Analytical Study of a Pin-Loaded Hole in Unidirectional Laminated Composites With Triangular and Circular Fibers

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The problem of stress concentrations in the vicinity of pin-loaded holes is of particular importance in the design of multilayered composite structures made of triangular or circular glass fibers. It is assumed that all of the fibers in the laminate lie in one direction while loaded by a force p_0 at infinity, parallel to the direction of the fibers. According to the shear lag model, equilibrium equations are derived for both types of fibers. A rectangular arrangement is postulated in either case. Upon the proper use of boundary and bondness conditions, stress fields are derived within the laminate, along with the surrounding pinhole. The analytical results are compared to those of the finite element values. A very good agreement is observed between the two methods. According to the results, composite structures made of triangular glass fibers result in lower values of stress concentrations around the pin, as opposed to those of circular glass fibers. [DOI: 10.1115/1.4007212]

1 Introduction

Fiber-reinforced polymer composites have been widely used in various kinds of structural applications. In general, composites can be joined through a variety of methods. Among those, one can point out bolts and pins (rivets) as mechanical elements. The improper design of any joint may lead to structural problems or affect its integrity. In addition, any conservative design may indirectly lead to the structure being overweight. On a microscopic scale, there are possible advantages to the increased rigidity of individual fibers, leading to improved fiber alignment within the composite and, therefore, a potential for increased compressive strength. Gains in rigidity are provided by the increase in the second moment of an area for a given mass of glass reinforcement. Microcomposite compression testing (using resin bonded tows of 12–15 filaments) has revealed that triangular fibers have a compression strength 60% greater than circular fibers [1]. Mechanical testing under tensile load has shown that the triangular glass fiber reinforced plastic performs marginally 20% better than that of the circular fiber, for the same fiber volume fractions [1]. Many authors have tried to investigate tensile or compressive stress distribution around a pin-loaded hole through experimental and/or analytical methods, using different assumptions. De Jong [2] gives one approach to the analytical solution of stresses around a pin-loaded hole in an orthotropic plate. Zhang and Ueng [3] also studied the stress distribution around a pin-loaded hole. In their study, displacement fields satisfying the required conditions were first postulated. With the proper use of deduced stress functions, the normal and shear stresses along the edge of the hole were determined. Waszczak and Cruse [4] studied the bolt load on the strength and failure of a composite plate using the method of complex functions. Yavari et al. [5] studied the induced stresses around a pin-loaded hole in a symmetrically stacked composite laminate of finite size. In their model they used the friction and clearance between the pin and hole edge. Wong and Matthews [6] used a 2D finite element analysis to solve the problem of single and two-hole bolted joints. They compared their solutions with

experimental data. In Ref. [7], a progressive failure analysis of a pin-loaded composite plate was performed to predict the bearing stress deformation curve of the joint up to the point of its failure. Camanho and Lambert [8] used a numerical technique, in conjunction with an analytical solution, to study the stress field in a composite in the presence of a pin. They used complex functions to solve the deduced equations. Echavarria and his co-authors [9] studied the presence of a pin on the deduced stresses in an orthotropic plate using complex functions. In their work, they used a harmonic function to simulate the compressive stresses around the pin. They neglected any clearance around the pin and its hole. Turvey and Wang [10] used the finite element method to find the effect of a pin and other geometrical parameters on the deduced stresses in an orthotropic plate. They used contact elements in their finite element model. Additionally, a certain clearance was selected between the pin and its hole. Grüber et al. [11] developed an analytical method to calculate the notch stresses in a multilayered anisotropic structure due to the presence of a pin. Their work was based on some assumptions on the load distribution around the pin. Li et al. [12] used fiber steering technique around bolt holes in carbon fiber reinforced epoxy composite laminates to locally enhance the bearing strength of the bolted joint. The shear lag concept, which was introduced by Hedgepeth to analyze the stress concentration in a unidirectional fibrous composite, has been widely used by many authors in their investigations [14]. It was shown that [13–16] this model gives relatively accurate results on normal stresses developed in composites with a low extensional stiffness in the matrix. Shishesaz and Attar [15] studied the effect of the pin load around two pins in series, which were used to join two composite plates. In addition, the effects of geometric parameters such as the edge distance to the pin hole diameter e/d , the pin hole diameters, and the center to center distance of the two pins were examined on the stress distribution in the joint. Shishesaz et al. [16] calculated the stress distribution and the displacement field around a pin joint for two arrangements of fibers (hexagonal and rectangular) were calculated.

