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A detector response function design in pinhole SPECT including geometrical calibration

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Abstract

Clinical single photon emission computed tomography (SPECT) equipped with pinhole collimators have a magnification factor that results in high spatial resolution images for small animal imaging. Using Monte Carlo simulations to model the acquisition process and the propagation of the photons from their point of emission to their detection point then integrating the model into an iterative reconstruction algorithm improves the signal-to-noise ratio, the contrast and the spatial resolution in the reconstructed images. However, pinhole SPECT systems are known to be very sensitive to geometrical misalignments. Geometrical misalignments are defined as the radial or axial shift of the collimator pinhole and/or twist and tilt of the detector heads and are introduced in the system each time the collimation device is changed (pinhole to parallel holes or vice versa). In this work, we present a flexible detector response function table (DRFT) design that takes into account the geometrical misalignments and avoids performing new Monte Carlo simulations for each exam in order to calculate a geometrical study-dependent system matrix. The utilization of the DRFT for the calculation of the system matrix speeds up its computation time by two orders of magnitude making it acceptable for preclinical and clinical applications.

1. Introduction

Pinhole single photon emission computed tomography (SPECT) is an imaging modality well known for its ability to provide high resolution images thanks to its magnification factor. Apart from using multi-pinhole geometries, improvement in spatial resolution is at the cost of a detection efficiency decrease when compared to parallel hole collimation. Furthermore, high resolution pinhole imaging is hindered by two major factors: (i) the physical processes that the photons undergo in the imaged subject and the detector during the acquisition process leading to false estimation of the radioactivity concentration value and/or its location (Hutton *et al* 2011), (ii) the need to correct geometrical misalignment responsible for artifacts appearance and loss of resolution (Bequé *et al* 2003). One way to correct for the physical processes undergone by photons is to use Monte Carlo simulations that are well known for their ability to mimic the acquisition process in SPECT (Lazaro *et al* 2004) and positron emission tomography (PET) (Lamare *et al* 2006).

Many groups have proposed the use of Monte Carlo simulations for the calculation of the system matrix (SM) in PET (Rafecas *et al* 2003) and in both parallel hole SPECT (Lazaro *et al* 2005, El Bitar *et al* 2006) and pinhole SPECT (El Bitar *et al* 2011). Once the system

matrix is calculated, it can be integrated within an iterative reconstruction algorithm such as Maximum Likelihood Expectation Maximization (MLEM) (Shepp and Vardi 1982) or Ordered Subset Expectation Maximization (OSEM) (Hudson and Larkin 1994) in order to implicitly correct for the physical processes within the reconstruction procedure. One major drawback of Monte Carlo simulations is that they are time consuming. In addition, utilization of Monte Carlo simulations can be easily criticized if a calculation of a system matrix is required for each study; this is the case when the camera geometry varies from one study to another or when a subject-dependent image reconstruction is needed in order to achieve high resolution images.

In order to tackle the time consumption drawback, the utilization of Monte Carlo simulation was often replaced by a hybrid approach where analytical and Monte Carlo methods were combined in order to speed-up the computation (Gieles *et al* 2002, De Jong *et al* 2001, De Beenhouwer *et al* 2008). Other groups also proposed to speed-up the simulations for parallel hole collimator SPECT applications by using precomputed and stored detector response function calculated by Monte Carlo simulations (Song *et al* 2005, Staelens *et al* 2007, Descourt *et al* 2010). In these works, the authors have made the assumptions that the collimator-detector response function can be considered to be spatially invariant over the collimator unit cell and that a photon interacting with the collimator is detected at its geometric intersection point with the crystal. However previous assumptions made in parallel hole SPECT do not hold in pinhole SPECT because: (i) spatial resolution is spatial-dependent in pinhole SPECT, (ii) the importance of inter-crystal scattering due to the incident angles of the photons which are not limited to small values around the perpendicular to the surface of the crystal. Recently (Wietholt *et al* 2006) developed a ray-driven based system matrix that accounts for detector blur, the geometric response and edge penetration of the pinhole, using Gate toolkit (Jan *et al* 2004). In this study, each imaging detector element was replaced with a point source and photons emitted from these sources were collected on the opposite side of the aperture. The system matrix was first calculated for one angle of view and then generated for the remaining views by applying rotation and interpolation in the object space. However, the system matrix calculated in the work of (Wietholt *et al* 2006) accounts only for edge penetration and geometric response but did not take into account the physical processes inside the crystal such as scattering and the depth of interaction. (El Bitar *et al* 2011) have shown the importance of modelling the scatter in the detector by studying the impact of the number of pixels of interest considered in the point spread function (PSF). It was shown that truncating the PSF by not considering all the pixels of interest introduces artefacts in the reconstructed images.

