Mohammad Reza Khoshnavan Azar

Associate Professor
Mechanical engineering Department,
University of Tabriz, Tabriz, Iran
e-mail: rkhosh@tabrizu.ac.ir

Ali Akbar Emami Satellou¹

Student
e-mail: aa_emami@nigc-dist8.ir

Mohammad Shishesaz

Professor
e-mail: shishesaz@yahoo.com

Islamic Azad University, Ahvaz Branch,
Mechanical engineering Department,
Ahvaz, Iran

Bahram Salavati

Iranian Gas Engineering
and Development Company,
Tehran, Iran
e-mail: Salavati_b@yahoo.com

Calculating the Optimum Angle of Filament-Wound Pipes in Natural Gas Transmission Pipelines Using Approximation Methods

Given the increasing use of composite materials in various industries, oil and gas industry also requires that more attention should be paid to these materials. Furthermore, due to variation in choice of materials, the materials needed for the mechanical strength, resistance in critical situations such as fire, costs and other priorities of the analysis carried out on them and the most optimal for achieving certain goals, are introduced. In this study, we will try to introduce appropriate choice for use in the natural gas transmission composite pipelines. Following a 4-layered filament-wound (FW) composite pipe will consider an offer our analyses under internal pressure. The analyses' results will be calculated for different combinations of angles 15 deg, 30 deg, 45 deg, 55 deg, 60 deg, 75 deg, and 80 deg. Finally, we will compare the calculated values and the optimal angle will be gained by using the Approximation methods. It is explained that this layering is as the symmetrical. [DOI: 10.1115/1.4007189]

Keywords: composite material, natural gas, transmission pipeline, approximation method, optimum angle

1 Introduction

At present, tend to use of FW composite pipes is increasing in different industries and structures. FW composite pipes made of fiber-reinforced plastics have many potential advantages over pipes made from conventional materials, such as high specific stiffness and strength, good corrosion resistance and thermal insulation [1]. With the development of manufacturing technology to produce FW pipes, there has been a growing interest in application of the FW fiber-reinforced cylindrical composite structures [2].

Numerous studies, including bending [3], transverse loading [4], axial compression [5], and internal pressure loading conditions [6], has been conducted on the mechanical properties and Failure of FW pipes. The elasticity solution of isotropic cylindrical shells subjected to uniform radial linear loads has been studied by Klosner et al. [7]. Based on the solution of Lekhnitskii [8], Wild and Vickers [9] have developed an analytical procedure for the orthotropic cylindrical sheets. Xia et al. [6] have developed an analytical procedure to assess the multilayered FW composite pipes under internal pressure. Also, other people such as Wahab [10], Bakaiyan [11] and Ansari [12] have done extensive research in various load conditions on composite pipes.

Laney [2] has described the use of composite pipe materials in transportation of natural gas. Note that natural gas is dangerous and incendiary material, so you can use a substance for preventing the spread of fire. Mouritz [13] has introduced the Carbon-Epoxy as the most often used composite in load-bearing aircraft structures, which is flammable and readily decomposes when exposed to heat and fire. Funucci [14] has reported that the thermal conductivity of a solid char at room temperature is $0.17 \text{ Wm}^{-1} \text{ K}^{-1}$,

whereas the in-plane conductivity for a virgin carbon-epoxy laminate is so higher at about $8\text{--}12 \text{ Wm}^{-1} \text{ K}^{-1}$, depending on fiber content. Low density and highly porous chars tend to provide the best thermal insulation [13].

2 Analysis Procedure

2.1 Characterization of a Unidirectional Lamina. Consider a four-layered FW laminated composite cylindrical pipe subjected to an internal pressure loading as shown in Fig. 1. We denote by r the radial, θ the hoop, and z the axial direction in cylindrical coordinates.

Cracks in composite materials are divided into internal cracks and interlaminar cracks. Eshraghi [15] has shown when increase the value of E_x/E_y , decreases the likelihood of internal cracks (see Fig. 2). Due to this we can write:

$$\begin{cases} E_x = E_1^f V^f + E^m V^m \\ \frac{1}{E_y} = \frac{V^f}{E_2^f} + \frac{V^m}{E^m} \end{cases} \quad (1)$$

$$V^f + V^m = 1 \quad (2)$$

If we obtain

$$H = \frac{E_x}{E_y} = [E_1^f V^f + E^m (1 - V^f)] \left[\frac{V^f}{E_2^f} + \frac{(1 - V^f)}{E^m} \right] \quad (3)$$

Since the Eq. (3) v_f is variable; therefore, the ratio of E_x/E_y or, in other words, the value of H , be maximum, it must be established with following:

¹Corresponding author.

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Fig. 1 Four-layer FW composite pipe in cylindrical coordinates

$$\frac{dH}{dV^f} = 0 \quad (4)$$

Thus, by solving Eq. (3)

$$V^f = \frac{\left(2 - \frac{E_1^f}{E^m} - \frac{E^m}{E_2^f}\right)}{2\left(\frac{E_1^f}{E_2^f} - \frac{E_1^f}{E^m} - \frac{E^m}{E_2^f} + 1\right)} \quad (5)$$

(x,y,z): Material principal coordinate system (r,θ,z): Cylindrical coordinate system

2.2. Stress and Deformation Analyses. See Fig. 1 again. When pipes are subjected to axisymmetric loading ($\partial/\partial\theta = 0$), the stresses and strain are independent of θ . In addition, the radial and axial displacement does not depend on the axial (z) and radial (r) directions, respectively.