Stress concentrations in filamentary composites are usually affected by many factors such as fiber spacing, the characteristics of the fiber/matrix interface, the ratio of the fiber to matrix elastic moduli, the fiber arrangements, the fiber's cross sectional shape, the method of joining, etc. Hence, the main objective of this paper

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Fig. 1 Laminate with triangular fibers

is to evaluate the effect of such factors on the stress concentrations in a laminated fibrous composite with a finite width dimension. Here, two types of cross sections, namely, triangular and circular fibers, are selected for filaments. The laminated composite is fastened to its surrounding medium through a pin. The resulting pin reactions are modeled through a series of spring elements, accounting for any possible pin elasticity.

2 Derivation of Field Equations

To derive the equilibrium equations, two models are considered, which will be separately discussed. In the derivation of the equations, the shear lag model is used. In this model, it is assumed that all fibers sustain a normal load while the matrix can take only pure shear. Moreover, it is assumed that there is a perfect bond between each fiber and its neighboring bays. All fibers behave as linearly elastic, up to the point of fracture. Linear elasticity is used to derive all field equations. Although, in practice, fibers may be irregularly dispersed within the laminate, throughout the modeling, a defined arrangement and spacing is selected between the filaments. This assumption is used by many authors throughout their analytical investigations. Let us consider the cross section of a laminated composite plate wherein the fibers are arranged according to Fig. 1. Based on this figure, i represents each column of fibers with numbers ranging from 1 to M , while j corresponds to any row of fibers in which the fibers are numbered from 1 to Ψ (Ψ is assumed to be even). For this arrangement of filaments, it can be shown that the volume fraction of fibers v_f is given by

$$v_f = \frac{0.25M\Psi\nu^2}{\{M\nu + (M-1)\eta\sin\alpha\}\{\mu(\Psi-1) + 0.5\nu\Psi\tan\alpha + 2\mu\cos\alpha\}} \quad (1)$$

Parameters η , μ , ν , and α are shown in Fig. 1. To write the equilibrium equations, the fibers are grouped into three groups as follows (see Fig. 1)

- Group I: those fibers surrounded by three filaments
- Group II: those fibers surrounded by one filament (corner fibers)
- Group III: those fibers surrounded by two filaments (edge fibers)

Figure 2 depicts the rectangular arrangement of fibers within the laminate. Here, m represents the number of fibers in each layer while γ corresponds to the total number of layers. Here, α_0 , ρ , and ζ are the fiber dimensions and spacing, respectively. For such an arrangement of filaments, one can show that the volume fraction of fibers is given by

Fig. 2 Fibers in a rectangular arrangement

$$v_f = \frac{m\gamma\alpha_0^2}{\{m\alpha_0 + \zeta(m-1)\}\{\gamma\alpha_0 + \rho(\gamma-1)\}} \quad (2)$$

Fibers in a rectangular arrangement are categorized into three groups (see Fig. 2) as follows

- Group I: those fibers surrounded by four filaments
- Group II: corner fibers which are surrounded by two filaments
- Group III: the edge fibers, surrounded by only three filaments

For a typical triangular fiber in group I, the equilibrium equation of forces results in (see Fig. 3)

$$\frac{dp_{ij}}{dx} + \{\tau_{i+1,j} - \tau_{i-1,j}\}dA_1 - \{\tau_{i,j+1}\}dA_2 = 0 \quad (3)$$

where p_{ij} and τ_{ij} correspond to the normal load in the (ij) th fiber and the corresponding shear stress in its neighboring matrix bays, respectively. In addition, x represents the axial coordinate acting along the direction of the fibers. For other fibers categorized in groups II and III, similar expressions can be written. We now introduce the following nondimensional parameters as

$$\zeta = \sqrt{\frac{G_m h_0}{E_f A_f v}} x, \quad S_t = \frac{p_{ij}}{p_0}, \quad U_{ij} = \sqrt{\frac{E_f A_f G_m h_0}{p_0^2 v}} u_{ij},$$

$$\Omega = \frac{v}{\eta \sin \alpha}, \quad \Lambda = \frac{2v}{\mu \tan \alpha} \quad (4)$$

Here, G_m represents the matrix shear modulus, p_0 is the applied load at infinity, E_f and u_{ij} correspond to the elastic modulus of the fibers and their axial displacement, respectively. Using Eq. (4), one may write the equilibrium equations in a nondimensional form as