In this paper, we propose a flexible detector response function table (DRFT) based on Monte Carlo simulations for pinhole SPECT. The main objective of the proposed DRFT is to enable the calculation of a system matrix that takes into account the geometrical misalignments in pinhole SPECT providing a geometrical study-dependent reconstruction without having to perform Monte Carlo simulations again.

2. Materials and Methods

2.1. Concept

The main purpose of a detector response function table (DRFT) in pinhole SPECT is to be able to know, for an incident gamma ray hitting the entry of the collimator, in which crystal's pixels the detection will occur, if any. The input of the DRFT should then be the point where the gamma ray intersects the entry plane of the collimator and the direction vector of the considered gamma ray. We first sampled the pinhole entry plane into $n \times n$ square bins and then we define L_{ij} to be the set of lines going into the bin i at the entry of the collimator and aiming at crystal pixel j (figure 1). Due to scatter in the collimator and in the crystal, gamma rays belonging to the set of lines L_{ij} can be detected not only in pixel p_j but also in any neighboring pixel p_k . Hence, the DRFT should store the probabilities of a gamma ray belonging to a set of lines L_{ij} and being detected in any crystal pixel p_k including the crystal pixel p_j . This probability is given by the following equation:

$$P_{ijk} = \frac{N_{ijk}}{N_{ij}} \quad (1)$$

where N_{ijk} is the number of photons whose directions belong to L_{ij} and were detected in the pixel p_k and N_{ij} is the number of photons whose directions belong to L_{ij} . The position of emission of a gamma ray and of its position in the crystal were randomly selected in the emission bin and the detection pixel.

Geant4 (Agostinelli *et al* 2003) Monte Carlo simulation toolkit was used to perform the simulations throughout this paper. All physical processes corresponding to gamma rays were activated: Photoelectric, Compton and Rayleigh scattering. Optical photons were not tracked.

2.2. Point spread function calculation using the DRFT

Once the DRFT is calculated, the Point Spread Function (PSF) of the detector for any point inside the FOV of the pinhole SPECT can be calculated by knowing the points of entry in the collimator and the crystal, respectively. The point of entry in the crystal, as well as in the collimator, can be considered as the geometric intersections between the ray modeling the photon emission direction and the collimator and the crystal entry planes, respectively. A set of parametric equations can be used to connect any gamma ray emission point $S = (x_s, y_s, z_s)$ with any point at the entry of the collimator $P = (x_p, y_p, z_p)$ (figure 1):

$$x = x_s + t\Delta x, \quad (2)$$

$$y = y_s + t\Delta y, \quad (3)$$

$$z = z_s + t\Delta z, \quad (4)$$

where

$$\Delta x = x_s - x_p, \Delta y = y_s - y_p, \Delta z = z_s - z_p. \quad (5)$$

The direction vector of the gamma ray will then be $\vec{v}(\Delta x, \Delta y, \Delta z)$. If $\vec{n}$ is the normal vector of the crystal plane and C is the center of the crystal, then the point of intersection I between the gamma ray line and the crystal should satisfy the following condition:

$$\vec{n} \cdot \vec{CI} = 0, \quad (6)$$

$$\vec{n} \cdot (\vec{SI} - \vec{SC}) = 0. \quad (7)$$

Replacing $\vec{SI}$ by $t\vec{v}$, we obtain:

$$\vec{n} \cdot (t\vec{v} - \vec{SC}) = 0, \quad (8)$$

$$t = \frac{\vec{n} \cdot \vec{SC}}{\vec{n} \cdot \vec{v}}. \quad (9)$$

Given the direction vector $\vec{v}$ of the gamma ray and replacing t by its value in equations (2), (3) and (4), we can calculate the coordinates of the intersection point of the incident photon with the collimator and crystal entry planes. This will enable the identification of the emission bin through which the photon goes, the aimed crystal pixel and so the family of lines to which the emitted photon belongs. The crystal pixels corresponding to the identified family of lines were then incremented by the pre-calculated values stored in the DRFT. This will give the PSF of the considered point (x_s, y_s, z_s) .