According to Xia et al. [6] studies

$$\begin{Bmatrix} \sigma_z \\ \sigma_\theta \\ \sigma_r \\ \tau_{\theta r} \\ \tau_{zr} \\ \tau_{z\theta} \end{Bmatrix}^{(k)} = \begin{bmatrix} \bar{C}_{11} & \bar{C}_{12} & \bar{C}_{13} & 0 & 0 & \bar{C}_{16} \\ \bar{C}_{12} & \bar{C}_{22} & \bar{C}_{23} & 0 & 0 & \bar{C}_{26} \\ \bar{C}_{13} & \bar{C}_{23} & \bar{C}_{33} & 0 & 0 & \bar{C}_{36} \\ 0 & 0 & 0 & \bar{C}_{44} & \bar{C}_{45} & 0 \\ 0 & 0 & 0 & \bar{C}_{45} & \bar{C}_{55} & 0 \\ \bar{C}_{16} & \bar{C}_{26} & \bar{C}_{36} & 0 & 0 & \bar{C}_{66} \end{bmatrix}^{(k)} \begin{Bmatrix} \varepsilon_z \\ \varepsilon_\theta \\ \varepsilon_r \\ \gamma_{\theta r} \\ \gamma_{zr} \\ \gamma_{z\theta} \end{Bmatrix}^{(k)} \quad (6)$$

and

$$\begin{cases} \varepsilon_r^{(k)} = \frac{du_r^{(k)}}{dr} \\ \varepsilon_\theta^{(k)} = \frac{u_r^{(k)}}{r} \\ \varepsilon_z^{(k)} = \frac{du_z^{(k)}}{dz} = \varepsilon_0 \end{cases} \quad \begin{cases} \gamma_{z\theta}^{(k)} = \frac{du_\theta^{(k)}}{dz} = \gamma_0 r \\ \gamma_{zr}^{(k)} = 0 \\ \gamma_{\theta r}^{(k)} = \frac{du_\theta^{(k)}}{\partial r} - \frac{u_\theta^{(k)}}{r} \end{cases} \quad (7)$$

For the anisotropic materials used in this study, there exist $\bar{C}_{22}^{(k)}/\bar{C}_{33}^{(k)} > 0$ and $\bar{C}_{22}^{(k)}/\bar{C}_{33}^{(k)} \neq 1$. If $\beta^{(k)} = \sqrt{\bar{C}_{22}^{(k)}/\bar{C}_{33}^{(k)}}$ we have

$$u_r^{(k)} = D^{(k)} r^{\beta^{(k)}} + E^{(k)} r^{-\beta^{(k)}} + \alpha_1^{(k)} \varepsilon_0 r + \alpha_2^{(k)} \gamma_0 r^2 \quad (8)$$

Fig. 2 Relation of coordinate system between principal material axis and cylindrical axes

where

$$\alpha_1^{(k)} = \frac{\bar{C}_{12}^{(k)} - \bar{C}_{13}^{(k)}}{\bar{C}_{33}^{(k)} - \bar{C}_{22}^{(k)}} \quad (9)$$

$$\alpha_2^{(k)} = \frac{\bar{C}_{26}^{(k)} - 2\bar{C}_{36}^{(k)}}{4\bar{C}_{33}^{(k)} - \bar{C}_{22}^{(k)}}$$

And $D^{(k)}$ and $E^{(k)}$ are integration constants.

2.3. 3D Laminated Plate Properties. To define the 3D alternate-ply material properties, the material modulus matrix elements $C_{ij}(i,j=x,y,z)$ and $G_{ij}(i=x,y,z)$ are needed. These values are calculated by following equations.

$$E_x = E_z, \quad G_{yy} = G_{zz}, \quad \nu_{zx} = \nu_{yx} \quad (10)$$

$$G_{xx} = \frac{E_y}{2(1 + \nu_{zy})} \quad (11)$$

$$\begin{bmatrix} C_{xx} & C_{xy} & C_{xz} \\ C_{yy} & C_{yz} \\ \text{Sym} & C_{zz} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_x} & \frac{-\nu_{yx}}{E_x} & \frac{-\nu_{yz}}{E_x} \\ \frac{1}{E_y} & \frac{-\nu_{zy}}{E_y} \\ \text{Sym} & \frac{1}{E_z} \end{bmatrix}^{-1} \quad (12)$$

The off-axis stiffness constants in Eq. (6), $\{\bar{C}_{ij}^{(k)}\}$, can be calculated from the on-axis stiffness constants, $\{C_{ij}^{(k)}\}$, by using a stiffness transformation matrix $[A_{kl}]$, written as:

Table 1 carbon (fiber) and epoxy 105/206 (matrix) engineering constants

E_1^f	E_2^f	G_{12}^f	E^m	ν_{12}^f	ν_{32}^f	ν^m
233 (GPa)	23.1 (GPa)	8.96 (GPa)	4.62 (GPa)	0.200	0.400	0.360

Table 2 Engineering constants for lamia made of carbon/epoxy

E_x (GPa)	E_y (GPa)	E_z (GPa)	G_{xx} (GPa)	G_{yy} (GPa)	G_{zz} (GPa)	ν_{xy}	ν_{yz}	ν_{zx}
$\nu^f = 0.6149$								
145.0475	9.0928	9.0928	3.25	3.25	3.3857	0.2616	0.400	0.2616

Table 3 values of ε_0 , γ_0 for $(\pm 15)_3$, $(\pm 30)_3$, $(\pm 45)_3$, $(\pm 55)_3$, $(\pm 60)_3$, $(\pm 75)_3$, and $(\pm 80)_3$

$\emptyset$	γ_0	ε_0
$(\pm 15)_s$	1.8244×10^{-7}	-2.4×10^{-3}
$(\pm 30)_s$	4.1472×10^{-7}	-11.7×10^{-3}
$(\pm 45)_s$	-4.1529×10^{-7}	-12.5×10^{-3}
$(\pm 55)_s$	-1.2574×10^{-6}	1.8×10^{-3}
$(\pm 60)_s$	-1.5116×10^{-6}	9.0×10^{-3}
$(\pm 75)_s$	-1.9453×10^{-6}	21×10^{-3}
$(\pm 80)_s$	-2.0254×10^{-6}	22.6×10^{-3}

