \right\} \\ &= \frac{1}{(\omega^2 + \delta^2)^\gamma} (I_{c1} + I_{c2} + I_{c3} + I_{c4}) \end{aligned} \quad (54)$$

We now apply Cauchy's integral formula $\oint_C \frac{f(z)}{(z - z_o)} dz = 2\pi i f(z_o)$ and ignore all upfront constants (because we are interested only in the proportionality), to give:

$$I_{c1} = \frac{(z_1 - u_1)(z_1 - u_2)(z_1 - u_3)(z_1 - u_4)}{(z_1 - z_2)(z_1 - z_3)(z_1 - z_4)(z_1 - Z_1)(z_1 - Z_2)(z_1 - Z_3)(z_1 - Z_4)}, \text{ and from Eqs. (49, 50, 51) this is}$$

$$= \frac{\left(r^{\frac{1}{2}} e^{i\frac{\theta}{2}} - \varrho^{\frac{1}{2}} e^{i\frac{\varphi}{2}} \right) \left(r^{\frac{1}{2}} e^{i\frac{\theta}{2}} + \varrho^{\frac{1}{2}} e^{-i\frac{\varphi}{2}} \right) \left(r^{\frac{1}{2}} e^{i\frac{\theta}{2}} + \varrho^{\frac{1}{2}} e^{i\frac{\varphi}{2}} \right) \left(r^{\frac{1}{2}} e^{i\frac{\theta}{2}} + \varrho^{\frac{1}{2}} e^{-i\frac{\varphi}{2}} \right)}{\left(2r^{\frac{1}{2}} \cos \frac{\theta}{2} \right) \left(2r^{\frac{1}{2}} e^{i\frac{\theta}{2}} \right) \left(2ir^{\frac{1}{2}} \sin \frac{\theta}{2} \right) \left(r^{\frac{1}{2}} e^{i\frac{\theta}{2}} - R^{\frac{1}{2}} e^{i\frac{\Theta}{2}} \right) \left(r^{\frac{1}{2}} e^{i\frac{\theta}{2}} + R^{\frac{1}{2}} e^{-i\frac{\Theta}{2}} \right) \left(r^{\frac{1}{2}} e^{i\frac{\theta}{2}} + R^{\frac{1}{2}} e^{i\frac{\Theta}{2}} \right) \left(r^{\frac{1}{2}} e^{i\frac{\theta}{2}} - R^{\frac{1}{2}} e^{-i\frac{\Theta}{2}} \right)}$$

$$\propto \frac{1}{r^{\frac{3}{2}} \cos \frac{\theta}{2} \sin \frac{\theta}{2}} \frac{1}{e^{i\frac{\theta}{2}}} \frac{(re^{i\theta} - \varrho e^{i\varphi})(re^{i\theta} - \varrho e^{-i\varphi})}{(re^{i\theta} - Re^{i\Theta})(re^{i\theta} - Re^{-i\Theta})} \quad (55)$$

$$I_{c2} = \frac{(z_2 - u_1)(z_2 - u_2)(z_2 - u_3)(z_2 - u_4)}{(z_2 - z_1)(z_2 - z_3)(z_2 - z_4)(z_2 - Z_1)(z_2 - Z_2)(z_2 - Z_3)(z_2 - Z_4)}$$

$$= \frac{\left(-r^{\frac{1}{2}}e^{-\frac{i\theta}{2}} - \varrho^{\frac{1}{2}}e^{i\frac{\varphi}{2}}\right)\left(-r^{\frac{1}{2}}e^{-\frac{i\theta}{2}} + \varrho^{\frac{1}{2}}e^{-\frac{i\varphi}{2}}\right)\left(-r^{\frac{1}{2}}e^{-\frac{i\theta}{2}} + \varrho^{\frac{1}{2}}e^{\frac{i\varphi}{2}}\right)\left(-r^{\frac{1}{2}}e^{-\frac{i\theta}{2}} - \varrho^{\frac{1}{2}}e^{-\frac{i\varphi}{2}}\right)}{\left(-2r^{\frac{1}{2}}\cos\frac{\theta}{2}\right)\left(2ir^{\frac{1}{2}}\sin\frac{\theta}{2}\right)\left(-2r^{\frac{1}{2}}e^{-i\frac{\theta}{2}}\right)\left(-r^{\frac{1}{2}}e^{-\frac{i\theta}{2}} - R^{\frac{1}{2}}e^{i\frac{\Theta}{2}}\right)\left(-r^{\frac{1}{2}}e^{-\frac{i\theta}{2}} + R^{\frac{1}{2}}e^{-\frac{i\Theta}{2}}\right)\left(-r^{\frac{1}{2}}e^{-\frac{i\theta}{2}} + R^{\frac{1}{2}}e^{\frac{i\Theta}{2}}\right)\left(-r^{\frac{1}{2}}e^{-\frac{i\theta}{2}} - R^{\frac{1}{2}}e^{-\frac{i\Theta}{2}}\right)}$$

$$\propto \frac{1}{r^{\frac{3}{2}} \cos \frac{\theta}{2} \sin \frac{\theta}{2}} \frac{1}{e^{-i\frac{\theta}{2}}} \frac{(re^{-i\theta} - \varrho e^{i\varphi})(re^{-i\theta} - \varrho e^{-i\varphi})}{(re^{-i\theta} - Re^{i\Theta})(re^{-i\theta} - Re^{-i\Theta})} \quad (56)$$