Very similar approaches for calculating the point of intersection with a plane and a line of intersection with a collimator can be found in (Smith *et al* 1998, Accorsi *et al* 2004).

2.3. The pinhole SPECT system used for evaluation

The LBNL pinhole SPECT system used for the evaluation of the DRFT design is composed of two pinhole collimators (Nuclear Fields, Vortum-Mullen, The Netherlands) mounted on a GE Millenium VG3 dual head SPECT system with slip ring detector technology. The pinhole collimators as well as the shielding were fabricated with a design that is not a standard item and is not included neither in the product sheets of Nuclear Fields nor that of GE's. We used two sets of pinholes made of tungsten and having $2 \times 1.5 \text{ mm}^2$ and $0.6 \times 0.4 \text{ mm}^2$ respectively aperture areas. The $45 \times 45 \text{ cm}^2$ detectors each have a block of sodium iodine crystal doped with thallium having a thickness of 9.525 mm, providing an intrinsic spatial resolution of 3.9 mm and an energy resolution of 9.8% at an energy of 140 keV. The studies performed for this paper were done with a rotation radius of 50 mm and a focal distance of 250 mm. The opening angles of the pinholes were 52° . Reader can find more

details in an internal laboratory report of the Lawrence Berkeley National Laboratory (Hu *et al* 2008).

2.4. Calculation of the DRFT

In order to calculate the DRFT, the entry of the collimator was segmented into 100×100 square bins each of $10^4 \mu\text{m}^2$. 10^3 photons were emitted from each bin at the entry of the collimator in the direction of each crystal pixel (i.e., photons with detection probability less than 10^{-3} were not considered). The position of emission of a gamma ray and its target position in the crystal were randomly selected in the emission bin and the aimed pixel. When a photon is detected, its detection position was convoluted with a Gaussian having the intrinsic spatial resolution (3.9 mm) as value of its full width half measurement. This was done to model the intrinsic spatial resolution of the system. Only the sparse DRFT was stored, in other words, the zero value probabilities were not stored. Geant4 simulations required for the calculation of the DRFT were parallelized on a cluster comprised of 1 head node and 10 compute nodes that are Apple Xserve rack-mounted servers with 2 quad-core Intel Xeon 2.93 GHz processors. Each of the compute nodes has 96 GBytes of memory and 1 TBytes of internal hard drive.

2.5. Projections calculation using the DRFT

In this work, we propose a method of calculating projections using the PSFs obtained from the DRFT as explained in section 2.2. The tracking of photons from their emission point to the surface of the collimator was performed using an analytical standalone C++ code where the position of emission of the photon is generated inside the activity distribution to be projected while its direction vector is defined as the vector linking the emission point and a point belonging to the 100×100 square bins used for the calculation of the DRFT. In the remaining of the paper, the proposed method will be referred as proj-DRFT.

For accuracy assessment, the proj-DRFT method was used to project three 0.5 mm diameter sphere sources located at (-20 mm, 0, 0), (0, 0, 0) and (20 mm, 0, 0) and the resulting projection was compared with the one obtained using a Geant4 simulation. In both methods, the spheres and the field of view was filled with air and none of the two methods included any scatter or attenuation correction in the object. As in the Geant4 simulation, when using the proj-DRFT method, the position of emission inside the three spheres and the target position at the entry of the collimator were randomly selected. The calculation using the proj-DRFT method was performed on a single intel Xeon 2.93 GHz core while the Geant4 simulation was parallelized on 100 intel Xeon 2.93 GHz cores. A total of 10^8 photons were used for the calculation of the projection of the three points and this for both methods. The considered number of photons guaranteed a low noise level in the projection resulting from the Geant4 simulation so that it may be comparable to the projection obtained using the proj-DRFT method. In order to evaluate the speed-up factor, we calculated projections of a 2 cm diameter sphere located in the center of the field of view using the Geant4 full Monte Carlo simulation and the proj-DRFT method. Computation time is compared for projections having the same level of signal to noise ratio (SNR). The SNR was defined as the ratio of the mean pixel value in the projection over its standard deviation value.

