Sensitivity Analysis of Fitness-for-Service Assessment Based on Reliability for Cylindrical Pressure Vessels With Local Metal Loss

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Concern about fitness-for-service (FFS) assessments using stochastic analyses for aged pressure equipment with local metal loss has been growing. When a decision must be made regarding whether to run or repair equipment with local metal loss, a structural integrity assessment based on reliability helps. In analyses of failure probability, it is important to identify which variables strongly affect the structural integrity. The stochastic properties of influential parameters must be clarified, but few data have been published regarding the quantitative analysis of the sensitivity of the parameters in FFS assessments of components with local metal loss. Here, we investigated the effects of parameters on the plastic collapse of a damaged cylindrical pressure vessel with local metal loss, in an evaluation of parameter sensitivity. We also analyzed sensitivity indices for the component with several shapes of local metal loss. We found that the corrosion rate has a major influence on the probability of failure. We propose a practical stochastic analysis procedure for components with local metal loss. In this procedure, the parameter that has consistently low sensitivity to the limit state is used as a constant value. [DOI: 10.1115/1.4024455]

1 Introduction

Over the past few decades, a considerable number of studies have examined FFS assessment techniques for aged industrial plants. The metal loss assessment procedure was developed based on the results of many actual burst tests and simulations using finite element methods [1,2]. Typical FFS standards, i.e., API 579-1/ASME FFS-1 [3], ASME B31G [4], and DNV RP F101 [5] provide closed form equations for the burst pressure or remaining strength of pressure equipment with metal loss. Generally, the structural integrity of equipment with metal loss is evaluated based on the deterministic approach using the equation for burst pressure. Several past studies have validated the estimate equations provided by several standards [6–8].

Concern about the probabilistic approach for metal loss assessment has been growing over the last several years. Ahmad et al. presented a probabilistic methodology for the estimation of the remaining life of a corroded pipeline [9]. Caley et al. carried out a reliability assessment of corroded pipeline using several published failure pressure models [10]. Maes et al. studied the reliability associated with the evolution of corrosion between normal and high-strength pipe material [11]. Regarding general pressure vessels, Osage and Janelle proposed a procedure to determine the probability of failure using the FFS models and the limit state function for the metal loss in a cylindrical shell [12]. The probabilistic approach may be relevant to the design of a maintenance plan based on risk assessment, i.e., risk-based-maintenance.

Stochastic properties of all random variables in a limit state function are needed to quantitatively analyze the safety margin using the probabilistic approach. The details of all random properties (i.e., the mean value, dispersion, and distribution) are usually

not easy to obtain due to a lack of statistical data. Some of the prior studies of the reliability of pressure equipment with metal loss assumed stochastic properties for some random variables.

In order to apply a reasonably probabilistic FFS assessment to actual equipment, the limit state function should consist of the minimum necessary number of random variables. If the influence of a variable on failure probability is low, the variable may be treated as a constant value. It is important to determine which variables greatly affect the structural integrity. However, few studies have quantitatively analyzed the sensitivity of the parameters used in the FFS assessments of components with local metal loss.

In this study, we propose a procedure to establish a practical limit state model based on a sensitivity analysis. The procedure was applied to the FFS assessment of pressure equipment with metal loss. A simplified limit state model for metal loss assessment was established, and a metal loss assessment curve based on reliability using the limit state model was developed.

2 Methodology

2.1 Procedure of Assessment of Probability of Failure. The flow chart for the assessment of the probability of failure is shown in Fig. 1. First, the limit state and parameters used in the FFS assessment of pressure equipment with local metal loss are defined. In this study, the limit state was defined as the bursting of the cylindrical vessel. Second, the probability of failure is calculated in cases in which all variables are treated as random variables. Generally, some of the stochastic properties of variables are not sufficiently clear. It is important to know which variables have significant effects on the probability of failure. Third, the sensitivity indices are analyzed. The sensitivity index is defined as a partial derivative of the probability of failure.

In the first-order reliability method (FORM), the sensitivity index is related to the gradient vector of the limit state function in the space of standard normal variables. The probability of failure is then calculated in cases in which the variables with low

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Fig. 1 Flow chart for development of practical limit state model

sensitivity are treated as deterministic variables. Simplifying problems by reducing the size of variables with random fields helps make the reliability assessment easier. Finally, the calculation results for the probability of failure using the simplified model are verified. The results using the original model and those obtained using the simplified model are compared. If the probability of failure obtained with the simplified model gives a safer side estimation than that using the original model, the original FFS assessment model is calibrated by adopting a correction factor.