Table 5 Radial displacements for $(\pm 15)_3$, $(\pm 30)_3$, $(\pm 45)_3$, $(\pm 55)_3$, $(\pm 60)_3$, $(\pm 75)_3$, and $(\pm 80)_3$

R (mm)	498	500.5	503	505.5	508
$(\pm 15)_s$	24.6808	24.6314	24.5829	24.5353	24.4885
$(\pm 30)_s$	22.2106	22.1759	22.1419	22.1087	22.0763
$(\pm 45)_s$	10.5468	10.5367	10.5272	10.5184	10.5101
$(\pm 55)_s$	2.7945	2.7856	2.7773	2.7695	2.7622
$(\pm 60)_s$	1.1825	1.1695	1.1571	1.1453	1.1340
$(\pm 75)_s$	0.9142	0.8895	0.8654	0.8419	0.8189
$(\pm 80)_s$	1.1693	1.1425	1.1163	1.0906	1.0655

Table 4 values of ε_0 , γ_0 for different combinations of angles 15 deg, 30 deg, 45 deg, 55 deg, 60 deg, 75 deg, and 80 deg

$\emptyset$	γ_0	ε_0
$(+30, -15)_s$	-1.75×10^{-2}	-3.2×10^{-3}
$(+45, -15)_s$	-2.45×10^{-2}	-2.6×10^{-3}
$(+55, -15)_s$	-2.57×10^{-2}	-1.9×10^{-3}
$(+60, -15)_s$	-2.54×10^{-2}	-1.6×10^{-3}
$(+75, -15)_s$	-1.62×10^{-2}	3.98×10^{-4}
$(+80, -15)_s$	-7.1×10^{-3}	1.7×10^{-3}
$(+45, -30)_s$	-1.37×10^{-2}	-7.0×10^{-3}
$(+55, -30)_s$	-1.41×10^{-2}	-4.2×10^{-3}
$(+60, -30)_s$	-1.26×10^{-2}	-2.7×10^{-3}
$(+75, -30)_s$	7.29×10^{-4}	2.6×10^{-3}
$(+80, -30)_s$	9.9×10^{-3}	5.1×10^{-3}
$(+55, -45)_s$	-3.6×10^{-3}	-3.4×10^{-3}
$(+60, -45)_s$	-2.2×10^{-3}	-2.42×10^{-3}
$(+75, -45)_s$	1.05×10^{-2}	7.9×10^{-3}
$(+80, -45)_s$	1.8×10^{-2}	10.6×10^{-3}
$(+60, -55)_s$	9.0×10^{-4}	5.3×10^{-3}
$(+75, -55)_s$	1.2×10^{-2}	12.8×10^{-3}
$(+80, -55)_s$	1.84×10^{-2}	14.9×10^{-3}
$(+75, -60)_s$	1.08×10^{-2}	15.2×10^{-3}
$(+80, -75)_s$	1.68×10^{-2}	16.9×10^{-3}
$(+80, -75)_s$	5.7×10^{-3}	21.7×10^{-3}

Table 6 Radial displacements for different combinations of angles 15 deg, 30 deg, 45 deg, 55 deg, 60 deg, 75 deg, and 80 deg

R (mm)	498	500.5	503	505.5	508
$(+30, -15)_s$	16.2141	16.1828	16.1520	16.1218	16.0928
$(+45, -15)_s$	9.7329	9.7141	9.6957	9.6776	9.6609
$(+55, -15)_s$	7.0149	7.0011	6.9876	6.9743	6.9624
$(+60, -15)_s$	5.9555	5.9435	5.9318	5.9204	5.9101
$(+75, -15)_s$	3.1598	3.5110	3.5025	3.4942	3.4868
$(+80, -15)_s$	3.0195	3.0106	3.0019	2.9934	2.9857
$(+45, -30)_s$	10.0432	10.0283	10.0140	10.0000	9.9872
$(+55, -30)_s$	6.1769	6.1671	6.1578	6.1487	6.1407
$(+60, -30)_s$	4.9108	4.9022	4.8940	4.8861	4.8792
$(+75, -30)_s$	2.7916	2.7822	2.7732	2.7643	2.7563
$(+80, -30)_s$	2.6572	2.6457	2.6345	2.6235	2.6132
$(+55, -45)_s$	4.7667	4.7591	4.7521	4.7454	4.7395
$(+60, -45)_s$	3.4506	3.4425	3.4349	3.4278	3.4213
$(+75, -45)_s$	2.0929	2.0796	2.0667	2.0541	2.0421
$(+80, -45)_s$	2.2714	2.2550	2.2391	2.2236	2.2084
$(+60, -55)_s$	1.9411	1.9305	1.9204	1.9108	1.9018
$(+75, -55)_s$	1.5437	1.5264	1.5097	1.4935	1.4776
$(+80, -55)_s$	1.8769	1.8569	1.8375	1.8185	1.7999
$(+75, -60)_s$	1.2859	1.2666	1.2478	1.2296	1.2117
$(+80, -75)_s$	1.6600	1.6384	1.6172	1.5966	1.5763
$(+80, -75)_s$	1.1899	1.1640	1.1387	1.1140	1.0898

$$\{\bar{C}_{ij}^{(k)}\} = [A_{kl}]\{C_{ij}^{(k)}\} \quad (13)$$

$$\{\bar{C}_{ij}^{(k)}\} = \{\bar{C}_{11}^{(k)}, \bar{C}_{12}^{(k)}, \bar{C}_{13}^{(k)}, \bar{C}_{16}^{(k)}, \bar{C}_{22}^{(k)}, \bar{C}_{23}^{(k)}, \bar{C}_{26}^{(k)}, \bar{C}_{33}^{(k)}, \bar{C}_{36}^{(k)}, \bar{C}_{44}^{(k)}, \bar{C}_{45}^{(k)}, \bar{C}_{66}^{(k)}\}^T \quad (14)$$