2.6. The forced detection technique

In order to improve the efficiency of Monte Carlo simulations, (Beck *et al* 1982, Ljungberg M and Strand SE 1991) have proposed to use a variance reduction technique (VRT) based on a forced detection (FD) approach where the directions of the emitted photons are forced in the direction of the collimator. Since the pinhole aperture has a rectangular shape, in this work we used the formula of the solid angle for a plane triangle proposed by (Van Oosterom A and Strackee J 1983) and we considered the rectangular entry of the collimator as the union of two triangles. We used the solid angle factor to take into account the geometrical weight factor and we applied the forced detection technique for all the studies except the calculation of the DRFT itself.

2.7. Considering for geometrical misalignments within the proj-DRFT method

Besides the detector response function modelling of the conventional geometry of a planar rotating-camera, utilization of the proj-DRFT method should enable the consideration of parameters related to mechanical misalignments. Figure 2 represents the acquisition geometry of a conventional pinhole SPECT system where the object was defined in the xyz system with the z-axis along the rotation axis. To consider misalignments due to mechanical defects, the geometry of the acquisition can be completely described using seven parameters (Bequé *et al* 2003). For a given rotation angle θ , the angle between the detector surface and the x-axis, the activity distribution $A(x,y,z)$ is projected through the pinhole onto the detector as $P_\theta(u, v)$, where the origin of the uv-system is the perpendicular projection of the origin of the xyz-system on the detector. The radius (r) is the distance between the focal point and the center of the object. The focal length (f) is the distance between the focal point and the detector. The mechanical offset (m) is the distance between the rotation axis and the pinhole projection ray orthogonal to the detector. The tilt is the angle between the detector and the rotation axis. The twist describes a rotation of the detector around an axis parallel to the central ray. The electrical shifts (e_u and e_v) describe a collective translation of the projection image caused by drift of the detector hardware.

Figure 3 shows one example of correction of a geometrical misalignment represented by a collimator mechanical shift. Figure 3 (a) illustrates a collimator shift $\vec{m}$. The sequential steps for correcting the collimator shift are illustrated in figure 3 (b) and are as follows:

1. A shift $-\vec{m}$ is applied on the line representing the direction of photon 1 (solid line).
2. The pixel of intersection j' of the new direction of photon 1 (dashed line) with the crystal plane is calculated and the detector response at pixel j' is given by the DRFT.
3. The detector response function at pixel j' is then considered to be the detector response function at pixel j which is the pixel of intersection between the original photon 1 direction (solid line) and the crystal plane. The calculated detector response function at pixel j takes into account a collimator shift $\vec{m}$.

In the event that the detector is tilted or twisted, the equations of the collimator and the crystal entry planes will take into account the tilt and the twist by applying a rotation

transformation. The equations of the rotated collimator and crystal entry planes will be used to calculate the point spread functions from the DRFT.

2.8. System matrix calculation using the DRFT

The DRFT was used to calculate the system matrix used for image reconstruction. For this, the field of view (FOV) was segmented into $96 \times 96 \times 96$ voxels each of $0.6 \times 0.6 \times 0.6 \text{ mm}^3$. From the centre of each voxel, a photon was emitted towards the centre of every bin at the entry of the collimator. The tracking in the object was hence only based on geometry and did not model for neither attenuation nor scatter. The pixel of intersection of the photon with the crystal plane was calculated given the direction vector of the photon and the crystal plane equation as described in section 2.2. Geometrical parameters of the acquisition including misalignments were also considered using the strategy proposed in section 2.7. The calculation of the system matrix was parallelized on the same cluster used for the calculation of the DRFT. The parallelization was performed in a way that a system matrix corresponding to a slice of the FOV was calculated on an independent core. The proposed method of calculating the system matrix will be referred to as the SM-DRFT in the remaining of the paper. The tomographic performance of a system matrix calculated using the SM-DRFT method was evaluated by reconstructing 60 projections of the 2 cm diameter sphere calculated using Geant4 Monte Carlo simulation with a system matrix calculated using the SM-DRFT method.