2.2 First-Order Reliability Method. In the present study, we used the Hasofer-Lind (H-L) method to evaluate the reliability of components with local metal loss [13]. First, the reduced variables are defined below

$$X'_i = \frac{X_i - \mu_{X_i}}{\sigma_{X_i}} \quad (1)$$

where μ_{X_i} is a mean, σ_{X_i} is a standard deviation, and X'_i is a random variable with zero mean and unit standard deviation. Equation (1) is used to transform the original limit state $g(\mathbf{X}) = 0$ to the reduced limit state, $g(\mathbf{X}') = 0$. In the reduced coordinate system, the limit state $g(\mathbf{X}) = 0$ is basically a nonlinear function. The point of minimum distance from the origin to the limit state surface represents the worst combination of the stochastic variables, named the design point or the most probable point (MPP) of failure. The computation of the minimum distance is an optimization problem

$$\text{Minimize } D = \sqrt{\mathbf{x}'^T \cdot \mathbf{x}'} \quad (2)$$

$$\text{Subject to the constant } g(\mathbf{x}) = g(\mathbf{x}') = 0$$

where $\mathbf{x}'$ represents the coordinates of the checking point on the limit state function in the reduced coordinates to be estimated. The reliability index β_{HL} is expressed by Eq. (3) using the method of Lagrange multipliers.

$$\beta_{HL} = - \frac{\sum_{i=1}^n x'_i \left(\frac{\partial g}{\partial X'_i} \right)^*}{\sqrt{\sum_{i=1}^n \left(\frac{\partial g}{\partial X'_i} \right)^{2*}}} \quad (3)$$

where $\mathbf{x}'^*$ is a vector of design point. The design point is given below

$$x'_i = -\alpha_i \beta_{HL} \quad (i = 1, 2, \dots, n) \quad (4)$$

where

$$\alpha_i = \frac{\left(\frac{\partial g}{\partial X'_i} \right)^*}{\sqrt{\sum_{i=1}^n \left(\frac{\partial g}{\partial X'_i} \right)^{2*}}} \quad (5)$$

The α_i is the direction cosines along the coordinate axes X'_i . In this study, the failure probability P_f is expressed by Eq. (6) using a first-order approximation of limit state function

$$P_f \approx \Phi(-\beta) \quad (6)$$

where $\Phi(\cdot)$ is the cumulative distribution function (CDF) of the standard normal variable.

It must be noted that the H-L method is applicable only for normal random variables. In cases in which all variables do not show a normal distribution, it is necessary to transform the non-normal variables into equivalent normal variables. In this study, we used the two-parameter equivalent normal transformation method suggested by Rackwitz and Fiessler to address this issue. We used a Newton-Raphson-type recursive algorithm, suggested by Rackwitz and Fiessler, to find the design point [14].

2.3 Analysis of Sensitivity Indices. For the reliability analysis, three sensitivities were defined as follows:

$$\eta_i = \frac{\partial P_f}{\partial X_i} \quad \text{Physical sensitivity} \quad (7)$$

$$\frac{\partial P_f}{\partial X_i} \frac{\sigma_{X_i}}{\sigma_{P_f}} = \eta_i \frac{\sigma_{X_i}}{\sigma_{P_f}} \quad \text{Probabilistic sensitivity} \quad (8)$$

$$\frac{\partial P_f}{\partial X_i} \frac{\mu_{X_i}}{\mu_{P_f}} = \eta_i \frac{\mu_{X_i}}{\mu_{P_f}} \quad \text{Rate sensitivity} \quad (9)$$

where P_f is the probability of failure, which is a function of random variables represented by the vector $\mathbf{X}(X_1, X_2, \dots, X_n)$. μ_{X_i} are means of X_i and σ_{X_i} are standard deviations.

Shinozuka performed sensitivity studies for a transport aircraft using the rate sensitivity [15]. In the present study, the probabilistic sensitivity was applied to verify that the parameter which has consistently low sensitivity to limit state was used as constant value.