We can see by replacing z for Z and vice versa (or r for R , and vice versa, and similarly for the angles) that the integrals I_{c3} and I_{c4} have a form similar to I_{c1} and I_{c2} :

$$I_{c3} \equiv I_{c1 \text{ replacing: } Z \rightarrow z, z \rightarrow Z} = I_{c1 \text{ replace: } R \rightarrow r, r \rightarrow R, \Theta \rightarrow \theta, \theta \rightarrow \Theta} \quad (57)$$

$$I_{c4} \equiv I_{c2 \text{ replacing: } Z \rightarrow z, z \rightarrow Z} = I_{c2 \text{ replace: } R \rightarrow r, r \rightarrow R, \Theta \rightarrow \theta, \theta \rightarrow \Theta} \quad (58)$$

The $\omega \rightarrow \infty$ limit of Eq. (54) is:

$$\lim_{\omega \rightarrow \infty} S_{\text{ROT}}(\omega) \propto \lim_{\omega \rightarrow \infty} \frac{1}{(\omega^2 + \delta^2)^\gamma} (I_{c1} + I_{c2} + I_{c3} + I_{c4}) \quad (59)$$

We now apply the $\omega \rightarrow \infty$ limit on the variables and functions in $I_{c1, 2, 3, 4}$:

$$\lim_{\omega \rightarrow \infty} r = \lim_{\omega \rightarrow \infty} \left[(\beta\omega^2 - \alpha - \omega\xi)^2 + \omega^2 \right]^{1/2} = \lim_{\omega \rightarrow \infty} \omega^2 \left[\left(\beta - \frac{\alpha}{\omega^2} - \frac{\xi}{\omega} \right)^2 + \frac{1}{\omega^2} \right]^{1/2} \propto \omega^2 \quad (60)$$

Similarly $\lim_{\omega \rightarrow \infty} R \propto \omega^2$, and $\lim_{\omega \rightarrow \infty} \varrho \propto \omega^2$

$$\lim_{\omega \rightarrow \infty} \theta = \lim_{\omega \rightarrow \infty} \tan^{-1} \frac{\omega}{\beta\omega^2 - \alpha - \omega\xi} = \lim_{\omega \rightarrow \infty} \tan^{-1} \frac{1}{\omega(\beta - \frac{\alpha}{\omega^2} - \frac{\xi}{\omega})} \propto \frac{1}{\omega} \quad \text{Similarly } \lim_{\omega \rightarrow \infty} \Theta \propto \frac{1}{\omega}, \text{ and } \lim_{\omega \rightarrow \infty} \varphi \propto \frac{1}{\omega} \quad (61)$$

$$\lim_{\omega \rightarrow \infty} \sin \theta = \lim_{\omega \rightarrow \infty} \sin \frac{1}{\omega} \propto \frac{1}{\omega} \text{ and } \lim_{\omega \rightarrow \infty} \cos \theta = \lim_{\omega \rightarrow \infty} \cos \frac{1}{\omega} \sim 1 \quad \text{Similarly } \lim_{\omega \rightarrow \infty} \sin \Theta \propto \frac{1}{\omega} \text{ and } \lim_{\omega \rightarrow \infty} \cos \Theta \sim 1 \quad (62)$$

Thus, the angles hyperbolically decrease as $\frac{1}{\omega}$ with $\omega \rightarrow \infty$. Taylor expanding the exponential of the angles gives:

$$\lim_{\omega \rightarrow \infty} e^{i \operatorname{sign}(\theta)|\theta|} = 1 + \lim_{\omega \rightarrow \infty} \sum_{n=1}^{\infty} \frac{(i \operatorname{sign}(\theta)|\theta|)^n}{n!} = 1 + \lim_{\omega \rightarrow \infty} \sum_{n=1}^{\infty} \frac{(i \operatorname{sign}(\theta))^n}{n! \omega^n} \rightarrow 1 \quad (63)$$

Limits for r , ϱ and R in Eq. (60), and the exponential Taylor expansion in Eq. (63) imply

$$\lim_{\omega \rightarrow \infty} \frac{r e^{i \operatorname{sign}(\theta)|\theta|} - \varrho e^{i \operatorname{sign}(\varphi)|\varphi|}}{r e^{i \operatorname{sign}(\theta)|\theta|} - R e^{i \operatorname{sign}(\Theta)|\Theta|}} \rightarrow 1 \quad (64)$$