$$\{C_{ij}^{(k)}\} = \{C_{xx}^{(k)}, C_{yy}^{(k)}, C_{zz}^{(k)}, C_{xy}^{(k)}, C_{xz}^{(k)}, C_{yz}^{(k)}, G_{xx}^{(k)}, G_{yy}^{(k)}, G_{zz}^{(k)}\}^T \quad (15)$$

$$[A_{kl}] = \begin{bmatrix} m^4 & n^4 & 0 & 2m^2n^2 & 0 & 0 & 0 & 0 & 4m^2n^2 \\ m^2n^2 & m^2n^2 & 0 & m^4 + n^4 & 0 & 0 & 0 & 0 & -4m^2n^2 \\ 0 & 0 & 0 & 0 & m^2 & n^2 & 0 & 0 & 0 \\ m^3n & -mn^3 & 0 & -m^3n + mn^3 & 0 & 0 & 0 & 0 & -2m^3n + 2mn^3 \\ n^4 & m^4 & 0 & 2m^2n^2 & 0 & 0 & 0 & 0 & 4m^2n^2 \\ 0 & 0 & 0 & 0 & n^2 & m^2 & 0 & 0 & 0 \\ mn^3 & -m^3n & 0 & m^3n - mn^3 & 0 & 0 & 0 & 0 & 2m^3n - 2mn^3 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & mn & -mn & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & m^2 & n^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -mn & mn & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & n^2 & m^2 & 0 \\ m^2n^2 & m^2n^2 & 0 & -2m^2n^2 & 0 & 0 & 0 & 0 & (m^2 - n^2)^2 \end{bmatrix} \quad (16)$$

Fig. 3 Average radial displacement for $(\pm 15)_3$, $(\pm 30)_3$, $(\pm 45)_3$, $(\pm 55)_3$, $(\pm 60)_3$, $(\pm 75)_3$, and $(\pm 80)_3$

where $m = \cos\phi$ and $n = \sin\phi$, and ϕ is the cylindrical angle of the filament from the pipe axis.

2.4. Boundary Condition. All the unknown integration constants in Eq. (8) may be determined by substituting these equations into boundary conditions and solving the algebraic equations. These boundary conditions are

$$\begin{cases} \sigma_r^{(1)}(r_0) = -P_0 \\ \sigma_r^{(n)}(r_a) = 0 \end{cases} \quad \begin{cases} \tau_{\theta r}^{(1)}(r_0) = \tau_{zr}^{(1)}(r_0) = 0 \\ \tau_{\theta r}^{(n)}(r_a) = \tau_{zr}^{(n)}(r_a) = 0 \end{cases} \quad (17)$$

$$\begin{cases} u_r^{(k)}(r_k) = u_r^{(k+1)}(r_k) \\ u_\theta^{(k)}(r_k) = u_\theta^{(k+1)}(r_k) \end{cases} \quad \begin{cases} \sigma_r^{(k)}(r_k) = \sigma_r^{(k+1)}(r_k) \\ \tau_{zr}^{(k)}(r_k) = \tau_{zr}^{(k+1)}(r_k) \\ \tau_{\theta r}^{(k)}(r_k) = \tau_{\theta r}^{(k+1)}(r_k) \end{cases} \quad (18)$$

Fig. 4 Average radial displacement for different combinations of angles 15 deg, 30 deg, 45 deg, 55 deg, 60 deg, 75 deg, and 80 deg

where P_0 is the internal pressure, r_0 and r_a are the inner and outer radius, respectively. In addition, the two integral conditions can be expressed as

$$\begin{cases} 2\pi \sum_{k=1}^n \int_{r_{k-1}}^{r_k} \sigma_z^{(k)}(r) r dr = \pi r_0^2 P_0 \\ 2\pi \sum_{k=1}^n \int_{r_{k-1}}^{r_k} \tau_{z\theta}^{(k)}(r) r^2 dr = 0 \end{cases} \quad (19)$$

The first integral condition satisfies the axial equilibrium for a cylinder with closed ends, and the second equation is the zero torsion condition.

The integration constants' $D^{(k)}$, $D^{(k)}$, and ε_0 , γ_0 for four-layered pipe can be obtained from the solution of the simultaneous equation as follows:

$$\begin{pmatrix} D^{(1)} \\ D^{(2)} \\ D^{(3)} \\ D^{(4)} \\ E^{(1)} \\ E^{(2)} \\ E^{(3)} \\ E^{(4)} \\ \varepsilon_0 \\ \gamma_0 \end{pmatrix} = \begin{bmatrix} d_{11} & 0 & 0 & 0 & e_{11} & 0 & 0 & 0 & a_{11} & a_{12} \\ d_{21} & d_{22} & 0 & 0 & e_{21} & e_{22} & 0 & 0 & a_{21} & a_{22} \\ 0 & d_{32} & d_{33} & 0 & 0 & e_{32} & e_{33} & 0 & a_{31} & a_{32} \\ 0 & 0 & d_{43} & d_{44} & 0 & 0 & e_{43} & e_{44} & a_{41} & a_{42} \\ d_{51} & d_{52} & 0 & 0 & e_{51} & e_{52} & 0 & 0 & a_{51} & a_{52} \\ 0 & d_{62} & d_{63} & 0 & 0 & e_{62} & e_{63} & 0 & a_{61} & a_{62} \\ 0 & 0 & d_{73} & d_{74} & 0 & 0 & e_{73} & e_{74} & a_{71} & a_{72} \\ 0 & 0 & 0 & d_{84} & 0 & 0 & 0 & e_{84} & a_{81} & a_{82} \\ d_{91} & d_{92} & d_{93} & d_{94} & e_{91} & e_{92} & e_{93} & e_{94} & a_{91} & a_{92} \\ d_{01} & d_{02} & d_{03} & d_{04} & e_{01} & e_{02} & e_{03} & e_{04} & a_{01} & a_{02} \end{bmatrix}^{-1} \begin{pmatrix} -P_0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ r_0^2 P_0 / 2 \\ 0 \end{pmatrix} \quad (20)$$

where parameters of d, e, a are given in Appendix [6].