In calculations of the probability of failure using FORM, the sensitivity index is related to the gradient vector of the limit state function in the space of standard normal variables. α_i is the unit vector in the direction of this gradient vector, and it is expressed as Eq. (10) [16]. The unit vector α_i is shown in Fig. 2.

$$\alpha_i = - \frac{\partial \beta}{\partial X'_i} \quad (10)$$

Kiureghian and Ke indicated the relation between the sensitivity measures with respect to the probability of failure and the derivatives of β [17]. For any set of parameters p , the sensitivities of the first-order probability are given by

$$\nabla_p P_f = -\phi(\beta) \nabla_p \beta \quad (11)$$

where $\phi(\cdot)$ is the standard normal probability density function (PDF). It is found from Eqs. (10) and (11) that the derivatives of

Fig. 2 Sensitivity indices in standard normal space

P_f in the space of standard normal variables, which is probabilistic sensitivity, are directly related to α_i . Here, we used α_i for the assessment of sensitivities. These sensitivity measures are easily computed, since calculating α_i is part of the H-L method; additional calculations are not needed.

3 Results

3.1 Definition of Limit State and Summary of the Variables for the Assessment. The limit state function is defined by Eq. (12). The term of the load on the structure is the applied pressure, and that of the resistance is the burst pressure.

$$g = P_{bc} - P_{applied} \quad (12)$$

For the calculation of burst pressure, several FFS standards for pressure equipment describe the pressure capacity of the damaged component. API 579-1/ASME FFS-1 Part 5 provides a remaining strength factor (RSF) of the component with metal loss. The concept of the RSF is utilized to define the acceptability of a component with metal loss for continued service. The RSF is defined below

$$RSF = \frac{L_{DC}}{L_{UC}} \quad (13)$$

where L_{DC} is the limit or plastic collapse load of the component with metal loss, and L_{UC} is the limit or plastic collapse load of the undamaged component. Osage and Janelle proposed a procedure to determine the burst pressure of a cylindrical shell with metal loss [12]. That burst pressure defined by below equation

$$P_{bc} = RSF \cdot P_{bi} \quad (14)$$

where P_{bi} is the burst pressure of the undamaged cylindrical vessel. Svensson proposed an estimated equation of the burst pressure [18], which is given below

$$P_{bi} = \left(\frac{e}{n} \right)^n \left(\frac{0.25}{n + 0.227} \right) \ln \left(1 + \frac{2t_c}{D} \right) \sigma_u \quad (15)$$

where n is the work hardening coefficient and σ_u is the tensile strength. Janelle et al. performed the validation of the burst pressure of a cylindrical shell with metal loss [1].

The RSF depends on the dimensions of the cylindrical vessel and the metal loss shape. The dimensions of the longitudinal section at the region of metal loss are shown in Fig. 3. D is the internal diameter of the cylindrical shell, s is the longitudinal length at the region of the metal loss, t_{mm} is the minimum measured thickness, and t_{rd} is the uniform thickness evaluated at the location away from the local metal loss location, established by thickness

Fig. 3 Longitudinal section at the region of metal loss

measurements at the time of the assessment. The parameters of thickness using an FFS assessment are given below

$$t_c = t_{rd} - FCA \quad (16)$$

where FCA is the future corrosion allowance. FCA is defined below

$$FCA = C_v \cdot T_i \quad (17)$$

where C_v is the corrosion rate and T_i is the time until the next inspection term. This corrosion rate is established based on previous thickness measurements, information obtained from equipment in similar service, or corrosion design curves. In the present study, corrosion under insulation (CUI) was assumed to be found in the cylindrical pressure vessel, and the corrosion rate was established based on the inspection data from equipment in similar service.

RSF is given by two nondimensional parameters calculated using thickness data at the time of the next inspection. One is the remaining thickness ratio R_t related to the ligament at the location of metal loss, and the other is the shell parameter λ related to the longitudinal length of the metal loss area based on the shell theorem. R_t and λ are given by below equations, respectively.