Fig. 3 Example of three dimensions of the mesh and elements

$$\text{Group I: } \frac{d^2 U_{ij}}{d\zeta^2} + \Omega U_{i-1,j} - (2\Omega + \Lambda) U_{ij} + \Omega U_{i+1,j} + \Lambda U_{i,j+1} = 0 \quad (5)$$

$$\text{Group II: } \frac{d^2 U_{ij}}{d\zeta^2} - \Omega U_{ij} + \Omega U_{i+1,j} = 0 \quad (6)$$

$$\text{Group III: } \frac{d^2 U_{ij}}{d\zeta^2} + \Omega U_{i-1,j} - 2\Omega U_{ij} + \Omega U_{i+1,j} = 0 \quad (7)$$

Using the linear elasticity along with the shear lag model, the equilibrium equations in a nondimensional form for a rectangular arrangement of fibers reduce to

$$\text{Group I: } \frac{d^2 \hat{U}_{ij}}{d\zeta^2} + \phi \{ \hat{U}_{i+1,j} + \hat{U}_{i-1,j} - 2\hat{U}_{ij} \} + \varphi \{ \hat{U}_{i,j+1} + \hat{U}_{i,j-1} - 2\hat{U}_{ij} \} = 0 \quad (8)$$

For the fibers in groups II and III, similar expression can be written (please see Ref. [16]). Here, the small hat symbol “^” is used to designate the parameters associated with circular fibers. The following expressions are used to write the preceding equations

$$\varphi = \frac{\alpha_0}{\rho}, \quad \phi = \frac{\alpha_0}{\zeta}, \quad \zeta = \sqrt{\frac{G_m}{E_f A_f}} x, \quad \hat{U}_{ij} = \sqrt{\frac{E_f A_f G_m}{p_0^2}} \hat{u}_{ij} \quad (9)$$

The equilibrium Eqs. (5)–(7), along with Eq. (8), may be written in a matrix notation as

$$[U''] - [L][U] = [0] \quad (10)$$

where U is equal to $[U_{1,1}, U_{1,2}, \dots, U_{M,\Psi}]^T$ for triangular fibers and $\hat{U}$ is equal to $[\hat{U}_{1,1}, \hat{U}_{1,2}, \dots, \hat{U}_{m,\gamma}]^T$ for a rectangular arrangement of fibers. To solve the equilibrium equations, the laminated plate is divided into regions I and II, as shown in Fig. 4. According to Fig. 4, l and q represent the outermost broken fibers bonding the hole, while d corresponds to the hole diameter. Parameter e corresponds to the distance between the center of the pinhole to the free edge of the laminate. Equation (10) can be solved for the complete displacement and load in all of the fibers of regions I and II. One can show that the solution for the displacement of the fibers in region I may be written in terms of the eigenvalues $\pm \lambda_k$ and the eigenvectors $[R]^k$, while for region II, the positive values of the eigenvectors must be discarded due to the bondness condition as $P_{ij} = 1$ as $\zeta \rightarrow \infty$.

Fig. 4 Division of the laminate into two regions

For a rectangular arrangement of filaments, the displacement field in all of the fibers of regions I and II are written as

$$\begin{cases} \text{Reg. I: } \hat{U}_{ij} = \sum_{k=1}^N \hat{D}_k \hat{R}_{i+m(j-1)}^k e^{\lambda_k \zeta} + \sum_{k=1}^N \hat{E}_k \hat{R}_{i+m(j-1)}^k e^{-\lambda_k \zeta} \\ \text{Reg. II: } \hat{U}_{ij} = \sum_{k=1}^N \hat{C}_k \hat{R}_{i+m(j-1)}^k e^{\lambda_k \zeta} + \zeta \end{cases} \quad (11)$$

For triangular fibers one may write

$$\begin{cases} \text{Reg. I: } U_{ij}^* = \sum_{k=1}^N D_k R_{i+M(j-1)}^k e^{\lambda_k \zeta} + \sum_{k=1}^N E_k R_{i+M(j-1)}^k e^{-\lambda_k \zeta} \\ \text{Reg. II: } U_{ij} = \sum_{k=1}^N C_k R_{i+m(j-1)}^k e^{\lambda_k \zeta} + \zeta \end{cases} \quad (12)$$

In the preceding equations, a term such as $R_{i+m(j-1)}^k$ refers to a value on the $i + m(j - 1)i + m(j - 1)$ row of the k th eigenvector associated with the k th eigenvalue. Additionally, N refers to the total number of fibers ($m \times \gamma$ or $M \times \Psi$). In Eqs. (11) and (12), $C_k, D_k, E_k, \hat{C}_k, \hat{D}_k$ and $\hat{E}_k$ are constants yet to be determined, using proper boundary and continuity conditions for each case.