3 Numerical Results

In this analysis, all pipes are made with four plies of pipes with carbon fiber-epoxy with inner radius 498 mm and outer radius 508 mm (40 in. outer diameter). Internal pressure is 9 MPa. Table 1 shows carbon (fiber) and epoxy 105/206 (matrix) engineering constants.

According to Eq. (5), V_f is equal to 0.615. Therefore, engineering constants for lamina has shown in Table 2.

With solving Eqs. (13) and (20) results for values of ε_0 , γ_0 for different combinations of angles 15 deg, 30 deg, 45 deg, 55 deg, 60 deg, 75 deg, and 80 deg have been shown in Tables 3 and 4.

Tables 5 and 6 according Eq. (8) have shown radial displacements for different combinations of angles 15 deg, 30 deg, 45 deg, 55 deg, 60 deg, 75 deg, and 80 deg.

That Figs. 3–6 shows these values in the graphs.

4 Calculation of Optimum Angle

Figures 3 and 4 shows a comparison between the average radial displacements calculated for different angles. Given these graphs from the result Tables 3 and 4, as is clear when we use the same angle with opposite directions, the minimum radial displacement occurs at angles $(\pm 75)_s$. But at different angles as well, wherever it used 75 deg and more than it (for example, we solved for 80 deg), greatly reduced the amount of radial displacements. However, only low amount of displacement is not a criterion for design.

Fig. 5 Values of ε_0 for $(\pm 15)_3$, $(\pm 30)_3$, $(\pm 45)_3$, $(\pm 55)_3$, $(\pm 60)_3$, $(\pm 75)_3$, and $(\pm 80)_3$

Fig. 6 Values of ε_0 for different combinations of angles 15 deg, 30 deg, 45 deg, 55 deg, 60 deg, 75 deg, and 80 deg

As it can be seen in Tables 3 and 4, when using from two different angles, relative to that of same angle with opposite directions, increasing the amount of γ_0 or the twist to a very high, about a thousand even ten thousand. In addition, according to the Table 4, how to use a smaller angle in the opposite direction, decreases the value of γ_0 . This result is exactly the opposite of the previous conclusions, which tells matter how to use the larger angles, the radial displacement decreases.

However, the third factor that can be important in pipelines especially natural gas transmission pipelines, is the value of ε_0 . This amount, according to Eq. (7), is an elongation in the longitudinal direction of the pipe. This value is important when it becomes clear that, with respect to the pipe by connecting multiple joints to each other, this amount should be kept the minimum in order to tension and compression at the pipe. Because, it can cause bending or disjoining. But this is always tried on the design of pipelines; the pipes always maintain the tensile mode under various loads. Therefore, all compounds of angles were calculated with value of ε_0 is negative, they are unsuitable for use in transmission pipelines.

Therefore, according to the description provided, where axial force is considered, the best angle is the $[\pm 75]_3$ for the pipe. However, since the pipelines are required to use the joints, according to observations of Tables 3 and 4, and Fig. 5, the best angle is $(\pm 55)_3$ for the pipe.

If the ε_0 values in Fig. 5 continuously be connected together and this curve be approximated by using the least squares or Gram-Schmidt approximation methods with order 5, we can write

$$\varepsilon_0 = (1.061 \times 10^{-6})\varphi^5 - (2.764 \times 10^{-4})\varphi^4 + (2.662 \times 10^{-2})\varphi^3 - (1.1443)\varphi^2 + (21.2721)\varphi - 140.6713 \quad (21)$$

where φ is the angle in degrees, If we show Eq. (21) in graph, we have Fig. 7.

If we take this equation equal to zero, the equation will be answered 53.994 deg, and the error compared with the Actual value that is 53.829 deg, is 0.307%.

However, despite the desire in the 53.829 deg the values of ε_0 toward zero, due to manufacturing considerations and prevent of

Fig. 7 Approximation graph for Eq. (21)

the disjoining, composite pipe always should be under small tensions, so it seems more logical to use 55 deg for the pipe.

5 Conclusions

- (1) Carbon-epoxy composite material with good heat conductivity and good resistance to ignition and fire growth is suitable material for use in natural gas composite transmission pipelines.
- (2) The minimum radial displacement occurs at angles $(\pm 75)_3$.
- (3) At different angles as well, wherever it used 75 deg and more than it, greatly reduced the amount of radial displacements.
- (4) All compounds of angles were calculated with value of ε_0 is negative, they are unsuitable for use in transmission pipelines.
- (5) Where axial force is considered, the best angle is the $(\pm 75)_3$ for the pipe. However, since the pipelines are required to use the joints, according to observations of Tables 3 and 4, and Fig. 5, the best angle is $(\pm 55)_3$ for the pipe.
- (6) Although the desire in the 53.829 deg the values of ε_0 toward zero, due to manufacturing considerations and prevent of the disjoining, composite pipe always should be under small tensions, so it seems more logical to use 55 deg for the pipe.