Applying the limits in Eqs. (60, 62, 63, 64) on each of Eqs. (55, 56, 57, 58), reduces Eq. (59) to:

$$\lim_{\omega \rightarrow \infty} S_{\text{ROT}}(\omega) \propto \frac{1}{\omega^{2\gamma} \left(1 + \frac{\delta^2}{\omega^2}\right)} \left(\frac{1}{(\omega^2)^{\frac{3}{2}}} \frac{1}{\left(\frac{1}{\omega}\right)} \right) \propto \omega^{-2-2\gamma} \quad (65)$$

Thus, $S_{\Omega=0}(\omega) \sim \omega^{-2-2\gamma}$ as $\omega \rightarrow \infty$, and also $S_{\text{ROT}}(\omega) \propto \omega^{-2-2\gamma}$. To obtain the power law in τ we Fourier transform $\omega^{-2-2\gamma}$ to obtain $S(\tau)$

$$S(\tau) \propto \int d\omega e^{-i\omega\tau} \omega^{-2-2\gamma}$$

Defining $h = \omega\tau$ we get:

$$S(\tau) \propto \int \frac{dh}{\tau} e^{-ih} \frac{h^{-2-2\gamma}}{\tau^{-2-2\gamma}} \propto \tau^{1+2\gamma} \quad (66)$$

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Fig. 1.

A subject standing on the axis of rotation in a rotating (Ω) reference frame experiences a movement-contingent ($\mathbf{v}$) Coriolis force ($\mathbf{F}_{\text{cor}} = -2m\boldsymbol{\Omega} \times \mathbf{v}$) during body sway. The quiet stance dynamics is equivalently modeled by the dynamics of a flexible string with a mass density (ρ), a charge density ($q_{\text{eff}} = 2\rho$), under tension (T), immersed in a magnetic field ($\mathbf{B}_{\text{eff}} = \boldsymbol{\Omega}$), and embedded in a visco-elastic bath that provides frictional drag, elastic restoration, and stochastic fluctuations. See text for details

Fig. 2.

Top panels are for AP direction, bottom for ML. **a** Plot of the two-point correlation function, $C(\tau)$ versus τ , for $0.01 \leq \tau \leq 10$ s. This is the mean square CoP displacement (MSD) as a function of the time interval τ between CoP position comparisons. The quiet stance MSD profiles include an initial short period with positive slope, followed by a plateau or decrease in slope with or without oscillatory profile. **b** Enlargement of $C(\tau)$ versus τ for $\tau < 400$ ms. **c** Log-log plot of $C(\tau)$ versus τ . The autocorrelation functions calculated from the data give an average scaling of $C(\tau) = \tau^{1+0.7}$ for $\tau \rightarrow 0$ limit. The straight line in the bottom panel is the same line of slope = 1.7 shown in the top panel

Fig. 3.

Top panels are for AP direction, bottom for ML. **a** Residual $C = C_{\text{rot}} - C_0$ where C_{rot} and C_0 are the correlation functions defined for rotating and stationary reference frames, respectively. $C_{\text{rot}} - C_0$ should have the same scaling as the individual correlation functions.

b C versus τ plotted on log-log scales. The exponent scaling goes as $\propto \tau^{1+0.9}$. The straight line in the bottom panel is the same line of slope = 1.9 shown in the top panel

Fig. 4.

a A histogram of the velocity magnitudes shows a peak near 0 for all conditions, but the widths are larger after onset of room rotation, then return to baseline distribution when rotation stops. *Top panel* is for AP direction, *bottom* for ML. **b** The *inset bar graph* shows the half-widths of the velocity component distributions in the initial per-rotation period are about 70–80 % larger compared to baseline pre, diminish to <40 % wider by final per-rotation, and post-rotation return to near baseline levels

Fig. 5.

Cross-covariances between **a** AP and ML sways, **b** the AP and ML velocity components, and **c** the AP velocity component and the ML sway. The covariances are minimal for the stationary room condition, while relatively small magnitudes arise during rotation for short time lags, <2 s

Fig. 6.

The cross-spectral coherence **a** for the AP and ML sway, computed at 0.1 Hz bin resolution, over a range 0–50 Hz. **b** Enlargement of the baseline (no rotation) and the rotation conditions at lower frequency range where the coherence is maximal. Room rotation leads to an emergence of additional coherence at a frequency of ≈ 1 Hz. *Inset* in **b** shows the difference of cross-coherence between the initial room rotation condition and the baseline condition before room rotation. This shows the coherence introduced by the Coriolis coupling during room rotation, over and above any inherent coherence from the stochastic forcing present in stationary room conditions