3 Boundary and Continuity Conditions

Using the following boundary and continuity conditions, the solution to the equilibrium equations may be obtained as follows: For all fibers in region I, at the free edge of the laminate (see Fig. 5), one may write the following relation for both types of fiber arrangements as

$$\begin{cases} P_{ij}^* = 0 & \text{as } \zeta_{ij} = -Q_{ij} \\ \hat{P}_{ij}^* = 0 & \text{as } \zeta_{ij} = -\hat{Q}_{ij} \end{cases} \quad (13)$$

The nondimensional parameters Q_{ij} and $\hat{Q}_{ij}$ are defined as $\hat{Q} = \sqrt{G_m/E_f A_f} e$ and $Q = \sqrt{(G_m h/E_f A_f v)} e$. Using Figs. 4 and 5, for triangular fibers, a force balance on any cut fibers in direct contact with the pin results in a nondimensional form for the fibers in group I

$$\frac{dU_{ij}^*}{d\zeta} + \tan \theta_{ij} \{ \Delta_2 U_{ij+1}^* + \Delta_1 (3U_{i+1,j}^* + U_{i-1,j}^*) - (\Delta_2 + 4\Delta_1) U_{ij}^* \} + \Gamma U_{ij}^* = 0 \quad (14)$$

Fig. 5 Loads at the end of a typical cut fiber by the hole (spring element are not shown)

Table 1 Dimensions and mechanical properties of selected materials

$v_f = 0.471$ $m = M = 17, \Psi = \gamma = 4$		Triangular fibers		Rectangular arrangement	
E_f (GPa)	G_m (GPa)	v (mm)	μ, η (mm)	α_0 (mm)	ζ, ρ (mm)
80	1.6	1	0.5	0.65	0.39

Table 2 Comparison of the results based on the analytical solution and those of the FEM

Tensile stress concentration factor, S_t		
Type of fiber cross section	Analytical solution	FEM
Rectangular arrangement of fibers	3.88	3.76
Triangular fibers	3.42 ($\alpha = 30$ deg)	3.34

Similarly, for a rectangular arrangement, a force balance on any cut fibers in direct contact with the pin in the nondimensional form results in

$$\hat{P}_{ij} + A_0 \phi \{ \hat{U}_{i-1,j} - \hat{U}_{ij} \} \tan \theta_{ij} + 0.5 A_0 \phi \tan \theta_{ij} \{ \hat{U}_{ij-1}^* + \hat{U}_{ij+1}^* - 2 \hat{U}_{ij}^* \} + \hat{\Gamma} \hat{U}_{ij}^* = 0 \quad (15)$$

In the preceding equations, Δ_1 , Δ_2 , and Γ are defined as

$$\Delta_1 = \sqrt{\frac{G_m}{8E_f}} \frac{1}{\sin \alpha} \frac{v}{\eta}, \quad \Delta_2 = \sqrt{\frac{2G_m}{E_f}} \frac{1}{\tan \alpha} \frac{v}{\mu}, \quad \Gamma = \frac{(k_0)_n}{\sqrt{E_f A_f G_m^*}},$$

$$\hat{\Gamma} = \frac{(k_0)_n}{\sqrt{G_m E_f A_f}}, \quad A_0 = \alpha \sqrt{\frac{G_m}{E_f A_f}} \quad (16)$$

In both models, on the free surface of all cut fibers in region two, the nondimensional equilibrium equations are similar to Eqs. (15) and (16) with $\Gamma = \hat{\Gamma} = 0$ (please see Refs. [15,16]). In both models, the continuity in the load and displacements for all intact fibers in regions I and II at $\zeta = 0$ or $\varsigma = 0$ can be written. Upon the proper substitution of the preceding boundary and continuity equations into the field equations, the complete load and displacement fields are determined within the laminate. It is worth mentioning that S_t , S_c , and S_{yx} correspond to the tensile stress concentration, the compressive stress concentration, and the nondimensional shear stress within the laminate and are defined as follows:

Triangular fibers

$$S_t = \frac{P_{ij}}{p_0}, \quad S_c = -\frac{P_{ij}}{p_0}, \quad S_{yx} = \frac{\nu}{\eta} \{ U_{ij} - U_{i-1,j} \} \quad (17)$$

Rectangular arrangement of fibers

$$S_t = \frac{\hat{P}_{ij}}{p_0}, \quad S_c = -\frac{\hat{P}_{ij}^*}{p_0}, \quad S_{yx} = \{ \hat{U}_{ij} - \hat{U}_{i-1,j} \} \quad (18)$$

4 Discussion of Results

In order to investigate the effect of pin stiffness on tensile and compressive stresses, and the peak shear stress developed within the laminate, the following values in Table 1 were used. Due to nature of the loading, compressive stresses are produced behind the pin in region I, while tensile stresses are developed in both regions. The number of broken fibers “ r ” (along with the cut matrix bays in between) controls the pinhole diameter. This number is selected to be the same in all of the layers. To check the accuracy of the results obtained through the analytical solution, a finite

Table 3 The effect of Γ on the maximum tensile or compressive stress concentration factors within the fibers behind the pin

i Γ	S_t		S_c			S_t
	5	6	7	9	10	11
∞	3.58	4.50	1.29	1.29	4.49	3.58
40	3.58	4.50	1.30	1.30	4.50	3.58
20	3.58	4.45	1.32	1.32	4.45	3.58
8	3.59	4.35	1.38	1.38	4.35	3.59
4	3.60	4.23	1.44	1.44	4.23	3.60

Fig. 6 The effect of broken fibers on the maximum tensile stress concentration in the laminate

element model (FEM) of the laminate was prepared. In order to investigate the effect of the pin stiffness on the stress concentration in the laminate, the parameter Γ was allowed to vary from ∞ to 4. A rectangular arrangement of fibers was used for the analysis, while r was set equal to 7. For the typical data given in Table 2, these two values correspond to $k_0 = \infty$ MN/m (rigid support) and 45.2 MN/m, respectively. An examination of the results in Table 3 shows that for any typical fiber in direct contact with the pin (region I), tensile or compressive stress concentrations (S_t or S_c) do not seem to be very dependent on the pin stiffness for the previously selected values. Figure 6 depicts the effect of e/d on the tensile stress concentration factors in the first intact fiber bonding the hole. The results in this figure are for triangular fibers. As the e/d ratio is decreased, the stress concentration in both types of fibers is increased. This means that for a fixed value of d , for holes closer to the free edge, fibers will experience higher values of tensile stress concentrations.

The effect of e/d on S_t becomes almost negligible as its value approaches that of 4. Figure 7 shows the effect of M (the number of fibers in each layer), on the tensile stress concentration factors in the first intact triangular fiber bonding the hole. Parameter “ w ” corresponds to the width of the laminate. As “ M ” increases, the stress concentration in the first intact fiber decreases. This shows the effect of the laminate edge on the tensile stress concentrations.

Figure 8 demonstrates the distribution of the compressive stresses behind the pin, which are based on $r = 7$. A similar behavior is observed for other values of “ r ”. Here, an increase in the e/d ratio results in a decrease in the compressive stresses developed within the fibers (laminate) in direct contact with the pin. Broken fibers further away from the x -axis seem to sustain higher values of compressive stress concentration.

Fig. 7 The effect of e and w on the maximum stress concentration produced within the laminate

Fig. 8 Compressive stress distribution within the fibers behind the pin

According to Fig. 9, for triangular fibers, the peak value of the shear stress within the laminate occurs in a bay bonded by the last broken fiber and the first intact fiber bonding the hole (point b). This value seems to be independent of the e/d ratio as the size of the hole is increased.

Figure 10 shows the effect of the side angle α (see Fig. 1) on the maximum tensile stress concentration factors, which occurs in the first intact fiber bonding the hole. An increase in α causes a rise in the stress concentration in the first intact fiber bonding the hole. Here, " r " corresponds to the total number of broken fibers cut by the pinhole.