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Nomenclature

- $\bar{C}_{ij}$ = stiffness matrix element ($i, j = 1, 2, \dots, 6$)
 E_x = the Young's modulus of the lamina in the longitudinal direction of fiber
 E_y = the Young's modulus of the lamina in the transverse direction
 E_1^f = the Young's modulus of the fiber in the longitudinal direction
 E_2^f = the Young's modulus of the fiber in the transverse direction
 E_x = the Young's modulus of the matrix
 G = shear Modulus
 u_i = displacement along i direction ($i = r, \theta, z$)
 V_f = fiber volume fraction
 V_m = matrix volume fraction
 γ_{ij} = shear strain in i, j plane ($i, j = r, \theta, z$)
 ε_i = normal strain in i direction ($i = r, \theta, z$)
 ν = the Poisson's Ratio
 σ_i = normal stress in i direction ($i = r, \theta, z$)
 τ_{ij} = shear stress (the first index i indicates that the stress acts on a plane normal to the x_i axis, and the second index j denotes the direction in which the stress acts. $i = r, \theta, z$)

$$d_{11} = \left(\bar{C}_{23}^{(1)} + \beta^{(1)} \bar{C}_{33}^{(1)} \right) r_0^{\beta^{(1)}-1}$$

$$a_{11} = \bar{C}_{13}^{(1)} + \alpha_1^{(1)} \left(\bar{C}_{23}^{(1)} + \bar{C}_{33}^{(1)} \right)$$

$$d_{21} = r_1^{\beta^{(1)}}$$

$$e_{21} = r_1^{-\beta^{(1)}}$$

$$a_{21} = \left(\alpha_1^{(1)} - \alpha_1^{(2)} \right) r_1$$

$$d_{32} = r_2^{\beta^{(2)}}$$

$$e_{32} = r_2^{-\beta^{(2)}}$$

$$a_{31} = \left(\alpha_1^{(2)} - \alpha_1^{(3)} \right) r_2$$

$$d_{43} = r_3^{\beta^{(3)}}$$

$$e_{43} = r_3^{-\beta^{(3)}}$$

$$a_{41} = \left(\alpha_1^{(3)} - \alpha_1^{(4)} \right) r_3$$

$$d_{51} = \left(\bar{C}_{23}^{(1)} + \beta^{(1)} \bar{C}_{33}^{(1)} \right) r_1^{\beta^{(1)}-1}$$

$$e_{51} = \left(\bar{C}_{23}^{(1)} - \beta^{(1)} \bar{C}_{33}^{(1)} \right) r_1^{-\beta^{(1)}-1}$$

$$a_{51} = \left(\bar{C}_{13}^{(1)} - \bar{C}_{13}^{(2)} \right) + \alpha_1^{(1)} \left(\bar{C}_{23}^{(1)} + \bar{C}_{33}^{(1)} \right) - \alpha_1^{(2)} \left(\bar{C}_{23}^{(2)} + \bar{C}_{33}^{(2)} \right)$$

$$a_{52} = \left[\left(\bar{C}_{36}^{(1)} - \bar{C}_{36}^{(2)} \right) + \alpha_2^{(1)} \left(\bar{C}_{23}^{(1)} + 2\bar{C}_{33}^{(1)} \right) - \alpha_2^{(2)} \left(\bar{C}_{23}^{(2)} + 2\bar{C}_{33}^{(2)} \right) \right] r_1$$

$$d_{62} = \left(\bar{C}_{23}^{(2)} + \beta^{(2)} \bar{C}_{33}^{(2)} \right) r_2^{\beta^{(2)}-1}$$

$$e_{62} = \left(\bar{C}_{23}^{(2)} - \beta^{(2)} \bar{C}_{33}^{(2)} \right) r_2^{-\beta^{(2)}-1}$$

$$a_{61} = \left(\bar{C}_{13}^{(2)} - \bar{C}_{13}^{(3)} \right) + \alpha_1^{(2)} \left(\bar{C}_{23}^{(2)} + \bar{C}_{33}^{(2)} \right) - \alpha_1^{(3)} \left(\bar{C}_{23}^{(3)} + \bar{C}_{33}^{(3)} \right)$$

$$a_{62} = \left[\left(\bar{C}_{36}^{(2)} - \bar{C}_{36}^{(3)} \right) + \alpha_2^{(2)} \left(\bar{C}_{23}^{(2)} + 2\bar{C}_{33}^{(2)} \right) - \alpha_2^{(3)} \left(\bar{C}_{23}^{(3)} + 2\bar{C}_{33}^{(3)} \right) \right] r_2$$

$$d_{73} = \left(\bar{C}_{23}^{(3)} + \beta^{(3)} \bar{C}_{33}^{(3)} \right) r_3^{\beta^{(3)}-1}$$

$$e_{73} = \left(\bar{C}_{23}^{(3)} - \beta^{(3)} \bar{C}_{33}^{(3)} \right) r_3^{-\beta^{(3)}-1}$$

$$a_{71} = \left(\bar{C}_{13}^{(3)} - \bar{C}_{13}^{(4)} \right) + \alpha_1^{(3)} \left(\bar{C}_{23}^{(3)} + \bar{C}_{33}^{(3)} \right) - \alpha_1^{(4)} \left(\bar{C}_{23}^{(4)} + \bar{C}_{33}^{(4)} \right)$$

$$a_{72} = \left[\alpha_2^{(3)} \left(\bar{C}_{23}^{(3)} + 2\bar{C}_{33}^{(3)} \right) - \alpha_2^{(4)} \left(\bar{C}_{23}^{(4)} + 2\bar{C}_{33}^{(4)} \right) \right] r_3$$

$$d_{84} = \left(\bar{C}_{23}^{(4)} + \beta^{(4)} \bar{C}_{33}^{(4)} \right) r_a^{\beta^{(4)}-1}$$

$$a_{81} = \bar{C}_{13}^{(4)} + \alpha_1^{(4)} \left(\bar{C}_{23}^{(4)} + \bar{C}_{33}^{(4)} \right)$$

$$d_{91} = \frac{\bar{C}_{12}^{(1)} + \beta^{(1)} \bar{C}_{13}^{(1)}}{1 + \beta^{(1)}} \left[r_1^{\beta^{(1)}+1} - r_0^{\beta^{(1)}+1} \right]$$