$$R_t = \frac{t_{mm} - FCA}{t_c} \quad (18)$$

$$\lambda = \frac{1.285s}{\sqrt{D \cdot t_c}} \quad (19)$$

The RSF is calculated using the above parameters as follows:

$$RSF = \frac{R_t}{1 - \frac{R_t}{M_t}} \quad (20)$$

where M_t is the geometrical factor called the “Folias factor,” defined for the cylindrical shell as follows:

$$M_t = 1.0010 - 0.014195\lambda + 0.29090\lambda^2 - 0.096420\lambda^3 + 0.020890\lambda^4 - 0.0030540\lambda^5 + 2.9570(10^{-4})\lambda^6 - 1.8462(10^{-5})\lambda^7 + 7.1553(10^{-7})\lambda^8 - 1.531(10^{-8})\lambda^9 + 1.4656(10^{-10})\lambda^{10} \quad (21)$$

The RSF is based on API 579-1/ASME FFS-1 Part 5 Level 1 procedure [3]. All of the variables used in the reliability analysis and the causes of randomness are summarized in Table 1. The stochastic properties of eight variables except for T_i need to be provided. For some variables, it is not always easy to establish the mean value, standard deviation and distribution due to a lack of statistical data. A sensitivity analysis provides the information needed to determine which parameters are to be treated as deterministic.

Table 1 Variables in the limit state function

Variable	Cause of randomness
P_{applied}	Variation of working conditions
σ_u	Material composition, Heat treatment
n	Material composition, Heat treatment
D	Dimensional tolerance
t_{rd}	Dimensional tolerance, Inspection accuracy
C_v	Environment, Material, Surface condition
s	Inspection accuracy
t_{mm}	Inspection accuracy
T_i	Deterministic variable given by plant engineer

3.2 Calculation of Probability of Failure Using the Full Model. We calculated the probability of failure using the full model, in which all variables in the FFS model are treated as random variables. The stochastic properties are shown in Table 2.

Regarding the corrosion rate C_v , Kurihara et al. investigated the corrosion rate of CUI using actual inspection data [19]. They found that the distribution of corrosion rate for CUI fitted the Type I extreme value distribution of the largest value. The CDF of the distribution is given below

$$F_{Y_n} = \exp \left[-\exp \left[-\frac{y_n - u_n}{\eta_n} \right] \right] \quad (22)$$

where u_n is the location parameter and η_n is the scale parameter. This investigation's results for the shell of a tower indicated $u_n = 0.0725$ and $\eta_n = 0.0775$. In the present study, the mean value and standard deviation of the corrosion rate were calculated by using the stochastic parameters based on these actual data. Both the mean value and the standard deviation are shown as follows:

$$C_v^{\text{mean}} = u_n + 0.5772\eta_n \simeq 0.12[\text{mm/y}] \quad (23)$$

$$C_v^{\text{STD}} = \frac{\pi\eta_n}{\sqrt{6}} \simeq 0.10[\text{mm/y}] \quad (24)$$

The standard deviation of t_{mm} , which is the minimum thickness at the location of the metal loss, is assumed to be the value that depends on the inspection accuracy. In this case, ultrasonic wall thickness measurements are assumed to be used. The absolute sizing accuracy is assumed to be ± 0.5 mm, and the confidence level is assumed to be $80\% = 0.8$. The sizing accuracy and confidence level are based on partial safety factor (PSF) method for corroded pipeline described in DNV RP F101 [5]. This recommended practice provides the table of sizing accuracy for ultrasonic wall thickness. The standard deviation is given below

Table 2 Stochastic properties of random variables in the full model

Variable	Mean	STD	COV ^a	Dist.
P_{applied}	1.08 (MPa)	—	0.03	Type I
σ_u	400 (MPa)	—	0.06	Normal
n	0.2	—	0.06	Normal
D	2400 (mm)	—	0.03	Normal
t_{rd}	13 (mm)	—	0.03	Normal
C_v	0.12 (mm/y)	0.10 (mm/y)	—	Type I
s	68.7–1374.6 (mm)	—	0.05	Normal
t_{mm}	5.2–11.7 (mm)	0.4 (mm)	—	Normal
T_i	6 yr	—	—	—

^aCoefficient of variation.

$$\begin{aligned} \text{Standard Deviation} &= \frac{\text{Size Accuracy}}{\Phi^{-1} \left(0.5 + \frac{\text{Confidence Level}}{2} \right)} \\ &\Rightarrow \frac{0.5}{\Phi^{-1} \left(0.5 + \frac{0.8}{2} \right)} \\ &\Rightarrow 0.39 \simeq 0.4[\text{mm}] \end{aligned} \quad (25)$$

where $\Phi^{-1}(\cdot)$ is the inverse function of the standard normal cumulative distribution.