2.9. Validation of geometrical misalignments correction

The correction method for mechanical shift presented in section 2.7 was validated using simulated data. For this, two sets of 90 projections of a 2 mm diameter sphere over 360° were modelled using Geant4. In the first set of projections, P_1 , no collimator shift was modelled while in the second one, P_2 , a 2 mm collimator mechanical shift was introduced in the radial direction. We then calculated two system matrices using the SM-DRFT method as explained in section 2.8, the first one SM_1 corresponded to an acquisition with no collimator shift while the second one SM_2 took into account a 2 mm collimator mechanical radial shift. Three cases were studied:

1. Projection set P_1 was reconstructed using SM_1 . The reconstructed images were considered as reference images.
2. Projection set P_2 was reconstructed using SM_1 . This case evaluated the impact of not modelling the mechanical shift in the system matrix on the reconstructed images.
3. Projection set P_2 was reconstructed using SM_2 . This case evaluated the capability of correcting for misalignments when it is taken into account in the system matrix.

2.10. Evaluation of penetration impact on image reconstruction using simulation data

The impact of modelling the penetration within the system matrix on a reconstructed image was evaluated using a micro Jaszczack phantom. The micro Jaszczack phantom has a 3 cm inner diameter cylinder that includes six quadrants of equal sized rods. The rods within the quadrants have the following diameters 1.2, 1.6, 2.4, 3.2, 4.0 and 4.8 mm, respectively

(figure 9 (a)). Distance between center of adjacent rods belonging to the same quadrant is two times the diameter of the rod. We define a target area (TA) as an area at the collimator entry towards which the photons directions are forced. Each TA corresponds to a set of bins at the entry of the collimator. For example, a TA of $2 \times 2 \text{ mm}^2$ corresponds to the 20×20 central square bins each of $10^4 \mu\text{m}^2$ at the entry of the collimator. When penetration is not modelled, the photons are forced towards a TA exactly equal to the area of the pinhole aperture (figure 4(a)). Penetration is modelled when photons are allowed to go in the direction of the pinhole knife edges (figure 4(b)). We define two target areas at the entry of the collimator:

1. TA_1 corresponding exactly to the aperture of the collimator pinhole. In this case penetration is not modelled. The DRFT size for this case was equal to 108 MBytes.
2. TA_2 of $6 \times 6 \text{ mm}^2$ centered at the centre of the pinhole aperture. In this case penetration is modelled. The DRFT size for this case was equal to 567 MBytes.

Photons hitting the collimator entry outside the $6 \times 6 \text{ mm}^2$ surface have extremely low probability of reaching the crystal. For this, when modelling penetration, only the $6 \times 6 \text{ mm}^2$ target area was considered and this was used for experiments in this paper.

The notations $P(\text{TA}_i)$ and $\text{SM}(\text{TA}_i)$ where $i \in \{1, 2\}$ refer respectively to the projections and the system matrices obtained with photon directions forced towards the TA_i . The impact of penetration was evaluated when the projections were reconstructed using a system matrix calculated using a TA that mismatched the one used to calculate the projections. During this study, the projections and the system matrices were computed using respectively the proj-DRFT and the SM-DRFT methods.