$$d_{93} = \frac{\bar{C}_{12}^{(3)} + \beta^{(3)} \bar{C}_{13}^{(3)}}{1 + \beta^{(3)}} \left[r_3^{\beta^{(3)}+1} - r_2^{\beta^{(3)}+1} \right]$$

$$e_{91} = \frac{\bar{C}_{12}^{(1)} - \beta^{(1)} \bar{C}_{13}^{(1)}}{1 - \beta^{(1)}} \left[r_1^{-\beta^{(1)}+1} - r_0^{-\beta^{(1)}+1} \right]$$

$$e_{93} = \frac{\bar{C}_{12}^{(3)} - \beta^{(3)} \bar{C}_{13}^{(3)}}{1 - \beta^{(3)}} \left[r_3^{-\beta^{(3)}+1} - r_2^{-\beta^{(3)}+1} \right]$$

$$a_{91} = \sum_{k=1}^4 \left[\bar{C}_{11}^{(k)} + \alpha_1^{(k)} \left(\bar{C}_{12}^{(k)} + \bar{C}_{13}^{(k)} \right) \right] \frac{(r_k^2 - r_{k-1}^2)}{2}$$

$$e_{11} = \left(\bar{C}_{23}^{(1)} - \beta^{(1)} \bar{C}_{33}^{(1)} \right) r_0^{-\beta^{(1)}-1}$$

$$a_{12} = \left[\bar{C}_{36}^{(1)} + \alpha_2^{(1)} \left(\bar{C}_{23}^{(1)} + 2\bar{C}_{33}^{(1)} \right) \right] r_0$$

$$d_{22} = -r_1^{\beta^{(2)}}$$

$$e_{22} = -r_1^{-\beta^{(2)}}$$

$$a_{22} = \left(\alpha_2^{(1)} - \alpha_2^{(2)} \right) r_1^2$$

$$d_{33} = -r_2^{\beta^{(3)}}$$

$$e_{32} = -r_2^{-\beta^{(3)}}$$

$$a_{32} = \left(\alpha_2^{(2)} - \alpha_2^{(3)} \right) r_2^2$$

$$d_{44} = -r_3^{\beta^{(4)}}$$

$$e_{44} = -r_3^{-\beta^{(4)}}$$

$$a_{42} = \left(\alpha_2^{(3)} - \alpha_2^{(4)} \right) r_3^2$$

$$d_{52} = -\left(\bar{C}_{23}^{(2)} + \beta^{(2)} \bar{C}_{33}^{(2)} \right) r_1^{\beta^{(2)}-1}$$

$$e_{52} = -\left(\bar{C}_{23}^{(2)} - \beta^{(2)} \bar{C}_{33}^{(2)} \right) r_1^{-\beta^{(2)}-1}$$

$$d_{63} = -\left(\bar{C}_{23}^{(3)} + \beta^{(3)} \bar{C}_{33}^{(3)} \right) r_2^{\beta^{(3)}-1}$$

$$e_{63} = -\left(\bar{C}_{23}^{(3)} - \beta^{(3)} \bar{C}_{33}^{(3)} \right) r_2^{-\beta^{(3)}-1}$$

$$d_{74} = -\left(\bar{C}_{23}^{(4)} + \beta^{(4)} \bar{C}_{33}^{(4)} \right) r_3^{\beta^{(4)}-1}$$

$$e_{74} = -\left(\bar{C}_{23}^{(4)} - \beta^{(4)} \bar{C}_{33}^{(4)} \right) r_3^{-\beta^{(4)}-1}$$

$$e_{84} = \left(\bar{C}_{23}^{(4)} - \beta^{(4)} \bar{C}_{33}^{(4)} \right) r_a^{-\beta^{(4)}-1}$$

$$a_{82} = \left[\bar{C}_{36}^{(4)} + \alpha_2^{(4)} \left(\bar{C}_{23}^{(4)} + 2\bar{C}_{33}^{(4)} \right) \right] r_a$$

$$d_{92} = \frac{\bar{C}_{12}^{(2)} + \beta^{(2)} \bar{C}_{13}^{(2)}}{1 + \beta^{(2)}} \left[r_2^{\beta^{(2)}+1} - r_1^{\beta^{(2)}+1} \right]$$

$$d_{94} = \frac{\bar{C}_{12}^{(4)} + \beta^{(4)} \bar{C}_{13}^{(4)}}{1 + \beta^{(4)}} \left[r_a^{\beta^{(4)}+1} - r_3^{\beta^{(4)}+1} \right]$$

$$e_{92} = \frac{\bar{C}_{12}^{(2)} - \beta^{(2)} \bar{C}_{13}^{(2)}}{1 - \beta^{(2)}} \left[r_2^{-\beta^{(2)}+1} - r_1^{-\beta^{(2)}+1} \right]$$

$$e_{94} = \frac{\bar{C}_{12}^{(4)} - \beta^{(4)} \bar{C}_{13}^{(4)}}{1 - \beta^{(4)}} \left[r_a^{-\beta^{(4)}+1} - r_3^{-\beta^{(4)}+1} \right]$$

$$a_{92} = \sum_{k=1}^4 \left[\bar{C}_{16}^{(k)} + \alpha_2^{(k)} \left(\bar{C}_{12}^{(k)} + 2\bar{C}_{13}^{(k)} \right) \right] \frac{(r_k^3 - r_{k-1}^3)}{3}$$

$$d_{01} = \frac{\bar{C}_{26}^{(1)} + \beta^{(1)} \bar{C}_{36}^{(1)}}{2 + \beta^{(1)}} \left[r_1^{\beta^{(1)}+2} - r_0^{\beta^{(1)}+2} \right]$$