The stochastic properties of the other parameters were determined by the standard [5] or an engineering judgment. The sensitivity study is useful in order to validate these stochastic properties.

Based on the calculation results, the contours of the probability of failure in various metal loss shapes are shown in Fig. 4. The horizontal axis λ^{mean} and the vertical axis R_t^{mean} are calculated in reference to Eqs. (18) and (19) using the mean value of each parameter.

$$\lambda^{\text{mean}} = \frac{1.285s^{\text{mean}}}{\sqrt{D^{\text{mean}} \cdot t_c^{\text{mean}}}} \quad (26)$$

$$R_t^{\text{mean}} = \frac{t_{\text{mm}}^{\text{mean}} - C_v^{\text{mean}} \cdot T_i}{t_c^{\text{mean}}} \quad (27)$$

It was found from Fig. 4 that the assessment criteria based on low target reliability index allowed the component to be in service of the pressure vessel with deeper metal loss (low R_t^{mean}).

3.3 Sensitivity Study. We conducted probabilistic sensitivity studies using the gradient vector of the limit state function in the space of standard normal variables, given by Eq. (5). We performed the sensitivity studies for applied pressure, tensile strength, work hardening coefficient, diameter, thickness, corrosion rate, minimum remaining thickness, and length of metal loss. The analysis results of sensitivities are shown in Figs. 5–7. These figures show the relation between metal loss shape and sensitivity indices. The results indicate that

- (1) The variable that was most sensitive to the probability of failure was the corrosion rate C_v , and the second most sensitive was the minimum thickness at the location of metal loss t_{mm} .

Fig. 4 Probability of failure assessment curve in case in which all variables are treated as random variables

Fig. 5 Calculation results of sensitivity indices: $P_f = 1 \times 10^{-4}$

Fig. 6 Calculation results of sensitivity indices: $P_f = 1 \times 10^{-5}$

Fig. 7 Calculation results of sensitivity indices: $P_f = 1 \times 10^{-6}$

- (2) The sensitivity indices of all variables except C_v and t_{mm} were nearly zero.
- (3) The differences in the probability of failure had little influence on the sensitivity indices.

The results indicate that the corrosion rate was the most important variable for an FFS assessment. Thus, accurate stochastic properties of the corrosion rate and corroded thickness are needed. For the other variables, it might be not necessary to prepare perfectly stochastic properties. In this study, we determined the stochastic properties of both the corrosion rate and the minimum thickness at the location of metal loss by actual corrosion data and

Table 3 Stochastic properties of random variables in simplified model

Variable	Mean	StD	COV ^a	Dist.
C_v	0.12 (mm/y)	0.10 (mm/y)	—	Type I
t_{mm}	5.2–11.7 (mm)	0.4 (mm)	—	Normal
$P_{applied}$	1.08 (MPa)	—	—	—
σ_u	400 (MPa)	—	—	—
N	0.2	—	—	—
D	2400 (mm)	—	—	—
t_{rd}	13 (mm)	—	—	—
S	68.7–1374.6 (mm)	—	—	—
T_i	6 yr	—	—	—

^aCoefficient of variation.

inspection accuracy. The sources of other properties, however, were not clear, because engineering judgments were included and it was difficult to prepare sufficiently actual data. If a parameter has little effect on the probability of failure, the parameter can be treated as a constant value, and it is not necessary to prepare the stochastic properties. Sensitivity indices provide rational reasons to determine whether a variable is to be treated as random or not.

3.4 Calculation of the Probability of Failure Using the Simplified Model. The probability of failure was calculated based on the simplified model. In this model, the variables that have low sensitivity to the probability of failure are treated as deterministic variables. Based on the sensitivity analysis, the variables other than the corrosion rate and the minimum thickness at the location of metal loss were treated as deterministic variables. The stochastic properties in the simplified model are shown in Table 3, and Fig. 8 shows the 1×10^{-6} contour of probability of failure in various metal loss shapes. A comparison of Figs. 4 with 8 reveals that the contour lines were almost the same, because the parameters which were treated as a constant value had little influence on the probability of failure.

3.5 Verification and Calibration. Using the simplified model, the probability of failure can be easily calculated by using the minimum required number of random parameters. Actually, the parameters which would be treated as deterministic variables show a little scatter. The following two case studies were performed to verify the results of the simplified limit state model.