2.11. Phantom studies

The adequacy of the utilization of the SM-DRFT method including geometrical misalignments was evaluated using experimental data. For this, we assessed the tomographic performance of the micro SPECT system by calculating a system matrix using the SM-DRFT method and using it for the reconstruction of data acquired with a micro Jaszczack phantom. We evaluated the impact of turning on and off the correction of the penetration at the edges of the collimators and/or the geometrical misalignment. Two sets of measurements were performed using $2 \times 1.5 \text{ mm}^2$ and $0.6 \times 0.4 \text{ mm}^2$ pinhole aperture collimators. A set of 90 projections was performed over 360 degrees with an acquisition time of 50 seconds per projection. Projections were sampled into 128×88 pixels of $4.42 \times 4.42 \text{ mm}^2$ each. The phantom was filled with a solution of $^{99\text{m}}\text{Tc}$ having an activity of 544 MBq (14.7 mCi). The geometrical misalignment parameters were estimated using the method presented in (Bequé *et al* 2003) modified in a way that the calibration parameters vary with the angle of view of the SPECT detector head (Hu *et al* 2008). Three hot micro sphere phantoms were used for the calibration. Images were reconstructed using the Maximum Likelihood - Expectation Maximization (ML-EM). Images were sampled into $96 \times 96 \times 96$ voxels each of $0.6 \times 0.6 \times 0.6 \text{ mm}^3$.

2.12. Preclinical studies

A cardiac imaging study of a rat was performed on a clinical GE Millennium VG3 SPECT scanner. A Wistar Kyoto (WKY) rat (21 months old, 506 grams) was imaged using ^{201}Tl to image cardiac perfusion and iodine-123-meta-iodobenzylguanidine (^{123}I -MIBG) to image cardiac sympathetic neuronal function. Lead collimators with $1.5 \times 2 \text{ mm}^2$ rectangular knife edge tungsten pinholes were mounted on each of the two detector heads. The animal was anesthetized with 2% isofluorene and oxygen and was placed on a cardboard support holder. This set-up allowed minimal attenuation and maximum magnification by allowing the pinholes to approach close to the animal. First, the animal was injected with 718 MBq (19.4 mCi) of ^{201}Tl . A dynamic acquisition of 1 sec per projection began immediately at the time of the injection, acquiring 90 projections per gantry rotation. The animal was imaged for twelve 90-second time frames (individual gantry rotations). Then, the animal was injected with 234 MBq (6.33 mCi) of ^{123}I -MIBG and imaged for an additional 60 90-second time frames. Injections were performed through an intravenous route. The geometry of the imaging system was calibrated by acquiring pinhole projections of three hot microsphere phantoms. The protocol was approved by the LBNL Animal Welfare and Research Committee at Lawrence Berkeley National Laboratory.

3. Results and discussion

3.1. Validation of the DRFT

Calculation time was respectively 18 hours for the DRFT and 30 minutes for the system matrix using the SM-DRFT method. The size of the system matrix was 6.3 GBytes.

Figure 5 illustrates how a geometrical misalignment can be corrected by using a system matrix calculated using the SM-DRFT method. Figure 6 presents the comparison of profiles drawn through the centre of the projection of three points using the Geant4 method and the proj-DRFT method. The profiles drawn in the central lines of the two projections fit very well. Figure 7 shows projections of a 2 cm diameter sphere calculated with Geant4 full Monte Carlo simulation and with the proj-DRFT method. The projection calculated with the Geant4 method using 10^6 photons and requiring a computation time of 96 s has a signal to noise ratio equivalent to the one calculated with the proj-DRFT method using 5×10^5 photons and requiring 0.4 s. In this study where the projection of a 2 cm diameter sphere was calculated, the speed up factor is hence 240 when using the proj-DRFT method compared to when a full Geant4 Monte Carlo simulation is performed.

Figure 8 shows that a system matrix calculated using the SM-DRFT method can be used to perform reconstruction of projections calculated using a full Geant4 Monte Carlo simulation. We have adopted this validation strategy and did not calculate a system matrix using Geant4 simulations because it will require excessive computing time in order to have a system matrix with a signal to noise level equivalent to the one obtained with the SM-DRFT method. This result is thus an indirect validation of the system matrix calculated using the SM-DRFT method since the reconstructed images were uniform and presented no artefacts.