Figure 11 shows the effect of α on the peak value of the shear stress, which is developed at point b within the laminate. According to this figure, an increase in α results in an increase in the peak shear stress. This rise seems to be almost independent of r (or pinhole diameter). Figure 12 depicts the peak values of the tensile stress concentrations versus r (or pinhole diameter). Here, the fibers are assumed to have a circular cross-section and are arranged in a rectangular fashion (see Fig. 2). Due to symmetry, all fibers lying in the y - z plane and symmetric with respect to the x - y surface behave in the same manner.

Fig. 9 Variation of peak shear stresses at point b

Fig. 10 The effect of α on S_t within the laminate

Fig. 11 Variation of the peak shear stress at point b as a function of α

Fig. 12 Tensile stress variation within the laminate as a function of r ($m = 17$)

5 Conclusions

The effect of the pin load and its elasticity on the stress distribution in a unidirectional laminate subjected to a tensile load was studied for two models wherein two types of cross sections, namely, circular and triangular, were considered for the fibers. The analytical results on the maximum tensile stresses developed in both models were compared to those of the finite element solution. A very close agreement was observed between the two methods. According to the results, the pin stiffness does not seem to have much effect on the compressive stresses developed in the fibers behind the pin. The values of these stresses seem to be less in a laminate with triangular fibers. The percentage decrease is about 28% for $e/d = 1$. Although the maximum tensile stresses seem to be less in laminates with triangular fibers (about 17.5% at $e/d = 1$), the effect of e/d on this stress becomes almost negligible (in both models) as the ratio approaches that of 4. In a laminate with triangular fibers, the side angle α seems to have a noticeable effect on the peak values of the stresses induced in all fibers. In both models, the cut fibers behind the pin and closest to the intact filaments bonding the hole seem to take most of the compressive stress developed in the laminate. A comparison of the results reveals that, for similar values of the e/d ratio, a rectangular arrangement of fibers results in higher values of stress concentrations within the laminate as compared to those of triangular fibers.

Nomenclature

A_f = cross-sectional areas of the triangular and circular fibers
 A_0 = nondimensional parameter defined in Eq. (16)
 $\hat{C}_k, C_k$ = unknown constants
 d = pinhole diameter
 $\hat{D}_k, D_k$ = unknown constants
 dA_1, dA_2 = differential areas of triangular fibers over which the shear stress τ_{ij} acts
 e = distance from laminate edge to pinhole center
 E_f = fiber elastic modulus
 $\hat{E}_k, E_k$ = unknown constants
 G_m = matrix shear modulus
 h_0 = height of a triangular fiber
 k_0 = pin stiffness
 L = coefficient matrix
 M, m = total number of triangular (circular) fibers in each layer

N = total number of fibers in the laminate
 p_o = normal load applied to the laminate at infinity
 p_{ij} = localized load in each fiber
 $P_{ij}, \hat{P}_{ij}$ = nondimensional loads in fibers
 $Q, \hat{Q}$ = nondimensional parameters (Eq. (13))
 R = eigenvectors
 r = total number of broken fibers in each layer
 S_t = tensile stress concentration
 S_c = compressive stress concentration
 S_{yx} = nondimensional shear stress in each matrix bay
 $u_{ij}, \hat{u}_{ij}$ = fiber displacements
 $U_{ij}, \hat{U}_{ij}$ = nondimensional fiber displacements
 w = laminate width
 α = side angle of each triangular fiber
 α_0 = diameter of each circular fiber
 γ = number of layers in a laminate with circular fibers
 Γ = nondimensional spring constant (Eq. (14))
 $\hat{\Gamma}$ = nondimensional spring constant (Eq. (15))
 Δ_1 = nondimensional parameter (Eq. (14))
 Δ_2 = nondimensional parameter (Eq. (14))
 ζ = distance between any two successive columns of circular fibers
 η = fiber spacing in triangular fibers (see Fig. 2)
 θ_{ij} = position of each cut fiber in polar coordinates
 λ_k = k th eigenvalue
 Λ = nondimensional parameter (Eq. (5))
 μ = triangular fiber spacing (see Fig. 1)
 ν = size of each side of triangular fiber
 ξ = nondimensional coordinate along the direction of circular fibers
 ρ = distance between any two successive rows for circular fibers
 ς = nondimensional coordinate along direction of circular fibers
 τ_{ij} = shear stress in each matrix bay
 v_f = fiber volume fraction
 ϕ = nondimensional parameter (Eq. (8))
 φ = nondimensional parameter (Eq. (8))
 ψ = total number of layers with triangular fibers
 Ω = nondimensional parameter (Eq. (5))

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