Fig. 8 Assessment curve of probability of failure in case that C_v and t_{mm} are treated as random variables

Table 4 Stochastic properties for verification of high COV case

Variable	Mean	StD	COV ^a	Dist.
C_v	0.12 (mm/y)	0.10 (mm/y)	—	Type I
t_{mm}	5.2–11.7 (mm)	0.4 (mm)	—	Normal
P_{applied}	1.08 (MPa)	—	0.1	—
σ_u	400 (MPa)	—	0.1	—
N	0.2	—	0.1	—
D	2400 (mm)	—	0.1	—
t_{rd}	13 (mm)	—	0.1	—
S	68.7–1374.6 (mm)	—	0.1	—
T_i	6 (yr)	—	—	—

^aCoefficient of variation.

Table 5 Verification results

Case No.	$\lambda = 0.5$	$\lambda = 5.0$	$\lambda = 10.0$
1	1.04×10^{-6}	1.08×10^{-6}	1.09×10^{-6}
2	1.57×10^{-6}	1.94×10^{-6}	1.99×10^{-6}

- (1) Calculating the probability of failure by applying the parameters in Table 2 to the assessment curve of Fig. 8.
- (2) Calculating the probability of failure by applying the high coefficient of variation (COV) in Table 4 to the assessment curve of Fig. 8.

A case of high COVs is shown in Table 4. The calculations of three points were performed for verification: $\lambda = 0.5$, $\lambda = 5.0$, and $\lambda = 10.0$. The verification results are shown in Table 5. In case No. 1, the probability of failure results were almost the same as the target reliability, 1×10^{-6} . In case No. 2, the values were slightly larger than the target values.

To ensure that the calculation results using the simplified model satisfy the target reliability, we modified the simplified model. We propose the modified limit state function as follows:

$$g_m = \frac{P_{bc}}{X_m} - P_{\text{applied}} \quad (28)$$

where X_m is the correction factor for calibrating the resistance term of the limit state function. X_m was investigated to satisfy the target reliability, $P_f = 1 \times 10^{-6}$, with respect to any metal loss shape.

Fig. 9 Probability of failure assessment curve utilizing the modified limit state model, $X_m = 1.2$ **Table 6 Verification results (modified limit state model $X_m = 1.2$)**

Case No.	$\lambda = 0.5$	$\lambda = 5.0$	$\lambda = 10.0$
2	7.43×10^{-7}	5.51×10^{-7}	5.29×10^{-7}

Figure 9 shows the assessment curve of the probability of failure using the modified limit state model. Here, the correction factor X_m was 1.2. Table 6 shows the verification results applying the case No. 2 to the modified limit state model shown in Fig. 9. Since all results were less than the target reliability, it is appropriate to use the modified limit state model and the correction factor $X_m = 1.2$.

4 Discussion

Our proposed procedure (summarized in Fig. 1) can be used not only for the metal loss of a cylindrical pressure vessel but also for general problems of structural integrity. In cases in which the differences of sensitivities among variables are high, the proposed procedure may also be appropriate. In such cases, the sensitivity indexes of some variables are nearly zero. In the present study's metal loss assessment, the high-sensitivity variable was C_v , and the low-sensitivity variables were all of the variables except C_v and t_{mm} . The correction factor X_m becomes a high value if high-sensitivity variables are treated as constant. The simplified model with high X_m provides too-conservative results.

We performed a sensitivity study about distributions of high sensitive random variable, which was corrosion rate C_v . Here, the shape parameter and slope parameter of Weibull distribution were assumed to be 1.23 and 0.125, respectively. CDF of Type I and Weibull distributions for corrosion rate are shown in Fig. 10. These CDFs are seemed to be close for C_v greater than 0.05. Analysis conditions for the assessment of the Type I distribution assumption for C_v are shown in Table 7. In the cases A1–A3, the probability of failure is 1×10^{-4} . In the cases A4–A6, that is 1×10^{-6} by assumption of the Type I distribution for C_v . The results of calculation of probability of failure using Weibull distribution for C_v are shown in Table 8. It was found from the results that the cases A1–A3 (1×10^{-4}) have larger deviation from the results using Type I distribution than the cases A4–A6 (1×10^{-6}). The differences result from characteristic of exceedance probability. The exceedance probabilities of Type I and