$$d_{03} = \frac{\bar{C}_{26}^{(3)} + \beta^{(3)} \bar{C}_{36}^{(3)}}{2 + \beta^{(3)}} \left[r_3^{\beta^{(3)}+2} - r_2^{\beta^{(3)}+2} \right]$$

$$e_{01} = \frac{\bar{C}_{26}^{(1)} - \beta^{(1)} \bar{C}_{36}^{(1)}}{2 - \beta^{(1)}} \left[r_1^{-\beta^{(1)}+2} - r_0^{-\beta^{(1)}+2} \right]$$

$$e_{03} = \frac{\bar{C}_{26}^{(3)} - \beta^{(3)} \bar{C}_{36}^{(3)}}{2 - \beta^{(3)}} \left[r_3^{-\beta^{(3)}+2} - r_2^{-\beta^{(3)}+2} \right]$$

$$a_{01} = \sum_{k=1}^4 \left[\bar{C}_{16}^{(k)} + \alpha_1^{(k)} \left(\bar{C}_{26}^{(k)} + \bar{C}_{36}^{(k)} \right) \right] \frac{(r_k^3 - r_{k-1}^3)}{3}$$

$$a_{02} = \sum_{k=1}^4 \left[\bar{C}_{66}^{(k)} + \alpha_2^{(k)} \left(\bar{C}_{26}^{(k)} + 2\bar{C}_{36}^{(k)} \right) \right] \frac{(r_k^4 - r_{k-1}^4)}{4}$$

$$d_{02} = \frac{\bar{C}_{26}^{(2)} + \beta^{(2)} \bar{C}_{36}^{(2)}}{2 + \beta^{(2)}} \left[r_2^{\beta^{(2)}+2} - r_1^{\beta^{(2)}+2} \right]$$

$$d_{04} = \frac{\bar{C}_{26}^{(4)} + \beta^{(4)} \bar{C}_{36}^{(4)}}{2 + \beta^{(4)}} \left[r_a^{\beta^{(4)}+2} - r_3^{\beta^{(4)}+2} \right]$$

$$e_{02} = \frac{\bar{C}_{26}^{(2)} - \beta^{(2)} \bar{C}_{36}^{(2)}}{2 - \beta^{(2)}} \left[r_2^{-\beta^{(2)}+2} - r_1^{-\beta^{(2)}+2} \right]$$

$$e_{04} = \frac{\bar{C}_{26}^{(4)} - \beta^{(4)} \bar{C}_{36}^{(4)}}{2 - \beta^{(4)}} \left[r_a^{-\beta^{(4)}+2} - r_3^{-\beta^{(4)}+2} \right]$$

References

- [1] Mohitpour, M., Golshan, H., and Murray A., 2007, *Pipeline Design and Construction: A Practical Approach*, 3rd ed., ASME Order number: 802574, New York.
- [2] Laney, P., 2002, *Use of Composite Pipe Materials in the Transportation of Natural Gas*, Idaho International Engineering and Environmental Laboratory, Bechtel BWXT Idaho, LLC.
- [3] Akkus, N., and Kawahara, M., 2000, "Bending Behaviors of Thin Composite Pipes With Reinforcing Nodes," *Mater. Sci. Res. Int.*, **6**, pp. 131–135.
- [4] Alderson, K. L., and Evans, K. E., 1992, "Failure Mechanics During the Transverse Loading of Filament-Wound Pipes Under Static and Low Velocity Impact Conditions," *Composites*, **23**, pp. 167–173.
- [5] Smerdov, A. A., 2000, "A Computational Study in Optimum Formations of Optimization Problems on Laminated Cylindrical Shells for Buckling: I. Shells Under Axial Compression," *Compos. Sci. Technol.*, **60**, pp. 2057–2066.
- [6] Xia, M., Takayanagi, H., and Kemmochi, K., 2001, "Analysis of Multi-Layered Filament-Wound Composite Pipes Under Internal Pressure," *Compos. Struct.*, **53**, pp. 483–491.
- [7] Klosner, J. M., and Kempner, J., 1966, "Comparison of Elasticity and Shell Theory Solutions," *AIAA J.*, **4**(3), pp. 467–480.
- [8] Lekhnitskii, S. G., 1981, *Theory of Elasticity of an Anisotropic Body*, MIR Publishers, Moscow.
- [9] Wild, P. M., and Vickers, G. W., 1997, "Analysis of Filament-Wound Cylindrical Hells Under Combined Centrifugal Pressure and Axial Loading," *Composites, Part A*, **28**, pp. 47–55.
- [10] Wahab, M. A., Alam, M. S., Pang, S. S., Peck, J. A., and Jones, R. A., 2007, "Stress Analysis of Non-Conventional Composite Pipes," *Compos. Struct.*, **79**, pp. 125–132.
- [11] Bakaiyan, H., Hosseini, H., and Ameri, E., 2009, "Analysis of Multi-Layered Filament-Wound Composite Pipes Under Combined Internal Pressure and Thermomechanical Loading With Thermal Variations," *Compos. Struct.*, **88**, pp. 532–541.
- [12] Ansari, R., Alisafaei, F., and Ghaedi, P., 2010, "Dynamic Analysis of Multi-Layered Filament-Wound Composite Pipes Subjected to Cyclic Internal Pressure and Cyclic Temperature," *Compos. Struct.*, **92**, pp. 1100–1109.
- [13] Mouritz, A. P., and Gibson, A. G., 2006, *Fire Properties of Polymer Composite Materials*, Springer, New York.
- [14] Fanucci, J. P., 1987, "Thermal Response of Radiantly Heated Kevlar and Graphite/Epoxy Composites," *J. Compos. Mater.*, **21**, pp. 129–139.
- [15] Eshraghi, M. R., 2003, *Analysis of Various Cracks and Provide a Suitable Layer Arrangement Using the Integral J Methods*, Tehran University, **37**(1), pp. 85–95.