During this evaluation work, the choice of the collimator square bin size ($10^4 \mu\text{m}^2$) was based on geometrical arguments assuming the closest point to the collimator to be at 5 mm

and that its projection through one collimator bin on the crystal surface still fall within the intrinsic spatial resolution of the system. As results were satisfactory, we did not undergo further investigations to optimize the choice of the bin size. Hence, the impact of the collimator bin size on the robustness of the DRFT was not studied. More investigations about the impact of the collimator bin size may be required mainly if the shape of the collimator is modified (keel edge instead of knife edge, opening angle, aperture diameter, etc.).

Concerning the choice of the number of photons used to calculate the DRFT, it could be possible to simulate more photons but we estimate that it is not necessary for this study. In all cases, the noise level in a system matrix calculated using the SM-DRFT method is much more lower than the one obtained when the system matrix is calculated directly from simulations performed with Geant4 full Monte Carlo.

3.2. Evaluation of the penetration impact on image reconstruction using simulation data

Figure 9(a) and figure 9(d) represent the case where the TA used to calculate the projections matches the one used to calculate the system matrices. Figure 9(b) and figure 9(c) show the impact of the penetration by showing images reconstructed using system matrices calculated with a TA that mismatches the one used to calculate the projections. The time required for the calculation of the system matrix using the SM-DRFT method is dependent on the target area used (i.e., of the number of bins within the target area). In this particular case, the time required to calculate the system matrix using TA₁ (2×2 mm², 20×20 bins) compared to when using the TA₂ (6×6 mm², 60×60 bins) was accelerated by a factor of 9 (60×60/20×20). However, it is obvious that a lower quality reconstruction is obtained when penetration is not modelled. In general, the size of the target area should be studied when the energy of emitted gamma ray differs from one study to another; this will be needed to reflect more exact range of penetration within the collimator as well as photon incidence on crystal's surface. Moreover, this simulation study has enabled to evaluate the penetration impact independently from other effects like geometrical misalignments, attenuation or scatter that were not modelled. The artefact observed in figure 9(b) corresponds to the case when the penetration is under-estimated in the system matrix. In this case, the activity is over-estimated in the centre of the field of view. This artefact can compensate the attenuation effect in case where the phantom is filled with an attenuation media. Another interesting observation is the case of figure 9(c) where for example the penetration artefact could be attributed to attenuation effect which is not the case because the phantom did not contain any attenuation media.

Finally, we would like to mention that this is a very preliminary study of the effects of penetration to demonstrate the capability of the SM-DRFT method. A more carefully designed study investigating the impact of penetration on both image quality and quantitative accuracy is required for future research.

3.3. Phantom studies

Figures 10 and 11 show the reconstruction improvement when the correction of penetration and/or geometrical misalignment is applied. We can notice that collimators with large

aperture are less sensitive to penetration effect and geometrical misalignment. Furthermore, we can see that small inserts are more sensitive to geometrical misalignment than large inserts. This means that more precaution in correcting the collimator edge penetration effect and the geometrical misalignment should be provided when performing reconstruction with small collimator pinholes and when high spatial resolution is required.

3.4. Preclinical studies

Figure 12 shows a reconstructed slice of a rat heart imaged with ^{123}I -MIBG. As shown in figure 13, the profile drawn through the heart (dashed line in figure 12) shows that the heart's wall can be easily distinguished and that the DRFT method is applicable in preclinical studies.

The main purpose of presenting preclinical results is to show the feasibility of the proposed method for preclinical application. The method presented in this paper was developed independently of the animal studies at LBNL. Once the method developed, we searched within the database of the animal studies for data that could be reconstructed in order to evaluate the feasibility of the DRFT method for realistic cases. Even though the previous studies performed at LBNL use ^{123}I -MIBG and ^{201}Tl , the system matrix was calculated only for iodine and for the 159 keV energy window. We then reconstructed the acquisitions obtained with ^{123}I -MIBG knowing that the data was contaminated by some high-energy gamma emissions from ^{201}Tl (135 keV, 167 keV) and from ^{123}I (350 keV and 538 keV: 3%). We estimate that the study of the impact of contamination is beyond the scope of this paper.