Fig. 10 Cumulative density function of Type I and Weibull distribution for C_v

Table 7 Analysis conditions for the assessment of the Type I distribution assumption for C_v

Case No.	λ^{mean}	R_t^{mean}	Probability of failure
A1	1.0	0.458	1.0×10^{-4}
A2	5.0	0.584	1.0×10^{-4}
A3	10.0	0.615	1.0×10^{-4}
A4	1.0	0.653	1.0×10^{-6}
A5	5.0	0.763	1.0×10^{-6}
A6	10.0	0.790	1.0×10^{-6}

Table 8 Comparison between probability using Type I distribution and Weibull distribution

Case No.	Probability of failure (Weibull distribution for C_v)
A1	7.2×10^{-5}
A2	7.4×10^{-5}
A3	7.4×10^{-5}
A4	2.8×10^{-7}
A5	2.9×10^{-7}
A6	2.9×10^{-7}

Fig. 11 Exceedance probabilities of Type I and Weibull distributions for C_v

Weibull in large C_v are shown in Fig. 11. The figure shows that the ratio between the exceedance probabilities of these distributions is increased in large C_v . Thus, it is important for a sensitive variable to clear the deviation of probability due to assumption of distribution for high sensitive random variables.

The correction factor X_m depends on the result of a case study of high COVs for verification. In the present study, all of the COVs of the low-sensitivity values were assumed to be 0.1 in the verification study with high COVs. This assumption shows that the simplified model can be used in cases in which the COVs of low-sensitivity values are less than 0.1. Regarding low-sensitivity variables in the proposed approach, accurate COVs are not needed, and it is only necessary to decide the maximum of the COVs by engineering judgment.

5 Conclusion

In this study, we have proposed a procedure to establish a practical limit state model using sensitivity analysis. The procedure

was applied to an FFS assessment of pressure equipment with metal loss. The following results were obtained:

- (1) The sensitivity study for metal loss assessment revealed that the corrosion rate and the minimum thickness at the location of metal loss were the most important variables for fitness-for-service assessments. Therefore, accurate stochastic properties of the corrosion rate and the corroded thickness were needed.
- (2) We proposed a simple assessment procedure for metal loss assessment using important random variables, which are the corrosion rate and the minimum thickness at the location of metal loss. The assessment curve was almost the same as original one.
- (3) A modified assessment procedure for assessing metal loss that uses a correction factor was proposed. The proposed assessment curve satisfied target reliability, $P_f = 1 \times 10^{-6}$, by applying 1.2 as the correction factor. The correction factor 1.2 has the limitation that the COV cannot be greater than 0.1.

Nomenclature

- α_i = gradient vector of the limit state function in the space of standard normal variables
- β_{HL} = reliability index
- C_v = corrosion rate
- D = internal diameter of the cylinder
- D^{mean} = mean value of inside diameter of the cylinder
- FCA = future corrosion allowance
- g = limit state function for assessment of local metal loss
- g_m = modified limit state function for assessment of local metal loss
- λ = longitudinal flaw length parameter
- M_t = folias factor based on the longitudinal length of the local metal loss
- L_{DC} = limit or plastic collapse load of the component with metal loss
- L_{UC} = limit or plastic collapse load of the undamaged component
- n = work hardening coefficient
- P_{applied} = applied pressure
- P_{bi} = burst pressure of undamaged cylindrical shell
- P_f = probability of failure
- RSF = remaining strength factor
- R_t = remaining thickness ratio
- s = longitudinal length of the region of the local metal loss at the time of the inspection
- s^{mean} = mean value of longitudinal length of the region of the local metal loss at the time of the inspection
- σ_u = tensile strength
- t_c = corroded wall thickness evaluated at the location away from the region of local metal loss
- t_c^{mean} = mean value of corroded wall thickness evaluated at the location away from the region of local metal loss
- T_i = time until next inspection term
- t_{mm} = minimum remaining thickness determined at the time of the assessment
- $t_{\text{mm}}^{\text{mean}}$ = mean value of minimum remaining thickness determined at the time of the assessment
- t_{rd} = uniform thickness evaluated at the location away from the local metal loss location established by thickness measurements at the time of the assessment
- X_m = correction factor for modified limit state function

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