4. Conclusion

In this paper we presented a new methodology of sampling that allows for the storage of a detector response function table (DRFT) for human SPECT camera equipped with pinhole collimators. The same methodology is applicable in dedicated small animal pinhole and multi-pinhole SPECT. The stored DRFT avoids the resimulation of the photons trajectories in the detector when a calibration is required for each study. When modelling the detector only, the DRFT allows one to take into account all the physical processes that occur in the detector. The calculated projections are the same as those obtained when full Monte Carlo simulation is performed with the advantage of reducing the computation time by two orders of magnitude. The utilization of the DRFT for the calculation of the system matrix speeds up its computation time by two orders of magnitude making it acceptable for preclinical and clinical applications.

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Figure 1.

A two dimensional illustration of the family of lines L_{ij} (dashed zone) entering the collimator at bin i and aiming at pixel j . The figure also shows the point of emission S , the point of intersection P with the collimator entrance plane, the point of intersection I with the crystal plane, the center of the crystal C and the vector normal to the surface of the crystal $\vec{n}$.

Figure 2. Illustration of the parameters describing the acquisition geometry of a pinhole SPECT system.

Figure 3.

Example of correction in case of mechanical shift. (a) Illustrates a shift $\vec{m}$ applied on the collimator and (b) illustrates correcting for the shift $\vec{m}$ by applying an inverse shift $-\vec{m}$ to the gamma ray with respect to the centre of the plane of the crystal.

Figure 4.
Illustration of emission from a point source in case of (a) TA₁ and (b) TA₂.

Figure 5.

Reconstructed slice of a 2 mm diameter sphere (MLEM, 20 iterations: 1 minute): (a) projections acquired with no collimator shift, (b) projections acquired with 2 mm collimator radial shift w/o correction in the system matrix, (c) projections acquired with 2 mm collimator radial shift w/correction in the system matrix.

Figure 6.
Profile drawn through the centre line of the projection of three 0.5 mm diameter sphere sources located at $P_1(-20, 0, 0)$, $P_2(0, 0, 0)$ and $P_3(20, 0, 0)$.

Figure 7.

Projection of a 2 cm diameter sphere located in the centre of the field of view. Top line: Geant4 method, bottom line: proj-DRFT method. Numbers at the bottom of each column correspond to the number of photons used for the projection modelling. Signal to noise ratio (SNR) and required computation time are represented respectively on top and bottom of each individual figure.

Figure 8.

Reconstruction of the 2 cm diameter sphere whose projections were calculated using Geant4 Monte Carlo simulations: (a) ideal image, (b) image reconstructed using the system matrix calculated with the SM-DRFT method (MLEM, 20 iterations: 1 minute). The figure at the bottom show the profiles drawn through the centre line of the images (a) and (b).

Figure 9.

Penetration impact on reconstructed images evaluated by matching and mismatching projections and system matrices (MLEM, 100 iterations: 4 minutes): (a) $P(TA_1)/SM(TA_1)$, (b) $P(TA_1)/SM(TA_2)$, (c) $P(TA_2)/SM(TA_1)$ and (d) $P(TA_2)/SM(TA_2)$.

Figure 10.

Slice of the reconstructed image (MLEM, 100 iterations: 4 minutes) of the micro Jaszczack phantom showing the effect of turning on/off the (penetration correction, geometrical misalignment correction): (a) (off, off), (b) (off, on), (c) (on, off), (d) (on, on). Acquisitions were performed using the $2.0 \times 1.5 \text{ mm}^2$ pinhole aperture collimator.

Figure 11.

Slice of the reconstructed image (MLEM, 100 iterations: 4 minutes) of the micro Jaszczack phantom showing the effect of turning on/off the (penetration correction, geometrical misalignment correction): (a) (off, off), (b) (off, on), (c) (on, off), (d) (on, on). Acquisitions were performed using the $0.6 \times 0.4 \text{ mm}^2$ pinhole aperture collimator.

Figure 12.

Reconstructed slice of a rat heart using ^{123}I -MIBG (MLEM, 100 iterations: 4 minutes).

(Contrast between the heart's wall and the background is more preserved on the web version of the article.)

Figure 13.
A profile drawn through the heart showing clearly the wall.